# On the Design of Price Caps as Sanctions

by

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**Abstract**

A ceiling has been imposed on the price at which Russian producers can sell oil. The price cap is intended to reduce Russian government tax revenue without increasing the world price of oil excessively. We show that such price caps can have counterintuitive effects. A price cap can induce sanctioned producers to increase their output, thereby increasing their revenue. This increased output can also reduce the world price of the homogeneous product supplied by sanctioned and non-sanctioned producers. The welfare-maximizing price cap, which is often well below the unrestricted world price, can increase welfare substantially.

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## 1 Introduction.

In response to Russia's military operations in Ukraine, G7 countries and other allies ("the Alliance") have imposed a cap on the price at which Russian firms can sell the oil they supply using key Alliance inputs (e.g., shipping and insurance).[1] The price cap is intended to reduce the (tax) revenue that Russia has available to finance its operations in Ukraine[2] without causing the sharp increase in the world price of oil that would likely arise if the Alliance were to withhold its inputs from Russian oil suppliers altogether.[3]

Using price caps to reduce the (tax) revenue that accrues to a sanctioned nation is a relatively novel undertaking,[4] and so has received little formal analysis to date. The primary analysis of this issue has (appropriately) examined the effects of price caps on non-renewable resources. Johnson, Rachel, and Wolfram et al. (2023a) (hereinafter JRW) demonstrate that an exogenous price reduction often encourages a producer to increase its supply of a non-renewable resource. A lower price reduces the value of the remaining reserves, thereby enhancing incentives for current extraction and sale of the resource.[5] It follows that the imposition of a binding cap on the price at which a supplier can sell a non-renewable product can induce the firm to increase its current supply of the product.

The present research is intended to complement JRW's important work by examining the effects of imposing a price ceiling on a product supplied by a "rogue" supplier ($R$), even if the product is not a non-renewable resource. Historically, restrictions have been imposed on many different types of exports. For example, the U.S. has restricted the flows of a broad spectrum of goods and services to and from many countries, including Cuba, Iran, Libya,

[1] See Wolfram et al. (2022), Baumeister (2023), Horwich (2023), and Johnson et al. (2023a,b) for details.

[2] Johnson et al. (2023b, p. 1245) observe that "The price cap ... is an integral part of a broader sanctions package designed to reduce Russia's foreign exchange revenues and reduce its capacity to wage war in Ukraine. ... Reducing the revenue from oil exports provides a key potential lever to reduce Russia's ability to wage war." Wessel (2022) reports that a key imperative of the price cap is "to reduce the flow of oil revenues that are financing Russia's war machine." Fishman (2022) notes that the "price cap marks the first major attempt by the US and its allies to cut Russia's oil revenues."

[3] The U.S. Department of the Treasury (2022) states that "The price cap is an important tool to restrict the revenue Russia receives to fund its illegal war in Ukraine, while also maintaining a reliable supply of oil onto global markets." Horwich (2023, p. 1) notes that "if Russian oil doesn't get to the market somewhere, then there's a global shortfall that would have significant ramifications for the price."

[4] Neil Mehrotra, one of the architects of the cap on the price at which Russian oil can be sold, observes that "The price cap is an entirely novel effort. Typically, U.S. sanctions have been just outright prohibitions on certain types of business with certain entities. The price cap is novel in that we are trying to facilitate trade, but only under certain terms. ... I think this is definitely a new front in the tools of economic statecraft" (Horwich, 2023, p. 5). Johnson et al. (2023a, p. 16) observe that "The price cap on Russian oil reflects a novel approach to sanctions and the world is just beginning to understand its impacts on Russian oil revenues, geopolitical alignments, and oil trade."

[5] Also see Johnson et al. (2023b).

North Korea, and South Africa.[6] In principle, corresponding future restrictions might take the form of price restrictions rather than quantity restrictions. Therefore, it is important to understand the likely effects of price restrictions on a wide variety of products.

If all prices were exogenous in our static model, a binding ceiling on the price at which $R$ can sell the product it supplies using an Alliance input would induce $R$ to reduce its supply of this product.[7] Thus, there is no natural tendency for a binding price cap to induce expanded supply in our model, in contrast to JRW's model. Nevertheless, when prices are endogenous in our model,[8] the imposition of a price cap can induce $R$ to increase its supply of the product, and thereby reduce the (unrestricted, endogenous) world price of the product.[9]

These potentially counterintuitive findings arise because a binding cap on the price at which $R$ sells a portion of its output ensures that the price at which this output is sold does not decline as $R$'s output increases. When this standard drawback to output expansion by a firm with market power is eliminated, $R$ finds it profitable to increase its output, *ceteris paribus*.[10] The increased output can both increase $R$'s revenue and reduce the world price of oil. Consequently, a price cap can have two effects that differ from the effects typically recognized by policymakers. First, a price cap can reduce, not increase, the world price of the product in question. Second, a price cap on a portion of a sanctioned supplier's output can increase, not reduce, the supplier's revenue. These findings imply that the optimal design of a price cap entails important subtleties even in the absence of the intertemporal considerations in JRW's analysis.[11]

We show that the potentially counterintuitive qualitative effects we identify can be economically significant under arguably plausible conditions. Specifically, modest reductions in

[6] See U.S. Senate and House of Representatives (1986) and U.S. Government Accountability Office (1987, 1988, 2007, 2010, 2015).

[7] See Turner and Sappington (2024, Part D) for a formal proof of this conclusion.

[8] The uncapped equilibrium price of the product is affected by the strategic output decisions of industry suppliers in our model.

[9] The unrestricted world price is the price at which suppliers other than $R$ sell the product. It is also the price at which $R$ sells the output that it produces without using an Alliance input.

[10] In this respect, a price cap functions much like forward contracting (i.e., arranging to deliver future output at a fixed price that does not vary with the (spot) price that ultimately prevails). Allaz and Vila (1993) demonstrate that forward contracting can enhance incentives for output expansion by Cournot competitors. It can be shown that a corresponding effect arises in our model even if $R$ is a monopolist.

[11] We focus on the effects of a price cap on $R$'s revenue because the tax that the Russian government imposes on oil exporters has historically been based primarily on oil export revenues. Goldsworthy and Zakharova (2010, p. 9) observe that "Russia's petroleum sector is governed primarily by a tax & royalty system that relies on petroleum revenue as a tax base." Reuters (2019) reports that "In Russia, oil-industry taxes are based on revenues and output." In recent years, the Russian government has begun to impose profit-based taxes on some oil exports. However, oil export revenue remains the primary tax base (Reuters, 2021). As the discussion in Section 7 reports, a price cap affects $R$'s profit and revenue in similar fashion.

the price cap below the prevailing world price of the product can cause $R$'s revenue to increase substantially. Consequently, relatively stringent price caps can be required to reduce $R$'s revenue.

We also characterize the price cap that maximizes the difference between consumer surplus and a multiple ($r > 0$) of $R$'s revenue. We demonstrate that this "welfare-maximizing price cap" ($\overline{p}^*$) often is well below the uncapped price of the product. We also demonstrate that the imposition of $\overline{p}^*$ can increase welfare substantially under arguably plausible conditions. In addition, we identify conditions under which welfare is higher when $\overline{p}^*$ prevails than when the Alliance refuses to supply its input to $R$. For example, when $r$ is small, welfare increase little when lack of access to the Alliance input reduces $R$'s output, and thus its revenue. In contrast, consumer surplus declines considerably (thereby reducing welfare) when $R$'s significantly reduced output causes the uncapped price to increase substantially.

Our analysis and JRW's analysis are related to Becko (2024)'s analysis of the design of tariffs and taxes that maximize the welfare of a home country for any level of welfare reduction imposed on a sanctioned country.[12] However, our work differs substantially from Becko's analysis in part because the suppliers in Becko's model are price takers.[13] Consequently, the key considerations that underlie our primary findings do not arise in Becko's model.[14]

The analysis proceeds as follows. Section 2 describes our model. Section 3 identifies conditions under which a binding price cap increases $R$'s revenue and reduces the uncapped price of the sanctioned product. Section 4 considers the practical relevance of these potentially counterintuitive effects. Section 5 examines the welfare-maximizing choice of a price

[12] We share JRW's focus on the effects of a price cap rather than the effects of tariffs and taxes. However, we abstract from the stochastic prices, risk aversion, and degree of intertemporal elasticity of substitution that underlie JRW's key findings. We focus on the strategic interaction between the sanctioned supplier and a non-sanctioned supplier, both of which have market power. JRW explain that they "do not model the strategic interaction between Russia and other global producers, [although their] model features parameters that reflect the responsiveness of other producers, such as OPEC, to shocks originating from Russia or elsewhere" (p. 4) In contrast to JRW, we also examine the design of a welfare-maximizing price cap.

[13] Wachtmeister et al. (2022) also abstract from strategic oligopolistic interactions among suppliers. The authors compare the effects of price restrictions and quantity restrictions after estimating prevailing demand and supply functions. They find that price discounts often are better able than quantity restrictions to reduce the profits of Russian oil producers without reducing unduly the surplus secured by oil consumers. Ehrhart and Schlecht (2022) also do not model formally the strategic interactions among industry suppliers. The authors identify conditions under which a sanctioned supplier will accept the price cap imposed by buyers of its product.

[14] Furthermore, we examine the effects of a price cap on some of $R$'s output, rather than a tax on all of $R$'s output. Sturm (2022) extends the analysis in Becko (2024) in part to examine the design of tariffs that maximize the difference between the welfare of the home country and a multiple of the welfare of the sanctioned country. Sturm (2022) also considers retaliatory tariffs by the sanctioned country. Sturm (2023) examines how the presence of non-sanctioning countries that can either purchase the sanctioned product or supply substitute products affects the optimal design of sanctions.

cap. Section 6 examines selected extensions of the analysis in sections 2 – 5.[15] Section 7 summarizes our key findings and suggests directions for future research. The Appendix provides the proofs of all formal conclusions in the text.

## 2 The Model.

We consider a setting in which $R$ and a rival producer supply a homogeneous product. The rival's cost of producing $q$ units of output is $C(q)$. $R$ produces $q_A \geq 0$ units of output using an input (e.g., shipping and/or insurance) supplied by an (Alliance) input owner ("$A$"). $R$ also produces $q_N \geq 0$ units of output without employing this input.[16] $R$'s total cost of producing these outputs is $C^R(q_A, q_N)$. We assume that costless arbitrage supports a single market for the homogeneous product with a corresponding single aggregate demand curve.[17] This aggregate (inverse) demand curve is $P(Q)$, where $Q = q_A + q_N + q$ is aggregate output and $P(\cdot)$ denotes price.

The activity in our static model proceeds as follows. First, $A$ specifies the maximum price, $\overline{p}$, at which $R$ can sell the output it produces using $A$'s input ($q_A$). Then $R$ chooses $q_A$ and $q_N$, and the rival chooses $q$ (simultaneously and noncooperatively). The resulting total output, $Q$, gives rise to a market-clearing price, $P(Q)$. Finally, $R$ sells $q_N$ and the rival sells $q$ at price $P(Q)$. $R$ also sells $q_A$ at this price if the price cap does not bind, i.e., if $\overline{p} > P(Q)$. Otherwise, $R$ sells $q_A$ at the lower, capped price, $\overline{p}$. Excess demand for $q_A$ arises at this capped price when $\overline{p} < P(Q)$. We assume the excess demand is rationed efficiently, so the marginal consumer valuation of each unit of $q_A$ that is sold at price $\overline{p} < P(Q)$ is at least $P(Q)$.[18]

Define $p_m \equiv \min\{\, P(Q),\, \overline{p}\,\}$. Then $R$'s problem, given $\overline{p}$ and $q$, is:

$$\underset{q_A \geq 0,\; q_N \geq 0}{\text{Maximize}} \;\; \Pi^R(q_A, q_N) \;\equiv\; p_m\, q_A + P\,(\, q_A + q_N + q\,)\; q_N \;-\; C^R(q_A, q_N)\,. \tag{1}$$

The rival's problem, given $q_A$ and $q_N$, is:

[15] The model extensions include an analysis of the effects of a price cap on $R$'s profit. We find that a binding price cap can increase $R$'s profit, just as it can increase $R$'s revenue.

[16] $R$ might either produce a substitute input itself, or procure a substitute, but potentially more costly, input from an alternative supplier other than $A$.

[17] We thereby assume that all consumers – including those who reside in "Alliance territories" – can purchase the product at a market-clearing price, $P(Q)$, that exceeds $\overline{p}$. Thus, like JRW, we abstract from any additional considerations that might arise if the price controls we analyze were supplemented by quantity controls. In practice, some Alliance countries have precluded non-pipeline imports of Russian oil (Kaniecki et al., 2023).

[18] $R$ and the rival each chooses its output(s) to maximize its profit, taking the other firm's output as given and fully anticipating how $P(Q)$ will be determined and how any excess demand for $q_A$ will be rationed.

$$\underset{q \geq 0}{\text{Maximize}} \;\; \Pi(q) \;\equiv\; P\,(\,q_A + q_N + q\,)\;q \,-\, C(q)\,. \tag{2}$$

(1) implies that the rates at which $R$'s profit increases as $q_A$ and $q_N$ increase are:

$$\frac{\partial \Pi^R(\cdot)}{\partial q_A} \;=\; p_m + q_A\,\frac{\partial p_m}{\partial q_A} + P'(Q)\,q_N \,-\, \frac{\partial C^R(\cdot)}{\partial q_A}\,;$$

$$\frac{\partial \Pi^R(\cdot)}{\partial q_N} \;=\; P\,(Q) + q_A\,\frac{\partial p_m}{\partial q_N} + P'(Q)\,q_N \,-\, \frac{\partial C^R(\cdot)}{\partial q_N}\,. \tag{3}$$

In practice, Russian oil exporters relied heavily, but not exclusively, on shipping and insurance services supplied by the Alliance before the price cap was imposed. The exporters reduced, but did not eliminate, this reliance after the cap was imposed.[19] The limited supply of ships that can transport crude oil efficiently has prevented Russian oil exporters from eliminating their reliance on Alliance shipping services. To capture a setting in which $R$ often supplies output both using $A$'s input ($q_A > 0$) and without employing $A$'s input ($q_N > 0$), we assume that $R$'s cost function is

$$C^R(q_A, q_N) \;=\; c_A\,q_A + \frac{k_A}{2}\,[\,q_A\,]^2 + c_N\,q_N + \frac{k_N}{2}\,[\,q_N\,]^2 + \frac{k^R}{2}\,[\,q_A + q_N\,]^2\,, \tag{4}$$

where $c_A > 0$, $c_N$ ($\geq c_A$), $k_A > 0$, $k_N$ ($\geq k_A$), and $k^R > 0$ are parameters. This cost function implies that $R$ operates with increasing marginal costs,[20] which ensures that $q_A > 0$ and $q_N > 0$ in equilibrium, barring an exceptionally stringent price cap.[21]

$R$ can be viewed as incurring both manufacturing (e.g., extraction) costs and delivery (e.g., shipping and insurance) costs. Manufacturing costs are costs that do not vary with the presence or absence of $A$'s input. The marginal cost of extracting oil typically increases with output because less efficient wells are brought into service as output increases. Delivery costs are costs that vary according to whether $R$'s output is supplied using $A$'s input. As noted above, the marginal cost of shipping oil often rises as output increases due to the limited supply of ships that can transport crude oil efficiently.[22] The parameter $k^R$ scales

[19] Lin and Perkins (2023) report that "the total share of non-Western tankers moving Russian crude ... stood at 48% of total exports by November 2022, a month before the price cap came into effect." Levi et al. (2024) report that as of December 2023, "46% of Russian oil and its products were transported by tankers subject to the oil price cap. The remainder was shipped by 'shadow' tankers that are not subject to the price cap policy."

[20] Equation (4) implies that $R$ operates with increasing marginal costs because $\frac{\partial^2 C^R(\cdot)}{\partial (q_A)^2} = k_A + k^R > 0$, $\frac{\partial^2 C^R(\cdot)}{\partial (q_N)^2} = k_N + k^R > 0$, and $\frac{\partial^2 C^R(\cdot)}{\partial q_A \partial q_N} = k^R > 0$.

[21] Turner and Sappington (2024, Part E) show that if $k_A = k_N = 0$ and $c_N > c_A$, then $R$ will either set $q_A = 0$ or set $q_N = 0$ in equilibrium.

[22] Parker (2024) reports that "between 2021 and 2022 the price index for second-hand tankers jumped to the highest level in 15 years. In ordinary circumstances, some of these vessels would have been mothballed

the nonlinear component of $R$'s manufacturing costs. The parameters $k_A$ and $k_N$ scale the nonlinear component of $R$'s delivery costs, whereas the parameters $c_A$ and $c_N$ scale the linear component of these costs.

We also take the rival's cost function to be quadratic, i.e., $C(q) = c\,q + \frac{k}{2}\,q^2$, where $c > 0$ and $k > 0$ are parameters.[23] In addition, unless otherwise noted, we assume that the inverse demand function for the homogeneous product is linear, i.e., $P(Q) = a - b\,Q$, where $a > 0$ and $b > 0$ are parameters.[24]

In the presence of linear demand and quadratic costs, the rival will produce output (i.e., $q > 0$) and $R$ will supply output without using $A$'s input (i.e., $q_N > 0$) as long as: (i) market demand is sufficiently pronounced relative to cost; and (ii) $R$'s marginal cost of supplying $q_A$ increases sufficiently rapidly as $q_A$ increases.[25] In addition, as long as the price cap is not too stringent, $R$ will both supply output using $A$'s input (i.e., $q_A > 0$) and supply output without using $A$'s input (i.e., $q_N > 0$) if $R$'s marginal costs rise sufficiently rapidly as output increases.[26] These conditions are assumed to hold throughout the ensuing analysis.

# 3 A Price Cap Can Reduce $P(Q)$ and Increase $R$'s Revenue.

Proposition 1 reports that the impacts of a price cap ($\overline{p}$) on $R$'s output using $A$'s input ($q_A$) and on the equilibrium unrestricted price ($P(Q)$) vary with the level of the price cap.

**Proposition 1.** *There exist values of the price cap, $0 < \overline{p}_0 < \overline{p}_d < \overline{p}_b$, such that, in equilibrium, $q_A = 0$ if and only if $\overline{p} \leq \overline{p}_0$. Furthermore: (i) $\overline{p} < P(Q)$ if $\overline{p} \leq \overline{p}_d$; (ii) $\overline{p} = P(Q)$ if $\overline{p} \in (\overline{p}_d, \overline{p}_b]$; and (iii) $\overline{p} > P(Q)$ if $\overline{p} > \overline{p}_b$.*[27]

or broken up, but they have been snapped up for the shadow fleet" that Russian oil exporters employ to evade the price cap.

[23] The parameters $c$ and $k$ can be viewed as pertaining to both the rival's manufacturing costs and its delivery costs. For expositional ease, we abstract from fixed production costs for both $R$ and the rival. Observe that the rival operates with increasing marginal cost because $\frac{\partial^2 C(\cdot)}{\partial q^2} = k > 0$.

[24] As explained further in section 6, our key qualitative conclusions are not an artifact of the assumption that demand is linear, which is maintained to facilitate a complete characterization of equilibrium outcomes.

[25] Formally, to ensure that $q > 0$ in equilibrium, we assume that $a > c$ and $\overline{p}_d > c$, where $\overline{p}_d$ (which is an increasing function of $a$) is defined in equation (8) in the Appendix. In addition, to ensure that $q_N > 0$ in equilibrium, we assume that $a > c_N$ and $k_A\,[\,(a - c_N)\,(2\,b + k) - b\,(a - c)\,] > [\,c_N - c_A\,]\,[\,3\,b^2 + 2\,b\,(k + k^R) + k\,k^R\,]$.

[26] Formally, we assume that $k_A\,[\,2\,b + k_N\,] + k^R\,[\,k_A + k_N\,] > b^2$. This condition holds when $b$ is sufficiently small (so the uncapped market price is sufficiently insensitive to industry output) and when $k_A$, $k_N$, and $k^R$ are sufficiently large (so $R$'s marginal costs increase with output sufficiently rapidly). This condition ensures that the second-order condition in $R$'s profit-maximization problem is satisfied when a nontrivial region of price caps ($\overline{p}$) exists in which $\overline{p} = P(Q)$.

[27] The values of $\overline{p}_0$, $\overline{p}_d$, and $\overline{p}_b$ are specified in equations (7) – (9) in the Appendix.

Proposition 1 implies that the set of possible price caps can be divided into four distinct regions, as illustrated in Figure 1. The first region, $\overline{p} > \overline{p}_b$, consists of price caps that strictly exceed the the market-clearing price that prevails in equilibrium in the absence of a price cap. The price cap does not bind, and so does not affect equilibrium outcomes, when $\overline{p} > \overline{p}_b$.[28] Consequently, imposing a price cap that exceeds $\overline{p}_b$ is effectively equivalent to imposing no sanctions at all.

The second region of price caps, $\overline{p} \in (\overline{p}_d, \overline{p}_b\,]$, is comprised of the least stringent price caps that bind, in the sense that they affect equilibrium outcomes. In this region, $R$ produces some output using $A$'s input and, perhaps surprisingly, the equilibrium unrestricted price declines at exactly the same rate that the binding price cap declines. Formally, $q_A > 0$ and $P(Q) = \overline{p}$ for all $\overline{p} \in (\overline{p}_d, \overline{p}_b\,]$, as depicted in Figure 1.[29]

[**Figure 1 about Here**]

The third region of price caps, $\overline{p} \in (\overline{p}_0, \overline{p}_d\,]$, consists of binding price caps that are strictly below the equilibrium unrestricted price and that induce $R$ to supply some output using $A$'s input. Formally, $q_A > 0$ and $\overline{p} < P(Q)$ for all $\overline{p} \in (\overline{p}_0, \overline{p}_d\,]$. The fourth region of price caps, $\overline{p} \leq \overline{p}_0$, consists of binding price caps that are so stringent that they induce $R$ to supply no output using $A$'s input. Formally, $q_A = 0$ and $\overline{p} < P(Q)$ for all $\overline{p} \leq \overline{p}_0$, as illustrated in Figure 1.[30]

To explain the presence of an entire range of price caps ($\overline{p} \in (\overline{p}_d, \overline{p}_b\,]$) for which the cap coincides with the unrestricted equilibrium price, it is helpful to determine how $R$'s equilibrium outputs ($q_A$ and $q_N$) change as the level of the binding price cap changes.

**Proposition 2.** *In equilibrium: (i)* $\frac{dq_A}{d\overline{p}} < 0$ *and* $\frac{dq_N}{d\overline{p}} < 0$ *for* $\overline{p} \in (\overline{p}_d, \overline{p}_b)$ *; and (ii)* $\frac{dq_A}{d\overline{p}} > 0$ *and* $\frac{dq_N}{d\overline{p}} < 0$ *for* $\overline{p} \in (\overline{p}_0, \overline{p}_d)$ *.*

Conclusion (i) in Proposition 2 reports that $q_A$ and $q_N$ both increase as $\overline{p}$ declines in $(\overline{p}_d, \overline{p}_b)$. The increased output causes $P(Q)$ to decline at the same rate that $\overline{p}$ declines. (See Figure 1.) This finding reflects the net impact of two countervailing effects of a binding price cap: the *compensation reduction effect* and the *output enhancement effect*. The compensation reduction effect arises because a reduction in $\overline{p}$ reduces the unit compensation that $R$ derives from selling $q_A$. The reduced unit compensation induces $R$ to reduce $q_A$, *ceteris paribus*.

[28] Thus, $\overline{p}_b$ is the highest price cap that "binds." $\overline{p}_b$ is also the equilibrium price, $P(Q)$, in the absence of a price cap.

[29] Thus, $\overline{p}_d$ is the highest price cap for which the binding price cap and the uncapped price "diverge."

[30] Thus, $\overline{p}_0$ is the highest price cap for which $R$ sets $q_A = 0$.

The (countervailing) output enhancement effect of a binding price cap is more subtle. To understand the effect, observe that in the absence of a binding price cap, $R$ sells its entire output ($Q^R = q_A + q_N$) at the unrestricted equilibrium price $P(Q)$. This price declines at the rate $P'(Q)$ as $R$ increases its output. When the price cap binds, $R$ sells only a portion ($q_N$) of its output at a price that declines as its output ($Q^R$) increases. The (capped) price ($\overline{p}$) at which $R$ sells $q_A$ does not decline as $Q^R$ increases. Consequently, for this output ($q_A$), $R$ avoids the standard drawback to output expansion that a firm with market power faces (i.e., an associated reduction in the price at which output is sold). The elimination of this drawback to output expansion induces $R$ to increase its output, *ceteris paribus*.

$R$'s enhanced incentive to increase its output in the presence of a binding price cap is reflected in the second term to the right of the equality in each of the equations in expression (3). $p_m = \overline{p}$ and $\frac{\partial p_m}{\partial q_A} = \frac{\partial p_m}{\partial q_N} = 0$ when the price cap binds. Consequently, the identified terms in expression (3) are 0. In contrast, these terms are negative ($\frac{\partial p_m}{\partial q_A} = \frac{\partial p_m}{\partial q_N} = P'(Q) < 0$) in the absence of a binding price cap. Therefore, the presence of a binding price cap serves to increase the rate at which $R$'s profit increases as $Q^R$ increases, which induces $R$ to increase its output, *ceteris paribus*.[31]

When $\overline{p}$ is set marginally below the unrestricted equilibrium price ($\overline{p}_b$), the impact of the compensation reduction effect is relatively limited. Specifically, the marginally lower price that $R$ secures for $q_A$ induces a relatively small reduction in $q_A$, *ceteris paribus*. The predominant effect of reducing $\overline{p}$ marginally below $\overline{p}_b$ is to increase $R$'s output, reflecting the output enhancement effect.[32] The expanded output reduces $P(Q)$.[33] Conceivably, the output enhancement effect might be so pronounced as to drive $P(Q)$ below $\overline{p}$. However, if this were the case, then the price cap would no longer bind, so the output enhancement effect would be eliminated. Consequently, as $\overline{p}$ declines marginally below $\overline{p}_b$, the combined impact of the (relatively pronounced) output enhancement effect and the (relatively minor) compensation reduction effect is to induce $R$ to increase its output up to, but not beyond, the point at which $P(Q)$ declines at the same rate that $\overline{p}$ declines.

[31] A monopolist's incentive to expand its output is also enhanced when a portion of its output is sold at a (capped) price that does not decline as the monopolist's aggregate output increases. This conclusion follows directly from expression (3) which, when $q = 0$, specifies the rates at which a monopolist's profit increases as its outputs ($q_A$ and $q_N$) increase. The analysis in Part C of the Appendix demonstrates that the output enhancement effect often is more pronounced in a duopoly setting than in a monopoly setting due in part to the strategic competitive advantage that a binding price cap can provide in a duopoly setting.

[32] Miller (2023) reports that Russian oil exports increased after the Alliance imposed its price cap.

[33] Recall that $R$'s incentive to expand its output, thereby reducing $P(Q)$, is particularly pronounced when $R$ faces a rival. In this case, $R$'s expanded output induces an accommodating output reduction from the rival.

$q_A$ increases further as $\overline{p}$ declines farther below $\overline{p}_b$. The increase in $q_A$ increases the magnitude of the compensation reduction effect, which causes $R$'s profit from supplying $q_A$ to decline more rapidly or increase more slowly as $\overline{p}$ declines.[34] Eventually, the compensation reduction effect outweighs the output enhancement effect, inducing $R$ to reduce $q_A$ as $\overline{p}$ declines below $\overline{p}_d$, as reported in conclusion (ii) in Proposition 2 and illustrated in Figure 1. The corresponding increase in $P(Q)$ causes $P(Q)$ to exceed $\overline{p}$ when $\overline{p} < \overline{p}_d$.[35]

Having established how a binding price cap affects equilibrium outputs and prices, we can determine the corresponding impact on $R$'s revenue, $V(\cdot)$, where:

$$V(\overline{p}) \;\equiv\; \overline{p}\, q_A(\cdot) + P(Q(\cdot))\, q_N(\cdot)\,. \tag{5}$$

Proposition 3 considers this impact for the highest binding price caps.

**Proposition 3.** *$R$'s equilibrium revenue, $V(\overline{p})$ is a strictly concave function of $\overline{p}$ for $\overline{p} \in (\overline{p}_d, \overline{p}_b)$. Furthermore, $\left.\frac{\partial V(\overline{p})}{\partial \overline{p}}\right|_{\overline{p}=\overline{p}_b} < 0$.*

Proposition 3 reports that as the price cap ($\overline{p}$) declines below $\overline{p}_b$, a more stringent price cap *increases* $R$'s revenue. Furthermore, $R$'s revenue increases at a decreasing rate as $\overline{p}$ declines below $\overline{p}_b$, as illustrated in Figure 2.[36]

[**Figure 2 about Here**]

$R$'s revenue increases as $\overline{p}$ declines marginally below $\overline{p}_b$ because the relatively pronounced output enhancement effect of a reduction in $\overline{p}$ induces $R$ to increase $q_A$ relatively rapidly. $R$ continues to increase $q_A$ as $\overline{p}$ declines further below $\overline{p}_b$. (Recall conclusion (i) in Proposition 2.) The higher level of $q_A$ enhances the revenue-reducing compensation reduction effect of a binding price cap, which causes $R$'s revenue to increase more slowly as $\overline{p}$ declines in $(\overline{p}_d, \overline{p}_b)$. Consequently, $V(\overline{p})$ is a concave function of $\overline{p}$ when $\overline{p} \in (\overline{p}_d, \overline{p}_b)$.

If $q_A$ and $q_N$ increase sufficiently rapidly as $\overline{p}$ declines in $(\overline{p}_d, \overline{p}_b)$, the compensation reduction effect can outweigh the output expansion effect, so a reduction in $\overline{p}$ can reduce $R$'s

[34] The output expansion effect can cause $R$'s profit to increase as $\overline{p}$ declines. See the discussion in section 6.

[35] Proposition 2 reports that $q_N$ increases systematically as a binding price cap becomes more stringent. This increase in $q_N$ when $\overline{p} \in (\overline{p}_0, \overline{p}_d)$ reflects in part the output enhancement effect. This increase also arises when $\overline{p} \in (\overline{p}_0, \overline{p}_d)$ because the reduction in $q_A$ as $\overline{p}$ declines reduces $R$'s marginal cost of supplying $q_N$. (Recall that $\frac{\partial C^R(\cdot)}{\partial q_A \partial q_N} = k^R > 0$, from equation (4).)

[36] Figure 2 is drawn to roughly approximate $R$'s revenue (and consumer surplus) in the baseline setting that is analyzed in section 4. Figure A1 in Turner and Sappington (2024) illustrates the (minor) variations in $R$'s revenue (and consumer surplus) that can arise in other settings.

revenue as $\overline{p}$ declines toward $\overline{p}_d$.[37] Alternatively, $R$'s revenue can continue to increase as $\overline{p}$ declines for all $\overline{p} \in [\,\overline{p}_d, \overline{p}_b\,]$, as illustrated in Figure 2.[38]

Propositions 1 – 3 establish that a price cap can introduce two effects that are not commonly recognized in policy discussions. First, $R$'s output and its revenue can increase as the cap declines below the level at which it first binds ($\overline{p}_b$). Second, the increase in $R$'s output can cause $P(Q)$ to decline.[39]

## 4 Practical Importance of Findings

To assess the practical importance of the findings reported in section 3, it is useful to consider the following *baseline setting*. Although our analysis abstracts from the intertemporal considerations associated with non-renewable resources, the parameters in the baseline setting are chosen to reflect selected elements of Russia's activity in the oil sector, given the world's focus on the cap that has been imposed on the price of oil sold by Russian suppliers that employ Alliance inputs.[40]

We choose demand parameters $a$ and $b$ to ensure that in the absence of a price cap, the equilibrium price is 70 (dollars) and equilibrium total output is 90 million units (e.g., barrels of oil per day) when the price elasticity of demand is $-\,0.75$.[41] This elasticity, which exceeds common estimates of the price elasticity of demand for oil,[42] helps to ensure that the specified equilibrium price and output prevail in our duopoly model when arguably plausible values for cost parameters are adopted.[43] These considerations imply that $a = 163.33$ and $b = 1.03703 \times 10^{-6}$ because:

[37] See Figure A1 in Turner and Sappington (2024). It can be shown that $V(\overline{p})$ attains its maximum value on $[\,\overline{p}_d, \overline{p}_b\,]$ at some interior price cap $\overline{p} \in (\,\overline{p}_d, \overline{p}_b)$ if and only if $\Phi_1 < 0$ where $\Phi_1 \equiv \left[\,k^R + \frac{b^2}{2\,b+k}\,\right][\,k_A + k_N\,]\,A + 2\,b\,[\,b+k\,]\,c_A\,[\,k_N + b\,] + [\,2\,b\,(b+k)\,c_N + A\,k_N\,]\,[\,k_A - b\,]$ and $A \equiv a\,[\,b+k\,] + b\,c$. (See the proof of Proposition 3.) It is apparent that $k_A < b$ when $\Phi_1 < 0$. When $k_A$ is relatively small, $q_A$ is relatively large. Consequently, $R$'s revenue from supplying $q_A$ declines relatively rapidly as $\overline{p}$ declines (reflecting a relatively pronounced compensation reduction effect).

[38] Proposition 4 (below) establishes that $\overline{p}_b - \overline{p}_d$ becomes smaller as $c_A$, $k_A$, or $k^R$ increases. It is apparent that $\Phi_1$ (defined in the preceding footnote) increases as $c_A$, $k_A$, or $k^R$ increases. Therefore, because $V(\overline{p})$ attains its maximum value on $[\,\overline{p}_d, \overline{p}_b\,]$ at $\overline{p}_d$ when $\Phi_1 > 0$, $V(\overline{p})$ increases as $\overline{p}$ declines throughout the entire $[\,\overline{p}_d, \overline{p}_b\,]$ interval when this interval is relatively small.

[39] In contrast, $P(Q)$ would increase if $R$ were denied all access to $A$'s input.

[40] The Appendix considers substantial variation of the parameters in the baseline setting.

[41] In 2021 (the year prior to Russia's invasion of Ukraine), the average Brent oil price was approximately \$71 per barrel (U.S. Energy Information Administration, 2023). The average daily world production of oil in 2021 was approximately 89.9 million barrels (bp, 2022, p. 15).

[42] Caldara et al. (2016)'s review of studies of the short-run price elasticity of demand for oil reports an average elasticity of $-\,0.22$.

[43] The identified equilibrium price and output can arise when equilibrium demand is substantially less elastic if the number of industry suppliers is sufficiently large. We consider duopoly competition for analytic ease.

$$\frac{\partial Q}{\partial p}\,\frac{p}{Q} \;=\; -\,\frac{1}{b}\left[\,\frac{70}{90,000,000}\,\right] \;=\; -\,0.75 \quad\Rightarrow\quad b \;=\; 1.03703\times 10^{-6}\,;\;\text{ and}$$

$$P(Q) \;=\; a - b\,[\,90,000,000\,] \;=\; 70 \quad\Rightarrow\quad a \;=\; 70 + 1.03703\,[\,90\,] \;\approx\; 163.33\,.$$

The cost parameters in our baseline setting are chosen so that, in the absence of a price cap, $R$'s equilibrium marginal cost when it employs $A$'s input is approximately 25 (dollars), and $R$'s corresponding average variable cost is approximately 15.[44] Furthermore, the rival's cost is presumed to parallel's $R$'s cost when $R$ employs $A$'s input (i.e., $c = c_A$ and $k = k_A + k^R$). In addition, we assume $c_A = \beta\, c_N$ and $k_A = \beta\, k_N$, and set $\beta = 0.5$ to capture $R$'s cost saving from employing $A$'s input.[45] Table 1 records the parameter values in the baseline setting.[46]

| Parameter | Parameter Value | | Parameter | Parameter Value |
|---|---|---|---|---|
| $a$ | 163.33 | | $c_N$ | 5 |
| $b$ | $1.03703 \times 10^{-6}$ | | $k_N$ | $1 \times 10^{-6}$ |
| $c_A$ | 2.5 | | $c$ | 2.5 |
| $k_A$ | $5 \times 10^{-7}$ | | $k$ | $6 \times 10^{-7}$ |
| $k^R$ | $1 \times 10^{-7}$ | | | |

**Table 1. Parameters in the Baseline Setting.**

The first column in Table 2 presents for the baseline setting the critical levels of the price cap identified in Proposition 1.[47] Recall that the price cap ($\overline{p}$) coincides with $P(Q)$, the equilibrium price of the output supplied without using $A$'s input, when $\overline{p} \in (\overline{p}_d, \overline{p}_b\,]$. Therefore, the first column in Table 2 implies that $P(Q) = \overline{p}$ as $\overline{p}$ declines from $\overline{p}_b = 71.52$ to $\overline{p}_d = 56.35$.[48] Consequently, as indicated in the first numerical entry in the last column

[44] Horwich (2023) estimates Russia's marginal cost of supplying oil to be approximately \$20 per barrel. The Center for Research on Energy and Clean Air (2023) estimates this cost to be between \$2.70 and \$25. Hausmann (2022) suggests that Russia's average variable cost may be less than \$6 per barrel. Kennedy (2022)'s corresponding estimate is between \$20 and \$25 per barrel.

[45] Table A2 in the Appendix reports the equilibrium outcomes that arise for different values of $\beta$, including $\beta = 1$, in which case $R$'s operating costs with and without using $A$'s input are symmetric.

[46] These parameters ensure that in the absence of a binding price cap, $P(Q) = 71.52$, $Q = 88.535$ million, $R$'s marginal cost ($c_A + k_A\, q_A + k^R\,[\,q_A + q_N\,]$) is 23.43, and $R$'s average variable cost ($c_A + \frac{1}{2}\, k_A\, q_A + \frac{1}{2}\, k^R \left[\, \frac{(q_A+q_N)^2}{q_A} \,\right]$) is 13.34.

[47] It can be verified that $R$'s revenue, $V(\overline{p})$, peaks at $\overline{p}_d$ (because $\Phi_1 > 0$) in the baseline setting. $V(\overline{p})$ would peak at some $\overline{p} \in (\overline{p}_d, \overline{p}_b)$ (because $\Phi_1 < 0$) if, for example, $k_A$ were reduced by 50% (to $2.5 \times 10^{-7}$) while all other parameters remained at their values in the baseline setting. This possibility is illustrated in Figure A1 in Turner and Sappington (2024).

[48] All entries in Table 2 (and subsequent tables) are rounded.

in Table 2, $P(Q)$ declines at the same rate that $\overline{p}$ declines as $\overline{p}$ declines by as much as 21% below $\overline{p}_b$. The middle column in Table 2 reports corresponding changes in $R$'s revenue. As illustrated in Figure 3 and as summarized in the second numerical entry in the last column in Table 2, $R$'s revenue increases by approximately 19% as $\overline{p}$ declines from $\overline{p}_b = 71.52$ to $\overline{p}_d = 56.35$.

| Price Cap | | $R$'s Revenue | | Variation |
|---|---|---|---|---|
| $\overline{p}_0 = 41.82$ | | $V(\overline{p}_0) = 2.70 \times 10^9$ | | $\frac{\overline{p}_b - \overline{p}_d}{\overline{p}_b} = 0.21$ |
| $\overline{p}_d = 56.35$ | | $V(\overline{p}_d) = 3.95 \times 10^9$ | | $\frac{V(\overline{p}_d) - V(\overline{p}_b)}{V(\overline{p}_b)} = 0.19$ |
| $\overline{p}_b = 71.52$ | | $V(\overline{p}_b) = 3.32 \times 10^9$ | | |

**Table 2. Equilibrium Outcomes in the Baseline Setting.**

[**Figure 3 about Here**]

Table 2 indicates that under arguably plausible conditions, $P(Q)$ declines at the same rate that $\overline{p}$ declines for a relatively broad range of $\overline{p}$ values. Furthermore, a more stringent price cap can increase $R$'s equilibrium revenue considerably. Tables A1 and A2 in the Appendix demonstrate that values of $\frac{\overline{p}_b - \overline{p}_d}{\overline{p}_b}$ and $\frac{V(\overline{p}_d) - V(\overline{p}_b)}{V(\overline{p}_b)}$ similar to those in Table 1 arise in equilibrium as parameter values diverge from their values in the baseline setting.[49]

Proposition 4 identifies how production costs influence $\overline{p}_b - \overline{p}_d$, the extent of the range of price caps for which $P(Q)$ declines at the same rate that $\overline{p}$ declines.

**Proposition 4.** *$\overline{p}_b - \overline{p}_d$ increases as: (i) $c_A$, $k_A$, or $k^R$ declines; (ii) $c$ or $c_N$ increases; or (iii) $k_N$ increases if $k_A - b$ is sufficiently small.*

Conclusion (i) in Proposition 4 holds because $q_A$ ($R$'s output using $A$'s input) increases as $R$'s cost of supplying $q_A$ declines (i.e., as $c_A$, $k_A$, or $k^R$ declines).[50] The higher level of $q_A$ increases the amount of $R$'s output that is sold at a fixed price ($\overline{p}$) that does not decline as expanded output reduces $P(Q)$. A binding price cap thereby provides $R$ with a relatively strong incentive to expand its output aggressively, which increases the range of $\overline{p}$'s for which $P(Q)$ declines at the same rate that $\overline{p}$ declines.

Similarly, conclusions (ii) and (iii) in Proposition 4 arise in part because $q_N$ (the output that $R$ supplies without using $A$'s input) declines as $R$'s cost of supplying $q_N$ increases (i.e., as $c_N$ or $k_N$ increases). The reduction in $q_N$ leads $R$ to increase $q_A$ for two reasons. First,

[49] Tables A1 and A2 also report equilibrium levels of welfare and welfare-maximizing price caps. These variables are defined in section 5.

[50] Recall that $R$'s cost function is presented in equation (4).

$R$'s marginal cost of supplying $q_A$ declines as $q_N$ declines. (Recall that $\frac{\partial C^R(\cdot)}{\partial q_A \partial q_N} = k^R > 0$, from equation (4).) Second, the amount of output that $R$ sells at price $P(Q)$ declines as $q_N$ declines. This reduced exposure to the profit-reducing effect of a reduction in $P(Q)$ enhances $R$'s incentive to increase $q_A$.[51] The increase in $q_A$ induced by these two effects exerts downward pressure on $P(Q)$, which increases the range of $\overline{p}$'s for which $P(Q)$ declines at the same rate that $\overline{p}$ declines.

Finally, observe that $q$ declines and $P(Q)$ increases as the rival's production cost, $c$, increases. The higher price and increased potential market share for $R$ enhance $R$'s incentive to increase its output aggressively when a binding price cap eliminates the exposure of some of $R$'s output to the corresponding reduction in $P(Q)$. Consequently, $P(Q)$ declines at the same rate that $\overline{p}$ declines over a broader range of price caps, i.e., $\overline{p}_b - \overline{p}_d$ increases, as $c$ increases.[52]

# 5 Welfare.

We now examine how a price cap is optimally set to limit $R$'s revenue without reducing consumer surplus unduly. To do so, we take welfare, $W(\cdot)$, to be the difference between consumer surplus, $S(\cdot)$, and a multiple ($r > 0$) of $R$'s revenue. Formally:

$$W(\overline{p}) \; = \; S(\overline{p}) - r \left[ \, \overline{p} \; q_A(\overline{p}) + P(Q(\overline{p})) \, q_N(\overline{p}) \, \right] \tag{6}$$

where $S(\overline{p})$ denotes equilibrium consumer surplus when the price cap is $\overline{p}$.[53] To characterize the welfare-maximizing price cap, $\overline{p}^* \equiv \arg\max \{ W(\overline{p}) \}$, we first examine the properties of consumer surplus when the equilibrium unrestricted price coincides with the price cap (so $P(Q) = \overline{p}$), i.e., when $\overline{p} \in (\overline{p}_d, \overline{p}_b)$.

**Lemma 1.** *Equilibrium consumer surplus, $S(\overline{p})$, is a strictly decreasing, strictly convex function of $\overline{p}$ for $\overline{p} \in (\overline{p}_d, \overline{p}_b)$.*

Lemma 1 establishes that consumer surplus increases at an increasing rate as $\overline{p}$ declines in $(\overline{p}_d, \overline{p}_b)$, as illustrated in Figure 2. This is the case because reductions in $\overline{p}$ and $P(Q)$ both

[51] This reduced exposure is relatively pronounced when $P(Q)$ is relatively sensitive to changes in output, i.e., when $b$ is relatively large (so $k_A - b$ is relatively small).

[52] Corresponding analytic conclusions about the impact of parameter values on $\frac{\overline{p}_b - \overline{p}_d}{\overline{p}_b}$ are not available. Numerical solutions reveal that $\frac{\overline{p}_b - \overline{p}_d}{\overline{p}_b}$ often increases as: (i) $a$, $c_A$, $k_A$, or $k^R$ declines; or (ii) $c_N$, $k_N$, $k$, or $b$ increases. Thus, $\frac{\overline{p}_b - \overline{p}_d}{\overline{p}_b}$ and $\overline{p}_b - \overline{p}_d$ tend to become relatively large as $c_N$ and $k_N$ increase relative to $c_A$ and $k_A$, i.e., as it becomes relatively costly for $R$ to "evade" the effects of the price cap. This is the case in the baseline setting, for example, and for substantial variation in parameters around their values in the baseline setting.

[53] Recall that efficient rationing of the excess demand for $q_A$ at the capped price $\overline{p} < P(Q)$ prevails, by assumption. Therefore, the marginal consumer valuation of each unit of $q_A$ that is sold is at least $P(Q)$.

increase consumer surplus. As $\overline{p}$ declines in $(\overline{p}_d, \overline{p}_b)$, equilibrium output increases, reflecting the output enhancement effect. (Recall conclusion (i) in Proposition 2.) The increased output causes consumer surplus to increase more rapidly as the prevailing price ($\overline{p} = P(Q)$) declines.

Proposition 3 and Lemma 1 imply that welfare is a strictly convex function of $\overline{p}$ for $\overline{p} \in (\overline{p}_d, \overline{p}_b\,]$. Consequently, the welfare-maximizing price cap, $\overline{p}^*$, is never in $(\overline{p}_d, \overline{p}_b)$. It can also be shown that a binding price cap always increases welfare, i.e., $\overline{p}^* < \overline{p}_b$. This conclusion reflects in part the fact that $R$'s revenue is lower when the price cap is so stringent that it induces $R$ to set $q_A = 0$ than when no price cap is imposed (i.e., $V(\overline{p}_0) < V(\overline{p}_b)$).[54] This finding implies that if $r$ is sufficiently large, then welfare is highest when a binding price cap is imposed (because the price cap reduces $R$'s revenue). A binding price cap also maximizes welfare when $r$ is small because consumer surplus increases as $\overline{p}$ declines below $\overline{p}_b$. (Recall Lemma 1 and Figure 2.) Because $\overline{p}^*$ is not in $(\overline{p}_d, \overline{p}_b\,]$, it follows that $\overline{p}^* \in [\,\overline{p}_0, \overline{p}_d\,]$.[55]

To further characterize the welfare-maximizing level of the price cap, we present two lemmas that explain how $R$'s revenue, $V(\overline{p})$, and consumer surplus, $S(\overline{p})$, vary with $\overline{p}$ when $\overline{p} \in [\,\overline{p}_0, \overline{p}_d\,]$, so $R$ produces output using $A$'s input (i.e., $q_A > 0$) and the binding price cap is below $P(Q)$.

**Lemma 2.** *$R$'s equilibrium revenue, $V(\overline{p})$, is a strictly convex function of $\overline{p}$ for $\overline{p} \in [\,\overline{p}_0, \overline{p}_d\,]$. Furthermore, $\left.\frac{\partial V(\overline{p})}{\partial \overline{p}}\right|_{\overline{p}=\overline{p}_d} > 0$. In addition, $\left.\frac{\partial V(\overline{p})}{\partial \overline{p}}\right|_{\overline{p}=\overline{p}_0} < 0$ if $c_N - c_A$ is sufficiently large.*[56]

Lemma 2 reports that $R$'s revenue declines as $\overline{p}$ declines below $\overline{p}_d$ (the highest level of $\overline{p}$ for which the binding price cap is strictly less than $P(Q)$), as illustrated in Figure 2. The revenue reduction reflects: (i) the lower unit compensation that $R$ receives for $q_A$ as $\overline{p}$ declines; and (ii) the reduction in $q_A$ that arises as $\overline{p}$ declines in $(\overline{p}_0, \overline{p}_d)$.[57]

The convexity of $V(\overline{p})$ reported in Lemma 2 implies that $V(\cdot)$ declines more slowly as $\overline{p}$ declines further below $\overline{p}_d$ (as depicted in Figure 2). This is the case because $R$'s output

[54] See Lemma A7 in the Appendix.

[55] See Proposition A1 in the Appendix. Observe that welfare is the same for all $\overline{p} \leq \overline{p}_0$. This is the case because equilibrium outcomes do not vary with $\overline{p}$ when $\overline{p} \leq \overline{p}_0$ because all output is sold at the unrestricted equilibrium price $P(Q)$ (since $q_A = 0$) whenever $\overline{p} \leq \overline{p}_0$.

[56] More precisely, $\left.\frac{\partial V(\overline{p})}{\partial \overline{p}}\right|_{\overline{p}=\overline{p}_0} < 0$ if $\Phi_2 > 0$, where $\Phi_2 \equiv \{\, k^R\,[\,2\,b + k\,]\,\big[\,k^R\,(2\,b + k) + 2\,b\,(3\,b + 2\,k)\,\big] + k_N\,[\,2\,b + k\,]\,\big[\,k^R\,(2\,b + k) + b^2\,\big] + b^2\,\big[\,5\,b^2 + 6\,b\,k + 2\,k^2\,\big]\,\}\; c_N - \{\, b\,[\,3\,b + 2\,k\,] + [\,2\,b + k\,]\,\big[\,k_N + k^R\,\big]^2 \}\, c_A - b\,\big[\,b^2 - k\,k_N + (2\,b + k)\,k^R\,\big]\,[\,a\,(b + k) + b\,c\,]$. It is apparent that $\Phi_2$ increases as $c_N - c_A$ increases.

[57] Recall from conclusion (ii) in Proposition 2 that $\frac{dq_A}{d\,\overline{p}} > 0$ when $\overline{p} \in (\overline{p}_0, \overline{p}_d)$, as illustrated in Figure 1.

using $A$'s input ($q_A$) declines as $\overline{p}$ declines in $(\overline{p}_0, \overline{p}_d)$, which diminishes the revenue-reducing compensation reduction effect of a more stringent price cap.

Lemma 2 also reports that $R$'s revenue declines as $\overline{p}$ increases above $\overline{p}_0$ when $c_N - c_A$ is sufficiently large. In this case, $R$ reduces $q_N$ relatively rapidly as $q_A$ increases in response to the increase in $\overline{p}$ above $\overline{p}_0$. The reduction in $q_N$ (sold at the relatively high price, $P(Q)$) reduces $R$'s revenue, despite the increase in $q_A$ (sold at the relatively low price, $\overline{p}$). When $R$'s revenue declines as $\overline{p}$ increases above $\overline{p}_0$ and $r$ is sufficiently large, welfare increases as $\overline{p}$ increases above $\overline{p}_0$.

Lemma 3 characterizes equilibrium consumer surplus, $S(\overline{p})$. The lemma refers to $\overline{p}_{SM}$, which is the level of $\overline{p}$ at which $S(\cdot)$ is maximized.

**Lemma 3.** *Equilibrium consumer surplus, $S(\overline{p})$, is a strictly concave function of $\overline{p}$ for $\overline{p} \in [\overline{p}_0, \overline{p}_d]$. Furthermore: (i) $\left.\frac{\partial S(\overline{p})}{\partial \overline{p}}\right|_{\overline{p}=\overline{p}_d} < 0$ when $\overline{p}_{SM} < \overline{p}_d$;*[58] *and (ii) $\left.\frac{\partial S(\overline{p})}{\partial \overline{p}}\right|_{\overline{p}=\overline{p}_0} > 0$.*

Lemma 3 reports that consumer surplus initially increases as $\overline{p}$ declines below $\overline{p}_d$ when $\overline{p}_{SM} < \overline{p}_d$ (as illustrated in Figure A1 in Turner and Sappington (2024)). The increase in $S(\cdot)$ reflects in part the lower $\overline{p}$ at which $q_A$ is sold. The concavity of $S(\cdot)$ reported in Lemma 3 implies that the rate at which consumer surplus increases as $\overline{p}$ declines diminishes as $\overline{p}$ declines further below $\overline{p}_d$ (when $\overline{p}_{SM} < \overline{p}_d$). The diminishing rate of increase in $S(\cdot)$ reflects the reduction in $q_A$ that $R$ implements as $\overline{p}$ declines in $(\overline{p}_0, \overline{p}_d)$.[59] Eventually, $S(\cdot)$ declines as $\overline{p}$ declines when $\overline{p}$ is sufficiently close to $\overline{p}_0$.[60] The reduction in consumer surplus arises because: (i) the reduction in $q_A$ induced by a reduction in $\overline{p}$ causes $P(Q)$ to increase; and (ii) the reduction in $\overline{p}$ reduces the price at which only a relatively small number of units are sold as $\overline{p}$ approaches $\overline{p}_0$.

Lemmas 2 and 3 allow us to determine when the welfare-maximizing price cap ($\overline{p}^*$) induces $R$ to supply output using $A$'s input.

**Proposition 5.** *$\overline{p}^* \in (\overline{p}_0, \overline{p}_d]$ if $c_N - c_A$ is sufficiently large to ensure that $\left.\frac{\partial V(\overline{p})}{\partial \overline{p}}\right|_{\overline{p}=\overline{p}_0} < 0$. In contrast, $\overline{p}^* = \overline{p}_0$ if $\left.\frac{\partial V(\overline{p})}{\partial \overline{p}}\right|_{\overline{p}=\overline{p}_0} > 0$ and $r$ is sufficiently large.*[61]

Proposition 5 reports that when $c_N - c_A$ is sufficiently large, the welfare-maximizing

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[58] When $\overline{p}_{SM} = \overline{p}_d$ (as in Figure 2), $\frac{\partial S(\overline{p})}{\partial \overline{p}} > 0$ as $\overline{p}$ approaches $\overline{p}_d$ from below.

[59] Recall the observation in footnote 57.

[60] In Figure 2, $S(\cdot)$ declines as $\overline{p}$ declines throughout the entire $(\overline{p}_0, \overline{p}_d)$ interval, as it does in the baseline setting.

[61] More precisely, $\overline{p}^* \in (\overline{p}_0, \overline{p}_d]$ if $\Phi_2 \geq 0$. In contrast, $\overline{p}^* = \overline{p}_0$ if $\Phi_2 < 0$ and $r$ is sufficiently large. Recall that $\Phi_2$ is defined in footnote 56.

price cap exceeds $\overline{p}_0$, so $R$ produces some output using $A$'s input (i.e., $q_A > 0$). This conclusion arises because consumer surplus increases as $\overline{p}$ increases above $\overline{p}_0$ (Lemma 3)[62] and because $R$'s revenue declines as $\overline{p}$ increases above $\overline{p}_0$ (Lemma 2). Consequently, the welfare-maximizing price cap generates a strictly higher level of welfare than does a refusal to supply any of $A$'s input to $R$ (which would induce $R$ to set $q_A = 0$). In contrast, such a refusal (or setting $\overline{p} \leq \overline{p}_0$) can maximize welfare when $R$'s revenue increases as $\overline{p}$ increases above $\overline{p}_0$ and society is primarily concerned with limiting $R$'s revenue (i.e., $r$ is sufficiently large). In this case, the optimal policy effectively implements a complete embargo on output supplied by $R$ using $A$'s input.

The welfare-maximizing price cap ($\overline{p}^*$) lies between the price cap at which $R$'s revenue is minimized (i.e., at $\overline{p}_{Vm}$, which coincides with $\overline{p}_0$ in Figure 2) and the price cap at which consumer surplus is maximized (i.e., at $\overline{p}_{SM}$, which coincides with $\overline{p}_d$ in Figure 2).[63] Furthermore, $\overline{p}^*$ approaches $\overline{p}_{SM}$ as the concern with reducing $R$'s revenue becomes negligible. In contrast, $\overline{p}^*$ approaches $\overline{p}_{Vm}$ when reducing $R$'s revenue is of paramount importance.[64]

Proposition 6 provides additional guidance on how $R$'s production costs ($c_A$, $k_A$, and $c_N$)[65] affect the level of the welfare-maximizing price cap ($\overline{p}^*$) when $\overline{p}^* \in (\overline{p}_0, \overline{p}_d)$.

**Proposition 6.** *When* $\overline{p}^* \in (\overline{p}_0, \overline{p}_d)$: *(i)* $\frac{d\overline{p}^*}{dc_A} > 0$; *(ii)* $\frac{d\overline{p}^*}{dk_A} > 0$; *and (iii)* $\frac{d\overline{p}^*}{dc_N} < 0$.

Proposition 6 states that the welfare-maximizing price cap ($\overline{p}^*$) increases as $q_A$ becomes more costly to produce (i.e., as $c_A$ or $k_A$ increases) or as $q_N$ becomes less costly to produce (i.e., as $c_N$ declines), *ceteris paribus*. These cost changes induce $R$ to reduce $q_A$ relative to $q_N$. The relative reduction in $q_A$ diminishes the potential welfare gain from reducing $\overline{p}$ for two reasons. First, when $q_A$ is small, the surplus of consumers that purchase $q_A$ increases relatively slowly as $\overline{p}$ declines. Second, when $q_A$ is small, $R$'s revenue from selling $q_A$ declines relatively slowly as $\overline{p}$ declines. Both sources of diminished benefit from reducing $\overline{p}$ imply that $\overline{p}^*$ increases (i.e., $\frac{d\overline{p}^*}{dc_A} > 0$ and $\frac{d\overline{p}^*}{dk_A} > 0$).[66]

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[62] Consumer surplus increases in part because as $\overline{p}$ increases above $\overline{p}_0$, there is no first-order effect on consumer surplus associated with $q_A$ (because $q_A \approx 0$). Furthermore, when $c_A$ is sufficiently small relative to $c_N$, the increase in $\overline{p}$ induces $R$ to increase $q_A$ by more than $q_N$ and $q$ decline, so $P(Q)$ declines.

[63] Formally, $\overline{p}^* \in [\overline{p}_{Vm}, \overline{p}_{SM}]$, where $\overline{p}_{Vm} \equiv \underset{\overline{p} \in [\overline{p}_0, \overline{p}_d]}{\arg\min} V(\overline{p})$ and $\overline{p}_{SM} \equiv \underset{\overline{p} \in [\overline{p}_0, \overline{p}_d]}{\arg\max} S(\overline{p})$. See Proposition A2 in the Appendix. Figure A1 in Turner and Sappington (2024) considers a setting in which $\overline{p}_{Vm} \in (\overline{p}_0, \overline{p}_d)$ and $\overline{p}_{SM} \in (\overline{p}_0, \overline{p}_d)$. Lemma 3 in Turner and Sappington (2024) establishes that $\overline{p}_{Vm} < \overline{p}_{SM}$.

[64] Formally, $\overline{p}^* \to \overline{p}_{SM}$ as $r \to 0$ and $\overline{p}^* \to \overline{p}_{Vm}$ as $r \to \infty$. See Proposition A2 in the Appendix.

[65] Recall the specification of $R$'s cost function in equation (4).

[66] It can also be shown that $\frac{d\overline{p}^*}{dc} > 0$ when $\overline{p}^* \in (\overline{p}_0, \overline{p}_d)$. Furthermore, numerical solutions reveal that $\overline{p}^*$ often increases as: (i) $a$, $k$, or $k^R$ increases; or (ii) $k_N$ or $b$ declines. This is the case, for example, in the baseline setting and for substantial variation in parameters around their values in the baseline setting.

Proposition 6 implies that the welfare-maximizing price cap becomes more stringent as access to $A$'s input reduces $R$'s marginal cost more substantially (i.e., $\overline{p}^*$ declines as $c_N - c_A$ increases). Intuitively, the welfare-maximizing price cap becomes more stringent as the value of $A$'s input increases. In essence, the price cap instrument is employed more extensively when it becomes more costly for $R$ to avoid the impact of the price cap by operating without $A$'s input.

Figure 4 illustrates how welfare varies with $\overline{p}$ in the baseline setting when $r = \frac{1}{2}$, so $W(\overline{p}) = S(\overline{p}) - \frac{1}{2} V(\overline{p})$.[67] As is apparent from Figure 4, the welfare-maximizing price cap strictly exceeds $\overline{p}_0$ in the baseline setting when $r = \frac{1}{2}$.[68] As $\overline{p}$ declines from $\overline{p}_b = 71.52$ to $\overline{p}^* = 54.31$, welfare increases by nearly 50%, from 2.40 (million dollars) to 3.58. Welfare then declines to 2.06 as $\overline{p}$ declines from $\overline{p}^*$ to $\overline{p}_0 = 41.86$.[69] Tables A1 and A2 in the Appendix report that the welfare-maximizing price cap generates corresponding increases in welfare as parameter values diverge substantially from the levels in the baseline setting. Table A3 in the Appendix reports how $\overline{p}^*$, $\frac{\overline{p}^*}{\overline{p}_b}$, $W(\overline{p}^*)$, and $\frac{W(\overline{p}^*) - W(\overline{p}_b)}{|W(\overline{p}_b)|}$ vary as $r$ varies in the baseline setting.[70]

[**Figure 4 about Here**]

# 6 Model Variations.

We now briefly extend the foregoing analysis to consider nonlinear demand functions, to examine $R$'s profit, and to illustrate the effects of an alternative welfare function.

To begin, consider the *modified baseline setting*, which parallels the baseline setting described in section 4 except that the inverse aggregate demand for the homogeneous product is $P(Q) = m\, Q^{-\frac{1}{\varepsilon}}$, where $m = 10^6$ and $\varepsilon = 2$. The parameters of this iso-elastic demand formulation are chosen to satisfy relevant second order conditions and to generate an equilibrium price near 70 in the absence of a price cap.[71] The equilibrium unrestricted market

[67] Figure A2 in Turner and Sappington (2024) illustrates how welfare, consumer surplus, and $R$'s revenue all vary with $\overline{p}$ in the baseline setting when $r = \frac{1}{2}$.

[68] $\Phi_2 < 0$ in the baseline setting, and for the variations in the baseline parameters identified in Table A1. $\Phi_2 > 0$ if, for example, $c_N$ exceeds 21 while all other parameters remain at their values in the baseline setting.

[69] Numerical solutions reveal that $W(\overline{p}^*)$ often increases as: (i) $c_A$, $k_A$, $k^R$, $c$, or $k$ declines; or (ii) $a$, $c_N$, $k_N$, or $b$ increases. This is the case, for example, as parameters vary (substantially) around their values in the baseline setting. These findings indicate in part that higher levels of welfare can often be achieved when it is more costly for $R$ to diminish the impact of a binding price cap by producing more of its output without employing the Alliance input.

[70] The absolute value sign in the denominator of the proportionate increase in welfare reflects the fact that welfare as defined in equation (6) can be negative if $r$ is suffiicently large.

[71] The second order condition for $R$'s maximization problem is violated if $\varepsilon$ is too small. Recall that a monopolist can always increase its profit by reducing its output when demand is inelastic.

price ($P(Q)$) declines at the same rate that the price cap ($\overline{p}$) declines as $\overline{p}$ declines from from $\overline{p}_b = 71.52$ to $\overline{p}_d = 67.59$ in the modified baseline setting. $R$'s revenue increases by 12.35% (from 7.84 billion to 8.97 billion) as the price cap declines in this range.[72]

The analysis to this point has focused on the impact of a binding price cap on $R$'s revenue because, as noted in the Introduction, the tax that the Russian government imposes on oil exporters is based primarily on oil export revenues. However, just as a price cap can increase the revenue of the sanctioned supplier ($R$), it can increase $R$'s profit. Figure 5 documents how $R$'s profit varies with the cap ($\overline{p}$) on the price at which $R$ can sell output ($q_A$) using the $A$'s input in the baseline setting. Much like $R$'s revenue, $R$'s profit ($\Pi^R$) increases as $\overline{p}$ declines below the equilibrium price in the absence of a price cap ($\overline{p}_b = 71.52$). $\Pi^R$ continues to increase as $\overline{p}$ declines from 71.52 to 62.38, which constitutes a 12.8% reduction in the price cap. $\Pi^R$ increases by 6.07%, from 2.80 billion to 2.97 billion, as $\overline{p}$ declines from 71.52 to 62.38.[73] $\Pi^R$ remains above the profit that $R$ secures in the absence of a price cap until the cap falls below 55.12, which constitutes nearly a 23% reduction in $\overline{p}$ below $\overline{p}_b$.[74]

[**Figure 5 about Here**]

The welfare analysis in section 5 implicitly treated the welfare of different consumers symmetrically. We now illustrate the changes that can arise when the designers of a price cap value differently the welfare of "A consumers" – consumers who purchase the output that is supplied using $A$'s input – and the welfare of "other consumers" – consumers who purchase the output that is supplied without using $A$'s input. Specifically, suppose the designers set the price cap ($\overline{p}$) to maximize $W_w(\overline{p}) = w_A S_A(\overline{p}) + [1 - w_A] S_{-A}(\overline{p}) - r V(\overline{p})$, where $w_A \in (0, 1)$ and $r > 0$ are parameters, $S_A(\cdot)$ is the surplus of A consumers, $S_{-A}(\cdot)$ is the surplus of other consumers, and $V(\cdot)$ is $R$'s revenue.[75]

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[72] Table TA1 in Turner and Sappington (2024, Part F) reports how equilibrium outcomes change as parameter values change in the modified baseline setting.

[73] This percentage increase in $R$'s profit is less than the correponding increase in $R$'s revenue (19%) that arises as $\overline{p}$ declines from $\overline{p}_b = 71.52$ to $\overline{p}_d = 56.35$ in the baseline setting. (Recall Table 2.) The smaller percentage increase in profit arises in part because $R$'s costs increase as the output expansion effect induces $R$ to increase its outputs ($q_A$ and $q_N$) as the price cap declines below $\overline{p}_b = 71.52$.

[74] Welfare effects similar to those reported in Section 5 arise if welfare is the difference between consumer surplus and one-half of $R$'s profit (rather than $R$'s revenue). This alternative measure of welfare increases by approximately 52%, from 2.66 billion to 4.07 billion, as $\overline{p}$ declines from $\overline{p}_b = 71.52$ to $\overline{p}_d = 56.35$ in the baseline setting. More generally, Sappington and Turner (2024, Part C) prove that when welfare is the difference between consumer surplus and a multiple $r > 0$ of $R$'s profit, the welfare-maximizing price cap lies in the interval $[\overline{p}_0, \overline{p}_d]$, just as it does in the analysis in section 5.

[75] Under the maintained assumption of efficient rationing, $S_A(\overline{p}) \equiv \int_0^{q_A(\overline{p})} [a - b\,\widetilde{Q}]\,d\widetilde{Q} - \overline{p}\,q_A(\overline{p})$ and $S_{-A}(\overline{p}) \equiv \int_{q_A(\overline{p})}^{Q(\overline{p})} [a - b\,\widetilde{Q}]\,d\widetilde{Q} - P(Q(\overline{p}))\,[q_N(\overline{p}) + q(\overline{p})]$.

Table 3 reports how outcomes in the baseline setting change as $w_A$ changes when $r = \frac{1}{2}$. Three effects warrant emphasis. First, $\overline{p}_w^*$ (the price cap that maximizes $W_w$) increases as $w_A$ (the weight placed on the surplus of A consumers) increases.[76] The increase in $\overline{p}_w^*$ induces $R$ to expand its output using $A$'s input ($q_A$).[77] The increase in $q_A$ increases the corresponding consumer surplus, $S_A(\cdot)$, despite the increase in the price at which $q_A$ is sold ($\overline{p}_w^*$).[78]

Second, the surplus of other consumers declines as $w_A$ increases. The reduction in $S_{-A}(\cdot)$ arises because the aforementioned increase in $q_A$ reduces the equilibrium price of the output that is supplied without using $A$'s input ($P(Q)$), which reduces the corresponding equilibrium outputs ($q_N$ and $q$) and consumer surplus ($S_{-A}(\cdot)$). Third, $R$'s revenue increases as $w_A$ increases. The increase in $V(\cdot)$ stems from the identified increases in $q_A$ and $\overline{p}_w^*$.

| $w_A$ | $\overline{p}_w^*$ | $q_A$ | $q_N$ | $q$ | $P(Q)$ | $S_A$ | $S_{-A}$ | $V$ | $W_w$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 41.82 | 0.00 | 3.46 | 4.67 | 78.98 | 0.00 | 3.43 | 2.73 | 1.72 |
| 0.3 | 42.59 | 0.25 | 3.40 | 4.60 | 77.84 | 0.29 | 3.32 | 2.75 | 1.04 |
| 0.5 | 51.52 | 3.23 | 2.60 | 3.75 | 63.89 | 3.07 | 2.09 | 3.33 | 0.92 |
| 0.7 | 55.76 | 4.66 | 2.23 | 3.35 | 57.26 | 3.88 | 1.61 | 3.87 | 1.27 |
| 0.9 | 56.35 | 4.85 | 2.18 | 3.29 | 56.35 | 3.97 | 1.55 | 3.96 | 1.75 |

**Table 3. Equilibrium Outcomes with Weighted Welfare in the Baseline Setting.**

## 7 Conclusions.

We have examined the design of price caps as an instrument to reduce the (tax) revenue available to a sanctioned nation without causing the world price of a key product to increase excessively. We have shown that a price cap on a portion of a supplier's output can have potentially counterintuitive effects. Specifically, the price cap can increase, not reduce, the supplier's revenue by inducing the supplier to increase its output. Furthermore, the sanctioned supplier's increased output can cause the world price of the product to decline, not increase.

The supplier's increased output stems from the output enhancement effect of a price cap. This effect arises because when output is sold at the capped price, the sales price does not decline as output increases. Consequently, the profit the supplier secures from the enhanced

[76] Numerical solutions reveal that this qualitative conclusion persists for wide variation in the parameters in the baseline setting.

[77] The welfare-maximizing price cap ($\overline{p}_w^*$) lies in the interval $[\overline{p}_0, \overline{p}_d]$, where $R$ increases $q_A$ as the price cap ($\overline{p}$) increases. The outputs recorded in the third, fourth, and fifth columns in Table 3 are in units of 10 million. All outputs are equilibrium outputs when the price cap is $\overline{p}_w^*$.

[78] The entries for consumer surplus, revenue, and welfare in the last four columns in Table 3 are reported in billions. All these outcomes are equilibrium outcomes when the price cap is $\overline{p}_w^*$.

output increases relatively rapidly, which induces the supplier to increase its output. The increased output reduces the price at which output that is not subject to the price cap is sold, *ceteris paribus*. The increased output can also increase the supplier's revenue.

We have also shown that the welfare-maximizing price cap often is well below the prevailing market price of the product, and that a price cap can enhance welfare considerably under arguably plausible conditions. In addition, we have shown that raising a price cap above the level that would eliminate sales by the sanctioned supplier at the capped price often can both increase consumer surplus and reduce the aggregate revenue of the sanctioned producer. Thus, moderately stringent price caps often outperform relatively lenient or particularly severe price caps.

Our streamlined duopoly model was designed to illustrate simply and clearly potentially counterintuitive effects of price caps as sanctions. Future research should consider additional demand and cost functions, alternative rationing rules, differentiated products, more than two suppliers, and different market interactions (e.g., bargaining among industry suppliers and large buyers). Future research might also consider alternative (e.g., nonlinear) welfare functions and allow the sanctioned supplier to act to reduce the cost it incurs when it operates without access to key (Alliance) inputs. Future research might also consider the coordination (and enforcement) problems that arise when the nations that impose the price cap differ in their valuations of the sanctioned product.

These model extensions likely will alter the extent to which a more stringent price cap increases the revenue (and the profit) of a sanctioned supplier, the level of the welfare-maximizing pice cap, and the potential welfare gains from a price cap. However, the model extensions seem unlikely to eliminate the output enhancement effect of a price cap that we have identified. Consequently, the model extensions seem unlikely to fundamentally alter the potentially counterintuitive effects that arise in our streamlined model of price caps as sanctions.

## Appendix

Part A of this Appendix illustrates how equilibrium outcomes change as parameter values diverge from their levels in the baseline setting. Part B presents the proofs of the formal conclusions in the text. Part C analyzes a benchmark setting in which $R$ is a monopoly supplier.

### A. Outcomes in Settings Other Than the Baseline Setting.

| Parameter Variation | $\frac{\overline{p}_b - \overline{p}_d}{\overline{p}_b}$ | $\frac{V(\overline{p}_d) - V(\overline{p}_b)}{V(\overline{p}_b)}$ | $\overline{p}^*$ | $\frac{\overline{p}^*}{\overline{p}_b}$ | $\frac{W(\overline{p}^*) - W(\overline{p}_b)}{W(\overline{p}_b)}$ |
|---|---|---|---|---|---|
| $1.50\,a$ | 0.21 | 0.19 | 81.08 | 0.76 | 0.48 |
| $0.50\,a$ | 0.21 | 0.21 | 27.55 | 0.75 | 0.52 |
| $1.50\,b$ | 0.26 | 0.19 | 49.57 | 0.74 | 0.51 |
| $0.50\,b$ | 0.13 | 0.19 | 59.56 | 0.72 | 0.65 |
| $1.50\,c_A$ | 0.20 | 0.20 | 55.24 | 0.77 | 0.46 |
| $0.50\,c_A$ | 0.22 | 0.19 | 53.39 | 0.75 | 0.51 |
| $1.50\,k_A$ | 0.15 | 0.18 | 57.86 | 0.79 | 0.34 |
| $0.5\,k_A$ | 0.35 | 0.15 | 45.36 | 0.65 | 0.87 |
| $1.50\,k^R$ | 0.20 | 0.20 | 55.47 | 0.77 | 0.46 |
| $0.50\,k^R$ | 0.23 | 0.19 | 53.06 | 0.75 | 0.52 |
| $1.50\,c_N$ | 0.22 | 0.20 | 53.58 | 0.75 | 0.52 |
| $0.50\,c_N$ | 0.20 | 0.19 | 55.05 | 0.77 | 0.46 |
| $1.50\,k_N$ | 0.24 | 0.21 | 51.03 | 0.71 | 0.60 |
| $0.50\,k_N$ | 0.17 | 0.15 | 58.81 | 0.83 | 0.34 |
| $1.50\,c$ | 0.21 | 0.19 | 54.51 | 0.76 | 0.50 |
| $0.50\,c$ | 0.21 | 0.19 | 54.12 | 0.76 | 0.48 |
| $1.50\,k$ | 0.22 | 0.17 | 56.22 | 0.75 | 0.63 |
| $0.50\,k$ | 0.20 | 0.23 | 51.78 | 0.77 | 0.36 |

**Table A1. The Effects of Changing Baseline Parameters.**

The first column in Table A1 identifies the single parameter that is changed in the baseline setting and the amount by which it is changed. All other parameters remain at their levels in the baseline setting.[79] The remaining columns in Table A1 identify outcomes that arise in equilibrium, including the outcomes reported above in Table 2 and the outcomes reported below (and defined) in Table A3. The welfare calculations in the last column assume that $r = \frac{1}{2}$.

[79] For example, the first row of data in Table A1 records the outcomes that arise in equilibrium when $a$ is increased by 50% above its level in the baseline setting, holding all other parameters at their values in the baseline setting.

| $\beta$ | $\frac{\overline{p}_b - \overline{p}_d}{\overline{p}_b}$ | $\frac{V(\overline{p}_d) - V(\overline{p}_b)}{V(\overline{p}_b)}$ | $\overline{p}^*$ | $\frac{\overline{p}^*}{\overline{p}_b}$ | $\frac{W(\overline{p}^*) - W(\overline{p}_b)}{W(\overline{p}_b)}$ |
|---|---|---|---|---|---|
| 0.25 | 0.36 | 0.14 | 44.23 | 0.64 | 0.91 |
| 0.50 | 0.21 | 0.19 | 54.31 | 0.76 | 0.49 |
| 0.75 | 0.14 | 0.18 | 58.69 | 0.80 | 0.32 |
| 1.00 | 0.11 | 0.16 | 61.34 | 0.83 | 0.23 |

**Table A2. Additional Effects of Changing Baseline Parameters.**

Table A2 reports the impact of changing $\beta$ in the baseline setting, where $c_A = \beta\, c_N$ and $k_A = \beta\, k_N$. The first column in Table A1 identifies the relevant value of $\beta$. All parameters other than $c_A$, $k_A$, and $\beta$ remain at their levels in the baseline setting (including $c_N = 5$ and $k_N = 1 \times 10^{-6}$). The remaining columns in Table A2 identify outcomes that arise in equilibrium, including the outcomes reported above in Table 2 and the outcomes reported below in Table A3. The welfare calculations in the last column assume that $r = \frac{1}{2}$.

| $r$ | $\overline{p}^*$ | $\frac{\overline{p}^*}{\overline{p}_b}$ | $W(\overline{p}^*)$ | $\frac{\lvert W(\overline{p}^*) - W(\overline{p}_b)\rvert}{\lvert W(\overline{p}_b)\rvert}$ |
|---|---|---|---|---|
| 0.00 | 56.35 | 0.79 | $5.518 \times 10^9$ | 0.36 |
| 0.25 | 56.26 | 0.79 | $4.529 \times 10^9$ | 0.40 |
| 0.50 | 54.31 | 0.76 | $3.579 \times 10^9$ | 0.49 |
| 0.75 | 52.79 | 0.74 | $2.688 \times 10^9$ | 0.70 |
| 1.00 | 51.56 | 0.72 | $1.837 \times 10^9$ | 1.46 |
| 2.00 | 48.36 | 0.68 | $-1.325 \times 10^9$ | 0.48 |
| 10.0 | 42.70 | 0.60 | $-23.873 \times 10^9$ | 0.18 |

**Table A3. The Effects of Changing $r$ in the Baseline Setting.**

The first column in Table A3 identifies the value of $r$ in the welfare function $W(\cdot) = S(\cdot) - r\,V(\cdot)$. The remaining columns report the corresponding welfare-maximizing price cap ($\overline{p}^*$), the ratio of this price cap to the equilibrium price in the absence of a price cap ($\overline{p}_b$), the maximized level of welfare ($W(\overline{p}^*)$), and the proportionate maximum welfare gain, respectively.[80]

[80] The relatively large welfare gain that arises when $r = 1$ arises in part because $W(\overline{p}_b)$ is relatively close to 0 in the baseline setting when $r = 1$.

## B. Proofs of Formal Conclusions in the Text[81]

Proof of Proposition 1. The proof follows directly from Lemmas A1 – A6 (below), which refer to the following definitions.[82]

$$\overline{p}_0 \;\equiv\; c_A + \frac{[\,a - c_N\,]\,[\,2\,b + k\,] - b\,[\,a - c\,]}{[\,2\,b + k_N + k^R\,]\,[\,2\,b + k\,] - b^2}\,\left[\,b + k^R\,\right]. \tag{7}$$

$$\overline{p}_d \;\equiv\; \frac{1}{D_2}\,\Big\{\,[\,a\,(b+k) + b\,c\,]\left[\,\left(b + k^R\right)(k_N + k_A) + k_N\,k_A - b\,k_N\,\right] + \; b\,[\,b+k\,]\,[\,k_A - b\,]\,c_N + b\,[\,k_N + b\,]\,[\,b + k\,]\,c_A\,\Big\}$$

$$\text{where}\;\; D_2 \;\equiv\; b\,[\,b+k\,]\,[\,k_N + k_A\,] + k_N\,[\,k_A - b\,]\,[\,2\,b + k\,] + \;[\,k_N + k_A\,]\,[\,2\,b + k\,]\left[\,b + k^R\,\right]. \tag{8}$$

$$\overline{p}_b \;\equiv\; \frac{1}{D_3}\,\Big\{\,[\,a\,(b+k) + b\,c\,]\left[\,\left(b + k^R\right)(k_N + k_A) + k_N\,k_A\,\right] + \; b\,c_N\,[\,b+k\,]\,k_A + b\,k_N\,[\,b+k\,]\,c_A\,\Big\}$$

$$\text{where}\;\; D_3 \;\equiv\; b\,[\,b+k\,]\,[\,k_N + k_A\,] + k_N\,k_A\,[\,2\,b + k\,] + \;[\,k_N + k_A\,]\,[\,2\,b + k\,]\left[\,b + k^R\,\right] \;=\; D_2 + b\,k_N\,[\,2\,b + k\,]. \tag{9}$$

**Lemma A1**. Suppose $\overline{p} \leq \overline{p}_0$. Then in equilibrium:

$$q_A \;=\; 0\,, \quad q_N \;=\; \frac{[\,a - c_N\,]\,[\,2\,b + k\,] - b\,[\,a - c\,]}{[\,2\,b + k_N + k^R\,]\,[\,2\,b + k\,] - b^2}\,,$$

$$q \;=\; \frac{[\,a - c\,]\left[\,2\,b + k_N + k^R\,\right] - b\,[\,a - c_N\,]}{[\,2\,b + k_N + k^R\,]\,[\,2\,b + k\,] - b^2}\,, \;\text{ and}$$

$$Q \;=\; q_A + q_N + q \;=\; \frac{[\,a - c\,]\left[\,b + k_N + k^R\,\right] + [\,a - c_N\,]\,[\,b + k\,]}{[\,2\,b + k_N + k^R\,]\,[\,2\,b + k\,] - b^2}\,. \tag{10}$$

**Lemma A2**. *Suppose* $\overline{p} \in (\overline{p}_0, \overline{p}_d\,]$. *Then in equilibrium*:

$$q_A \;=\; \frac{1}{D}\,\Big\{\left[\,3\,b^2 + 2\,b\left(k + k_N + k^R\right) + k\left(k_N + k^R\right)\right][\,\overline{p} - c_A\,] + \; b\left[\,b + k^R\,\right][\,a - c\,] - [\,2\,b + k\,]\left[\,b + k^R\,\right][\,a - c_N\,]\,\Big\}\,; \tag{11}$$

[81] Part B of this Appendix sketches the proofs of the formal conclusions in the text. Detailed proofs are available in Turner and Sappington (2024, Part A).

[82] The proofs of Lemmas A1, A2, and A4 – A6 employ relatively standard techniques, and so are omitted. Detailed proofs of these lemmas are available in Turner and Sappington (2024, Part A).

$$q_N = \frac{1}{D}\left\{ [2b+k]\left[k_A + k^R\right][a - c_N] - b\left[k_A + k^R\right][a - c] \right.$$

$$\left. - \left[b\left(b + 2k^R\right) + k\left(b + k^R\right)\right][\overline{p} - c_A] \right\}; \tag{12}$$

$$Q^R \equiv q_A + q_N = \frac{1}{D}\left\{ [2b+k][b + k_N][\overline{p} - c_A] + [2b+k][k_A - b][a - c_N] \right.$$

$$\left. - b[k_A - b][a - c] \right\}; \tag{13}$$

$$q = \frac{1}{D}\left\{ \left[k_N\left(k_A + k^R\right) + k_A k^R + 2b k_A - b^2\right][a - c] \right.$$

$$\left. - b[k_A - b][a - c_N] - b[b + k_N][\overline{p} - c_A] \right\}; \text{ and} \tag{14}$$

$$Q = q + q_A + q_N = \frac{1}{D}\left\{ [b+k][b + k_N][\overline{p} - c_A] + [b+k][k_A - b][a - c_N] \right.$$

$$\left. + \left[k^R(k_A + k_N) + k_A(b + k_N)\right][a - c] \right\} \tag{15}$$

where[83]

$$D \equiv [2b+k]\left[k_N\left(k_A + k^R\right) + k_A k^R\right] + b k_A[3b + 2k] - b^2[b+k] > 0. \tag{16}$$

**Lemma A3**. *Suppose* $\overline{p} \in (\overline{p}_d, \overline{p}_b]$, *where* $\overline{p}_d < \overline{p}_b$. *Then in equilibrium,* $P(Q) = \overline{p}$. *Furthermore*:

$$q_A = \frac{b[b+k][c_N - c_A] + k_N[a - \overline{p}][b+k] - b k_N[\overline{p} - c]}{b[b+k][k_N + k_A]};$$

$$q_N = \frac{k_A[b+k][a - \overline{p}] - b k_A[\overline{p} - c] - b[b+k][c_N - c_A]}{b[b+k][k_N + k_A]}; \quad q = \frac{\overline{p} - c}{b+k};$$

$$Q^R \equiv q_A + q_N = \frac{[b+k][a - \overline{p}] - b[\overline{p} - c]}{b[b+k]}; \text{ and } Q \equiv \frac{a - \overline{p}}{b}. \tag{17}$$

<u>Proof</u>. (1) implies that $R$'s problem can be written as:

$$\underset{q_A,\, Q^R}{\text{Maximize}} \ \Pi_R \equiv \left[P_A(q + Q^R) - c_A\right] q_A + \left[P(Q^R + q) - c_N\right]\left[Q^R - q_A\right]$$

$$- \frac{k_A}{2}[q_A]^2 - \frac{k_N}{2}\left[Q^R - q_A\right]^2 - \frac{k^R}{2}\left[Q^R\right]^2$$

$$\text{where } P_A(q + Q^R) = \begin{cases} \overline{p} & \text{if } P(q + Q^R) \geq \overline{p} \\ P(q + Q^R) & \text{if } \overline{p} > P(q + Q^R). \end{cases} \tag{18}$$

[83] As noted above, the inequality in (16) is ensured by the maintained assumption that $k_A[2b + k_N] + k^R[k_A + k_N] > b^2$.

(18) implies that when $q_A > 0$ and there exists a range of $\overline{p}$ for which $P(Q) = \overline{p}$, the necessary conditions for a solution to $R$'s problem are:

$$\frac{\partial \Pi_R}{\partial q_A} \;=\; P_A\left(q + Q^R\right) - c_A - k_A\, q_A - \left[\, P\left(q + Q^R\right) - c_N \,\right] + k_N \left[\, Q^R - q_A \,\right] \;=\; 0\,; \tag{19}$$

$$\frac{\partial^+ \Pi^R}{\partial Q^R} \;\leq\; 0 \;\text{ and }\; \frac{\partial^- \Pi^R}{\partial Q^R} \;\geq\; 0 \;\text{ for all }\; \overline{p} \in [\,\overline{p}_d, \overline{p}_b\,]\,, \tag{20}$$

where: (i) $\frac{\partial^- \Pi_R}{\partial Q^R}$ denotes the left-sided derivative of $\Pi_R$ with respect to $Q^R$, which is relevant when $P_A(\cdot) = \overline{p}$; and (ii) $\frac{\partial^+ \Pi_R}{\partial Q^R}$ denotes the right-sided derivative of $\Pi_R$ with respect to $Q^R$, which is relevant when $P_A(\cdot) = P(Q)$. The first inequality in (20) indicates that $R$'s profit declines if $R$ increases $Q^R$ so as to reduce $P(Q)$ below $\overline{p}$ (thereby rendering the cap nonbinding). The second inequality in (20) indicates that $R$'s profit declines if $R$ reduces $Q^R$ so as to increase $P(Q)$ above $\overline{p}$ (thereby causing the cap to bind). Together, the inequalities in (20) ensure that when $\overline{p} \in [\,\overline{p}_d, \overline{p}_b\,]$, $R$ cannot increase its profit by changing $Q^R$ so as to cause $P(Q)$ to differ from $\overline{p}$.

(2) implies that the rival's choice of $q$ is determined by:

$$\overline{p} - b\,q - c - k\,q \;=\; 0 \quad \Leftrightarrow \quad q \;=\; \frac{\overline{p} - c}{b + k}\,. \tag{21}$$

Because $\overline{p} \,=\, a - b\left[\, q + Q^R \,\right]$, (21) implies:

$$\overline{p} \;=\; a - b\left[\frac{\overline{p} - c}{b + k} + Q^R\right] \;\;\Leftrightarrow\;\; Q^R \;=\; \frac{[\, a - \overline{p}\,]\,[\, b + k\,] - b\,[\,\overline{p} - c\,]}{b\,[\, b + k\,]}\,. \tag{22}$$

Because $\overline{p} \,=\, P_A(q + Q^R)$ in equilibrium, (19) holds if:

$$\begin{aligned} &\overline{p} - c_A - k_A\, q_A - [\,\overline{p} - c_N\,] + k_N \left[\, Q^R - q_A \,\right] \;=\; 0 \\ \Leftrightarrow \quad & c_N - c_A - k_A\, q_A + k_N\, Q^R - k_N\, q_A \;=\; 0\,. \end{aligned} \tag{23}$$

(22) implies that (23) holds if:

$$q_A \;=\; \frac{b\,[\, b + k\,]\,[\, c_N - c_A\,] + k_N\,[\, a - \overline{p}\,]\,[\, b + k\,] - b\,k_N\,[\,\overline{p} - c\,]}{b\,[\, b + k\,]\,[\, k_N + k_A\,]}\,. \tag{24}$$

(22) and (24) imply:

$$q_N \;=\; Q^R - q_A \;=\; \frac{k_A\,[\, b + k\,]\,[\, a - \overline{p}\,] - b\,k_A\,[\,\overline{p} - c\,] \,-\, b\,[\, b + k\,]\,[\, c_N - c_A\,]}{b\,[\, b + k\,]\,[\, k_N + k_A\,]}\,. \tag{25}$$

(24) and (25) imply:

$$Q^R \;\equiv\; q_A \,+\, q_N \;=\; \frac{[\, b + k\,]\,[\, a - \,\overline{p}\,\,] - b\,[\,\overline{p} - c\,]}{b\,[\, b + k\,]}\,. \tag{26}$$

(21) and (26) imply:

$$Q \;\equiv\; Q^R + q \;=\; \frac{[\,b+k\,]\,[\,a-\overline{p}\,] - b\,[\,\overline{p}-c\,]}{b\,[\,b+k\,]} + \frac{b\,[\,\overline{p}-c\,]}{b\,[\,b+k\,]} \;=\; \frac{a-\overline{p}}{b}\,.$$

(18) implies:

$$\begin{aligned}\frac{\partial^+ \Pi_R}{\partial Q^R} \;&=\; a - 2\,b\,Q^R - b\,q - c_N - k_N\left[\,Q^R - q_A\,\right] - k^R\,Q^R \\ &=\; \overline{p} - b\,Q^R - c_N - k_N\,q_N - k^R\,Q^R \;=\; \overline{p} - \left[\,b + k^R\,\right]Q^R - c_N - k_N\,q_N\,; \end{aligned} \tag{27}$$

$$\begin{aligned}\frac{\partial^- \Pi_R}{\partial Q^R} \;&=\; a - 2\,b\,Q^R - b\,q - c_N + b\,q_A - k_N\left[\,Q^R - q_A\,\right] - k^R\,Q^R \\ &=\; \overline{p} - \left[\,b + k^R\,\right]Q^R - c_N + b\,q_A - k_N\,q_N\,. \end{aligned} \tag{28}$$

(27) and (28) imply that (20) can be written as:

$$\left[\,b + k^R\,\right]Q^R + c_N + k_N\,q_N - b\,q_A \;<\; \overline{p} \;\leq\; \left[\,b + k^R\,\right]Q^R + c_N + k_N\,q_N\,. \tag{29}$$

(9), (22), and (25) imply:

$$\begin{aligned} &\overline{p} \;\leq\; \left[\,b + k^R\right]Q^R + c_N + k_N\,q_N \\ \Leftrightarrow\quad &\left[\,b + k^R\,\right][\,a\,(b+k) + b\,c\,]\,[\,k_N + k_A\,] + c_N\,b\,[\,b+k\,]\,[\,k_N + k_A\,] \\ &\qquad + \; k_N\,[\,k_A\,(b\,+\,k)\,a + b\,k_A\,c - b\,(b+k)\,(c_N - c_A)\,] \\ &\quad \geq\; \overline{p}\;[\,b\,(b+k)\,(k_N + k_A) + k_N\,k_A\,(2\,b + k) \\ &\qquad + \;(k_N + k_A)\,(2\,b + k)\,\big(b + k^R\big)\,] \;=\; \overline{p}\;D_3. \end{aligned} \tag{30}$$

(30) implies:

$$\overline{p} \;\leq\; \left[\,b + k^R\,\right]Q^R + c_N + k_N\,q_N \quad\Leftrightarrow\quad \overline{p} \;\leq\; \overline{p}_b\,. \tag{31}$$

(8), (22), (24), and (25) imply:

$$\begin{aligned} &\left[\,b + k^R\,\right]Q^R + c_N + k_N\,q_N - b\,q_A \;<\; \overline{p} \\ \Leftrightarrow\quad &\left[\,b + k^R\,\right][\,a\,(b+k) + b\,c\,]\,[\,k_N + k_A\,] + c_N\,b\,[\,b+k\,]\,[\,k_N + k_A\,] \\ &\qquad + \; k_N\,[\,k_A\,(b+k)\,a + b\,k_A\,c - b\,(b+k)\,(c_N - c_A)\,] \\ &\qquad - \; b\,[\,b\,(b+k)\,(c_N - c_A) + k_N\,a\,(b+k) + b\,k_N\,c\,] \\ &\quad <\; \overline{p}\;[\,b\,(b+k)\,(k_N + k_A) + k_N\,(k_A - b)\,(2\,b + k) \\ &\qquad + \;(k_N + k_A)\,(2\,b + k)\,\big(b + k^R\big)\,] \;=\; \overline{p}\;D_2\,. \end{aligned} \tag{32}$$

(32) implies:

$$\left[\,b + k^R\,\right]Q^R + c_N + k_N\,q_N - b\,q_A \;<\; \overline{p}$$

$$\Leftrightarrow \quad \overline{p} \; > \; \frac{1}{D_2} \left\{ \left[ a\,(b+k) + b\,c \right] \left[ \left( b + k^R \right) (k_N + k_A) + k_N\,k_A - b\,k_N \right] \right.$$
$$\left. + \; b\,[\,b+k\,]\,[\,k_A - b\,]\,c_N + b\,[\,k_N + b\,]\,[\,b+k\,]\,c_A \right\} \; \equiv \; \overline{p}_d\,. \qquad (33)$$

(9), (27), (28), (31), and (33) imply:

$$\overline{p}_d \;=\; \left[\,b + k^R\,\right] Q^R + c_N + k_N\,q_N - b\,q_A \quad \text{and}$$
$$\overline{p}_b \;=\; \left[\,b + k^R\,\right] Q^R + c_N + k_N\,q_N\,. \qquad (34)$$

(34) implies that $\overline{p}_d < \overline{p}_b$ because $q_A > 0$ when $\overline{p} > \overline{p}_0$. $\square$

**Lemma A4**. *Suppose* $\overline{p} > \overline{p}_b$. *Then in equilibrium*:

$$q_A \;=\; \frac{1}{D_3} \left\{ [\,a - c_A\,] \left[ 2\,b\,k + 2\,b\,k_N + 2\,b\,k^R + k\,k_N + k\,k^R + 3\,b^2 \right] \right.$$
$$\left. - \; [\,a - c_N\,] \left[ 2\,b\,k + 2\,b\,k^R + k\,k^R + 3\,b^2 \right] - b\,k_N\,[\,a - c\,] \right\}; \qquad (35)$$

$$q_N \;=\; \frac{1}{D_3} \left\{ [\,a - c_N\,] \left[ 2\,b\,k + 2\,b\,k_A + 2\,b\,k^R + k\,k_A + k\,k^R + 3\,b^2 \right] \right.$$
$$\left. - \; [\,a - c_A\,] \left[ 2\,b\,k + 2\,b\,k^R + k\,k^R + 3\,b^2 \right] - b\,k_A\,[\,a - c\,] \right\}; \qquad (36)$$

$$q \;=\; \frac{1}{D_3} \left\{ [\,a - c\,] \left[ 2\,b\,k_A + 2\,b\,k_N + k_A\,k_N + k_A\,k^R + k_N\,k^R \right] \right.$$
$$\left. - \; b\,k_A\,[\,a - c_N\,] - b\,k_N\,[\,a - c_A\,] \right\}; \quad \text{and} \qquad (37)$$

$$Q^R \;\equiv\; q_A + q_N \;=\; \frac{1}{D_3} \left\{ [\,a - c_A\,]\,k_N\,[\,2\,b + k\,] + [\,a - c_N\,]\,k_A\,[\,2\,b + k\,] \right.$$
$$\left. - \; b\,[\,k_A + k_N\,]\,[\,a - c\,] \right\} \qquad (38)$$

*where* $D_3$ *is as specified in (9).*

<u>Definitions</u>

$q_{A1}(\overline{p}_0)$, $q_{N1}(\overline{p}_0)$, and $q_1(\overline{p}_0)$, respectively, denote the values of $q_A(\cdot)$, $q_N(\cdot)$, and $q(\cdot)$ specified in Lemma A1, where $\overline{p} \leq \overline{p}_0$.

$q_{A2}(\overline{p}_0)$, $q_{N2}(\overline{p}_0)$, and $q_2(\overline{p}_0)$, respectively, denote the values of $q_A(\cdot)$, $q_N(\cdot)$, and $q(\cdot)$ specified in Lemma A2, where $\overline{p} \in (\,\overline{p}_0, \overline{p}_d\,]$.

**Lemma A5**. $\lim_{\overline{p} \to \overline{p}_0} q_{A2}(\overline{p}) = q_{A1}(\overline{p}_0)$, $\lim_{\overline{p} \to \overline{p}_0} q_{N2}(\overline{p}) = q_{N1}(\overline{p}_0)$, *and* $\lim_{\overline{p} \to \overline{p}_0} q_2(\overline{p}) = q_1(\overline{p}_0)$.

**Lemma A6**. $0 < \overline{p}_0 < \overline{p}_d < \overline{p}_b$. $\blacksquare$

Proof of Proposition 2. The conclusions in the proposition follow directly from Lemmas A2 and A3. ■

Proof of Proposition 3. (22) implies that for $\overline{p} \in (\overline{p}_d, \overline{p}_b)$, $R$'s revenue is:

$$V(\overline{p}) \;=\; \overline{p} \left[ \frac{a\,(b+k) + b\,c - \overline{p}\;(2\,b+k)}{b\,[\,b+k\,]} \right] \;=\; \frac{[\,a\,(b+k) + b\,c\,]\,\overline{p} - [\,2\,b+k\,]\,\overline{p}^2}{b\,[\,b+k\,]}\,. \tag{39}$$

The value of $\overline{p}$ at which $V(\overline{p})$ in (39) is maximized is determined by:

$$a\,[\,b+k\,] + b\,c - 2\,[\,2\,b+k\,]\,\overline{p} \;=\; 0 \;\;\Rightarrow\;\; \overline{p} \;=\; \frac{a\,[\,b+k\,] + b\,c}{2\,[\,2\,b+k\,]} \;\equiv\; \overline{p}_{VM}\,. \tag{40}$$

(9) and (40) imply that $\overline{p}_{VM} < \overline{p}_b$ if:

$$\frac{a\,[\,b+k\,] + b\,c}{2\,[\,2\,b+k\,]} \;<\; \frac{\left[\,\left(\,b+k^R\,\right)(\,k_N + k_A\,) + k_N\,k_A\,\right]\,\frac{a\,[\,b+k\,]+b\,c}{b\,[\,b+k\,]} + c_N\;k_A + k_N\,c_A}{k_N + k_A + [\,(\,b+k^R\,)\,(k_N + k_A\,) + k_N\,k_A\,]\,\frac{2\,b+k}{b\,[\,b+k\,]}}$$

$$\Leftrightarrow\;\; \frac{[\,2\,b+k\,]\,\left[\,b+k^R\,\right] - b\,[\,b+k\,]}{b\,[\,b+k\,]}\;[\,k_N + k_A\,] + k_N\,k_A \left[\frac{2\,b+k}{b\,(b\,+k)}\right] \;>\; 0\,. \tag{41}$$

It is readily verified that the inequality in (41) always holds, so $\overline{p}_{VM} < \overline{p}_b$.

(39) and (40) imply that for $\overline{p} \in (\overline{p}_d, \overline{p}_b)$, $V(\overline{p})$ is a strictly concave function that attains its maximum at $\overline{p}_{VM}$. Therefore, $\frac{\partial V(\overline{p})}{\partial \overline{p}} < 0$ for $\overline{p} \in (\overline{p}_{VM}, \overline{p}_b)$.

(8) and (40) imply that $\overline{p}_d \geq \overline{p}_{VM}$ if and only if:

$$\frac{1}{b\,[\,b+k\,]\,[\,k_N + k_A\,] + [\,k_A\,k_N - k_N\,b\,]\,[\,2\,b+k\,] + [\,k_N + k_A\,]\,[\,2\,b+k\,]\,[\,b+k^R\,]}$$

$$\cdot\;\{\,[\,(b+k)\,a + b\,c\,]\,\left[\,\left(b+k^R\right)(k_N + k_A) + k_N\,k_A - b\,k_N\,\right]$$

$$+\;\; b\,[\,b+k\,]\,[\,k_A - b\,]\,c_N + b\,[\,k_N + b\,]\,[\,b+k\,]\,c_A\,\}$$

$$\geq\;\; \frac{a\,[\,b+k\,] + b\,c}{2\,[\,b+k\,]}$$

$$\Leftrightarrow\;\; \left[\frac{(b+k)\,a + b\,c}{2\,b+k}\right]\left[\,\left(b^2 + k^R\,[\,2\,b+k\,]\right)(k_N + k_A) + k_N\,k_A\,(2\,b+k) - b\,k_N\,(2\,b+k)\,\right]$$

$$+\;2\,b\,[\,b+k\,]\,[\,k_A\,c_N + k_N\,c_A - b\,(c_N - c_A)\,] \;\geq\; 0\,. \tag{42}$$

It is readily verified that:

$$\left[\,b^2 + k^R\,(2\,b+k)\,\right][\,k_N + k_A\,] + k_N\,k_A\,[\,2\,b+k\,] - b\,k_N\,[\,2\,b+k\,] \;=\; -\,2\,b\,[\,b+k\,]\,[\,k_N + k_A\,] + D_2.$$

Therefore, (42) implies that $\overline{p}_d \geq \overline{p}_{VM} \;\Leftrightarrow\; \widetilde{\Phi}_1 \geq 0$, where:

$$\widetilde{\Phi}_1 \;\equiv\; \left[\frac{(b+k)\,a + b\,c}{2\,b+k}\right]\{\,D_2 - 2\,b\,[\,b+k\,]\,[\,k_N + k_A\,]\,\}$$

$$+\ 2\,b\,[\,b+k\,]\,[\,k_A\,c_N + k_N\,c_A - b\,(c_N - c_A)\,]\,.$$

It is readily verified that $\widetilde{\Phi}_1 = \Phi_1$, where:

$$\Phi_1 \equiv \left[ k^R + \frac{b^2}{2\,b+k} \right] [\,k_A + k_N\,]\,A + 2\,b\,[\,b+k\,]\,c_A\,[\,k_N + b\,]$$

$$+\ [\,2\,b\,(b+k)\,c_N + A\,k_N\,]\,[\,k_A - b\,] \quad \text{and} \quad A \equiv a\,[\,b+k\,] + b\,c\,. \quad \blacksquare$$

Proof of Proposition 4. Let $q_A(\overline{p})$ denote $R$'s equilibrium output using $A$'s input when the price cap is $\overline{p} \in [\,\overline{p}_d, \overline{p}_b\,]$. Let $q_N(\overline{p})$ denote $R$'s corresponding output when $R$ does not employ $A$'s input. Also let $Q^R(\overline{p}) = q_A(\overline{p}) + q_N(\overline{p})$.

To prove that $\frac{\partial(\,\overline{p}_b - \overline{p}_d)}{\partial k^R} < 0$, observe that (34) implies:

$$\overline{p}_b \;=\; \left[\,b + k^R\,\right] Q^R(\overline{p}_b) + c_N + k_N\,q_N(\overline{p}_b)$$

where, from (17):

$$q_N(\overline{p}_b) \;=\; \frac{k_A\,[\,b+k\,]\,[\,a - \,\overline{p}_b\,] - b\,k_A\,[\,\overline{p}_b - c\,] - b\,[\,b+k\,]\,[\,c_N - c_A\,]}{b\,[\,b+k\,]\,[\,k_N + k_A\,]} \quad \text{and}$$

$$Q^R(\overline{p}_b) \;=\; \frac{[\,b+k\,]\,[\,a - \,\overline{p}_b\,] - b\,[\,\overline{p}_b - c\,]}{b\,[\,b+k\,]}\,. \tag{43}$$

(43) implies that $q_N(\overline{p}_b)$ and $Q^R(\overline{p}_b)$ vary with $k^R$ only through $\overline{p}_b$. Therefore:

$$\frac{\partial\,\overline{p}_b}{\partial k^R} \;=\; Q^R(\overline{p}_b) + \left[\,b + k^R\,\right] \frac{\partial Q^R(\overline{p}_b)}{\partial \overline{p}_b}\,\frac{\partial \overline{p}_b}{\partial k^R} + k_N\,\frac{\partial q_N(\overline{p}_b)}{\partial \overline{p}_b}\,\frac{\partial \overline{p}_b}{\partial k^R}\,,$$

$$\frac{\partial q_N(\overline{p}_b)}{\partial \overline{p}_b} \;=\; -\,\frac{k_A\,[\,b+k\,] + b\,k_A}{b\,[\,b+k\,]\,[\,k_N + k_A\,]} \;\equiv\; D_N \;<\; 0\,, \text{ and } \;\frac{\partial Q^R(\overline{p}_b)}{\partial \overline{p}_b} \;=\; -\,\frac{2\,b+k}{b\,[\,b+k\,]} \;\equiv\; D_R \;<\; 0$$

$$\Rightarrow\;\; \frac{\partial\,\overline{p}_b}{\partial k^R} \;=\; \frac{Q^R(\overline{p}_b)}{1 - [\,b + k^R\,]\,D_R - k_N\,D_N} \;>\; 0\,. \tag{44}$$

(17) and (34) imply:

$$\overline{p}_d \;=\; \left[\,b + k^R\,\right] Q^R(\overline{p}_d) + c_N + k_N\,q_N(\overline{p}_d) - b\,q_A(\overline{p}_d)$$

$$\text{where}\;\; q_A(\overline{p}_d) \;=\; \frac{b\,[\,b+k\,]\,[\,c_N - c_A\,] + k_N\,[\,a - \overline{p}\,]\,[\,b+k\,] - b\,k_N\,[\,\overline{p} - c\,]}{b\,[\,b+k\,]\,[\,k_N + k_A\,]}\,;$$

$$q_N(\overline{p}_d) \;=\; \frac{k_A\,[\,b+k\,]\,[\,a - \,\overline{p}_d\,] - b\,k_A\,[\,\overline{p}_d - c\,] - b\,[\,b+k\,]\,[\,c_N - c_A\,]}{b\,[\,b+k\,]\,[\,k_N + k_A\,]}\,; \;\text{ and}$$

$$Q^R(\overline{p}_d) \;=\; \frac{[\,b+k\,]\,[\,a - \,\overline{p}_d\,] - b\,[\,\overline{p}_d - c\,]}{b\,[\,b+k\,]}\,. \tag{45}$$

(45) implies that $q_A(\overline{p}_d)$, $q_N(\overline{p}_d)$, and $Q^R(\overline{p}_d)$ vary with $k^R$ only through $\overline{p}_d$. Therefore:

$$\frac{\partial \overline{p}_d}{\partial k^R} = Q^R(\overline{p}_d) + \left[ b + k^R \right] \frac{\partial Q^R(\overline{p}_d)}{\partial \overline{p}_d} \frac{\partial \overline{p}_d}{\partial k^R} + k_N \frac{\partial q_N(\overline{p}_d)}{\partial \overline{p}_d} \frac{\partial \overline{p}_d}{\partial k^R} - b \frac{\partial q_A(\overline{p}_d)}{\partial \overline{p}_d} \frac{\partial \overline{p}_d}{\partial k^R} ;$$

$$\frac{\partial q_A(\overline{p}_d)}{\partial \overline{p}_d} = - \frac{k_N [b + k] + b\, k_N}{b [b + k] [k_N + k_A]} \equiv D_A < 0 ;$$

$$\frac{\partial q_N(\overline{p}_d)}{\partial \overline{p}_d} = - \frac{k_A [b + k] + b\, k_A}{b [b + k] [k_N + k_A]} \equiv D_N < 0 ;$$

$$\frac{\partial Q^R(\overline{p}_d)}{\partial \overline{p}_d} = - \frac{2\, b + k}{b [b + k]} \equiv D_R < 0 . \tag{46}$$

(46) implies:

$$\frac{\partial \overline{p}_d}{\partial k^R} = \frac{Q^R(\overline{p}_d)}{1 - [b + k^R] D_R - k_N D_N + b\, D_A} , \text{ and} \tag{47}$$

$$- b\, D_R + b\, D_A = b \left[ \frac{2\, b + k}{b\, (b + k)} \right] \left[ 1 - \frac{k_N}{k_N + k_A} \right] > 0 . \tag{48}$$

Because $D_N < 0$ and $D_R < 0$ from (46), (48) implies:

$$1 - \left[ b + k^R \right] D_R - k_N D_N + b\, D_A > 1 - k^R D_R - k_N D_N > 0 . \tag{49}$$

Because $D_A < 0$ from (46), (49) implies:

$$1 - \left[ b + k^R \right] D_R - k_N D_N > 0 . \tag{50}$$

(47) and (49) imply:

$$\frac{\partial \overline{p}_d}{\partial k^R} = \frac{Q^R(\overline{p}_d)}{1 - [b + k^R] D_R - k_N D_N + b\, D_A} > 0 . \tag{51}$$

(44) and (49) – (51) imply:

$$\frac{\partial \overline{p}_b}{\partial k^R} - \frac{\partial \overline{p}_d}{\partial k^R} < 0 \Leftrightarrow \frac{Q^R(\overline{p}_b)}{Q^R(\overline{p}_d)} < \frac{1 - \left[ b + k^R \right] D_R - k_N D_N}{1 - [b + k^R] D_R - k_N D_N + b\, D_A} . \tag{52}$$

(22) implies that $\frac{Q^R(\overline{p}_b)}{Q^R(\overline{p}_d)} < 1$. Furthermore, because $1 - \left[ b + k^R \right] D_R - k_N D_N + b D_A > 0$ from (49):

$$\frac{1 - \left[ b + k^R \right] D_R - k_N D_N}{1 - [b + k^R] D_R - k_N D_N + b\, D_A} > 1 \Leftrightarrow D_A < 0 . \tag{53}$$

(46) implies that the last inequality in (53) holds. Therefore, (52) holds. Consequently, because $\overline{p}_b > \overline{p}_d > 0$ from Proposition 1, (44) and (52) imply that $\frac{\partial (\overline{p}_b - \overline{p}_d)}{\partial k^R} < 0$.

To prove that $\frac{\partial (\overline{p}_b - \overline{p}_d)}{\partial c_N} > 0$, observe that (8) and (9) imply:

$$\frac{\partial \overline{p}_b}{\partial c_N} - \frac{\partial \overline{p}_d}{\partial c_N} = \frac{b [b + k] k_A}{D_2 + b\, k_N [2\, b + k]} - \frac{b [b + k] [k_A - b]}{D_2} > 0$$

$$\Leftrightarrow \quad D_2 - k_N \,[\, 2\, b + k \,]\,[\, k_A - b \,] \; > \; 0\,. \tag{54}$$

It is readily verified that the inequality in (54) holds.

To prove that $\frac{\partial(\overline{p}_b - \overline{p}_d)}{\partial c_A} < 0$, observe that (8) and (9) imply:

$$\frac{\partial \overline{p}_b}{\partial c_A} - \frac{\partial \overline{p}_d}{\partial c_A} \;=\; \frac{b\,[\, b + k \,]\, k_N}{D_2 + b\, k_N\,[\, 2\, b + k \,]} - \frac{b\,[\, b + k \,]\,[\, k_N + b \,]}{D_2} \; < \; 0$$

$$\Leftrightarrow \quad D_2 + k_N\,[\, 2\, b + k \,]\,[\, k_N + b \,] \; > \; 0\,. \tag{55}$$

It is readily verified that $D_2 > 0$, so the inequality in (55) holds.

To prove that $\frac{\partial(\overline{p}_b - \overline{p}_d)}{\partial c} > 0$, observe that (8) and (9) imply:

$$\frac{\partial\,(\,\overline{p}_b \,-\, \overline{p}_d)}{\partial c} \;\stackrel{s}{=}\; \frac{k_A \left[\, b + k^R \,\right] + k_N \left[\, k_A + k^R + b \,\right]}{D_3} - \frac{k_A \left[\, b + k^R \,\right] + k_N \left[\, k_A + k^R \,\right]}{D_2} \; > \; 0$$

$$\Leftrightarrow \quad b\,[\, b + k \,]\,[\, k_N + k_A \,] + k_N\,[\, k_A - b \,]\,[\, 2\, b + k \,] + \,[\, k_N + k_A \,]\,[\, 2\, b + k \,]\left[\, b + k^R \,\right]$$

$$> \; \left[\, \left(b + k^R\right)(k_A + k_N) + k_N\,(k_A - b) \,\right][\, 2\, b + k \,] \quad \Leftrightarrow \quad b\,[\, b + k \,]\,[\, k_N + k_A \,] \; > \; 0\,.$$

The proofs of the remaining conclusions are similar, but more tedious. See Turner and Sappington (2024, Part A) for details. ■

Recall that welfare is:

$$W(\overline{p}) \;\equiv\; S(\overline{p}) - r\,[\, \overline{p}\; q_A + (\, a - b\,[\, q_A + q_N + q \,]\,)\, q_N \,] \;=\; S(\overline{p}) - r\, V(\overline{p}) \tag{56}$$

where $r > 0$ is a parameter and $S(\cdot)$ denotes consumer surplus. The gross value that consumers derive from $Q$ units of output is:

$$\frac{1}{2}\,[\, a - P(Q) \,]\, Q + P(Q)\, Q \;=\; \frac{1}{2}\,[\, a + P(Q) \,]\, Q \;=\; \frac{1}{2}\,[\, a + a - b\, Q \,]\, Q \;=\; a\, Q - \frac{b}{2}\, Q^2.$$

Therefore, consumer surplus when the price cap is $\overline{p}$ is:

$$S(\overline{p}) \;=\; a\, Q - \frac{b}{2}\, Q^2 - \overline{p}\; q_A - P(Q)\,[\, q_N + q \,]\,. \tag{57}$$

<u>Proof of Lemma 1</u>. (17) implies that when $\overline{p} \in (\overline{p}_d, \overline{p}_b)$ (so $P(Q) = \overline{p}$), $Q = \frac{a - \overline{p}}{b} \Rightarrow \frac{\partial Q}{\partial \overline{p}} = -\frac{1}{b}$. Therefore, (57) implies:

$$\frac{\partial S(\overline{p})}{\partial \overline{p}} \;=\; -\,\frac{a - \overline{p}}{b} \; < \; 0 \quad \Rightarrow \quad \frac{\partial^2 S(\overline{p})}{\partial\,(\overline{p})^2} \;=\; \frac{1}{b} \; > \; 0\,. \;\blacksquare \tag{58}$$

**Lemma A7**. $V(\overline{p}_0) < V(\overline{p}_b)$.

<u>Proof</u>. Lemmas A1 and A3 imply that because $q_A(\overline{p}_0) = 0$ and $P(Q(\overline{p}_b)) = \overline{p}_b$:

$$V(\overline{p}_0) \;=\; \overline{p}_0\; q_N(\,\overline{p}_0) \;=\; \overline{p}_0\; \frac{[\, a - c_N \,]\,[\, 2\, b + k \,] - b\;[\, a - c \,]}{[\, 2\, b + k_N + k^R \,]\,[\, 2\, b + k \,] - b^2}\,;$$

$$V(\overline{p}_b) \;=\; \overline{p}_b\, Q^R(\,\overline{p}_b) \;=\; \overline{p}_b\, \frac{[\,b+k\,]\,[\,a-\overline{p}_b\,]-b\,[\,\overline{p}_b-c\,]}{b\,[\,b+k\,]}\,. \tag{59}$$

Definition. $D_N \;\equiv\; \left[\,2\,b+k_N+k^R\,\right][\,2\,b+k\,]-b^2\,.$ (60)

Because $\overline{p}_0 \,<\, \overline{p}_b$, (59) and (60) imply that $V(\overline{p}_0) \,<\, V(\overline{p}_b)$ if:

$$q_N(\overline{p}_0) \;=\; \frac{[\,a-c_N\,]\,[\,2\,b+k\,]-b\,[\,a-c\,]}{D_N} \;<\; \frac{[\,b+k\,]\,[\,a-\overline{p}_b\,]-b\,[\,\overline{p}_b-c\,]}{b\,[\,b+k\,]} \;=\; Q^R(\overline{p}_b)$$

$$\Leftrightarrow\; \frac{a\,[\,b+k\,]+b\,c-c_N\,[\,2\,b+k\,]}{D_N} \;<\; \frac{[\,b+k\,]\,a+b\,c-[\,2\,b+k\,]\,\overline{p}_b}{b\,[\,b+k\,]}$$

$$\Leftrightarrow\; \frac{[\,a\,(b+k)+b\,c\,]\left[\,b+k_N+k^R\,\right]+c_N\,b\,[\,b+k\,]}{D_N} \;>\; \overline{p}_b\,. \tag{61}$$

As established in the proof of Proposition 4, $\overline{p}_b$ is increasing in $k_A$. Therefore, (9) implies that because $k_A \leq k_N$ by assumption:

$$\overline{p}_b \;\leq\; \frac{[\,a\,(b+k)+b\,c\,]\left[\,2\,k_N\left(b+k^R\right)+(k_N)^2\,\right]+b\,c_N\,[\,b+k\,]\;k_N+b\,k_N\,[\,b+k\,]\;c_A}{2\,b\,[\,b+k\,]\,k_N\;+(k_N)^2\,[\,2\,b+k\,]+2\,k_N\,[\,2\,b+k\,]\,[\,b+k^R\,]}\,. \tag{62}$$

(9) implies that $\overline{p}_b$ is increasing in $c_A$. Therefore, because $c_A \leq c_N$ by assumption, (9) implies:

$$\begin{aligned}\overline{p}_b \;&\leq\; \frac{[\,a\,(b+k)+b\,c\,]\left[\,2\,k_N\left(b+k^R\right)+(k_N)^2\,\right]+2\,b\,c_N\,[\,b+k\,]\,k_N}{2\,b\,[\,b+k\,]\,k_N\;+(k_N)^2\;[\,2\,b+k\,]+2\,k_N\,[\,2\,b+k\,]\,[\,b+k^R\,]}\\ &=\; \frac{[\,a\,(b+k)+b\,c\,]\left[\,b+k^R+\frac{k_N}{2}\,\right]+b\,c_N\,[\,b+k\,]}{[\,2\,b+k\,]\left[\,2\,b+k^R+\frac{k_N}{2}\,\right]-b^2}\,.\end{aligned} \tag{63}$$

(60), (61), and (63) imply that the Lemma holds if:

$$\frac{[\,a\,(b+k)+b\,c\,]\left[\,b+k^R+\frac{k_N}{2}\,\right]+b\,c_N\,[\,b+k\,]}{[\,2\,b+k\,]\left[\,2\,b+k^R+\frac{k_N}{2}\,\right]-b^2}$$

$$<\; \frac{[\,a\,(b+k)+b\,c\,]\left[\,b+k^R+k_N\,\right]+b\,c_N\,[\,b+k\,]}{[\,2\,b+k\,]\,[\,2\,b+k^R+k_N\,]-b^2}\,.$$

It can be verified that this inequality holds. ■

**Proposition A1**. $\overline{p}^* \in [\,\overline{p}_0, \overline{p}_d\,]$.

Proof. Proposition 3 and Lemma 1 imply that $W(\cdot)$ is a strictly convex function of $\overline{p}$ for $\overline{p} \in (\overline{p}_d, \overline{p}_b)$. Therefore, $\overline{p}^* \notin (\overline{p}_d, \overline{p}_b)$. Lemma A1 implies that $W(\overline{p}) = W(\overline{p}_0)$ for all $\overline{p} < \overline{p}_0$. Lemma A4 implies that $W(\overline{p}) = W(\overline{p}_b)$ for all $\overline{p} > \overline{p}_b$. Therefore, $\overline{p}^* \in [\,\overline{p}_0, \overline{p}_d\,] \bigcup \overline{p}_b\,$.

It remains to show that $\overline{p}^* \neq \overline{p}_b$. The proof of Lemma A7 establishes that:

$$Q^R(\overline{p}_0) \ < \ Q^R(\overline{p}_b)\,. \tag{64}$$

Lemma A6 and Proposition 2 imply:

$$Q^R(\overline{p}_b) \ < \ Q^R(\overline{p}_d)\,. \tag{65}$$

(64) and (65) imply that $Q^R(\overline{p}_0) < Q^R(\overline{p}_b) < Q^R(\overline{p}_d)$. $Q^R(\overline{p})$ is continuous and monotonically increasing in $\overline{p}$ for $\overline{p} \in (\overline{p}_0, \overline{p}_d)$ (from Lemma A2). Therefore, the intermediate value theorem implies that there exists a $\overline{p}_E \in (\overline{p}_0, \overline{p}_d)$ such that:

$$Q^R(\overline{p}_E) \ = \ Q^R(\overline{p}_b)\,. \tag{66}$$

(2) implies that the rival's output $q$ is determined by:

$$a - b\left[\, Q^R(\overline{p}) + q(\overline{p})\,\right] - c - b\,q(\overline{p}) - k\,q(\overline{p}) \ = \ 0\,. \tag{67}$$

(66) and (67) imply:

$$q(\overline{p}_E) \ = \ q(\overline{p}_b)\,. \tag{68}$$

(66) and (68) imply:

$$Q(\overline{p}_E) \ = \ Q(\overline{p}_b) \ \text{ and } \ P(Q(\overline{p}_E)) \ = \ P(Q(\overline{p}_b))\,. \tag{69}$$

$R$'s revenue is:

$$\begin{aligned} V_2(\overline{p}_E) \ &= \ \overline{p}_E\, q_A(\overline{p}_E) + P(Q(\overline{p}_E))\, q_N(\overline{p}_E) \\ &< \ P(Q(\overline{p}_E))\, q_A(\overline{p}_E) + P(Q(\overline{p}_E))\, q_N(\overline{p}_E) \\ &= \ P(Q(\overline{p}_E))\, Q^R(\overline{p}_E) \ = \ P(Q(\overline{p}_b))\, Q^R(\overline{p}_b) \ = \ V_3(\overline{p}_b)\,. \end{aligned} \tag{70}$$

The inequality in (70) holds because $\overline{p}_E < P(Q(\overline{p}_E))$, since $\overline{p}_E \in (\overline{p}_0, \overline{p}_d)$. The penultimate equality in (70) reflects (69). The last equality in (70) holds because $P(Q(\overline{p}_b)) = \overline{p}_b$.

(57) and (69) imply:

$$\begin{aligned} S(\overline{p}_E) \ &= \ a\, Q(\overline{p}_E) - \frac{b}{2}\, Q(\overline{p}_E)^2 - P(Q(\overline{p}_E))\, [\, q(\overline{p}_E) + q_N(\overline{p}_E)\,] - \overline{p}_E\, q_A(\overline{p}_E) \\ &> \ a\, Q(\overline{p}_E) - \frac{b}{2}\, Q(\overline{p}_E)^2 - P(Q(\overline{p}_E))\, [\, q(\overline{p}_E) + q_N(\overline{p}_E) + q_A(\overline{p}_E)\,] \\ &= \ a\, Q(\overline{p}_E) - \frac{b}{2}\, Q(\overline{p}_b)^2 - P\left(Q(\overline{p}_b)\right)\, Q(\overline{p}_b) \ = \ S(\overline{p}_b)\,. \end{aligned} \tag{71}$$

The inequality in (71) holds because $\overline{p}_E < P\left(Q(\overline{p}_E)\right)$, since $\overline{p}_E \in (\overline{p}_0, \overline{p}_d)$. (70) and (71) imply that consumer surplus is higher and $R$'s revenue is lower when $\overline{p} = \overline{p}_E$ than when $\overline{p} = \overline{p}_b$. Therefore, $W(\overline{p}_E) > W(\overline{p}_b)$, so $\overline{p}^* \neq \overline{p}_b$. ■

Proof of Lemma 2.

$$\text{Define} \quad \widetilde{V}_2(\overline{p}) \ \equiv \ q_{A2}(\,\overline{p}\,)\,\overline{p} + q_{N2}(\,\overline{p}\,)\, P(Q_2(\,\overline{p}\,)) \tag{72}$$

where $q_{A2}(\overline{p})$ and $q_{N2}(\overline{p})$ are as defined in (11) and (12), respectively. Observe that $\widetilde{V}_2(\overline{p}) =$

$V(\overline{p})$ for $\overline{p} \in [\overline{p}_0, \overline{p}_d]$. Because $P(Q_2) = a - b\,Q_2$, (72) implies:

$$\frac{\partial \widetilde{V}_2(\overline{p})}{\partial \overline{p}} = q_{A2} + \overline{p}\,\frac{\partial q_{A2}}{\partial \overline{p}} + P(Q_2)\,\frac{\partial q_{N2}}{\partial \overline{p}} - b\,q_{N2}\,\frac{\partial Q_2}{\partial \overline{p}}\,. \tag{73}$$

(16) and Lemma A2 imply that $\frac{\partial^2 q_{A2}}{\partial(\overline{p})^2} = \frac{\partial^2 q_{N2}}{\partial(\overline{p})^2} = \frac{\partial^2 q_2}{\partial(\overline{p})^2} = \frac{\partial^2 Q_2}{\partial(\overline{p})^2} = 0$. Therefore, (73) implies:

$$\frac{\partial^2 \widetilde{V}_2(\overline{p})}{\partial\,(\overline{p})^2} = 2\,\frac{\partial q_{A2}}{\partial \overline{p}} - 2\,b\,\frac{\partial Q_2}{\partial \overline{p}}\,\frac{\partial q_{N2}}{\partial \overline{p}} > 0\,. \tag{74}$$

The inequality in (74) holds because $D > 0$ by assumption, so $\frac{\partial q_{A2}}{\partial \overline{p}} > 0$ from (11), $\frac{\partial Q_2}{\partial \overline{p}} > 0$ from (15), and $\frac{\partial q_{N2}}{\partial \overline{p}} < 0$ from (12).

$\overline{p}_{Vm} \equiv \arg\min\limits_{\overline{p}} \{\,\widetilde{V}_2(\overline{p})\,\}$ is unique and is determined by:

$$\frac{\partial \widetilde{V}_2(\overline{p}_{Vm})}{\partial \overline{p}} \equiv \left.\frac{\partial \widetilde{V}_2(\overline{p})}{\partial \overline{p}}\right|_{\overline{p}=\overline{p}_{Vm}} = 0. \tag{75}$$

This is the case because (16), (11) – (15), and (73) imply that $\frac{\partial \widetilde{V}_2(\overline{p})}{\partial \overline{p}}$ is a linear function of $\overline{p}$. Therefore, $\widetilde{V}_2(\,\overline{p}\,)$ is a quadratic function of $\overline{p}$. Consequently, (74) implies that $\widetilde{V}_2(\,\overline{p}\,)$ has a unique minimum that is determined by (75).

Recall that:

$$\begin{aligned}\Phi_2 \equiv\ & \{\,k^R\,[\,2\,b+k\,]\left[\,k^R\,(2\,b+k)+2\,b\,(3\,b+2\,k)\,\right] + k_N\,[\,2\,b+k\,]\left[\,k^R\,(2\,b+k)+b^2\,\right] \\ & + \ b^2\left[\,5\,b^2+6\,b\,k+2\,k^2\,\right]\}\ c_N\ - \{\,b\,[\,3\,b+2\,k\,]+[\,2\,b+k\,]\left[\,k_N+k^R\,\right]^2\}\,c_A \\ & - \,b\left[\,b^2-k\,k_N+(2\,b+k)\,k^R\,\right][\,a\,(b+k)+b\,c\,]\,.\end{aligned}$$

It is apparent that $\Phi_2$ increases as $c_N - c_A$ increases.

To establish that $\overline{p}_{Vm} > \overline{p}_0$ when $c_N - c_A$ is sufficiently large to ensure that $\Phi_2 > 0$, observe that $R$'s revenue is:

$$V(\overline{p}) = \overline{p}\,q_A + P(Q)\,q_N = \overline{p}\,q_A + [\,a - b\,Q\,]\,q_N\,. \tag{76}$$

(76) implies that $\left.\frac{\partial V(\overline{p})}{\partial\,\overline{p}}\right|_{\overline{p}=\overline{p}_0} < 0$ when $\Phi_2 > 0$ if:

$$\frac{\partial^+ V(\overline{p}_0)}{\partial \overline{p}} = q_A + \overline{p}_0\,\frac{\partial q_A}{\partial \overline{p}} - b\,\frac{\partial Q}{\partial \overline{p}}\,q_N + P(Q)\,\frac{\partial q_N}{\partial \overline{p}} < 0\,, \tag{77}$$

where: (i) $\frac{\partial^+ V(\overline{p}_0)}{\partial \overline{p}} = \left.\frac{\partial^+ V(\overline{p})}{\partial \overline{p}}\right|_{\overline{p}=\overline{p}_0}$ denotes the right-sided derivative of $V(\cdot)$; (ii) $\frac{\partial q_A}{\partial\,\overline{p}}$, $\frac{\partial q_N}{\partial\,\overline{p}}$, and $\frac{\partial Q}{\partial\,\overline{p}}$ pertain to the quantities identified in Lemma A2; and (iii) $q_A$, $q_N$, and $Q$ are as defined in Lemma A1.

Lemma A2 implies that when $\overline{p} \in (\overline{p}_0, \overline{p}_d)$:

$$\frac{\partial q_N}{\partial \overline{p}} = -\,\frac{b\,k+2\,b\,k^R+k\,k^R+b^2}{D}\,; \quad \frac{\partial q_A}{\partial \overline{p}} = \frac{E}{D}\,; \text{ and } \frac{\partial Q}{\partial \overline{p}} = \frac{[\,b+k\,]\;[\,b+k_N\,]}{D}$$

$$\text{where } E \ \equiv \ b\,[\,3\,b + 2\,k\,] + [\,2\,b + k\,]\left[\,k_N + k^R\,\right]. \tag{78}$$

Lemma A1 implies that when $\overline{p} \leq \overline{p}_0$:

$$q_N \ = \ \frac{[\,a - c_N\,]\,[\,2\,b + k\,] - b\,[\,a - c\,]}{E}\,, \ q \ = \ \frac{[\,a - c\,]\left[\,2\,b + k_N + k^R\,\right] - b\,[\,a - c_N\,]}{E}\,,$$

$$\text{and } P(Q) \ = \ \frac{a\,E - b\,[\,a - c_N\,]\,[\,b + k\,] - b\left[\,b + k_N + k^R\,\right][\,a - c\,]}{E}\,. \tag{79}$$

(77) – (79) imply that because $q_A = 0$ when $\overline{p} = \overline{p}_0$ (from Lemma A1):

$$\begin{aligned}\frac{\partial^+ V(\overline{p}_0)}{\partial \overline{p}} \ &= \ \frac{1}{D\,E}\,\{\,\overline{p}_0\,E^2 - b\,[\,b + k\,]\,[\,b + k_N\,]\,[\,(a - c_N)\,(2\,b + k) - b\,(a - c\,)\,] \\ &\qquad - \left[\,a\,E - b\,(a - c_N)\,(b + k) - b\,\left(b + k_N + k^R\right)(a - c\,)\,\right] \\ &\qquad \cdot \left[\,b\,k + 2\,b\,k^R + k\,k^R + b^2\,\right]\}\,. \end{aligned} \tag{80}$$

Tedious calculations reveal that the expression in (80) is strictly negative when $\Phi_2 > 0$.

It remains to prove that $\overline{p}_{Vm} < \overline{p}_d$, which is established by demonstrating that $\left.\frac{\partial^- V(\overline{p})}{\partial \overline{p}}\right|_{\overline{p} = \overline{p}_d}$ $> 0$. Define $V_2(\overline{p}) \equiv \overline{p}\,q_A(\cdot) + P(Q(\cdot))\,q_N(\cdot)$ for $\overline{p} \in (\overline{p}_0, \overline{p}_d)$. Because $P(Q) = a - b\,Q$:

$$\frac{\partial^- V_2(\overline{p}_d)}{\partial \overline{p}} \ = \ q_A + \overline{p}_d\,\frac{\partial q_A}{\partial \overline{p}} + P(Q)\,\frac{\partial q_N}{\partial \overline{p}} - b\,q_N\,\frac{\partial Q}{\partial \overline{p}} \tag{81}$$

where $q_A$, $q_N$, and $Q$ are as specified in Lemma A2, evaluated at $\overline{p} = \overline{p}_d$. Because $\overline{p}_d = P(Q)$, (81) implies:

$$\frac{\partial^- V_2(\overline{p}_d)}{\partial \overline{p}} \ = \ q_A + \overline{p}_d\left[\frac{\partial q_A}{\partial \overline{p}} + \frac{\partial q_N}{\partial \overline{p}}\right] - b\,q_N\,\frac{\partial Q}{\partial \overline{p}}\,. \tag{82}$$

(28) implies:

$$\overline{p}_d \ = \ \left[\,b + k^R\,\right]Q^R + c_N + k_N\,q_N - b\,q_A \ = \ k^R\,q_A + \left[\,b + k_N + k^R\,\right]q_N + c_N\,. \tag{83}$$

(82) and (83) imply:

$$\frac{\partial^- V_2(\,\overline{p}_d)}{\partial \overline{p}} \ = \ q_A + \left[\,k^R\,q_A + \left(k_N + k^R\right)\,q_N + c_N\,\right]\left[\frac{\partial q_A}{\partial \overline{p}} + \frac{\partial q_N}{\partial \overline{p}}\right] - b\,q_N\,\frac{\partial q}{\partial \overline{p}} \ > \ 0\,. \tag{84}$$

The inequality holds here because $\frac{\partial q_A}{\partial \overline{p}} + \frac{\partial q_N}{\partial \overline{p}} = \frac{\partial Q^R}{\partial \overline{p}} > 0$ (from (13)) and $\frac{\partial q}{\partial \overline{p}} < 0$ (from (14)). ■

Proof of Lemma 3. As in (57), define:

$$\widetilde{S}_2(\overline{p}) \ \equiv \ a\,Q_2(\overline{p}) - \frac{b}{2}\,Q_2(\overline{p})^2 - q_{A2}(\,\overline{p})\,\overline{p} - [\,q_2(\overline{p}) + q_{N2}(\overline{p})\,]\,P(Q_2(\overline{p})) \tag{85}$$

where $q_{A2}(\,\overline{p})$, $q_{N2}(\,\overline{p})$, $q_2(\,\overline{p})$, and $Q_2(\,\overline{p})$ are as defined in (11), (12), (14), and (15), respectively. Observe that $\widetilde{S}_2(\,\overline{p}) = S(\,\overline{p})$ for $\overline{p} \in [\overline{p}_0, \overline{p}_d]$.

(85) implies that because $P(Q_2) = a - b\,Q_2$ and $Q_2 = q_{A2} + q_{N2} + q_2$:

$$\frac{\partial \widetilde{S}_2(\overline{p})}{\partial \overline{p}} = [\, P(Q_2) - \overline{p}\,] \, \frac{\partial q_{A2}}{\partial \overline{p}} + b \, \frac{\partial Q_2}{\partial \overline{p}} \, [\, q_{N2} + q_2 \,] - q_{A2} \tag{86}$$

$$\Rightarrow \quad \frac{\partial^2 \widetilde{S}_2(\overline{p})}{\partial \left(\overline{p}\right)^2} = \left[ -\, b \, \frac{\partial Q_2}{\partial \overline{p}} - 1 \right] \frac{\partial q_{A2}}{\partial \overline{p}} + b \, \frac{\partial Q_2}{\partial \overline{p}} \left[ \frac{\partial q_{N2}}{\partial \overline{p}} + \frac{\partial q_2}{\partial \overline{p}} \right] - \frac{\partial q_{A2}}{\partial \overline{p}} < 0 \, . \tag{87}$$

The inequality in (87) holds because Lemma A2 implies that $\frac{\partial q_{A2}}{\partial \, \overline{p}} > 0$, $\frac{\partial Q_2}{\partial \, \overline{p}} > 0$, $\frac{\partial q_{N2}}{\partial \, \overline{p}} < 0$, and $\frac{\partial q_2}{\partial \, \overline{p}} < 0$.

$\overline{p}_{SM} \equiv \arg\max_{\overline{p}} \{ \widetilde{S}_2( \, \overline{p}) \}$ is unique and is determined by:

$$\frac{\partial \widetilde{S}_2( \, \overline{p}_{SM})}{\partial \overline{p}} \equiv \left. \frac{\partial \widetilde{S}_2( \, \overline{p})}{\partial \overline{p}} \right|_{\overline{p} = \overline{p}_{SM}} = 0 \, . \tag{88}$$

This is the case because (16), (11) – (15), and (86) imply that $\frac{\partial \widetilde{S}_2(\overline{p})}{\partial \, \overline{p}}$ is a linear function of $\overline{p}$. Therefore, $\widetilde{S}_2(\overline{p})$ is a quadratic function of $\overline{p}$. Consequently, (87) implies that $\widetilde{S}_2(\overline{p})$ has a unique maximum that is determined by (88).

To prove that $\overline{p}_{SM} > \overline{p}_{Vm}$, define $H( \, \overline{p}) \equiv a \, Q_2 - \frac{b}{2} \, Q_2^2 - [\, a - b \, Q_2 \,] \, q_2$. Observe that:

$$\frac{\partial H(\overline{p})}{\partial \overline{p}} \equiv [\, a - b \, Q_2 \,] \, \frac{\partial Q_2}{\partial \overline{p}} - [\, a - b \, Q_2 \,] \, \frac{\partial q_2}{\partial \overline{p}} + b \, \frac{\partial Q_2}{\partial \overline{p}} \, q_2 \tag{89}$$

$$\Rightarrow \quad \frac{\partial^2 H(\overline{p})}{\left(\partial \overline{p}\right)^2} \equiv -\, b \left( \frac{\partial Q_2}{\partial \overline{p}} \right)^2 + 2 \, b \, \frac{\partial Q_2}{\partial \overline{p}} \, \frac{\partial q_2}{\partial \overline{p}} < 0 \, , \tag{90}$$

where $q_2$ and $Q_2$ are defined in (14) and (15). The inequality in (90) holds because $\frac{\partial Q_2}{\partial \overline{p}} > 0$ and $\frac{\partial q_2}{\partial \overline{p}} < 0$, from (14) and (15). (89) implies:

$$\frac{\partial H(\overline{p}_d)}{\partial \overline{p}} \equiv \left. \frac{\partial H(\overline{p})}{\partial \overline{p}} \right|_{\overline{p} = \overline{p}_d} = \overline{p}_d \, \frac{\partial Q_2}{\partial \overline{p}} - \overline{p}_d \, \frac{\partial q_2}{\partial \overline{p}} + b \, \frac{\partial Q_2}{\partial \overline{p}} \, q_2(\overline{p}_d) > 0 \, . \tag{91}$$

The inequality in (91) holds because $\frac{\partial Q_2}{\partial \overline{p}} > 0$ and $\frac{\partial q_2}{\partial \overline{p}} < 0$, from (14) and (15). The concavity of $H(\overline{p})$ established in (90), along with (91), imply:

$$\frac{\partial H(\overline{p})}{\partial \overline{p}} > 0 \text{ for all } \overline{p} < \overline{p}_d \quad \Rightarrow \quad \frac{\partial H(\overline{p}_{Vm})}{\partial \overline{p}} > 0 \, . \tag{92}$$

The implication in (92) holds because $\overline{p}_{Vm} < \overline{p}_d$, from Lemma 2.

(73) and (88) imply:

$$\frac{\partial \widetilde{V}_2(\overline{p}_{Vm})}{\partial \overline{p}} = [\, a - b \, Q_2(\cdot) \,] \, \frac{\partial q_{N2}(\cdot)}{\partial \overline{p}} - b \, \frac{\partial Q_2(\cdot)}{\partial \overline{p}} \, q_{N2}(\cdot) + \, q_{A2}(\cdot) + \overline{p}_{Vm} \, \frac{\partial q_{A2}(\cdot)}{\partial \overline{p}} = 0 \tag{93}$$

where $q_{A2}(\cdot)$, $q_{N2}(\cdot)$, and $Q_2(\cdot)$ are defined in (11), (12), and (15), and evaluated at $\overline{p}_{Vm}$.

(86) implies:

$$\begin{aligned}\frac{\partial \widetilde{S}_2(\overline{p})}{\partial \overline{p}} &= [\, a - b\, Q_2 \,] \, \frac{\partial Q_2}{\partial \overline{p}} - [\, a - b\, Q_2 \,] \, \frac{\partial q_2}{\partial \overline{p}} + b \, \frac{\partial Q_2}{\partial \overline{p}} \, q_2 \\ &\quad - [\, a - b\, Q_2 \,] \, \frac{\partial q_{N2}}{\partial \overline{p}} + b \, \frac{\partial Q_2}{\partial \overline{p}} \, q_{N2} - q_{A2} - \overline{p} \, \frac{\partial q_{A2}}{\partial \overline{p}} \end{aligned} \tag{94}$$

where $q_{A2}$, $q_{N2}$, $q_2$, and $Q_2$ are defined in (11), (12), (14), and (15). (94) implies:

$$\begin{aligned}\frac{\partial \widetilde{S}_2(\overline{p}_{Vm})}{\partial \overline{p}} &= [\, a - b\, Q_2(\overline{p}_{Vm}) \,] \, \frac{\partial Q_2}{\partial \overline{p}} - [\, a - b\, Q_2(\overline{p}_{Vm}) \,] \, \frac{\partial q_2}{\partial \overline{p}} + b \, \frac{\partial Q_2}{\partial \overline{p}} \, q_2(\overline{p}_{Vm}) \\ &= \frac{\partial H(\overline{p}_{Vm})}{\partial \overline{p}} \; > \; 0 \, . \end{aligned} \tag{95}$$

The last equality in (95) reflects (93). The inequality in (95) reflects (92).

(87) implies that $\widetilde{S}_2(\overline{p})$ is a strictly concave function of $\overline{p}$. Therefore, $\overline{p}_{Vm} < \overline{p}_{SM}$ because: (i) $\frac{\partial \widetilde{S}_2(\overline{p}_{SM})}{\partial \overline{p}} = 0$ from (88); and (ii) $\frac{\partial \widetilde{S}_2(\overline{p}_{Vm})}{\partial \overline{p}} > 0$, from (95).

To prove that $\overline{p}_{SM} > \overline{p}_0$, it suffices to establish that $\frac{\partial^+ S_2(\overline{p}_0)}{\partial \overline{p}} \equiv \left. \frac{\partial^+ S_2(\overline{p}_0)}{\partial \overline{p}} \right|_{\overline{p} = \overline{p}_0} > 0$. Lemma A1 implies that $q_A = 0$ when $\overline{p} = \overline{p}_0$. Therefore, (57) implies:

$$\frac{\partial^+ \widetilde{S}_2(\overline{p}_0)}{\partial \overline{p}} = [\, P(Q) - \overline{p}_0 \,] \, \frac{\partial q_A}{\partial \overline{p}} + b \, [\, q_N + q \,] \, \frac{\partial Q}{\partial \overline{p}} \; > \; 0 \, . \tag{96}$$

The inequality in (96) holds because $\frac{\partial q_A}{\partial \overline{p}} > 0$ and $\frac{\partial Q}{\partial \overline{p}} > 0$ from Lemma A2, and because $P(Q) > \overline{p}_0$ when $\overline{p} \in (\overline{p}_0, \overline{p}_d)$. ■

**Proposition A2**. $\overline{p}^* \in [\, \overline{p}_{Vm}, \overline{p}_{SM} \,]$. Furthermore: (i) $\overline{p}^* < \overline{p}_{SM}$ when $\overline{p}_{SM} < \overline{p}_d$ and $d > 0$; (ii) $\overline{p}^* > \overline{p}_{Vm}$ when $\overline{p}_{Vm} > \overline{p}_0$; (iii) $\overline{p}^* \to \overline{p}_{SM}$ as $d \to 0$; and (iv) $\overline{p}^* \to \overline{p}_{Vm}$ as $d \to \infty$.

Proof. To prove that $\overline{p}^* \leq \overline{p}_{SM}$, suppose that $\overline{p}^* > \overline{p}_{SM}$. $\widetilde{S}_2(\overline{p})$ is a strictly concave function of $\overline{p}$, from Lemma 3. Therefore, because $\overline{p}^* > \overline{p}_{SM}$, (88) implies:

$$\frac{\partial \widetilde{S}_2(\overline{p}^*)}{\partial \overline{p}} \; < \; \frac{\partial \widetilde{S}_2(\overline{p}_{SM})}{\partial \overline{p}} \; = \; 0 \, . \tag{97}$$

$\widetilde{V}_2(\overline{p})$ is a strictly convex function of $\overline{p}$, from Lemma 2. Therefore, because $\overline{p}_{Vm} < \overline{p}_{SM}$ from Lemma 3 and because $\overline{p}^* > \overline{p}_{SM}$ by assumption, (75) implies:

$$\frac{\partial \widetilde{V}_2(\overline{p}^*)}{\partial \overline{p}} \; > \; \frac{\partial \widetilde{V}_2(\overline{p}_{SM})}{\partial \overline{p}} \; > \; \frac{\partial \widetilde{V}_2(\overline{p}_{Vm})}{\partial \overline{p}} \; = \; 0 \, . \tag{98}$$

(97) and (98) imply that $R$'s revenue declines and consumer surplus increases as $\overline{p}$ declines below $\overline{p}^*$. Therefore, $\overline{p}^*$ is not the welfare-maximizing value of $\overline{p}$. Hence, by contradiction, $\overline{p}^* \leq \overline{p}_{SM}$.

To prove that $\overline{p}^* \geq \overline{p}_{Vm}$, suppose that $\overline{p}^* < \overline{p}_{Vm}$. $\widetilde{V}_2(\overline{p})$ is a strictly convex function of $\overline{p}$, from Lemma 2. Therefore, because $\overline{p}_{Vm} < \overline{p}_{SM}$ from Lemma 3, (75) implies:

$$\frac{\partial \widetilde{V}_2(\overline{p}^*)}{\partial \overline{p}} < \frac{\partial \widetilde{V}_2(\overline{p}_{Vm})}{\partial \overline{p}} = 0. \tag{99}$$

$\widetilde{S}_2(\overline{p})$ is a strictly concave function of $\overline{p}$, from Lemma 3. Therefore, because $\overline{p}_{Vm} < \overline{p}_{SM}$ from Lemma 3 and because $\overline{p}^* < \overline{p}_{Vm}$ by assumption, (88) implies:

$$\frac{\partial \widetilde{S}_2(\overline{p}^*)}{\partial \overline{p}} > \frac{\partial \widetilde{S}_2(\overline{p}_{Vm})}{\partial \overline{p}} > \frac{\partial \widetilde{S}_2(\overline{p}_{SM})}{\partial \overline{p}} = 0. \tag{100}$$

(99) and (100) imply that $R$'s revenue declines and consumer surplus increases as $\overline{p}$ increases above $\overline{p}^*$. Therefore, $\overline{p}^*$ is not the welfare-maximizing value of $\overline{p}$. Hence, by contradiction, $\overline{p}^* \geq \overline{p}_{Vm}$.

To prove conclusion (i) in the Proposition, define $\widetilde{W}_2(\cdot) \equiv \widetilde{S}_2(\cdot) - d\,\widetilde{V}_2(\cdot)$ and observe that when $\overline{p}_{SM} < \overline{p}_d$ and $d > 0$:

$$\left.\frac{\partial \widetilde{W}_2(\overline{p})}{\partial \overline{p}}\right|_{\overline{p}=\overline{p}_{SM}} = -d\,\frac{\partial \widetilde{V}_2(\overline{p}_{SM})}{\partial \overline{p}} < -d\,\frac{\partial \widetilde{V}_2(\overline{p}_{Vm})}{\partial \overline{p}} = 0. \tag{101}$$

The inequality in (101) holds because: (i) $\overline{p}_{SM} > \overline{p}_{Vm}$, from Lemma 3; and (ii) $\widetilde{V}_2(\cdot)$ is a strictly convex function of $\overline{p}$, from Lemma 2. (101) implies that $\overline{p}_{SM} > \overline{p}^*$ because $\widetilde{W}_2(\cdot)$ is a strictly concave function of $\overline{p}$ (because $\widetilde{S}_2(\cdot)$ is a strictly concave function of $\overline{p}$ and $\widetilde{V}_2(\cdot)$ is a strictly convex function of $\overline{p}$).

To prove conclusion (ii) in the Proposition, observe that when $\overline{p}_{Vm} > \overline{p}_0$:

$$\left.\frac{\partial \widetilde{W}_2(\overline{p})}{\partial \overline{p}}\right|_{\overline{p}=\overline{p}_{Vm}} = \frac{\partial \widetilde{S}_2(\overline{p}_{Vm})}{\partial \overline{p}} > \frac{\partial \widetilde{S}_2(\overline{p}_{SM})}{\partial \overline{p}} = 0. \tag{102}$$

The inequality in (101) holds because: (i) $\overline{p}_{SM} > \overline{p}_{Vm}$, from Lemma 3; and (ii) $\widetilde{S}_2(\cdot)$ is a strictly concave function of $\overline{p}$, from Lemma 3. (102) implies that $\overline{p}^* > \overline{p}_{Vm}$ because $\widetilde{W}_2(\cdot)$ is a strictly concave function of $\overline{p}$.

Conclusions (iii) and (iv) in the Proposition follow immediately from (56) because $\overline{p}^* \in (\overline{p}_0, \overline{p}_d)$ is a non-increasing function of $r$. This is the case because (56) implies that when $\overline{p}^* \in (\overline{p}_0, \overline{p}_d)$:

$$\frac{\partial S(\overline{p}^*)}{\partial \overline{p}} - r\,\frac{\partial \widetilde{V}(\overline{p}^*)}{\partial \overline{p}} = 0 \;\Rightarrow\; \frac{\partial^2 \widetilde{S}(\overline{p}^*)}{\partial (\overline{p})^2}\,\frac{\partial \overline{p}^*}{\partial r} - \frac{\partial \widetilde{V}(\overline{p}^*)}{\partial \overline{p}} - r\,\frac{\partial^2 \widetilde{V}(\overline{p}^*)}{\partial (\overline{p})^2}\,\frac{\partial \overline{p}^*}{\partial r} = 0$$

$$\Rightarrow\; \frac{\partial \overline{p}^*}{\partial r} = \frac{\frac{\partial \widetilde{V}(\overline{p}^*)}{\partial \overline{p}}}{\frac{\partial^2 \widetilde{S}(\overline{p}^*)}{\partial (\overline{p})^2} - d\,\frac{\partial^2 \widetilde{V}(\overline{p}^*)}{\partial (\overline{p})^2}} = \frac{\frac{\partial \widetilde{V}(\overline{p}^*)}{\partial \overline{p}}}{\frac{\partial^2 \widetilde{W}(\overline{p}^*)}{\partial (\overline{p})^2}} \stackrel{s}{=} -\,\frac{\partial \widetilde{V}(\overline{p}^*)}{\partial \overline{p}}. \tag{103}$$

The last conclusion in (103) holds because Lemmas 2 and 3 imply that $\frac{\partial^2 \widetilde{W}(\overline{p}^*)}{\partial (\overline{p})^2} < 0$.

It remains to prove that $\frac{\partial \widetilde{V}_2(\overline{p}^*)}{\partial \overline{p}} \geq 0$. To do so, suppose that $\frac{\partial \widetilde{V}_2(\overline{p}^*)}{\partial \overline{p}} < 0$. Then:

$$\overline{p}^{*} \; < \; \overline{p}_{Vm} \,. \tag{104}$$

(104) holds because: (i) $\widetilde{V}_2(\overline{p})$ is a strictly convex function of $\overline{p}$, from Lemma 2; and (ii) $\frac{\partial \widetilde{V}_2(\overline{p}_{Vm})}{\partial \overline{p}} = 0$, from (75). Furthermore, because $\widetilde{S}_2(\overline{p})$ is a strictly concave function of $\overline{p}$, from Lemma 3:

$$\frac{\partial \widetilde{S}_2(\overline{p})}{\partial \overline{p}} \; > \; 0 \;\; \text{for all } \overline{p} \; < \; \overline{p}_{SM} \,. \tag{105}$$

Observe that:

$$\overline{p}^{*} \; < \; \overline{p}_{Vm} \; < \; \overline{p}_{SM} \,. \tag{106}$$

The first inequality in (106) reflects (104). The second inequality in (106) reflects Lemma 3. (88), (105), and (106) imply:

$$\frac{\partial \widetilde{S}_2\left(\overline{p}^{*}\right)}{\partial \overline{p}} \; > \; 0 \,. \tag{107}$$

Because $\frac{\partial \widetilde{S}_2(\overline{p}^{*})}{\partial \overline{p}} > 0$ (from (107)), $\frac{\partial \widetilde{V}_2(\overline{p}^{*})}{\partial \overline{p}} < 0$ (by assumption), and $\overline{p}^{*} \in (\overline{p}_0, \overline{p}_d)$ (by assumption), consumer surplus increases and $R$'s revenue declines as $\overline{p}$ increases above $\overline{p}^{*}$. Therefore, $\overline{p}^{*}$ cannot be the welfare-maximizing value of $\overline{p}$. Hence, by contradiction, $\frac{\partial \widetilde{V}_2(\overline{p}^{*})}{\partial \overline{p}} \geq 0$. Consequently, (103) implies that $\frac{\partial \overline{p}^{*}}{\partial r} \leq 0$. ■

Proof of Proposition 5. The first conclusion in the Proposition holds because (56) implies that when $\Phi_2 \geq 0$:

$$\frac{\partial^{+} W_2(\overline{p}_0)}{\partial \overline{p}} \; \equiv \; \left.\frac{\partial^{+} W_2(\overline{p})}{\partial \overline{p}}\right|_{\overline{p}=\overline{p}_0} \; = \; \frac{\partial^{+} S_2(\overline{p}_0)}{\partial \overline{p}} - r\, \frac{\partial^{+} V_2(\overline{p}_0)}{\partial \overline{p}} \; > \; 0 \,. \tag{108}$$

The inequality in (108) holds because when $\Phi_2 \geq 0$: (i) $\frac{\partial^{+} V_2(\overline{p}_0)}{\partial \overline{p}} \leq 0$ from the proof of Lemma 2; and (ii) $\frac{\partial^{+} S_2(\overline{p}_0)}{\partial \, \overline{p}} > 0$ from (96).

The second conclusion in the Proposition holds if $V(\overline{p}_0) < V(\overline{p})$ for all $\overline{p} > \overline{p}_0$ when $d$ is sufficiently large and $\Phi_2 < 0$. The proof of Lemma 2 establishes that:

$$\left.\frac{\partial^{+} V(\overline{p})}{\partial \overline{p}}\right|_{\overline{p}=\overline{p}_0} \; > \; 0 \;\text{ when }\; \Phi_2 \; < \; 0 \,. \tag{109}$$

$V(\overline{p})$ is a strictly convex function of $\overline{p}$ for $\overline{p} \in (\overline{p}_0, \overline{p}_d)$, from Lemma 2. Therefore, (109) implies that $V(\overline{p})$ is a strictly increasing function of $\overline{p}$ for $\overline{p} \in [\overline{p}_0, \overline{p}_d\,]$ under the maintained conditions. Consequently:

$$V(\overline{p}_0) \; < \; V(\overline{p}) \;\text{ for all }\; \overline{p} \, \in \, (\overline{p}_0, \overline{p}_d\,] \,. \tag{110}$$

Lemma A7 implies that under the maintained conditions:

$$V(\overline{p}_0) \; < \; V(\overline{p}_b) \,. \tag{111}$$

(39) implies that $V(\overline{p})$ is a strictly concave function of $\overline{p}$ for $\overline{p} \in (\overline{p}_d, \overline{p}_b\,)$. Therefore, (110) and (111) imply:

$$V(\overline{p}) \; > \; V(\overline{p}_0) \;\text{ for all }\; \overline{p} \, \in \, (\overline{p}_d, \overline{p}_b\,] \,. \tag{112}$$

The conclusion follows from (110), (112), and Proposition A2. ■

Proof of Proposition 6. (57) implies that consumer surplus is:

$$S \;=\; \frac{b}{2}\,Q^2 + [\,a - \overline{p}\,]\,q_A - b\,Q\,q_A\,. \tag{113}$$

(113) implies that $\overline{p}^*$ is the solution to:

$$\underset{\overline{p}}{\text{Maximize}} \;\; W \;=\; \frac{b}{2}\,Q^2 + [\,a - \overline{p}\,]\,q_A - b\,Q\,q_A - \,r\,\overline{p}\;q_A - r\,a\,q_N + r\,b\,Q\,q_N\,. \tag{114}$$

(114) imply that for $\overline{p} \in (\overline{p}_0, \overline{p}_d)$:

$$\begin{aligned}
\frac{dW}{d\overline{p}} \;=\; 0 \;\Leftrightarrow\; & \{\,b\,[\,b+k\,]\,[\,b+k_N\,] - b\left[\,3\,b^2 + 2\,b\left(k + k_N + k^R\right) + k\left(k_N + k^R\right)\right] \\
& \qquad - r\,b\left[\,b\left(b + 2\,k^R\right) + k\left(b + k^R\right)\right]\}\,Q \\
& - \,[\,D + b\,(b+k)\,(b+k_N) + r\,D\,]\;q_A + r\,b\,[\,b+k\,]\,[\,b+k_N\,]\;q_N \\
& - \{\,3\,b^2 + 2\,b\left[\,k + k_N + k^R\right] + k\left[\,k_N + k^R\right] \\
& \qquad + \;r\left[\,3\,b^2 + 2\,b\left(k + k_N + k^R\right) + k\;\left(k_N + k^R\right)\right]\}\,\overline{p} \\
& + \{\,3\,b^2 + 2\,b\left[\,k + k_N + k^R\right] + k\left[\,k_N + k^R\right] \\
& \qquad + \;r\;\left[\,b\left(b + 2\,k^R\right) + k\left(b + k^R\right)\right]\}\;a \;=\; 0\,.
\end{aligned} \tag{115}$$

The coefficient on $Q$ in (115) is readily shown to be:

$$-\,b\,[\,2\,b^2 + b\,k + b\,k_N + 2\,b\,k^R + k\,k^R + b^2 r + 2\,b\,r\,k^R + b\,r\,k + r\,k\,k^R\,] \;<\; 0\,. \tag{116}$$

The coefficient on $-\,q_A$ in (115) is readily shown to be:

$$\begin{aligned}
& [\,1+r\,]\left\{[\,2\,b+k\,]\left[\,k_N\left(k_A + k^R\right) + k_A\,k^R\right] + b\,k_A\,[\,3\,b + 2\,k\,] - b^2\,[\,b+k\,]\right\} \\
& \qquad\qquad + \,b\,[\,b+k\,]\,[\,b+k_N\,] \;>\; 0\,.
\end{aligned} \tag{117}$$

(115) – (117) imply that if $\overline{p}^* \in (\overline{p}_0, \overline{p}_d)$, $\overline{p}^*$ is determined by:

$$G - g\;\overline{p}^* \;=\; 0, \;\; \text{where} \tag{118}$$

$$\begin{aligned}
G \;\equiv\; & r\,b\,[\,b+k\,]\,[\,b+k_N\,]\;q_N \\
& + \{\,3\,b^2 + 2\,b\left[\,k + k_N + k^R\right] + k\left[\,k_N + k^R\right] \\
& \qquad\qquad + \;r\;\left[\,b\left(b + 2\,k^R\right) + k\left(b + k^R\right)\right]\}\;a \\
& - \,b\,[\,2\,b^2 + b\,k + b\,k_N + 2\,b\,k^R + k\,k^R + b^2 r \\
& \qquad\qquad + \;2\,b\,r\,k^R + b\,r\,k + r\,k\,k^R\,]\,Q
\end{aligned}$$

$$- \ \{ [ 1 + r ] D + b [ b + k ] [ b + k_N ] \} \, q_A \, , \ \text{ and}$$

$$g \ \equiv \ \{ 3 b^2 + 2 b \left[ k + k_N + k^R \right] + k \left[ k_N + k^R \right] + \ r \left[ 3 b^2 + 2 b \left( k + k_N + k^R \right) + k \ \left( k_N + k^R \right) \right] \} \ > \ 0 \, . \tag{119}$$

To prove that $\frac{d \overline{p}^*}{d c_A} > 0$, observe from (119) that $\frac{dg}{d\overline{p}} = 0$. Therefore, (118) implies that for parameter $x$:

$$[ G_x - \ \overline{p}^* \ g_x ] \, dx + [ G_{\overline{p}} - \ g ] \, d \overline{p}^* \ = \ 0 \quad \Rightarrow \quad \frac{d \overline{p}^*}{dx} \ = \ \frac{G_x - \ \overline{p} \ g_x}{g - G_{\overline{p}}} \, . \tag{120}$$

(16) and (119) imply that because $D > 0$:

$$\begin{aligned} G_{c_A} \ = \ & r \, b [ b + k ] [ b + k_N ] \, \frac{d q_N}{d c_A} \\ & - \ b [ 2 b^2 + b \, k + b \, k_N + 2 \, b \, k^R + k \, k^R + b^2 r + \ 2 \, b \, r \, k^R + b \, r \, k + r \, k \, k^R ] \, \frac{dQ}{d c_A} \\ & - \ \{ [ 1 + r ] D + b [ b + k ] [ b + k_N ] \} \, \frac{d q_A}{d c_A} \ > \ 0 \, . \end{aligned} \tag{121}$$

The inequality in (121) holds because Lemma A2 implies that $\frac{dq_A}{dc_A} < 0$ , $\frac{dq_N}{dc_A} > 0$ , and $\frac{dQ}{dc_A} < 0$.

(16) and (119) imply:

$$\begin{aligned} G_{\overline{p}} \ = \ & r \, b [ b + k ] [ b + k_N ] \, \frac{d q_N}{d \overline{p}} \\ & - \ b [ 2 b^2 + b \, k + b \, k_N + 2 \, b \, k^R + k \, k^R + b^2 r + 2 \, b \, r \, k^R + b \, r \, k + r \, k \, k^R ] \, \frac{dQ}{d \overline{p}} \\ & - \ \{ [ 1 + r ] D + b [ b + k ] [ b + k_N ] \} \, \frac{d q_A}{d \overline{p}} \ < \ 0 \, . \end{aligned} \tag{122}$$

The inequality in (121) holds because Lemma A2 implies that $\frac{dq_A}{d\overline{p}} > 0$, $\frac{dq_N}{d\overline{p}} < 0$, and $\frac{dQ}{d\overline{p}} > 0$.

(119) implies:

$$g_{c_A} \ = \ 0 \, . \tag{123}$$

(119) – (123) imply that $\frac{d \overline{p}^*}{d c_A} = \frac{G_{c_A}}{g - G_{\overline{p}}} > 0$.

The proofs of the remaining conclusions are similar, and so are omitted. ■

## C. The Benchmark Setting where $R$ is a Monopoly Supplier.

Now consider the benchmark setting in which $R$ is the sole supplier of the product. Continue to assume that $a > c_N \geq c_A > 0$ and $k_N \geq k_A \geq 0$. To ensure that the second-order condition in $R$'s problem is satisfied in this setting, further assume that:

$$\left[ k_A + k^R \right] \left[ 2 \, b + k_N + k^R \right] \ > \ \left[ b + k^R \right]^2 .$$

Turner and Sappington (2024, Part B) show that in this setting, there is an interval of price caps, $\overline{p} \in (\overline{p}_{dM}, \overline{p}_{bM}]$, in which the price of the product sold without using $A$'s input, $P(Q)$, coincides with the cap on the price of the product sold using $A$'s input, $\overline{p}$. In this interval of price caps, $R$'s total output is:

$$Q^R_{dbM}(\overline{p}) \;=\; \frac{a-\overline{p}}{b} \;\;\Rightarrow\;\; \frac{\partial Q^R_{dbM}(\overline{p})}{\partial \overline{p}} \;=\; -\frac{1}{b} \;<\; 0\,. \tag{124}$$

Recall from (22) that $R$'s corresponding total output when $\overline{p} \in (\overline{p}_d, \overline{p}_b]$ and $R$ faces a rival is:

$$Q^R_{db}(\overline{p}) \;=\; \frac{a\,[\,b+k\,]+b\,c-\overline{p}\,[\,2\,b+k\,]}{b\,[\,b+k\,]} \;\;\Rightarrow\;\; \frac{\partial Q^R_{db}(\overline{p})}{\partial \overline{p}} \;=\; -\frac{2\,b+k}{b\,[\,b+k\,]} \;<\; 0\,. \tag{125}$$

(124) and (125) imply that $R$'s total output increases more rapidly as $\overline{p}$ declines in the region where $P(Q)=\overline{p}$ when $R$ faces a rival than when $R$ is a monopolist because:

$$\left|\frac{\partial Q^R_{dbM}(\overline{p})}{\partial \overline{p}}\right| \;>\; \left|\frac{\partial Q^R_{db}(\overline{p})}{\partial \overline{p}}\right| \;\;\Leftrightarrow\;\; \frac{2\,b+k}{b\,[\,b+k\,]} \;>\; \frac{1}{b} \;\;\Leftrightarrow\;\; 2\,b+k \;<\; b+k \;\;\Leftrightarrow\;\; b \;>\; 0\,.$$

(124) implies that when $R$ is a monopolist, $R$'s revenue when $\overline{p} \in (\overline{p}_{dM}, \overline{p}_{bM}]$ is:

$$V_{dbM}(\overline{p}) \;=\; \overline{p}\; Q^R_{dbM}(\overline{p}) \;=\; \overline{p}\left[\frac{a-\overline{p}}{b}\right] \;\;\Rightarrow\;\; \frac{\partial V_{dbM}(\overline{p})}{\partial \overline{p}} \;=\; \frac{a-2\,\overline{p}}{b}\,. \tag{126}$$

(125) implies that when $R$ faces a rival, $R$'s revenue when $\overline{p} \in (\overline{p}_d, \overline{p}_b]$ is:

$$V_{db}(\overline{p}) \;=\; \overline{p}\left[\frac{a\,(b+k)+b\,c-\overline{p}\,(2\,b+k)}{b\,(b+k)}\right]$$

$$\Rightarrow\;\; \frac{\partial V_{db}(\overline{p})}{\partial \overline{p}} \;=\; \frac{a\,[\,b+k\,]+b\,c-2\,\overline{p}\,[\,2\,b+k\,]}{b\,[\,b+k\,]}\,. \tag{127}$$

(126) and (127) imply that for values of $\overline{p} \in (\overline{p}_d, \overline{p}_b]$ for which $R$'s revenue increases as $\overline{p}$ declines, the rate of increase is more pronounced when $R$ faces a rival than when $R$ is a monopolist because:

$$\frac{\partial V_{db}(\overline{p})}{\partial \overline{p}} \;<\; \frac{\partial V_{dbM}(\overline{p})}{\partial \overline{p}} \;\;\Leftrightarrow\;\; \frac{a\,[\,b+k\,]+b\,c-2\,\overline{p}\,[\,2\,b+k\,]}{b\,[\,b+k\,]} \;<\; \frac{a-2\,\overline{p}}{b}$$

$$\Leftrightarrow\;\; a\,[\,b+k\,]+b\,c-2\,\overline{p}\,[\,2\,b+k\,] \;<\; [\,b+k\,]\,[\,a-2\,\overline{p}\,]$$

$$\Leftrightarrow\;\; b\,c+2\,\overline{p}\,[\,b+k-(2\,b+k)\,] \;<\; 0 \;\;\Leftrightarrow\;\; b\,c \;<\; 2\,b\,\overline{p} \;\;\Leftrightarrow\;\; \overline{p} \;>\; \frac{c}{2}\,.$$

The last inequality here holds because, by assumption, $\overline{p} \geq \overline{p}_d > c$.

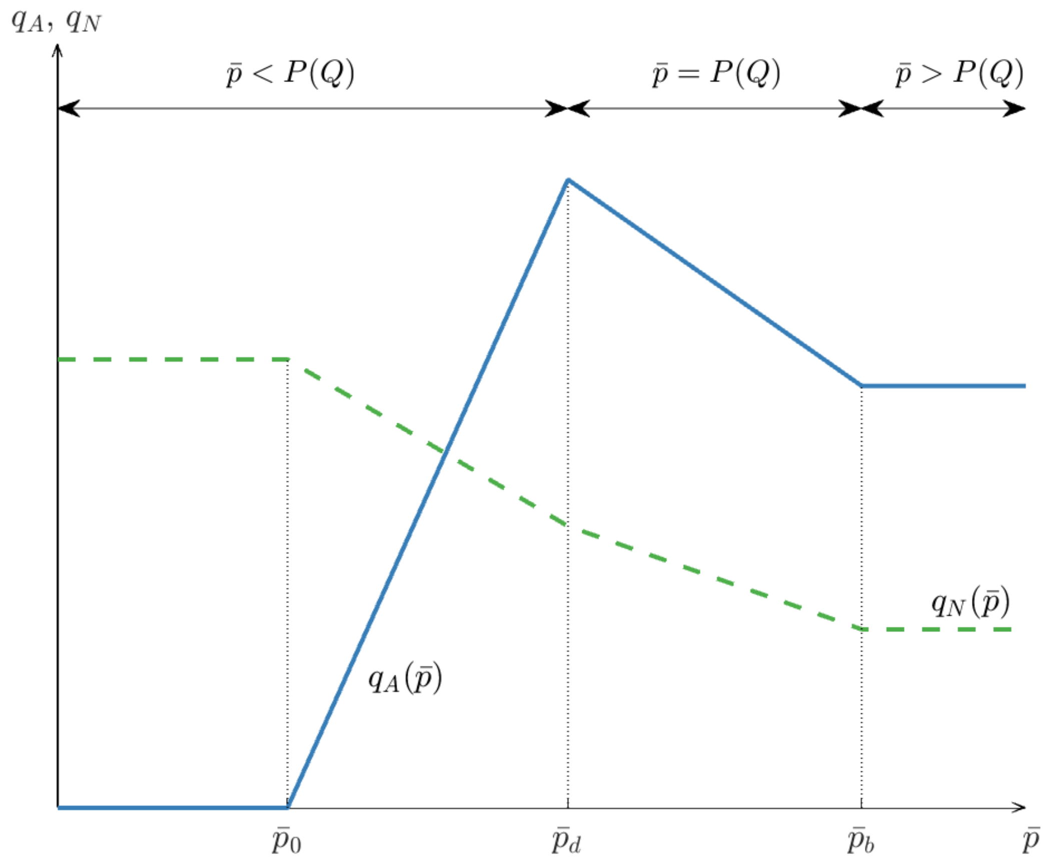


**Figure 1. *R*'s Equilibrium Outputs, $q_A$ and $q_N$, as a Function of $\bar{p}$.**

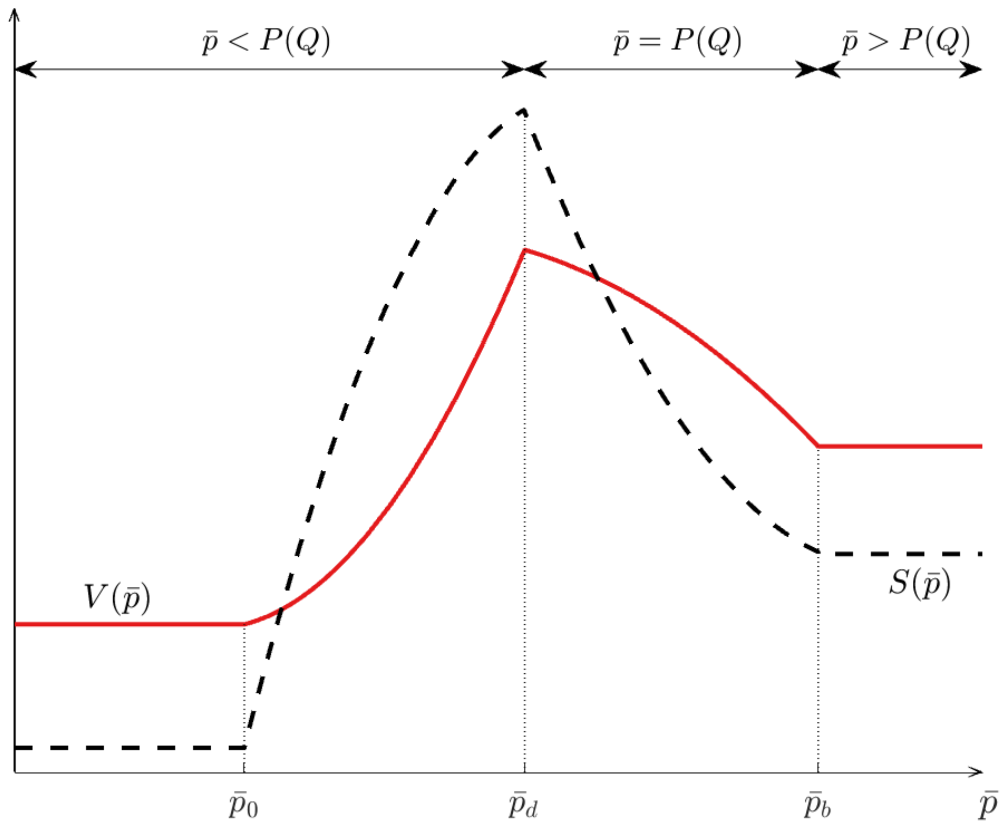


**Figure 2. Consumer Surplus, $S(\bar{p})$, and $R$'s Revenue, $V(\bar{p})$.**

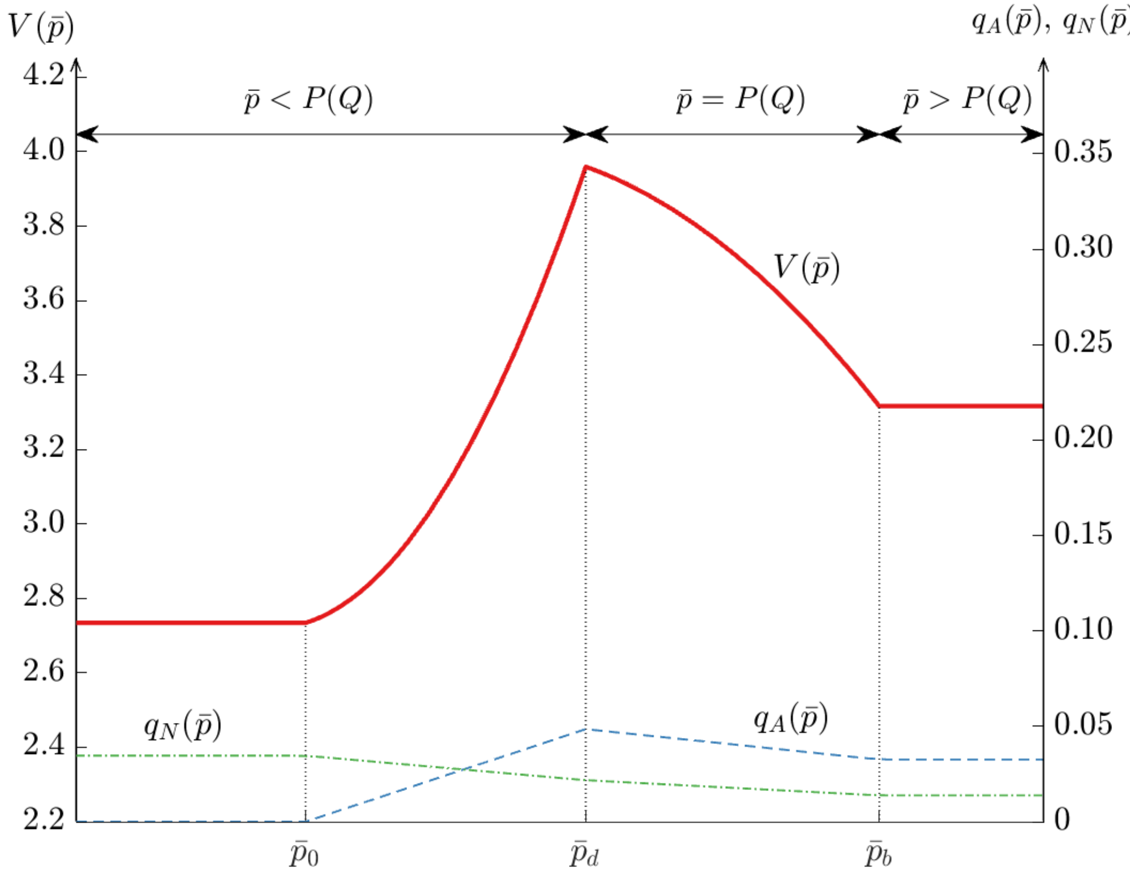


**Figure 3.** ***R*'s Revenue, $V(\overline{p})$, and Outputs, $q_A$ and $q_N$, in the Baseline Setting.***

* The numbers on the vertical axes in Figure 3 are in billions.

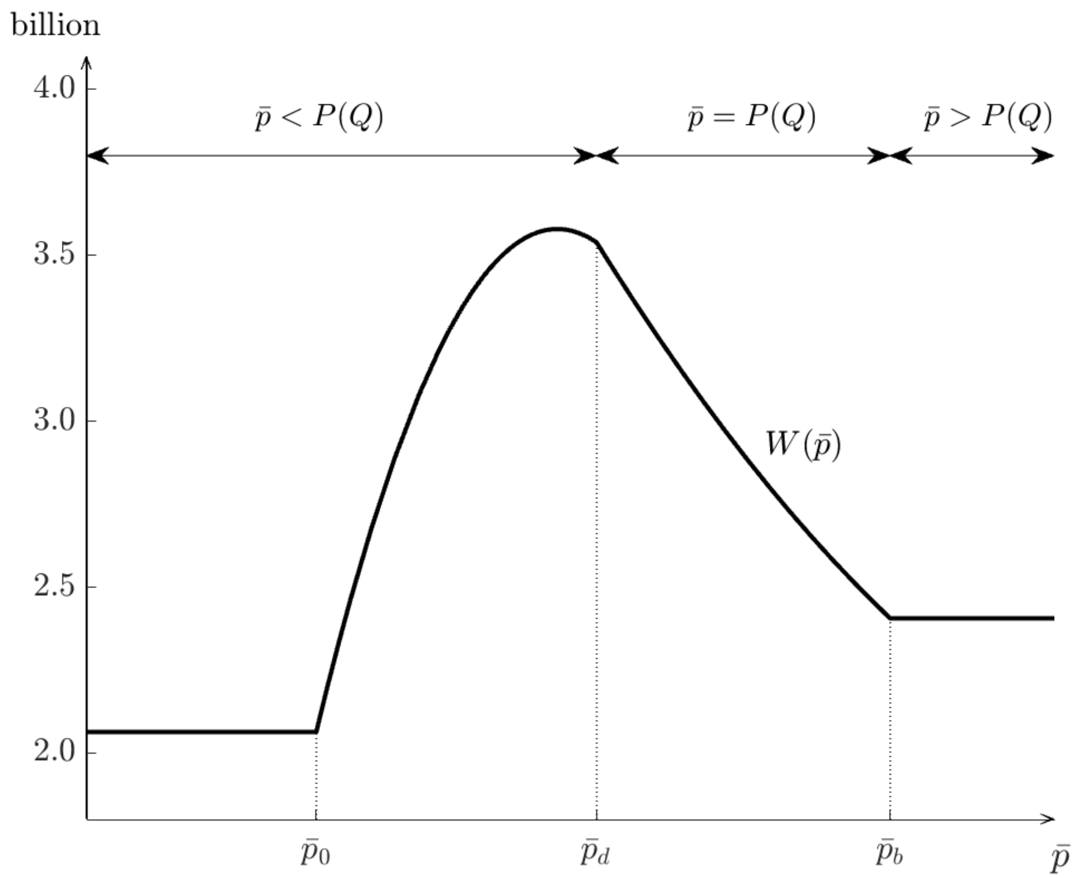


**Figure 4. Welfare $W(\overline{p})$ in the Baseline Setting when $r = \frac{1}{2}$.**

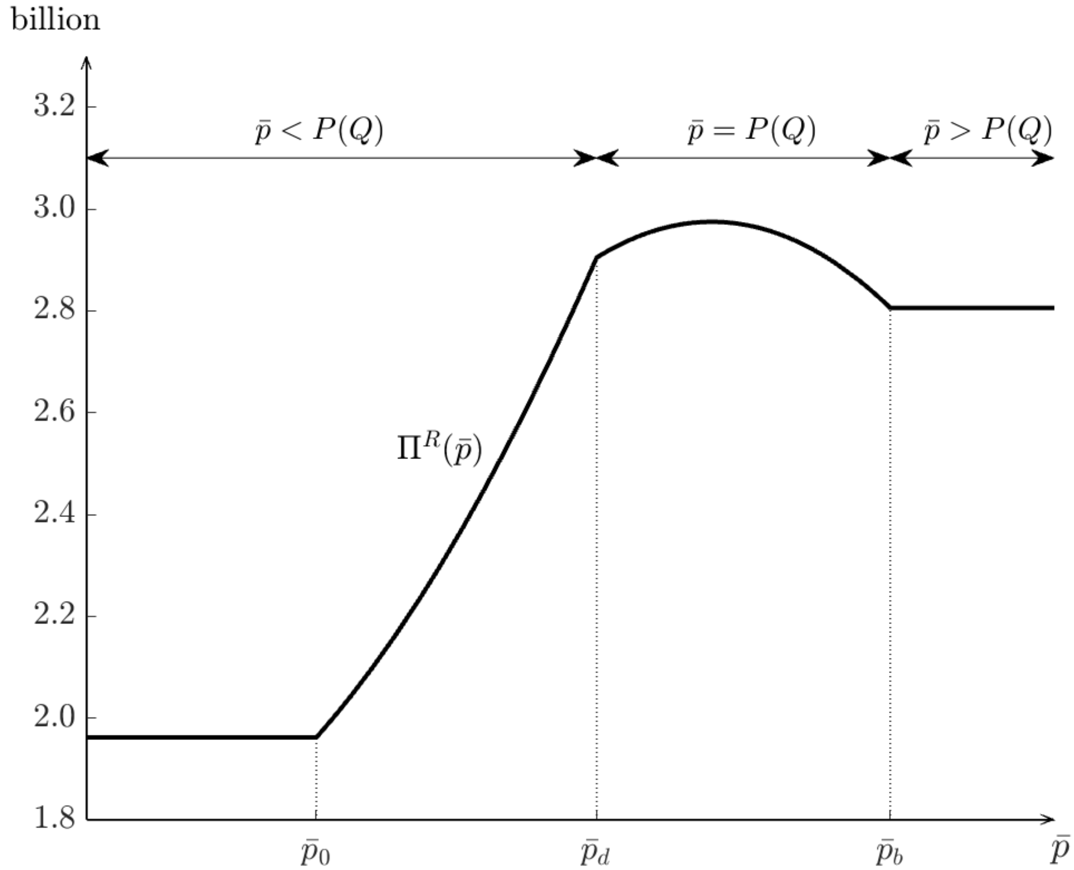


**Figure 5. *R*'s Profit, $\Pi^R(\overline{p})$, in the Baseline Setting.**

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