Technical Appendix to Accompany
"Vertical Integration and Capacity Investment in the Electricity Sector"
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Part I of this appendix provides the analysis that underlies the numerical solutions. Part II presents additional numerical solutions. Part III provides detailed proofs of Conclusions 7 and 9 in the text. Part IV presents the analysis of the benchmark setting where vertical integration does not eliminate a double marginalization problem.

I. Additional Analysis that Underlies the Numerical Solutions.

The analysis proceeds by backward induction. First, wholesale outputs are characterized, given capacity investments and retail prices. Then retail prices are characterized, given capacity investments. Finally, capacity investment decisions are characterized.

Characterizing the Choice of Wholesale Outputs

Recall from the text that:

$$Q_i^r(r_1, r_2) \;=\; a_i - b_i\, r_i + d_i\, r_j \;\text{ for }\; i, j \in \{1, 2\}\; (j \neq i)\,. \tag{1}$$

$$w(Q) \;=\; b^w[\, a^L + Q_1^r(\cdot) + Q_2^r(\cdot)\,] - b^w\, Q + \varepsilon\,. \tag{2}$$

$$\pi_i^G \;=\; [\, w - c_i\,]\, q_i \,-\, k_i\, K_i\,. \tag{3}$$

$$\pi_i^R \;=\; [\, r_i - c_i^r - w\,]\, Q_i^r(r_1, r_2)\,. \tag{4}$$

<u>The Setting where Neither Generator is Capacity-Constrained.</u>

(2) and (3) imply that when neither generator is capacity-constrained, generator Gi's choice of wholesale output (q_i) is determined by:

$$\frac{\partial \pi_i^G}{\partial q_i} \;=\; w'(Q)\left[\, q_i - \alpha_i^G\, Q_i^r\,\right] + w(Q) - c_i \;=\; 0$$

$$\Leftrightarrow \quad -\, b^w\left[\, q_i - \alpha_i^G\, Q_i^r\,\right] + b^w\left[\, a^L + Q_i^r + Q_j^r\,\right] - b^w\, q_i - b^w\, q_j + \varepsilon - c_i \;=\; 0$$

$$\Leftrightarrow \quad q_i \;=\; \frac{b^w\left[\, a^L + Q_j^r + Q_i^r\left(1 + \alpha_i^G\right)\,\right] + \varepsilon - c_i}{2\, b^w} - \frac{1}{2}\, q_j\,. \tag{5}$$

(5) implies that G1's equilibrium output in this case is:

$$q_1 \;=\; \frac{b^w\left[\, a^L + Q_2^r + Q_1^r\left(1 + \alpha_1^G\right)\,\right] + \varepsilon - c_1}{2\, b^w}$$
$$-\, \frac{1}{2}\left[\, \frac{b^w\left[\, a^L + Q_1^r + Q_2^r\left(1 + \alpha_2^G\right)\,\right] + \varepsilon - c_2}{2\, b^w} - \frac{1}{2}\, q_1\right]$$

$$\Rightarrow \quad \frac{3}{4}\, q_1 \;=\; \frac{1}{4\, b^w} \left\{ 2\, b^w \left[a^L + Q_2^r + Q_1^r \left(1 + \alpha_1^G\right) \right] + 2\, \varepsilon - 2\, c_1 \right.$$

$$\left. - \, b^w \left[a^L + Q_1^r + Q_2^r \left(1 + \alpha_2^G\right) \right] - \varepsilon + c_2 \right\}$$

$$\Rightarrow \quad q_1^*(\varepsilon) \;=\; \frac{b^w \left[a^L + \left(1 + 2\, \alpha_1^G\right) Q_1^r + \left(1 - \alpha_2^G\right) Q_2^r \right] + \varepsilon + c_2 - 2\, c_1}{3\, b^w} \, . \tag{6}$$

(5) and (6) imply:

$$q_2 \;=\; \frac{b^w \left[a^L + Q_1^r + Q_2^r \left(1 + \alpha_2^G\right) \right] + \varepsilon - c_2}{2\, b^w}$$

$$- \; \frac{b^w \left[a^L + Q_1^r \left(1 + 2\, \alpha_1^G\right) + Q_2^r \left(1 - \alpha_2^G\right) \right] + \varepsilon + c_2 - 2\, c_1}{6\, b^w}$$

$$= \; \frac{1}{6\, b^w} \left\{ 3\, b^w \left[a^L + Q_1^r + Q_2^r \left(1 + \alpha_2^G\right) \right] + 3\, \varepsilon - 3\, c_2 \right.$$

$$\left. - \; b^w \left[a^L + Q_1^r \left(1 + 2\, \alpha_1^G\right) + Q_2^r \left(1 - \alpha_2^G\right) \right] - \varepsilon - c_2 + 2\, c_1 \right\}$$

$$= \; \frac{1}{6\, b^w} \; \left\{ b^w \left[2\, a^L + \left(2 - 2\, \alpha_1^G\right) Q_1^r + \left(2 + 4\, \alpha_2^G\right) Q_2^r \right] + 2\, \varepsilon + 2\, c_1 - 4\, c_2 \right\}$$

$$\Rightarrow \quad q_2^*(\varepsilon) \;=\; \frac{b^w \left[a^L + \left(1 + 2\, \alpha_2^G\right) \; Q_2^r + \left(1 - \alpha_1^G\right) Q_1^r \right] + \varepsilon + c_1 - 2\, c_2}{3\, b^w} \, . \tag{7}$$

(2), (6), and (7) imply that total wholesale output ($Q^*(\varepsilon) = q_1^*(\varepsilon) + q_2^*(\varepsilon)$) and the corresponding wholesale price ($w(\varepsilon)$) when ε is realized in this case are:

$$Q^*(\varepsilon) \;=\; \frac{b^w \left[2\, a^L + \left(2 + \alpha_1^G\right) Q_1^r + \left(2 + \alpha_2^G\right) Q_2^r \right] + 2\, \varepsilon - c_1 - c_2}{3\, b^w} \, ; \tag{8}$$

$$w(\varepsilon) \;=\; b^w \left[a^L + Q_1^r + Q_2^r \right] - b^w \, Q^*(\varepsilon) + \varepsilon$$

$$= \; \frac{1}{3} \left\{ 3\, b^w \left[a^L + Q_1^r + Q_2^r \right] + 3\, \varepsilon \right.$$

$$\left. - \; b^w \left[2\, a^L + \left(2 + \alpha_1^G\right) Q_1^r + \left(2 + \alpha_2^G\right) Q_2^r \right] - 2\, \varepsilon + c_1 + c_2 \right\}$$

$$= \; \frac{1}{3} \left\{ b^w \left[a^L + \left(1 - \alpha_1^G\right) Q_1^r + \left(1 - \alpha_2^G\right) Q_2^r \right] + \varepsilon + c_1 + c_2 \right\} \, . \tag{9}$$

<u>The Setting where Only G2 is Capacity-Constrained.</u>

G2's equilibrium output in this case is $q_2^*(\varepsilon) = K_2$.

(5) implies that G1's equilibrium output is:

$$q_1^*(\varepsilon) \;=\; \frac{1}{2}\left[\, a^L + Q_2^r + Q_1^r \left(1 + \alpha_1^G\right) \right] + \frac{1}{2\, b^w} \left[\, \varepsilon - c_1 \,\right] - \frac{1}{2}\, K_2 \tag{10}$$

$$\Rightarrow \quad Q^*(\varepsilon) \;=\; q_1^*(\varepsilon) + q_2^*(\varepsilon) \;=\; \frac{1}{2}\left[\, a^L + Q_2^r + Q_1^r \left(1 + \alpha_1^G\right) + K_2 \,\right] + \frac{1}{2\, b^w} \left[\, \varepsilon - c_1 \,\right]. \tag{11}$$

(2) and (11) imply that when ε is realized in this case:

$$\begin{aligned} w(\varepsilon) \;&=\; b^w \left[\, a^L + Q_1^r + Q_2^r \,\right] - \frac{b^w}{2} \left[\, a^L + Q_2^r + Q_1^r \left(1 + \alpha_1^G\right) + K_2 \,\right] - \frac{1}{2} \left[\, \varepsilon - c_1 \,\right] + \varepsilon \\ &=\; \frac{1}{2}\, b^w \left[\, a^L + Q_1^r \left(1 - \alpha_1^G\right) + Q_2^r \,\right] + \frac{1}{2} \left[\, \varepsilon + c_1 - b^w\, K_2 \,\right]. \end{aligned} \tag{12}$$

<u>The Setting where Only G1 is Capacity-Constrained.</u>

G1's equilibrium output in this case is $q_1^*(\varepsilon) = K_1$.

(5) implies that G2's equilibrium output is:

$$q_2^*(\varepsilon) \;=\; \frac{1}{2}\left[\, a^L + Q_1^r + Q_2^r \left(1 + \alpha_2^G\right) \right] + \frac{1}{2\, b^w} \left[\, \varepsilon - c_2 \,\right] - \frac{1}{2}\, K_1 \tag{13}$$

$$\Rightarrow \quad Q^*(\varepsilon) \;=\; q_1^*(\varepsilon) + q_2^*(\varepsilon) \;=\; \frac{1}{2}\left[\, a^L + Q_1^r + Q_2^r \left(1 + \alpha_2^G\right) + K_1 \,\right] + \frac{1}{2\, b^w} \left[\, \varepsilon - c_2 \,\right]. \tag{14}$$

(2) and (14) imply that when ε is realized in this case:

$$\begin{aligned} w(\varepsilon) \;&=\; b^w \left[\, a^L + Q_1^r + Q_2^r \,\right] - \frac{b^w}{2} \left[\, a^L + Q_1^r + Q_2^r \left(1 + \alpha_2^G\right) + K_1 \,\right] - \frac{1}{2} \left[\, \varepsilon - c_2 \,\right] + \varepsilon \\ &=\; \frac{1}{2}\, b^w \left[\, a^L + Q_1^r + Q_2^r \left(1 - \alpha_2^G\right) \right] + \frac{1}{2} \left[\, \varepsilon + c_2 - b^w\, K_1 \,\right]. \end{aligned} \tag{15}$$

<u>The Setting where Both Generators are Capacity-Constrained.</u>

(2) implies that in this case:

$$\begin{aligned} q_1^*(\varepsilon) \;&=\; K_1\,;\;\; q_2^*(\varepsilon) \;=\; K_2\,;\;\; Q^*(\varepsilon) \;=\; K_1 + K_2\,;\;\; \text{and} \\ w(\varepsilon) \;&=\; b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] + \varepsilon\,. \end{aligned} \tag{16}$$

Recall that: (i) ε_0 is the largest realization of ε for which neither generator is capacity-constrained; and (ii) ε_{12} is the smallest realization of ε for which both generators are capacity-constrained. For $\varepsilon \in [\,\underline{\varepsilon}, \varepsilon_0\,]$, neither generator is capacity-constrained. For $\varepsilon \in (\,\varepsilon_0, \varepsilon_{12}\,)$, exactly one generator is capacity-constrained. For $\varepsilon \in [\,\varepsilon_{12}, \overline{\varepsilon}\,]$, both generators are capacity-constrained.

First suppose that G2 is capacity-constrained when $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$. (9), (12), and (16) imply that in this case:

$$E\{w(Q)\} = \frac{1}{3}\int_{\underline{\varepsilon}}^{\varepsilon_0} \left\{ b^w \left[a^L + \left(1-\alpha_1^G\right) Q_1^r + \left(1-\alpha_2^G\right) Q_2^r \right] + c_1 + c_2 + \varepsilon \right\} dH(\varepsilon)$$

$$+ \frac{1}{2}\int_{\varepsilon_0}^{\varepsilon_{12}} \left\{ b^w \left[a^L + Q_1^r \left(1-\alpha_1^G\right) + Q_2^r - K_2 \right] + c_1 + \varepsilon \right\} dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] + \varepsilon \right\} dH(\varepsilon). \tag{17}$$

(7) implies that in this case:

$$K_2 = \frac{b^w \left[a^L + \left(1 + 2\,\alpha_2^G\right) Q_2^r + \left(1-\alpha_1^G\right) Q_1^r \right] + \varepsilon_0 + c_1 - 2\,c_2}{3\,b^w}$$

$$\Rightarrow \frac{1}{3\,b^w}\left[\varepsilon_0 - 2\,c_2 + c_1\right] = K_2 - \frac{1}{3}\left[a^L + \left(1 + 2\,\alpha_2^G\right) Q_2^r + \left(1-\alpha_1^G\right) Q_1^r \right]$$

$$\Rightarrow \varepsilon_0 = 3\,b^w K_2 + 2\,c_2 - c_1 - b^w \left[a^L + \left(1 + 2\,\alpha_2^G\right) Q_2^r + \left(1-\alpha_1^G\right) Q_1^r \right]. \tag{18}$$

(1) and (18) imply that in this case:

$$\frac{d\varepsilon_0}{dr_1} = b^w\, b_1 \left[1-\alpha_1^G\right] - b^w\, d_2 \left[1 + 2\,\alpha_2^G\right];$$

$$\frac{d\varepsilon_0}{dr_2} = b^w\, b_2 \left[1 + 2\,\alpha_2^G\right] - b^w\, d_1 \left[1-\alpha_1^G\right]. \tag{19}$$

(5) implies that in this case:

$$K_1 = \frac{1}{2}\left[a^L + Q_2^r + Q_1^r \left(1+\alpha_1^G\right) \right] + \frac{\varepsilon_{12} - c_1}{2\,b^w} - \frac{1}{2} K_2$$

$$\Rightarrow 2\,b^w K_1 = b^w \left[a^L + Q_2^r + Q_1^r \left(1+\alpha_1^G\right) \right] + \varepsilon_{12} - c_1 - b^w K_2$$

$$\Rightarrow \varepsilon_{12} = 2\,b^w K_1 + b^w K_2 + c_1 - b^w \left[a^L + Q_1^r \left(1+\alpha_1^G\right) + Q_2^r \right]. \tag{20}$$

(1) and (20) imply that in this case:

$$\frac{d\varepsilon_{12}}{dr_1} = b^w\, b_1 \left[1+\alpha_1^G\right] - b^w\, d_2; \quad \frac{d\varepsilon_{12}}{dr_2} = b^w\, b_2 - b^w\, d_1 \left[1+\alpha_1^G\right]. \tag{21}$$

Now suppose G1 is capacity-constrained when $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$. (9), (15), and (16) imply that in this case:

$$\begin{aligned}
E\{w(Q)\} \;=\; & \frac{1}{3} \int_{\underline{\varepsilon}}^{\varepsilon_0} \left\{ b^w \left[a^L + \left(1-\alpha_1^G\right) Q_1^r + \left(1-\alpha_2^G\right) Q_2^r \right] + c_1 + c_2 + \varepsilon \right\} dH(\varepsilon) \\
& + \frac{1}{2} \int_{\varepsilon_0}^{\varepsilon_{12}} \left\{ b^w \left[a^L + Q_1^r + Q_2^r \left(1-\alpha_2^G\right) - K_1 \right] + c_2 + \varepsilon \right\} dH(\varepsilon) \\
& + \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] + \varepsilon \right\} dH(\varepsilon) . \qquad (22)
\end{aligned}$$

(6) implies that in this case:

$$\begin{aligned}
K_1 \;&=\; \frac{1}{3} \left[a^L + \left(1 + 2\,\alpha_1^G\right) Q_1^r + \left(1-\alpha_2^G\right) Q_2^r \right] + \frac{1}{3\,b^w} \left[\varepsilon_0 + c_2 - 2\,c_1 \right] \\
\Rightarrow \;\; \frac{1}{3\,b^w} \left[\varepsilon_0 - 2\,c_1 + c_2 \right] \;&=\; K_1 - \frac{1}{3} \left[a^L + \left(1 + 2\,\alpha_1^G\right) Q_1^r + \left(1-\alpha_2^G\right) Q_2^r \right] \\
\Rightarrow \;\; \varepsilon_0 \;&=\; 3\,b^w K_1 + 2\,c_1 - c_2 - b^w \left[a^L + \left(1 + 2\,\alpha_1^G\right) Q_1^r + \left(1-\alpha_2^G\right) Q_2^r \right] . \qquad (23)
\end{aligned}$$

(1) and (23) imply that in this case:

$$\begin{aligned}
\frac{d\varepsilon_0}{dr_1} \;&=\; b^w\, b_1 \left[1 + 2\,\alpha_1^G \right] - b^w\, d_2 \left[1 - \alpha_2^G \right] ; \\
\frac{d\varepsilon_0}{dr_2} \;&=\; b^w\, b_2 \left[1 - \alpha_2^G \right] - b^w\, d_1 \left[1 + 2\,\alpha_1^G \right] . \qquad (24)
\end{aligned}$$

(5) implies that in this case:

$$\begin{aligned}
K_2 \;&=\; \frac{1}{2} \left[a^L + Q_1^r + Q_2^r \left(1 + \alpha_2^G\right) \right] + \frac{\varepsilon_{12} - c_2}{2\,b^w} - \frac{1}{2} K_1 \\
\Rightarrow \;\; 2\,b^w K_2 \;&=\; b^w \left[a^L + Q_1^r + Q_2^r \left(1 + \alpha_2^G\right) \right] + \varepsilon_{12} - c_2 - b^w K_1 \\
\Rightarrow \;\; \varepsilon_{12} \;&=\; 2\,b^w K_2 + b^w K_1 + c_2 - b^w \left[a^L + Q_1^r + Q_2^r \left(1 + \alpha_2^G\right) \right] . \qquad (25)
\end{aligned}$$

(1) and (20) imply that in this case:

$$\frac{d\varepsilon_{12}}{dr_1} \;=\; b^w\, b_1 - b^w\, d_2 \left[1 + \alpha_2^G \right] ; \qquad \frac{d\varepsilon_{12}}{dr_2} \;=\; b^w\, b_2 \left[1 + \alpha_2^G \right] - b^w\, d_1 . \qquad (26)$$

Characterizing the Choice of Retail Prices

Case 1. Vertical Separation ($\alpha_1^R = \alpha_2^R = \alpha_1^G = \alpha_2^G = 0$).

The expected retail profit of Ri is:

$$E\left\{\, \pi_i^R(r_1, r_2) \,\right\} \;=\; [\, r_i - c_i^r - E\{\, w(\varepsilon)\,\}\,]\, Q_i^r(r_1, r_2)\,. \tag{27}$$

(1) and (27) imply that under vertical separation, R1's choice of r_1 is determined by:

$$\frac{\partial E\left\{\, \pi_1^R(\cdot)\,\right\}}{\partial r_1} \;=\; [\, a_1 - b_1\, r_1 + d_1\, r_2\,] \left[1 - \frac{\partial E\{\, w(\varepsilon)\,\}}{\partial\, r_1} \right]$$

$$-\; b_1\, [\, r_1 - c_1^r - E\{\, w(\varepsilon)\,\}\,] \;=\; 0\,. \tag{28}$$

Similarly, R2's choice of r_2 is determined by:

$$\frac{\partial E\left\{\, \pi_2^R(\cdot)\,\right\}}{\partial r_2} \;=\; [\, a_2 - b_2\, r_2 + d_2\, r_1\,] \left[1 - \frac{\partial E\{\, w(\varepsilon)\,\}}{\partial\, r_2} \right]$$

$$-\; b_2\, [\, r_2 - c_2^r - E\{\, w(\varepsilon)\,\}\,] \;=\; 0\,. \tag{29}$$

Case 1A. G2 is capacity-constrained when $\varepsilon \in (\, \varepsilon_0, \varepsilon_{12}\,)$.

(17) implies that under vertical separation in this case:

$$E\{w(\varepsilon)\} \;=\; \frac{1}{3} \int_{\underline{\varepsilon}}^{\varepsilon_0} \left\{\, b^w \left[\, a^L + Q_1^r + Q_2^r \,\right] + c_1 + c_2 + \varepsilon \,\right\}\, dH(\varepsilon)$$

$$+\; \frac{1}{2} \int_{\varepsilon_0}^{\varepsilon_{12}} \left\{\, b^w \left[\, a^L + Q_1^r + Q_2^r - K_2 \,\right] + c_1 + \varepsilon \,\right\}\, dH(\varepsilon)$$

$$+\; \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left\{\, b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] + \varepsilon \,\right\}\, dH(\varepsilon)\,. \tag{30}$$

(30) implies that when $H(\varepsilon)$ is the uniform distribution:

$$E\{w(\varepsilon)\} \;=\; \frac{1}{3} \left[\, b^w \left(\, a^L + Q_1^r + Q_2^r \,\right) + c_1 + c_2 \,\right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{6} \left[\frac{(\varepsilon_0)^2 - (\underline{\varepsilon})^2}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+\; \frac{1}{2} \left[\, b^w \left(\, a^L + Q_1^r + Q_2^r - K_2 \,\right) + c_1 \,\right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{4} \left[\frac{(\varepsilon_{12})^2 - (\varepsilon_0)^2}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \; b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{2} \left[\frac{(\overline{\varepsilon})^2 - (\varepsilon_{12})^2}{\overline{\varepsilon} - \underline{\varepsilon}} \right]. \tag{31}$$

Differentiating (31) provides:

$$\begin{aligned}
\frac{\partial E\{\,w(\varepsilon)\,\}}{\partial\, r_1} \;=\; & \frac{1}{3}\, b^w \left[d_2 - b_1 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{3\left[\overline{\varepsilon} - \underline{\varepsilon} \right]} \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right] \frac{d\varepsilon_0}{dr_1} \\
& + \frac{1}{3} \left[\frac{\varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_1} + \frac{1}{2}\, b^w \left[d_2 - b_1 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
& + \frac{1}{2\left[\overline{\varepsilon} - \underline{\varepsilon} \right]} \left[b^w \left(a^L + Q_1^r + Q_2^r - K_2 \right) + c_1 \right] \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1} \right] \\
& + \frac{1}{2\left[\overline{\varepsilon} - \underline{\varepsilon} \right]} \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0 \frac{d\varepsilon_0}{dr_1} \right] + b^w \left[d_2 - b_1 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
& - \frac{b^w}{\overline{\varepsilon} - \underline{\varepsilon}} \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \frac{d\varepsilon_{12}}{dr_1} - \left[\frac{\varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_1}\,;
\end{aligned} \tag{32}$$

$$\begin{aligned}
\frac{\partial E\{\,w(\varepsilon)\,\}}{\partial\, r_2} \;=\; & \frac{1}{3}\, b^w \left[d_1 - b_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{3\left[\overline{\varepsilon} - \underline{\varepsilon} \right]} \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right] \frac{d\varepsilon_0}{dr_2} \\
& + \frac{1}{3} \left[\frac{\varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_2} + \frac{1}{2}\, b^w \left[d_1 - b_2 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
& + \frac{1}{2\left[\overline{\varepsilon} - \underline{\varepsilon} \right]} \left[b^w \left(a^L + Q_1^r + Q_2^r - K_2 \right) + c_1 \right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right] \\
& + \frac{1}{2\left[\overline{\varepsilon} - \underline{\varepsilon} \right]} \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0 \frac{d\varepsilon_0}{dr_2} \right] + b^w \left[d_1 - b_2 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
& - \frac{b^w}{\overline{\varepsilon} - \underline{\varepsilon}} \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \frac{d\varepsilon_{12}}{dr_2} - \left[\frac{\varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_2}\,.
\end{aligned} \tag{33}$$

Case 1B. G1 is capacity-constrained when $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$.

(22) implies that under vertical separation in this case:

$$\begin{aligned} E\{w(\varepsilon)\} \;=\; & \frac{1}{3} \int\limits_{\underline{\varepsilon}}^{\varepsilon_0} \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_1 + c_2 + \varepsilon \right\} dH(\varepsilon) \\ & + \frac{1}{2} \int\limits_{\varepsilon_0}^{\varepsilon_{12}} \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 \right] + c_2 + \varepsilon \right\} dH(\varepsilon) \\ & + \int\limits_{\varepsilon_{12}}^{\overline{\varepsilon}} \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] + \varepsilon \right\} dH(\varepsilon) \,. \end{aligned} \tag{34}$$

(34) implies that when $H(\varepsilon)$ is the uniform distribution:

$$\begin{aligned} E\{w(\varepsilon)\} \;=\; & \frac{1}{3} \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{6} \left[\frac{(\varepsilon_0)^2 - (\underline{\varepsilon})^2}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\ & + \frac{1}{2} \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{4} \left[\frac{(\varepsilon_{12})^2 - (\varepsilon_0)^2}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\ & + b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{2} \left[\frac{(\overline{\varepsilon})^2 - (\varepsilon_{12})^2}{\overline{\varepsilon} - \underline{\varepsilon}} \right] . \end{aligned} \tag{35}$$

Differentiating (35) provides:

$$\begin{aligned} \frac{\partial E\{w(\varepsilon)\}}{\partial r_1} \;=\; & \frac{1}{3} b^w \left[d_2 - b_1 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{3\left[\overline{\varepsilon} - \underline{\varepsilon}\right]} \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right] \frac{d\varepsilon_0}{dr_1} \\ & + \frac{1}{3} \left[\frac{\varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_1} + \frac{1}{2} b^w \left[d_2 - b_1 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\ & + \frac{1}{2\left[\overline{\varepsilon} - \underline{\varepsilon}\right]} \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \right] \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1} \right] \\ & + \frac{1}{2\left[\overline{\varepsilon} - \underline{\varepsilon}\right]} \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0 \frac{d\varepsilon_0}{dr_1} \right] + b^w \left[d_2 - b_1 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\ & - \frac{b^w}{\overline{\varepsilon} - \underline{\varepsilon}} \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \frac{d\varepsilon_{12}}{dr_1} - \left[\frac{\varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_1} \,; \end{aligned} \tag{36}$$

$$
\begin{aligned}
\frac{\partial E\left\{ w(\varepsilon) \right\}}{\partial\, r_2} \;=\;& \frac{1}{3}\, b^w \left[\, d_1 - b_2 \,\right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{3\left[\, \overline{\varepsilon} - \underline{\varepsilon} \,\right]} \left[\, b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \,\right] \frac{d\varepsilon_0}{dr_2} \\
&+ \frac{1}{3} \left[\frac{\varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_2} + \frac{1}{2}\, b^w \left[\, d_1 - b_2 \,\right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ \frac{1}{2\left[\, \overline{\varepsilon} - \underline{\varepsilon} \,\right]} \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \,\right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right] \\
&+ \frac{1}{2\left[\, \overline{\varepsilon} - \underline{\varepsilon} \,\right]} \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0 \frac{d\varepsilon_0}{dr_2} \right] + b^w \left[\, d_1 - b_2 \,\right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&- \frac{b^w}{\overline{\varepsilon} - \underline{\varepsilon}} \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \frac{d\varepsilon_{12}}{dr_2} - \left[\frac{\varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_2} \,. \qquad (37)
\end{aligned}
$$

Case 2. Full Vertical Integration ($\alpha_1^R = \alpha_2^R = \alpha_1^G = \alpha_2^G = 1$) .

Under full vertical integration (VI), the combined expected profit of Ri and Gi when R1 sets retail price r_1 and R2 sets retail price r_2 is:

$$E\{\pi_i(r_1, r_2)\} = \int_{\underline{\varepsilon}}^{\varepsilon_0} \Pi_i^{0c}(\varepsilon)\, dH(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_{12}} \Pi_i^{1c}(\varepsilon)\, dH(\varepsilon) + \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \Pi_i^{2c}(\varepsilon)\, dH(\varepsilon) - k_i K_i \,,$$

where:

$$\Pi_i^{0c}(\varepsilon) \equiv w(\varepsilon)\,[\, q_i^*(\varepsilon) - Q_i^r(r_1, r_2)] - c_i\, q_i^*(\varepsilon) + [\, r_i - c_i^r\,]\, Q_i^r(r_1, r_2) \quad \text{for } \varepsilon \in [\, \underline{\varepsilon}, \varepsilon_0\,]\,;$$

$$\Pi_i^{1c}(\varepsilon) \equiv w(\varepsilon)\,[\, q_i^*(\varepsilon) - Q_i^r(r_1, r_2)] - c_i\, q_i^*(\varepsilon) + [\, r_i - c_i^r\,]\, Q_i^r(r_1, r_2) \quad \text{for } \varepsilon \in (\, \varepsilon_0, \varepsilon_{12}\,)\,;$$

$$\Pi_i^{2c}(\varepsilon) \equiv w(\varepsilon)\,[\, q_i^*(\varepsilon) - Q_i^r(r_1, r_2)] - c_i\, q_i^*(\varepsilon) + [\, r_i - c_i^r\,]\, Q_i^r(r_1, r_2) \quad \text{for } \varepsilon \in [\, \varepsilon_{12}, \overline{\varepsilon}\,]\,, \qquad (38)$$

where $Q_i^r(r_1, r_2) = a_i - b_i\, r_i + d_i\, r_l$ for $i, l \in \{1, 2\}$ $(l \neq i)$.

Definitions. For $j \in \{0, 1, 2\}$ and $i \in \{1, 2\}$:

$$Z_{i+}^{jc}(\varepsilon) \equiv \frac{\partial \Pi_i^{jc}(\varepsilon)}{\partial q_1^*} \frac{dq_1^*}{dr_i} + \frac{\partial \Pi_i^{jc}(\varepsilon)}{\partial q_2^*} \frac{dq_2^*}{dr_i} + \frac{\partial \Pi_i^{jc}(\varepsilon)}{\partial Q_i^r} \frac{\partial Q_i^r}{\partial r_i} + \frac{\partial \Pi_i^{jc}(\varepsilon)}{\partial r_i}\,.$$

$$Z_1^{jc}(\varepsilon) \equiv \frac{\partial \Pi_1^{jc}(\varepsilon)}{\partial q_2^*} \frac{dq_2^*}{dr_1} + \frac{\partial \Pi_1^{jc}(\varepsilon)}{\partial Q_1^r} \frac{\partial Q_1^r}{\partial r_1} + \frac{\partial \Pi_1^{jc}(\varepsilon)}{\partial r_1}\,.$$

$$Z_2^{jc}(\varepsilon) \equiv \frac{\partial \Pi_2^{jc}(\varepsilon)}{\partial q_1^*} \frac{dq_1^*}{dr_2} + \frac{\partial \Pi_2^{jc}(\varepsilon)}{\partial Q_2^r} \frac{\partial Q_2^r}{\partial r_2} + \frac{\partial \Pi_2^{jc}(\varepsilon)}{\partial r_2}\,. \qquad (39)$$

When it chooses r_1, R1 accounts for the impact of r_1 on retail profit and on wholesale profit via its impact on market demand, q_1^*, and q_2^*. Therefore, (38) implies that R1's choice of r_1 under VI is determined by:

$$\frac{dE\{\,\pi_1(r_1, r_2)\,\}}{dr_1} = \int_{\underline{\varepsilon}}^{\varepsilon_0} Z_{1+}^{0c}(\varepsilon)\, dH(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_{12}} Z_{1+}^{1c}(\varepsilon)\, dH(\varepsilon) + \int_{\varepsilon_{12}}^{\overline{\varepsilon}} Z_{1+}^{2c}(\varepsilon)\, dH(\varepsilon)$$

$$+ \left[\, \Pi_1^{0c}(\varepsilon_0) - \Pi_1^{1c}(\varepsilon_0)\,\right] h(\varepsilon_0)\, \frac{d\varepsilon_0}{dr_1} + \left[\, \Pi_1^{1c}(\varepsilon_{12}) - \Pi_1^{2c}(\varepsilon_{12})\,\right] h(\varepsilon_{12})\, \frac{d\varepsilon_{12}}{dr_1} = 0\,. \qquad (40)$$

G1 chooses $q_1^*(\varepsilon)$ to maximize its expected profit, so the envelope theorem implies that the first term in $Z_{1+}^{jc}(\varepsilon)$ can be ignored, and (40) can be written as:

$$\int_{\underline{\varepsilon}}^{\varepsilon_0} Z_1^{0c}(\varepsilon)\, dH(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_{12}} Z_1^{1c}(\varepsilon)\, dH(\varepsilon) + \int_{\varepsilon_{12}}^{\overline{\varepsilon}} Z_1^{2c}(\varepsilon)\, dH(\varepsilon)$$

$$+ \left[\, \Pi_1^{0c}(\varepsilon_0) - \Pi_1^{1c}(\varepsilon_0)\,\right] h(\varepsilon_0)\, \frac{d\varepsilon_0}{dr_1} + \left[\, \Pi_1^{1c}(\varepsilon_{12}) - \Pi_1^{2c}(\varepsilon_{12})\,\right] h(\varepsilon_{12})\, \frac{d\varepsilon_{12}}{dr_1} = 0\,. \qquad (41)$$

Case 2A. G2 is capacity-constrained when $\varepsilon \in (\,\varepsilon_0, \varepsilon_{12}\,)$.

(1), (6), and (7) imply that under VI, when $\varepsilon \in [\,\underline{\varepsilon}\,, \varepsilon_0\,]$:

$$q_1^*(\varepsilon) \;=\; \frac{1}{3}\left[\,a^L + 3\,Q_1^r\,\right] + \frac{1}{3\,b^w}\left[\,\varepsilon - 2\,c_1 + c_2\,\right] \;\equiv\; q_1^{*0c}(\varepsilon) \;\text{ and}$$

$$q_2^*(\varepsilon) \;=\; \frac{1}{3}[\,a^L + 3\,Q_2^r\,] + \frac{1}{3\,b^w}\left[\,\varepsilon - 2\,c_2 + c_1\,\right] \;\equiv\; q_2^{*0c}(\varepsilon) \tag{42}$$

$$\Rightarrow\; \frac{dq_1^{*0c}}{dr_2} \;=\; d_1 \quad\text{and}\quad \frac{dq_2^{*0c}}{dr_1} \;=\; d_2\,. \tag{43}$$

Furthermore, (9) implies that under VI, when $\varepsilon \in [\,\underline{\varepsilon}\,, \varepsilon_0\,]$:

$$w(\varepsilon) \;=\; \frac{1}{3}\left[\,b^w\,a^L + \varepsilon + c_1 + c_2\,\right] \;\equiv\; w^{0c}(\varepsilon)\,. \tag{44}$$

(10) implies that under VI, when $\varepsilon \in (\,\varepsilon_0, \varepsilon_{12}\,)$:

$$q_1^*(\varepsilon) \;=\; \frac{1}{2}\left[\,a^L + 2\,Q_1^r + Q_2^r - K_2\,\right] + \frac{1}{2\,b^w}\left[\,\varepsilon - c_1\,\right] \;\equiv\; q_1^{*1c}(\varepsilon)\,;$$

$$q_2^*(\varepsilon) \;=\; K_2\,. \tag{45}$$

Furthermore, (12) implies that under VI, when $\varepsilon \in (\,\varepsilon_0, \varepsilon_{12}\,)$:

$$w(\varepsilon) \;=\; \frac{1}{2}\,b^w\left[\,a^L + Q_2^r - K_2\,\right] + \frac{1}{2}\left[\,c_1 + \varepsilon\,\right] \;\equiv\; w^{1c}(\varepsilon)\,. \tag{46}$$

(16) implies that under VI, when $\varepsilon \in [\,\varepsilon_{12}, \overline{\varepsilon}\,]$:

$$q_1^*(\varepsilon) \;=\; K_1, \quad q_2^*(\varepsilon) \;=\; K_2, \quad\text{and}$$

$$w(\varepsilon) \;=\; b^w\left[\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right] + \varepsilon \;\equiv\; w^{2c}(\varepsilon)\,. \tag{47}$$

(1) and (2) imply:

$$\frac{\partial w(\varepsilon)}{\partial Q_1^r} \;=\; \frac{\partial w(\varepsilon)}{\partial Q_2^r} \;=\; b^w\,, \quad \frac{\partial w(\varepsilon)}{\partial q_2^*} \;=\; -\,b^w\,, \;\text{ and }\; \frac{\partial Q_2^r}{\partial r_1} \;=\; d_2\,. \tag{48}$$

(38) and (48) imply that, viewing $\Pi_1^{\bullet}(\varepsilon)$ as a function of q_1^*, q_2^*, Q_1^r, and r_1:

$$\frac{\partial \Pi_1^{\bullet}(\varepsilon)}{\partial q_2^*} \;=\; \frac{\partial w^{\bullet}(\varepsilon)}{\partial q_2^*}\,\left[\,q_1^{*\bullet}(\varepsilon) - Q_1^r\,\right] \;=\; -\,b^w\,\left[\,q_1^{*\bullet}(\varepsilon) - Q_1^r\,\right];$$

$$\frac{\partial \Pi_1^{\bullet}(\varepsilon)}{\partial Q_1^r} \;=\; \frac{\partial w^{\bullet}(\varepsilon)}{\partial Q_1^r}\,\left[\,q_1^{*\bullet}(\varepsilon) - Q_1^r\,\right] - \; w^{\bullet}(\varepsilon) + \, r_1 - c_1^r$$

$$= \; b^w \, [\, q_1^{*\bullet}(\varepsilon) - Q_1^r \,] + r_1 - c_1^r - w^\bullet(\varepsilon)\,;$$

$$\frac{\partial \Pi_1^\bullet(\varepsilon)}{\partial r_1} \;=\; Q_1^r + \frac{\partial w^\bullet(\varepsilon)}{\partial Q_2^r}\,\frac{\partial Q_2^r}{\partial r_1}\,[\, q_1^{*\bullet}(\varepsilon) - Q_1^r \,]$$

$$= \; b^w \, d_2 \, q_1^{*\bullet}(\varepsilon) + [\, 1 - b^w \, d_2 \,] \, Q_1^r \,. \tag{49}$$

Because $\frac{\partial Q_1^r}{\partial r_1} = -\, b_1$, $\frac{\partial Q_2^r}{\partial r_1} = d_2$, and $\frac{d q_2^*}{d r_1} = 0$ for $\varepsilon \in (\varepsilon_0, \overline{\varepsilon}\,]$ (since $q_2^* = K_2$ for $\varepsilon \in (\varepsilon_0, \overline{\varepsilon}\,]$), (39), (43), and (49) imply that (41) can be written as:

$$\int_{\underline{\varepsilon}}^{\varepsilon_0} \left[\, d_2 \, \frac{\partial \Pi_1^{0c}(\varepsilon)}{\partial q_2^*} - b_1 \, \frac{\partial \Pi_1^{0c}(\varepsilon)}{\partial Q_1^r} + \frac{\partial \Pi_1^{0c}(\varepsilon)}{\partial r_1} \,\right] dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \left[\, -\, b_1 \, \frac{\partial \Pi_1^{1c}(\varepsilon)}{\partial Q_1^r} + \frac{\partial \Pi_1^{1c}(\varepsilon)}{\partial r_1} \,\right] dH(\varepsilon) + \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left[\, -\, b_1 \, \frac{\partial \Pi_1^{2c}(\varepsilon)}{\partial Q_1^r} + \frac{\partial \Pi_1^{2c}(\varepsilon)}{\partial r_1} \,\right] dH(\varepsilon)$$

$$+ \left[\, \Pi_1^{0c}(\varepsilon_0) - \Pi_1^{1c}(\varepsilon_0) \,\right] h(\varepsilon_0) \, \frac{d\varepsilon_0}{dr_1} + \left[\, \Pi_1^{1c}(\varepsilon_{12}) - \Pi_1^{2c}(\varepsilon_{12}) \,\right] h(\varepsilon_{12}) \, \frac{d\varepsilon_{12}}{dr_1} \;=\; 0$$

$$\Leftrightarrow \quad -\, b^w \; d_2 \int_{\underline{\varepsilon}}^{\varepsilon_0} \left[\, q_1^{*0c}(\varepsilon) - (\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,) \,\right] dH(\varepsilon)$$

$$-\, b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} \left\{\, b^w \left[\, q_1^{*0c}(\varepsilon) - (\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,) \,\right] + r_1 - c_1^r - w^{0c}(\varepsilon) \,\right\} dH(\varepsilon)$$

$$+ \int_{\underline{\varepsilon}}^{\varepsilon_0} \left[\, b^w \, d_2 \, q_1^{*0c}(\varepsilon) + (\, 1 - b^w \, d_2 \,) \, (\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,) \,\right] dH(\varepsilon)$$

$$-\, b_1 \int_{\varepsilon_0}^{\varepsilon_{12}} \left\{\, b^w \left[\, q_1^{*1c}(\varepsilon) - (\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,) \,\right] + r_1 - c_1^r - w^{1c}(\varepsilon) \,\right\} \, dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \left[\, b^w \, d_2 \, q_1^{*1c}(\varepsilon) + (\, 1 - b^w \, d_2 \,) \, (\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,) \,\right] dH(\varepsilon)$$

$$-\, b_1 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left\{\, b^w \, [\, K_1 - (\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,) \,] + r_1 - c_1^r - w^{2c}(\varepsilon) \,\right\} \, dH(\varepsilon)$$

$$+ \, [\, 1 - H(\varepsilon_{12})] \, [\, b^w \, d_2 \, K_1 + (\, 1 - b^w \, d_2 \,) \, (\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,) \,]$$

$$+\ \left[\,\Pi_1^{0c}(\varepsilon_0)-\Pi_1^{1c}(\varepsilon_0)\,\right] h(\varepsilon_0)\,\frac{d\varepsilon_0}{dr_1}+\left[\,\Pi_1^{1c}(\varepsilon_{12})-\Pi_1^{2c}(\varepsilon_{12})\,\right] h(\varepsilon_{12})\,\frac{d\varepsilon_{12}}{dr_1}\ =\ 0\,. \tag{50}$$

(18) and (20) imply that in this case:

$$\begin{aligned}\varepsilon_0 &= 3\,b^w K_2+2\,c_2-c_1-b^w\left[\,a^L+3\,Q_2^r\,\right];\\ \varepsilon_{12} &= 2\,b^w K_1+b^w K_2+c_1-b^w\left[\,a^L+2\,Q_1^r+Q_2^r\,\right].\end{aligned} \tag{51}$$

Next we prove that $\Pi_1^{0c}(\varepsilon_0)=\Pi_1^{1c}(\varepsilon_0)$ and $\Pi_1^{1c}(\varepsilon_{12})=\Pi_1^{2c}(\varepsilon_{12})$. To do so, first observe that (42) and (51) imply:

$$\begin{aligned}q_1^{*0c}(\varepsilon_0) &= \frac{1}{3}\left[\,a^L+3\,Q_1^r\,\right]+\ \frac{1}{3\,b^w}\left[\,c_2-2\,c_1\,\right]\\ &\quad+\ \frac{1}{3\,b^w}\left[\,3\,b^w K_2+2\,c_2-c_1\,\right]-\ \frac{1}{3}\left[\,a^L+3\,Q_2^r\,\right]\\ &= Q_1^r-Q_2^r+\frac{1}{b^w}\left[\,c_2-c_1\,\right]+K_2\,.\end{aligned} \tag{52}$$

Furthermore, (45), (51), and (52) imply:

$$\begin{aligned}\lim_{\varepsilon\searrow\varepsilon_0} q_1^{*1c}(\varepsilon) &= \frac{1}{2}\left[\,a^L+2\,Q_1^r+Q_2^r\,\right]-\ \frac{1}{2\,b^w}\,c_1-\frac{1}{2}\,K_2\\ &\quad+\ \frac{1}{2\,b^w}\left[\,3\,b^w K_2+2\,c_2-c_1\,\right]-\frac{1}{2}\left[\,a^L+3\,Q_2^r\,\right]\\ &= Q_1^r-Q_2^r-\frac{1}{b^w}\,c_1+\frac{1}{b^w}\,c_2+K_2\ =\ q_1^{*0c}(\varepsilon_0)\,.\end{aligned} \tag{53}$$

Next observe that (45) and (51) imply:

$$\begin{aligned}\lim_{\varepsilon\nearrow\varepsilon_{12}} q_1^{*1c}(\varepsilon) &= \frac{1}{2}\left[\,a^L+2\,Q_1^r+Q_2^r\,\right]-\frac{1}{2\,b^w}\ c_1-\frac{1}{2}\,K_2\\ &\quad+\ \frac{1}{2\,b^w}\ \left[\,2\,b^w K_1+b^w K_2+c_1-\ b^w\left(\,a^L+2\,Q_1^r+Q_2^r\,\right)\right]\\ &= -\,\frac{1}{2\,b^w}\,c_1-\frac{1}{2}\,K_2+\frac{1}{2\,b^w}\left[\,2\,b^w K_1+b^w K_2+c_1\,\right]\ =\ K_1\,.\end{aligned} \tag{54}$$

Now observe that (44) and (51) imply:

$$\begin{aligned}w^{0c}(\varepsilon_0) &= \frac{1}{3}\left[\,b^w a^L+c_1+c_2\,\right]+\frac{1}{3}\left[\,3\,b^w K_2+2\,c_2-c_1\,\right]-\frac{1}{3}\,b^w\left[\,a^L+3\,Q_2^r\,\right]\\ &= b^w K_2+c_2-b^w Q_2^r\,.\end{aligned} \tag{55}$$

Furthermore, (46), (51), and (55) imply:

$$
\begin{aligned}
w^{1c}(\varepsilon_0) &= \frac{1}{2}\, b^w \left[\, a^L + Q_2^r - K_2 \,\right] + \frac{1}{2}\, c_1 \\
&\quad + \frac{1}{2}\left[\, 3\, b^w\, K_2 + 2\, c_2 - c_1 \,\right] - \frac{1}{2}\, b^w \left[\, a^L + 3\, Q_2^r \,\right] \\
&= -\, b^w\, Q_2^r + b^w\, K_2 + c_2 \;=\; w^{0c}(\varepsilon_0)\,. \qquad (56)
\end{aligned}
$$

Now observe that (47) and (51) imply:

$$
\begin{aligned}
w^{2c}(\varepsilon_{12}) &= b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] + 2\, b^w\, K_1 + b^w\, K_2 + c_1 \\
&\quad -\, b^w \left[\, a^L + 2\, Q_1^r + Q_2^r \,\right] \;=\; -\, b^w\, Q_1^r + b^w\, K_1 + c_1\,. \qquad (57)
\end{aligned}
$$

Furthermore, (46), (51), and (57) imply:

$$
\begin{aligned}
w^{1c}(\varepsilon_{12}) &= \frac{1}{2}\, b^w \left[\, a^L + Q_2^r - K_2 \,\right] + \frac{1}{2}\, c_1 + b^w\, K_1 + \frac{1}{2}\, b^w\, K_2 + \frac{1}{2}\, c_1 \\
&\quad - \frac{1}{2}\, b^w \left[\, a^L + 2\, Q_1^r + Q_2^r \,\right] \;=\; c_1 + b^w\, K_1 - b^w\, Q_1^r \;=\; w^{2c}(\varepsilon_{12})\,. \qquad (58)
\end{aligned}
$$

(38), (53), (54), (56), and (58) imply:

$$
\Pi_1^{0c}(\varepsilon_0) \;=\; \Pi_1^{1c}(\varepsilon_0) \quad \text{and} \quad \Pi_1^{1c}(\varepsilon_{12}) \;=\; \Pi_1^{2c}(\varepsilon_{12})\,. \qquad (59)
$$

(42) and (44) – (47) imply:

$$
\begin{aligned}
\int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon)\, dH(\varepsilon) &= \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right] \int_{\underline{\varepsilon}}^{\varepsilon_0} \left(\frac{1}{3}\left[\, a^L + 3\, Q_1^r \,\right] + \frac{1}{3\, b^w}\left[\, c_2 - 2\, c_1 \,\right] + \frac{1}{3\, b^w}\, \varepsilon \right) d\varepsilon \\
&= \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] \left\{ \frac{1}{3}\left[\, a^L + 3\,(a_1 - b_1\, r_1 + d_1\, r_2) \,\right] + \frac{1}{3\, b^w}\left[\, c_2 - 2\, c_1 \,\right] \right\} \\
&\quad + \frac{1}{6\, b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right];
\end{aligned}
$$

$$
\begin{aligned}
\int_{\underline{\varepsilon}}^{\varepsilon_0} q_2^{*0c}(\varepsilon)\, dH(\varepsilon) &= \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right] \int_{\underline{\varepsilon}}^{\varepsilon_0} \left(\frac{1}{3}\left[\, a^L + 3\, Q_2^r \,\right] + \frac{1}{3\, b^w}\left[\, c_1 - 2\, c_2 \,\right] + \frac{1}{3\, b^w}\, \varepsilon \right) d\varepsilon \\
&= \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] \left\{ \frac{1}{3}\left[\, a^L + 3\,(a_2 - b_2\, r_2 + d_2\, r_1) \,\right] + \frac{1}{3\, b^w}\left[\, c_1 - 2\, c_2 \,\right] \right\} \\
&\quad + \frac{1}{6\, b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right];
\end{aligned}
$$

$$\int_{\varepsilon_0}^{\varepsilon_{12}} q_1^{*1c}(\varepsilon)\, dH(\varepsilon) \;=\; \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \int_{\varepsilon_0}^{\varepsilon_{12}} \left(\frac{1}{2} \left[\, a^L + 2\, Q_1^r + Q_2^r - k_2 \,\right] - \frac{1}{2\, b^w}\, c_1 + \frac{1}{2\, b^w}\, \varepsilon \right) d\varepsilon$$

$$= \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ \frac{1}{2} \left[\, a^L + 2\, (a_1 - b_1\, r_1 + d_1\, r_2) + (a_2 - b_2\, r_2 + d_2\, r_1) - k_2 \,\right] \right.$$

$$\left. - \frac{1}{2\, b^w}\, c_1 \right\} + \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right];$$

$$\int_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon)\, dH(\varepsilon) \;=\; \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \int_{\underline{\varepsilon}}^{\varepsilon_0} \frac{1}{3} \left[\, b^w\, a^L + \varepsilon + c_1 + c_2 \,\right] d\varepsilon$$

$$= \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_1 + c_2 \,\right] + \frac{1}{6} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right];$$

$$\int_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon)\, dH(\varepsilon) \;=\; \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \int_{\varepsilon_0}^{\varepsilon_{12}} \left(\frac{1}{2}\, b^w \left[\, a^L + Q_2^r - K_2 \,\right] + \frac{1}{2}\, c_1 + \frac{1}{2}\, \varepsilon \right) d\varepsilon$$

$$= \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ \frac{1}{2}\, b^w \left[\, a^L + a_2 - b_2\, r_2 + d_2\, r_1 - K_2 \,\right] + \frac{1}{2}\, c_1 \right\}$$

$$+ \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right];$$

$$\int_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{2c}(\varepsilon)\, dH(\varepsilon) \;=\; \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left(b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] + \varepsilon \right) d\,\varepsilon$$

$$= \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\, a^L + a_1 - b_1\, r_1 + d_1\, r_2 + a_2 - b_2\, r_2 + d_2\, r_1 - K_1 - K_2 \,\right]$$

$$+ \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \,\right]. \tag{60}$$

(50) and (59) imply that under VI, if G2 is capacity-constrained when $\varepsilon \in (\, \varepsilon_0, \varepsilon_{12}\,)$, R1's choice of r_1 is determined by:

$$- b^w \; d_2 \int_{\underline{\varepsilon}}^{\varepsilon_0} \left[\, q_1^{*0c}(\varepsilon) - (\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,) \, \right] dH(\varepsilon)$$

$$- \, b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} \left\{ \, b^w \left[\, q_1^{*0c}(\varepsilon) - (\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,) \, \right] + r_1 - c_1^r - w^{0c}(\varepsilon) \, \right\} dH(\varepsilon)$$

$$+ \int_{\underline{\varepsilon}}^{\varepsilon_0} \left[\, b^w \, d_2 \, q_1^{*0c}(\varepsilon) + (\, 1 - b^w \, d_2 \,) \, (\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,) \, \right] dH(\varepsilon)$$

$$- \, b_1 \int_{\varepsilon_0}^{\varepsilon_{12}} \left\{ \, b^w \left[\, q_1^{*1c}(\varepsilon) - (\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,) \, \right] + r_1 - c_1^r - w^{1c}(\varepsilon) \, \right\} \, dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \left[\, b^w \, d_2 \, q_1^{*1c}(\varepsilon) + (\, 1 - b^w \, d_2 \,) \, (\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,) \, \right] dH(\varepsilon)$$

$$- \, b_1 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left\{ \, b^w \, [\, K_1 - (\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,) \,] + r_1 - c_1^r - w^{2c}(\varepsilon) \, \right\} \, dH(\varepsilon)$$

$$+ \, [\, 1 - H(\varepsilon_{12}) \,] \, [\, b^w \, d_2 \, K_1 + (\, 1 - b^w \, d_2 \,) \, (\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,) \,] \; = \; 0$$

$$\Rightarrow \quad - \; b^w \; d_2 \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon) \, dH(\varepsilon) + b^w \, d_2 \, [\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \; b^w \, b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon) \, dH(\varepsilon) + \; b^w \, b_1 \, [\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \; b_1 \, [\, r_1 - c_1^r \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon) \, dH(\varepsilon)$$

$$+ \; b^w \, d_2 \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon) \, dH(\varepsilon) + [\, 1 - b^w \, d_2 \,] \, [\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b^w b_1 \int_{\varepsilon_0}^{\varepsilon_{12}} q_1^{*1c}(\varepsilon)\, dH(\varepsilon) + b^w b_1 [\, a_1 - b_1 r_1 + d_1 r_2 \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b_1 [\, r_1 - c_1^r \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_1 \int_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon)\, dH(\varepsilon)$$

$$+ b^w d_2 \int_{\varepsilon_0}^{\varepsilon_{12}} q_1^{*1c}(\varepsilon)\, dH(\varepsilon) + [\, 1 - b^w d_2 \,] [\, a_1 - b_1 r_1 + d_1 r_2 \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b_1 \, \{ b^w [\, K_1 - (\, a_1 - b_1 r_1 + d_1 r_2 \,) \,] + r_1 - c_1^r \} \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_1 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{2c}(\varepsilon)\, dH(\varepsilon)$$

$$+ \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b^w d_2 K_1 + (\, 1 - b^w d_2 \,) (\, a_1 - b_1 r_1 + d_1 r_2 \,) \,] \; = \; 0$$

$$\Rightarrow \quad [\, b^w d_2 + b^w b_1 + 1 - b^w d_2 \,] [\, a_1 - b_1 r_1 + d_1 r_2 \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b^w b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon) dH(\varepsilon) - b_1 [\, r_1 - c_1^r \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon)\, dH(\varepsilon)$$

$$- b^w [\, b_1 - d_2 \,] \int_{\varepsilon_0}^{\varepsilon_{12}} q_1^{*1c}(\varepsilon)\, dH(\varepsilon) + [\, b^w b_1 + 1 - b^w d_2 \,] [\, a_1 - b_1 r_1 + d_1 r_2 \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b_1 [\, r_1 - c_1^r \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_1 \int_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon)\, dH(\varepsilon) - b_1 b^w K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ b_1 b^w [\, a_1 - b_1 r_1 + d_1 r_2 \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b_1 [\, r_1 - c_1^r \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_1 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{2c}(\varepsilon)\, dH(\varepsilon)$$

$$+ b^w d_2 K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + [\, 1 - b^w d_2 \,] [\, a_1 - b_1 r_1 + d_1 r_2 \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \; = \; 0$$

$$\Rightarrow \quad [\,1 + b^w\, b_1\,]\,[\,a_1 - b_1\, r_1 + d_1\, r_2\,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \ [\,1 - b^w\, d_2 + b^w\, b_1]\,[\,a_1 - b_1\, r_1 + d_1\, r_2\,] \left[\frac{\overline{\varepsilon} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \ b^w\, b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon)\, dH(\varepsilon) - b^w\,[\,b_1 - d_2\,] \int_{\varepsilon_0}^{\varepsilon_{12}} q_1^{*1c}(\varepsilon)\, dH(\varepsilon)$$

$$+ \ b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon)\, dH(\varepsilon) + b_1 \int_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon)\ dH(\varepsilon) + \ b_1 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{2c}(\varepsilon)\, dH(\varepsilon)$$

$$- \ b_1\,[\,r_1 - c_1^r\,] - [\,b_1 - d_2\,]\, b^w\, K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \ = \ 0 \qquad (61)$$

where $q_1^{*0c}(\varepsilon)$, $q_1^{*1c}(\varepsilon)$, $w^{0c}(\varepsilon)$, $w^{1c}(\varepsilon)$, and $w^{2c}(\varepsilon)$ are as specified in (42) – (47), the integral terms are defined in (60), and where ε_0 and ε_{12} are as specified in (51).

It remains to characterize R2's choice of r_2 in this case. G2 chooses $q_2^*(\varepsilon)$ to maximize its expected profit, so the envelope theorem implies that the first term in $Z_{2+}^{jc}(\varepsilon)$ can be ignored. Therefore, (39) implies that R2's choice of r_2 is given by:

$$\int_{\underline{\varepsilon}}^{\varepsilon_0} Z_2^{0c}(\varepsilon)\, dH(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_{12}} Z_2^{1c}(\varepsilon)\, dH(\varepsilon) + \int_{\varepsilon_{12}}^{\overline{\varepsilon}} Z_2^{2c}(\varepsilon)\, dH(\varepsilon)$$

$$+ \left[\, \Pi_2^{0c}(\varepsilon_0) - \Pi_2^{1c}(\varepsilon_0)\,\right] h(\varepsilon_0)\, \frac{d\varepsilon_0}{dr_2} + \left[\, \Pi_2^{1c}(\varepsilon_{12}) - \Pi_2^{2c}(\varepsilon_{12})\,\right] h(\varepsilon_{12})\, \frac{d\varepsilon_{12}}{dr_2} \ = \ 0\,. \qquad (62)$$

(1) and (2) imply:

$$\frac{\partial w(\varepsilon)}{\partial Q_2^r} \ = \ \frac{\partial w(\varepsilon)}{\partial Q_1^r} \ = \ b^w\,, \ \ \frac{\partial w(\varepsilon)}{\partial q_1^*} \ = \ -\, b^w\,, \ \text{ and } \ \frac{\partial Q_1^r}{\partial r_2} \ = \ d_1\,. \qquad (63)$$

(38) and (63) imply that, viewing $\Pi_2^{\bullet}(\varepsilon)$ as a function of q_1^*, q_2^*, Q_2^r, and r_2:

$$\frac{\partial \Pi_2^{\bullet}(\varepsilon)}{\partial q_1^*} \ = \ \frac{\partial w^{\bullet}(\varepsilon)}{\partial q_1^*}\ [\, q_2^{*\bullet}(\varepsilon) - Q_2^r\,] \ = \ -\, b^w\ [\, q_2^{*\bullet}(\varepsilon) - Q_2^r\,]\,;$$

$$\frac{\partial \Pi_2^{\bullet}(\varepsilon)}{\partial Q_2^r} \ = \ \frac{\partial w^{\bullet}(\varepsilon)}{\partial Q_2^r}\ [\, q_2^{*\bullet}(\varepsilon) - Q_2^r\,] - w^{\bullet}(\varepsilon) + \ r_2 - c_2^r$$

$$= b^w \left[q_2^{*\bullet}(\varepsilon) - Q_2^r \right] + r_2 - c_2^r - w^\bullet(\varepsilon) \, ;$$

$$\frac{\partial \Pi_2^\bullet(\varepsilon)}{\partial r_2} = Q_2^r + \frac{\partial w^\bullet(\varepsilon)}{\partial Q_1^r} \frac{\partial Q_1^r}{\partial r_2} \left[q_2^{*\bullet}(\varepsilon) - Q_2^r \right] = b^w \, d_1 \, q_2^{*\bullet}(\varepsilon) + \left[1 - b^w \, d_1 \right] Q_2^r \, . \tag{64}$$

(43), (45), and (47) imply:

$$\frac{dq_1^{*0c}}{dr_2} = d_1 \, , \quad \frac{dq_1^{*1c}}{dr_2} = d_1 - \frac{1}{2} \, b_2 \, , \text{ and } \frac{dq_1^{*2c}}{dr_2} = 0 \, . \tag{65}$$

Because $\frac{\partial Q_2^r}{\partial r_2} = -\, b_2$ and $\frac{\partial Q_1^r}{\partial r_2} = d_1$, (39) and (65) imply that (62) can be written as:

$$\int_{\underline{\varepsilon}}^{\varepsilon_0} \left[d_1 \frac{\partial \Pi_2^{0c}(\varepsilon)}{\partial q_1^*} - b_2 \frac{\partial \Pi_2^{0c}(\varepsilon)}{\partial Q_2^r} + \frac{\partial \Pi_2^{0c}(\varepsilon)}{\partial r_2} \right] dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \left[\left(d_1 - \frac{1}{2} \, b_2 \right) \frac{\partial \Pi_2^{1c}(\varepsilon)}{\partial q_1^*} - b_2 \frac{\partial \Pi_2^{1c}(\varepsilon)}{\partial Q_2^r} + \frac{\partial \Pi_2^{1c}(\varepsilon)}{\partial r_2} \right] dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left[-\, b_2 \frac{\partial \Pi_2^{2c}(\varepsilon)}{\partial Q_2^r} + \frac{\partial \Pi_2^{2c}(\varepsilon)}{\partial r_2} \right] dH(\varepsilon)$$

$$+ \left[\Pi_2^{0c}(\varepsilon_0) - \Pi_2^{1c}(\varepsilon_0) \right] h(\varepsilon_0) \frac{d\varepsilon_0}{dr_2} + \left[\Pi_2^{1c}(\varepsilon_{12}) - \Pi_2^{2c}(\varepsilon_{12}) \right] h(\varepsilon_{12}) \frac{d\varepsilon_{12}}{dr_2} = 0. \tag{66}$$

$w^{0c}(\varepsilon_0) = w^{1c}(\varepsilon_0)$ from (56). Also, $w^{1c}(\varepsilon_{12}) = w^{2c}(\varepsilon_{12})$ from (58). In addition, from (42) and (51):

$$\begin{aligned} q_2^{*0c}(\varepsilon_0) &= \frac{1}{3} \left[a^L + 3 \, Q_2^r \right] + \frac{1}{3 \, b^w} \left[c_1 - 2 \, c_2 \right] \\ &\quad + \frac{1}{3 \, b^w} \left[3 \, b^w \, K_2 + 2 \, c_2 - c_1 - b^w \left(a^L + 3 \, Q_2^r \right) \right] \\ &= \frac{1}{3 \, b^w} \left[c_1 - 2 \, c_2 \right] + \frac{1}{3 \, b^w} \left[3 \, b^w \, K_2 + 2 \, c_2 - c_1 \right] = K_2 \, . \end{aligned} \tag{67}$$

Therefore:

$$\Pi_2^{0c}(\varepsilon_0) = \Pi_2^{1c}(\varepsilon_0) \quad \text{and} \quad \Pi_2^{1c}(\varepsilon_{12}) = \Pi_2^{2c}(\varepsilon_{12}) \, . \tag{68}$$

(64), (65), (66), and (68) imply that under VI when G2 is capacity-constrained when $\varepsilon \in (\, \varepsilon_0, \varepsilon_{12} \,)$, R2's choice of r_2 is given by:

$$- b^w \; d_1 \int\limits_{\underline{\varepsilon}}^{\varepsilon_0} \left[q_2^{*0c}(\varepsilon) - (\, a_2 - b_2\, r_2 + d_2\, r_1 \,) \right] dH(\varepsilon)$$

$$- \; b_2 \int\limits_{\underline{\varepsilon}}^{\varepsilon_0} \left\{ b^w \left[q_2^{*0c}(\varepsilon) - (\, a_2 - b_2\, r_2 + d_2\, r_1 \,) \right] + r_2 - c_2^r - w^{0c}(\varepsilon) \right\} dH(\varepsilon)$$

$$+ \; \int\limits_{\underline{\varepsilon}}^{\varepsilon_0} \left[b^w\, d_1\, q_2^{*0c}(\varepsilon) + (\, 1 - b^w\, d_1 \,)\, (\, a_2 - b_2\, r_2 + d_2\, r_1 \,) \right] dH(\varepsilon)$$

$$- \; b^w \left[d_1 - \frac{1}{2}\, b_2 \right] [\, K_2 - (\, a_2 - b_2\, r_2 + d_2\, r_1 \,) \,]\, [\, H(\varepsilon_{12}) - H(\varepsilon_0) \,]$$

$$- \; b_2 \int\limits_{\varepsilon_0}^{\varepsilon_{12}} \left\{ b^w\, [\, K_2 - (\, a_2 - b_2\, r_2 + d_2\, r_1 \,) \,] + r_2 - c_2^r - w^{1c}(\varepsilon) \right\} \; dH(\varepsilon)$$

$$+ \; [\, b^w\, d_1\, K_2 + (\, 1 - b^w\, d_1 \,)\, (\, a_2 - b_2\, r_2 + d_2\, r_1 \,) \,]\, [\, H(\varepsilon_{12}) - H(\varepsilon_0) \,]$$

$$- \; b_2 \int\limits_{\varepsilon_{12}}^{\overline{\varepsilon}} \left\{ b^w\, [\, K_2 - (\, a_2 - b_2\, r_2 + d_2\, r_1 \,) \,] + r_2 - c_2^r - w^{2c}(\varepsilon) \right\} \; dH(\varepsilon)$$

$$+ \; [\, 1 - H(\varepsilon_{12}) \,]\, [\, b^w\, d_1\, K_2 + (\, 1 - b^w\, d_1 \,)\, (\, a_2 - b_2\, r_2 + d_2\, r_1 \,) \,] \; = \; 0$$

$$\Rightarrow \quad - b^w \; d_1 \int\limits_{\underline{\varepsilon}}^{\varepsilon_0} q_2^{*0c}(\varepsilon)\, dH(\varepsilon) + b^w\, d_1\, [\, a_2 - b_2\, r_2 + d_2\, r_1 \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \; b^w\, b_2 \int\limits_{\underline{\varepsilon}}^{\varepsilon_0} q_2^{*0c}(\varepsilon)\, dH(\varepsilon) + b^w\, b_2\, [\, a_2 - b_2\, r_2 + d_2\, r_1 \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \; b_2\, [\, r_2 - c_2^r \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_2 \int\limits_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon)\, dH(\varepsilon)$$

$$+ \; b^w\, d_1 \int\limits_{\underline{\varepsilon}}^{\varepsilon_0} q_2^{*0c}(\varepsilon)\, dH(\varepsilon) + [\, 1 - b^w\, d_1 \,]\, [\, a_2 - b_2\, r_2 + d_2\, r_1 \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \; b^w \left[d_1 - \frac{1}{2}\, b_2 \right] [\, K_2 - (\, a_2 - b_2\, r_2 + d_2\, r_1 \,) \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, b_2 \, \{ \, b^w \, [\, K_2 - (\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,) \,] + r_2 - c_2^r \, \} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_2 \int_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon) \, dH(\varepsilon)$$

$$+ \, [\, b^w \, d_1 \, K_2 + (\, 1 - b^w \, d_1 \,) \, (\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,) \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, b_2 \, \{ \, b^w \, [\, K_2 - (\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,) \,] + r_2 - c_2^r \} \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_2 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{2c}(\varepsilon) \, dH(\varepsilon)$$

$$+ \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b^w \, d_1 \, K_2 + (\, 1 - b^w \, d_1 \,) \, (\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,) \,] \; = \; 0$$

$$\Rightarrow \quad [\, b^w \; d_1 + b^w \; b_2 + 1 - b^w \, d_1 \,] \, [\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, b^w \, b_2 \int_{\underline{\varepsilon}}^{\varepsilon_0} q_2^{*0c}(\varepsilon) \, dH(\varepsilon) - b_2 \, [\, r_2 - c_2^r \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_2 \int_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon) \, dH(\varepsilon)$$

$$- \, b^w \left[d_1 - \frac{1}{2} \, b_2 \right] K_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b^w \left[d_1 - \frac{1}{2} \, b_2 \right] [\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \; b^w \, b_2 \, K_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b^w \, b_2 \, [\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b_2 \, [\, r_2 - c_2^r \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \, b_2 \int_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon) \, dH(\varepsilon) + b^w \, d_1 \, K_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \; [\, 1 - b^w \, d_1 \,] \, [\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w \, b_2 \, K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \, b^w \, b_2 \, [\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b_2 \, [\, r_2 - c_2^r \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \, b_2 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{2c}(\varepsilon) \, dH(\varepsilon) + b^w \, d_1 \, K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, 1 - b^w \, d_1 \,] \, [\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,] \; = \; 0$$

$$\Rightarrow \quad [\,1 + b^w\ b_2\,]\,[\,a_2 - b_2\,r_2 + d_2\,r_1\,]\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] - b^w\,b_2 \int\limits_{\underline{\varepsilon}}^{\varepsilon_0} q_2^{*0c}(\varepsilon)\,dH(\varepsilon)$$

$$+ \left[1 - b^w\,d_1 + b^w\,b_2 + b^w\left(d_1 - \frac{1}{2}\,b_2\right)\right][\,a_2 - b_2\,r_2 + d_2\,r_1\,]\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]$$

$$+ b_2 \int\limits_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon)\,dH(\varepsilon) + b_2 \int\limits_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon)\,dH(\varepsilon) + b_2 \int\limits_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{2c}(\varepsilon)\,dH(\varepsilon)$$

$$- b_2\,[\,r_2 - c_2^r\,]\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] - b_2\,[\,r_2 - c_2^r\,]\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right] - b_2\,[\,r_2 - c_2^r\,]\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]$$

$$- b^w\left[d_1 - \frac{1}{2}\,b_2\right]K_2\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right] - b^w\,b_2\,K_2\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right] + b^w\,d_1\,K_2\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]$$

$$- b^w\,b_2\,K_2\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] + b^w\,d_1\,K_2\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]$$

$$+ [\,1 - b^w\,d_1 + b^w\,b_2\,]\,[\,a_2 - b_2\,r_2 + d_2\,r_1\,]\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] \;=\; 0$$

$$\Rightarrow \quad [\,1 + b^w\,b_2\,]\,[\,a_2 - b_2\,r_2 + d_2\,r_1\,]\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] - b^w\,b_2 \int\limits_{\underline{\varepsilon}}^{\varepsilon_0} q_2^{*0c}(\varepsilon)\,dH(\varepsilon)$$

$$+ \left[1 + \frac{1}{2}\,b^w\,b_2\right][\,a_2 - b_2\,r_2 + d_2\,r_1\,]\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]$$

$$+ b_2 \int\limits_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon)\,dH(\varepsilon) + b_2 \int\limits_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon)\,dH(\varepsilon) + b_2 \int\limits_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{2c}(\varepsilon)\,dH(\varepsilon)$$

$$- b_2\,[\,r_2 - c_2^r\,] - \frac{1}{2}\,b^w\,b_2\,K_2\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right] - b^w\,[\,b_2 - d_1\,]\,K_2\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]$$

$$+ [\,1 - b^w\,d_1 + b^w\,b_2\,]\,[\,a_2 - b_2\,r_2 + d_2\,r_1\,]\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] \;=\; 0 \tag{69}$$

where $q_2^{*0c}(\varepsilon)$, $w^{0c}(\varepsilon)$, $w^{1c}(\varepsilon)$, and $w^{2c}(\varepsilon)$ are as specified in (42) – (47), the integral terms are defined in (60), and where ε_0 and ε_{12} are as specified in (51).

Case 2B. G1 is capacity-constrained when $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$.

(6), (7), and (9) imply that under VI, when $\varepsilon \in [\underline{\varepsilon}, \varepsilon_0]$:

$$q_1^*(\varepsilon) = \frac{1}{3}\left[a^L + 3\,Q_1^r\right] + \frac{1}{3\,b^w}\left[\varepsilon - 2\,c_1 + c_2\right] \equiv q_1^{*0c}(\varepsilon);$$

$$q_2^*(\varepsilon) = \frac{1}{3}\left[a^L + 3\,Q_2^r\right] + \frac{1}{3\,b^w}\left[\varepsilon - 2\,c_2 + c_1\right] \equiv q_2^{*0c}(\varepsilon);$$

$$w(\varepsilon) = \frac{1}{3}\left[b^w\,a^L + \varepsilon + c_1 + c_2\right] \equiv w^{0c}(\varepsilon). \tag{70}$$

(13) and (15) imply that when $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$ in this case:

$$q_1^*(\varepsilon) = K_1 \equiv q_1^{*1c}(\varepsilon);$$

$$q_2^*(\varepsilon) = \frac{1}{2}\left[a^L + Q_1^r + 2\,Q_2^r - K_1\right] + \frac{1}{2\,b^w}\left[\varepsilon - c_2\right] \equiv q_2^{*1c}(\varepsilon);$$

$$w(\varepsilon) = \frac{1}{2}\,b^w\left[a^L + Q_1^r - K_1\right] + \frac{1}{2}\left[c_2 + \varepsilon\right] \equiv w^{1c}(\varepsilon). \tag{71}$$

(16) implies that when $\varepsilon \in [\varepsilon_{12}, \overline{\varepsilon}]$ in this case:

$$q_1^*(\varepsilon) = K_1 \equiv q_1^{*2c}(\varepsilon), \quad q_2^*(\varepsilon) = K_2 \equiv q_2^{*2c}(\varepsilon), \quad \text{and}$$

$$w(\varepsilon) = b^w\left[a^L + Q_1^r + Q_2^r - K_1 - K_2\right] + \varepsilon \equiv w^{2c}(\varepsilon). \tag{72}$$

(70) – (72) imply that in this case:

$$\frac{dq_2^{*0c}(\varepsilon)}{dr_1} = d_2\,, \quad \frac{dq_2^{*1c}(\varepsilon)}{dr_1} = d_2 - \frac{1}{2}\,b_1\,, \quad \text{and} \quad \frac{dq_2^{*2c}(\varepsilon)}{dr_1} = 0\,. \tag{73}$$

Because $\frac{\partial Q_1^r}{\partial r_1} = -\,b_1$ and $\frac{\partial Q_2^r}{\partial r_1} = d_2$, (39), (49), and (73) imply that R1's choice of r_1 in this case is determined by:

$$\int_{\underline{\varepsilon}}^{\varepsilon_0}\left[d_2\,\frac{\partial \Pi_1^{0c}(\varepsilon)}{\partial q_2^*} - b_1\,\frac{\partial \Pi_1^{0c}(\varepsilon)}{\partial Q_1^r} + \frac{\partial \Pi_1^{0c}(\varepsilon)}{\partial r_1}\right] dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}}\left[\left(d_2 - \frac{1}{2}\,b_1\right)\frac{\partial \Pi_1^{1c}(\varepsilon)}{\partial q_2^*} - b_1\,\frac{\partial \Pi_1^{1c}(\varepsilon)}{\partial Q_1^r} + \frac{\partial \Pi_1^{1c}(\varepsilon)}{\partial r_1}\right] dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}}\left[-\,b_1\,\frac{\partial \Pi_1^{2c}(\varepsilon)}{\partial Q_1^r} + \frac{\partial \Pi_1^{2c}(\varepsilon)}{\partial r_1}\right] dH(\varepsilon)$$

$$+ \left[\Pi_1^{0c}(\varepsilon_0) - \Pi_1^{1c}(\varepsilon_0) \right] h(\varepsilon_0) \frac{d\varepsilon_0}{dr_1} + \left[\Pi_1^{1c}(\varepsilon_{12}) - \Pi_1^{2c}(\varepsilon_{12}) \right] h(\varepsilon_{12}) \frac{d\varepsilon_{12}}{dr_1} = 0 . \tag{74}$$

(23) and (25) imply that in this case:

$$\varepsilon_0 = 3\, b^w K_1 + 2\, c_1 - c_2 - b^w \left[a^L + 3\, Q_1^r \right] ;$$

$$\varepsilon_{12} = 2\, b^w K_2 + b^w K_1 + c_2 - b^w \left[a^L + Q_1^r + 2\, Q_2^r \right] . \tag{75}$$

Arguments that parallel those in (52) – (58) imply that in this case:

$$\Pi_1^{0c}(\varepsilon_0) = \Pi_1^{1c}(\varepsilon_0) \quad \text{and} \quad \Pi_1^{1c}(\varepsilon_{12}) = \Pi_1^{2c}(\varepsilon_{12}) . \tag{76}$$

(70) – (72) imply:

$$\begin{aligned} \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon)\, dH(\varepsilon) &= \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \int_{\underline{\varepsilon}}^{\varepsilon_0} \left(\frac{1}{3} \left[a^L + 3\, Q_1^r \right] + \frac{1}{3\, b^w} \left[\varepsilon - 2\, c_1 + c_2 \right] \right) d\varepsilon \\ &= \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\, (a_1 - b_1\, r_1 + d_1\, r_2) + \frac{1}{b^w} (c_2 - 2\, c_1) \right] \\ &\quad + \frac{1}{6\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right] ; \end{aligned}$$

$$\begin{aligned} \int_{\underline{\varepsilon}}^{\varepsilon_0} q_2^{*0c}(\varepsilon)\, dH(\varepsilon) &= \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \int_{\underline{\varepsilon}}^{\varepsilon_0} \left(\frac{1}{3} \left[a^L + 3\, Q_2^r \right] + \frac{1}{3\, b^w} \left[\varepsilon - 2\, c_2 + c_1 \right] \right) d\varepsilon \\ &= \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\, (a_2 - b_2\, r_2 + d_2\, r_1) + \frac{1}{b^w} (\, c_1 - 2\, c_2\,) \right] \\ &\quad + \frac{1}{6\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right] ; \end{aligned}$$

$$\begin{aligned} \int_{\varepsilon_0}^{\varepsilon_{12}} q_2^{*1c}(\varepsilon)\, dH(\varepsilon) &= \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \int_{\varepsilon_0}^{\varepsilon_{12}} \left(\frac{1}{2} \left[a^L + Q_1^r + 2\, Q_2^r - K_1 \right] + \frac{1}{2\, b^w} \left[\varepsilon - c_2 \right] \right) d\varepsilon \\ &= \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \Bigg[a^L + a_1 - b_1\, r_1 + d_1\, r_2 + 2\, (\, a_2 - b_2\, r_2 + d_2\, r_1\,) \\ &\qquad - K_1 - \frac{c_2}{b^w} \Bigg] + \frac{1}{4 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right] ; \end{aligned}$$

$$\int_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon)\, dH(\varepsilon) = \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \int_{\underline{\varepsilon}}^{\varepsilon_0} \frac{1}{3} \left[b^w\, a^L + \varepsilon + c_1 + c_2 \right] d\varepsilon$$

$$= \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w\, a^L + c_1 + c_2 \right] + \frac{1}{6} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right];$$

$$\int_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon)\, dH(\varepsilon) = \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \int_{\varepsilon_0}^{\varepsilon_{12}} \left(\frac{1}{2}\, b^w \left[a^L + Q_1^r - K_1 \right] + \frac{1}{2} \left[c_2 + \varepsilon \right] \right) d\varepsilon$$

$$= \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + a_1 - b_1\, r_1 + d_1\, r_2 - K_1 \right) + c_2 \right]$$

$$+ \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right];$$

$$\int_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{2c}(\varepsilon)\, dH(\varepsilon) = \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left(b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] + \varepsilon \right) d\varepsilon$$

$$= b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + a_1 - b_1\, r_1 + d_1\, r_2 + a_2 - b_2\, r_2 + d_2\, r_1 - K_1 - K_2 \right]$$

$$+ \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right]. \tag{77}$$

(49), (74), and (76) imply that R1's choice of r_1 in this case is determined by:

$$- b^w\ d_2 \int_{\underline{\varepsilon}}^{\varepsilon_0} \left[q_1^{*0c}(\varepsilon) - (\, a_1 - b_1\, r_1 + d_1\, r_2\,) \right] dH(\varepsilon)$$

$$- b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} \left\{ b^w \left[q_1^{*0c}(\varepsilon) - (\, a_1 - b_1\, r_1 + d_1\, r_2\,) \right] + r_1 - c_1^r - w^{0c}(\varepsilon) \right\} dH(\varepsilon)$$

$$+ \int_{\underline{\varepsilon}}^{\varepsilon_0} \left[b^w\, d_2\, q_1^{*0c}(\varepsilon) + (\, 1 - b^w\, d_2\,) \, (\, a_1 - b_1\, r_1 + d_1\, r_2\,) \right] dH(\varepsilon)$$

$$- b^w \left[d_2 - \frac{1}{2}\, b_1 \right] \left[K_1 - (\, a_1 - b_1\, r_1 + d_1\, r_2\,) \right] \left[H(\varepsilon_{12}) - H(\varepsilon_0) \right]$$

$$- b_1 \int_{\varepsilon_0}^{\varepsilon_{12}} \left\{ b^w \left[K_1 - (\, a_1 - b_1\, r_1 + d_1\, r_2\,) \right] + r_1 - c_1^r - w^{1c}(\varepsilon) \right\}\, dH(\varepsilon)$$

$$+ \left[b^w d_2 K_1 + (1 - b^w d_2)(a_1 - b_1 r_1 + d_1 r_2) \right] \left[H(\varepsilon_{12}) - H(\varepsilon_0) \right]$$

$$- b_1 \int_{\varepsilon_{12}}^{\bar{\varepsilon}} \left\{ b^w \left[K_1 - (a_1 - b_1 r_1 + d_1 r_2) \right] + r_1 - c_1^r - w^{2c}(\varepsilon) \right\} dH(\varepsilon)$$

$$+ \left[1 - H(\varepsilon_{12}) \right] \left[b^w d_2 K_1 + (1 - b^w d_2)(a_1 - b_1 r_1 + d_1 r_2) \right] = 0$$

$$\Rightarrow \quad - b^w d_2 \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon)\, dH(\varepsilon) + b^w d_2 \left[a_1 - b_1 r_1 + d_1 r_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b^w b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon)\, dH(\varepsilon) + b^w b_1 \left[a_1 - b_1 r_1 + d_1 r_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b_1 \left[r_1 - c_1^r \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}} \right] + b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon)\, dH(\varepsilon)$$

$$+ b^w d_2 \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon)\, dH(\varepsilon) + \left[1 - b^w d_2 \right] \left[a_1 - b_1 r_1 + d_1 r_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b^w \left[d_2 - \frac{1}{2} b_1 \right] \left[K_1 - (a_1 - b_1 r_1 + d_1 r_2) \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\bar{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b^w b_1 \left[K_1 - (a_1 - b_1 r_1 + d_1 r_2) \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\bar{\varepsilon} - \underline{\varepsilon}} \right] - b_1 \left[r_1 - c_1^r \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\bar{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ b_1 \int_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon)\, dH(\varepsilon) + \left[b^w d_2 K_1 + (1 - b^w d_2)(a_1 - b_1 r_1 + d_1 r_2) \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\bar{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b^w b_1 \left[K_1 - (a_1 - b_1 r_1 + d_1 r_2) \right] \left[\frac{\bar{\varepsilon} - \varepsilon_{12}}{\bar{\varepsilon} - \underline{\varepsilon}} \right] - b_1 \left[r_1 - c_1^r \right] \left[\frac{\bar{\varepsilon} - \varepsilon_{12}}{\bar{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ b_1 \int_{\varepsilon_{12}}^{\bar{\varepsilon}} w^{2c}(\varepsilon)\, dH(\varepsilon) + \left[\frac{\bar{\varepsilon} - \varepsilon_{12}}{\bar{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w d_2 K_1 + (1 - b^w d_2)(a_1 - b_1 r_1 + d_1 r_2) \right] = 0$$

$$\Rightarrow \quad \left[b^w d_2 + 1 - b^w d_2 + b^w b_1 \right] \left[a_1 - b_1 r_1 + d_1 r_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b^w b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon)\, dH(\varepsilon) - b_1 \left[r_1 - c_1^r \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}} \right]$$

$$
\begin{aligned}
&+ b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon)\, dH(\varepsilon) - b^w \left[d_2 - \frac{1}{2} b_1 \right] K_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ \left[b^w d_2 - \frac{1}{2} b^w b_1 + b^w b_1 + 1 - b^w d_2 \right] [\, a_1 - b_1 r_1 + d_1 r_2 \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&- b^w b_1 K_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b_1 [\, r_1 - c_1^r \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ b_1 \int_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon)\, dH(\varepsilon) + b^w d_2 K_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w b_1 K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ [\, b^w b_1 + 1 - b^w d_2 \,] [\, a_1 - b_1 r_1 + d_1 r_2 \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&- b_1 [\, r_1 - c_1^r \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_1 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{2c}(\varepsilon)\, dH(\varepsilon) + b^w d_2 K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \;=\; 0
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow \quad & [\, 1 + b^w b_1 \,] [\, a_1 - b_1 r_1 + d_1 r_2 \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon)\, dH(\varepsilon) \\
&+ b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon)\, dH(\varepsilon) + b_1 \int_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon)\, dH(\varepsilon) + b_1 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{2c}(\varepsilon)\, dH(\varepsilon) \\
&- b_1 [\, r_1 - c_1^r \,] - \frac{1}{2} b^w b_1 K_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ \left[1 + \frac{1}{2} b^w b_1 \right] [\, a_1 - b_1 r_1 + d_1 r_2 \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&- b^w [\, b_1 - d_2 \,] \; K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + [\, b^w b_1 + 1 - b^w d_2 \,] [\, a_1 - b_1 r_1 + d_1 r_2 \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \;=\; 0
\end{aligned}
\tag{78}
$$

where $q_1^{*0c}(\varepsilon)$, $w^{0c}(\varepsilon)$, $w^{1c}(\varepsilon)$, and $w^{2c}(\varepsilon)$ are as specified in (70) – (72), the integrals are defined in (77), and where ε_0 and ε_{12} are as specified in (75).

We now characterize R2's choice of r_2 in this case. (70) – (72) imply that in this case:

$$\frac{dq_1^{*0c}(\varepsilon)}{dr_2} = d_1 \quad \text{and} \quad \frac{dq_1^{*1c}(\varepsilon)}{dr_2} = \frac{dq_1^{*2c}(\varepsilon)}{dr_2} = 0\,. \tag{79}$$

Because $\frac{\partial Q_2^r}{\partial r_2} = -\,b_2$ and $\frac{\partial Q_1^r}{\partial r_2} = d_1$, (39) and (79) imply that (62) can be written as:

$$\int_{\underline{\varepsilon}}^{\varepsilon_0} \left[d_1 \frac{\partial \Pi_2^{0c}(\varepsilon)}{\partial q_1^*} - b_2 \frac{\partial \Pi_2^{0c}(\varepsilon)}{\partial Q_2^r} + \frac{\partial \Pi_2^{0c}(\varepsilon)}{\partial r_2} \right] dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \left[-\,b_2 \frac{\partial \Pi_2^{1c}(\varepsilon)}{\partial Q_2^r} + \frac{\partial \Pi_2^{1c}(\varepsilon)}{\partial r_2} \right] dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left[-\,b_2 \frac{\partial \Pi_2^{2c}(\varepsilon)}{\partial Q_2^r} + \frac{\partial \Pi_2^{2c}(\varepsilon)}{\partial r_2} \right] dH(\varepsilon)$$

$$+ \left[\Pi_2^{0c}(\varepsilon_0) - \Pi_2^{1c}(\varepsilon_0) \right] h(\varepsilon_0) \frac{d\varepsilon_0}{dr_2} + \left[\Pi_2^{1c}(\varepsilon_{12}) - \Pi_2^{2c}(\varepsilon_{12}) \right] h(\varepsilon_{12}) \frac{d\varepsilon_{12}}{dr_2} = 0. \tag{80}$$

As in the discussion that precedes (68), it can be shown that:

$$\Pi_2^{0c}(\varepsilon_0) = \Pi_2^{1c}(\varepsilon_0) \quad \text{and} \quad \Pi_2^{1c}(\varepsilon_{12}) = \Pi_2^{2c}(\varepsilon_{12})\,. \tag{81}$$

(64), (80), and (81) imply that R2's choice of r_2 in this case is determined by:

$$-\,b^w\, d_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} \left[q_2^{*0c}(\varepsilon) - (\, a_2 - b_2\, r_2 + d_2\, r_1 \,) \right] dH(\varepsilon)$$

$$-\, b_2 \int_{\underline{\varepsilon}}^{\varepsilon_0} \left\{ b^w \left[q_2^{*0c}(\varepsilon) - (\, a_2 - b_2\, r_2 + d_2\, r_1 \,) \right] + r_2 - c_2^r - w^{0c}(\varepsilon) \right\} dH(\varepsilon)$$

$$+ \int_{\underline{\varepsilon}}^{\varepsilon_0} \left[b^w\, d_1\, q_2^{*0c}(\varepsilon) + (\, 1 - b^w\, d_1 \,)\, (\, a_2 - b_2\, r_2 + d_2\, r_1 \,) \right] dH(\varepsilon)$$

$$-\, b_2 \int_{\varepsilon_0}^{\varepsilon_{12}} \left\{ b^w \left[q_2^{*1c}(\varepsilon) - (\, a_2 - b_2\, r_2 + d_2\, r_1 \,) \right] + r_2 - c_2^r - w^{1c}(\varepsilon) \right\} dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \left[b^w\, d_1\, q_2^{*1c}(\varepsilon) + (\, 1 - b^w\, d_1 \,)\, (\, a_2 - b_2\, r_2 + d_2\, r_1 \,) \right] dH(\varepsilon)$$

$$- \, b_2 \int\limits_{\varepsilon_{12}}^{\overline{\varepsilon}} \Big\{ \, b^w \, [\, K_2 - (\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,) \,] + r_2 - c_2^r - w^{2c}(\varepsilon) \, \Big\} \; dH(\varepsilon)$$

$$+ \; [\, 1 - H(\varepsilon_{12}) \,] \, [\, b^w \, d_1 \, K_2 + (\, 1 - b^w \, d_1 \,) \, (\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,) \,] \; = \; 0$$

$$\Rightarrow \quad - \, b^w \, d_1 \int\limits_{\underline{\varepsilon}}^{\varepsilon_0} q_2^{*0c}(\varepsilon) \, dH(\varepsilon) + b^w \, d_1 \, [\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, b^w \, b_2 \int\limits_{\underline{\varepsilon}}^{\varepsilon_0} q_2^{*0c}(\varepsilon) \, dH(\varepsilon) + \; b^w \, b_2 \, [\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, b_2 \, [\, r_2 - c_2^r \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_2 \int\limits_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon) \, dH(\varepsilon)$$

$$+ \, b^w \, d_1 \int\limits_{\underline{\varepsilon}}^{\varepsilon_0} q_2^{*0c}(\varepsilon) \, dH(\varepsilon) + [\, 1 - b^w \, d_1 \,] \, [\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, b^w \, b_2 \int\limits_{\varepsilon_0}^{\varepsilon_{12}} q_2^{*1c}(\varepsilon) \, dH(\varepsilon) + b^w \, b_2 \, [\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, b_2 \, [\, r_2 - c_2^r \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_2 \int\limits_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon) \, dH(\varepsilon)$$

$$+ \, b^w \, d_1 \int\limits_{\varepsilon_0}^{\varepsilon_{12}} q_2^{*1c}(\varepsilon) \, dH(\varepsilon) + [\, 1 - b^w \, d_1 \,] \, [\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, b^w \, b_2 \, K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b^w \, b_2 \, [\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, b_2 \, [\, r_2 - c_2^r \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_2 \int\limits_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{2c}(\varepsilon) \, dH(\varepsilon)$$

$$+ \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b^w \, d_1 \, K_2 + (\, 1 - b^w \, d_1 \,) \, (\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,) \,] \; = \; 0$$

$$\Rightarrow \quad [\, b^w \, d_1 + 1 - b^w \, d_1 + b^w \, b_2 \,] \, [\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$
\begin{aligned}
&- b^w b_2 \int_{\underline{\varepsilon}}^{\varepsilon_0} q_2^{*0c}(\varepsilon)\, dH(\varepsilon) - b_2 [\, r_2 - c_2^r \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_2 \int_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon)\, dH(\varepsilon) \\
&- b^w b_2 \int_{\varepsilon_0}^{\varepsilon_{12}} q_2^{*1c}(\varepsilon)\, dH(\varepsilon) + [\, b^w b_2 + 1 - b^w d_1 \,] [\, a_2 - b_2 r_2 + d_2 r_1 \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&- b_2 [\, r_2 - c_2^r \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_2 \int_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon)\, dH(\varepsilon) + b^w d_1 \int_{\varepsilon_0}^{\varepsilon_{12}} q_2^{*1c}(\varepsilon)\, dH(\varepsilon) \\
&- b^w b_2 K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + [\, b^w b_2 + 1 - b^w d_1 \,] [\, a_2 - b_2 r_2 + d_2 r_1 \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&- b_2 [\, r_2 - c_2^r \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_2 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{2c}(\varepsilon)\, dH(\varepsilon) + b^w d_1 K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \; = \; 0
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow \quad & [\, 1 + b^w b_2 \,] [\, a_2 - b_2 r_2 + d_2 r_1 \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w b_2 \int_{\underline{\varepsilon}}^{\varepsilon_0} q_2^{*0c}(\varepsilon)\, dH(\varepsilon) \\
&+ b_2 \int_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon)\, dH(\varepsilon) + b_2 \int_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon)\, dH(\varepsilon) + b_2 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{2c}(\varepsilon)\, dH(\varepsilon) \\
&- b_2 [\, r_2 - c_2^r \,] - b^w [\, b_2 - d_1 \,] \int_{\varepsilon_0}^{\varepsilon_{12}} q_2^{*1c}(\varepsilon)\, dH(\varepsilon) \\
&+ [\, b^w b_2 + 1 - b^w d_1 \,] [\, a_2 - b_2 r_2 + d_2 r_1 \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w [\, b_2 - d_1 \,] K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ [\, b^w b_2 + 1 - b^w d_1 \,] [\, a_2 - b_2 r_2 + d_2 r_1 \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \; = \; 0 \qquad (82)
\end{aligned}
$$

where $q_2^{*0c}(\varepsilon)$, $q_2^{*1c}(\varepsilon)$, $w^{0c}(\varepsilon)$, $w^{1c}(\varepsilon)$, and $w^{2c}(\varepsilon)$ are as specified in (70) – (72), the integrals are defined in (77), and where ε_0 and ε_{12} are as specified in (75).

Case 3. Partial Vertical Integration ($\alpha_1^R = \alpha_1^G = 1$; $\alpha_2^R = \alpha_2^G = 0$) .

Case 3A. G2 is capacity-constrained when $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$.

(6), (7), and (9) imply that in this case:

$$q_1^{*0c}(\varepsilon) = \frac{1}{3}\left[a^L + 3\,Q_1^r + Q_2^r\right] + \frac{1}{3\,b^w}\left[\varepsilon + c_2 - 2\,c_1\right];$$

$$q_2^{*0c}(\varepsilon) = \frac{1}{3}\left[a^L + Q_2^r\right] + \frac{1}{3\,b^w}\left[\varepsilon + c_1 - 2\,c_2\right];$$

$$w^{0c}(\varepsilon) = \frac{1}{3}\,b^w\left[a^L + Q_2^r\right] + \frac{1}{3}\left[\varepsilon + c_1 + c_2\right]. \tag{83}$$

(10) and (12) imply that in this case:

$$q_1^{*1c}(\varepsilon) = \frac{1}{2}\left[a^L + 2\,Q_1^r + Q_2^r - K_2\right] + \frac{1}{2\,b^w}\left[\varepsilon - c_1\right]; \quad q_2^{*1c}(\varepsilon) = K_2;$$

$$w^{1c}(\varepsilon) = \frac{1}{2}\,b^w\left[a^L + Q_2^r - K_2\right] + \frac{1}{2}\left[c_1 + \varepsilon\right]. \tag{84}$$

(16) implies that in this case:

$$q_1^{*2c}(\varepsilon) = K_1; \quad q_2^{*2c}(\varepsilon) = K_2;$$

$$w^{2c}(\varepsilon) = b^w\left[a^L + Q_1^r + Q_2^r - K_1 - K_2\right] + \varepsilon. \tag{85}$$

(18) and (20) imply that in this case:

$$\varepsilon_0 = 3\,b^w K_2 + 2\,c_2 - c_1 - b^w\left[a^L + Q_2^r\right];$$

$$\varepsilon_{12} = 2\,b^w K_1 + b^w K_2 + c_1 - b^w\left[a^L + 2\,Q_1^r + Q_2^r\right]. \tag{86}$$

(83) – (85) imply that in this case:

$$\frac{dq_2^{*0c}}{dr_1} = \frac{1}{3}\,d_2 \quad \text{and} \quad \frac{dq_2^{*1c}}{dr_1} = \frac{dq_2^{*2c}}{dr_1} = 0. \tag{87}$$

Because $\frac{\partial Q_1^r}{\partial r_1} = -\,b_1$ and $\frac{\partial Q_2^r}{\partial r_1} = d_2$, (39), (41), (49), and (87) imply that R1's choice of r_1 is given by:

$$\int_{\underline{\varepsilon}}^{\varepsilon_0}\left[\frac{1}{3}\,d_2\,\frac{\partial \Pi_1^{0c}(\varepsilon)}{\partial q_2^*} - b_1\,\frac{\partial \Pi_1^{0c}(\varepsilon)}{\partial Q_1^r} + \frac{\partial \Pi_1^{0c}(\varepsilon)}{\partial r_1}\right] dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \left[- b_1 \frac{\partial \Pi_1^{1c}(\varepsilon)}{\partial Q_1^r} + \frac{\partial \Pi_1^{1c}(\varepsilon)}{\partial r_1} \right] dH(\varepsilon) + \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left[- b_1 \frac{\partial \Pi_1^{2c}(\varepsilon)}{\partial Q_1^r} + \frac{\partial \Pi_1^{2c}(\varepsilon)}{\partial r_1} \right] dH(\varepsilon)$$

$$+ \left[\Pi_1^{0c}(\varepsilon_0) - \Pi_1^{1c}(\varepsilon_0) \right] h(\varepsilon_0) \frac{d\varepsilon_0}{dr_1} + \left[\Pi_1^{1c}(\varepsilon_{12}) - \Pi_1^{2c}(\varepsilon_{12}) \right] h(\varepsilon_{12}) \frac{d\varepsilon_{12}}{dr_1} \; = \; 0 \, . \tag{88}$$

Arguments that parallel those in (52) – (58) imply that in this case:

$$\Pi_1^{0c}(\varepsilon_0) \; = \; \Pi_1^{1c}(\varepsilon_0) \quad \text{and} \quad \Pi_1^{1c}(\varepsilon_{12}) \; = \; \Pi_1^{2c}(\varepsilon_{12}) \, . \tag{89}$$

(83) – (85) imply:

$$\int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon) \, dH(\varepsilon) \; = \; \int_{\underline{\varepsilon}}^{\varepsilon_0} \left(\frac{1}{3} \left[a^L + 3 \, Q_1^r + Q_2^r \right] + \frac{1}{3 \, b^w} \left[c_2 - 2 \, c_1 \right] + \frac{1}{3 \, b^w} \, \varepsilon \right) dH(\varepsilon)$$

$$= \; \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3 \, Q_1^r + Q_2^r \right] + \frac{1}{3 \, b^w} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[c_2 - 2 \, c_1 \right]$$

$$+ \; \frac{1}{6 \, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right] ;$$

$$\int_{\underline{\varepsilon}}^{\varepsilon_0} q_2^{*0c}(\varepsilon) \, dH(\varepsilon) \; = \; \int_{\underline{\varepsilon}}^{\varepsilon_0} \left(\frac{1}{3} \left[a^L + Q_2^r \right] + \frac{1}{3 \, b^w} \left[\varepsilon + c_1 - 2 \, c_2 \right] \right) dH(\varepsilon)$$

$$= \; \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_2^r \right] + \frac{1}{3 \, b^w} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[c_1 - 2 \, c_2 \right]$$

$$+ \; \frac{1}{6 \, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right] ;$$

$$\int_{\varepsilon_0}^{\varepsilon_{12}} q_1^{*1c}(\varepsilon) \, dH(\varepsilon) \; = \; \int_{\varepsilon_0}^{\varepsilon_{12}} \left(\frac{1}{2} \left[a^L + 2 \, Q_1^r + Q_2^r - K_2 \right] - \frac{1}{2 \, b^w} \, c_1 + \frac{1}{2 \, b^w} \, \varepsilon \right) dH(\varepsilon)$$

$$= \; \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 2 \, (a_1 - b_1 \, r_1 + d_1 \, r_2) + a_2 - b_2 \, r_2 + d_2 \, r_1 - K_2 \right]$$

$$- \; \frac{1}{2 \, b^w} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] c_1 + \frac{1}{4 \, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right] ;$$

$$\int_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon) \, dH(\varepsilon) \; = \; \int_{\underline{\varepsilon}}^{\varepsilon_0} \left(\frac{1}{3} \, b^w \left[a^L + Q_2^r \right] + \frac{1}{3} \left[c_1 + c_2 \right] + \frac{1}{3} \, \varepsilon \right) dH(\varepsilon)$$

$$= \frac{1}{3}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, b^w(a^L + a_2 - b_2\, r_2 + d_2\, r_1) + c_1 + c_2 \,\right]$$

$$+ \frac{1}{6}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right];$$

$$\int_{\varepsilon_0}^{\varepsilon_{12}} w^{*1c}(\varepsilon)\, dH(\varepsilon) = \int_{\varepsilon_0}^{\varepsilon_{12}} \left(\frac{1}{2}\, b^w\left[\, a^L + Q_2^r - K_2 \,\right] + \frac{1}{2}\, c_1 + \frac{1}{2}\, \varepsilon \right) dH(\varepsilon)$$

$$= \frac{1}{2}\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, b^w(a^L + a_2 - b_2\, r_2 + d_2\, r_1 - K_2) + c_1 \,\right]$$

$$+ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right];$$

$$\int_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{*2c}(\varepsilon)\, dH(\varepsilon) = \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left(b^w\left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] + \varepsilon \right) dH(\varepsilon)$$

$$= b^w\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, a^L + a_1 - b_1\, r_1 + d_1\, r_2 + a_2 - b_2\, r_2 + d_2\, r_1 - K_1 - K_2 \,\right]$$

$$+ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \,\right]. \qquad (90)$$

(49), (88), and (89) imply that under partial vertical integration, if G2 is capacity-constrained when $\varepsilon \in (\, \varepsilon_0, \varepsilon_{12}\,)$, R1's choice of r_1 is determined by:

$$- \frac{1}{3}\, b^w\ d_2 \int_{\underline{\varepsilon}}^{\varepsilon_0} \left[\, q_1^{*0c}(\varepsilon) - (\, a_1 - b_1\, r_1 + d_1\, r_2\,) \,\right] dH(\varepsilon)$$

$$- \, b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} \left\{\, b^w\left[\, q_1^{*0c}(\varepsilon) - (\, a_1 - b_1\, r_1 + d_1\, r_2\,) \,\right] + r_1 - c_1^r - w^{0c}(\varepsilon) \,\right\} dH(\varepsilon)$$

$$+ \int_{\underline{\varepsilon}}^{\varepsilon_0} \left[\, b^w\, d_2\, q_1^{*0c}(\varepsilon) + (\, 1 - b^w\, d_2\,)\,(\, a_1 - b_1\, r_1 + d_1\, r_2\,) \,\right] dH(\varepsilon)$$

$$- \, b_1 \int_{\varepsilon_0}^{\varepsilon_{12}} \left\{\, b^w\left[\, q_1^{*1c}(\varepsilon) - (\, a_1 - b_1\, r_1 + d_1\, r_2\,) \,\right] + r_1 - c_1^r - w^{1c}(\varepsilon) \,\right\}\ dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \left[b^w \, d_2 \, q_1^{*1c}(\varepsilon) + (1 - b^w \, d_2)(a_1 - b_1 \, r_1 + d_1 \, r_2) \right] dH(\varepsilon)$$

$$- \, b_1 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left\{ b^w \left[K_1 - (a_1 - b_1 \, r_1 + d_1 \, r_2) \right] + r_1 - c_1^r - w^{2c}(\varepsilon) \right\} dH(\varepsilon)$$

$$+ \left[1 - H(\varepsilon_{12}) \right] \left[b^w \, d_2 \, K_1 + (1 - b^w \, d_2)(a_1 - b_1 \, r_1 + d_1 \, r_2) \right] \; = \; 0$$

$$\Rightarrow \quad - \frac{1}{3} \, b^w \, d_2 \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon) \, dH(\varepsilon) + \frac{1}{3} \, b^w \, d_2 \left[a_1 - b_1 \, r_1 + d_1 \, r_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, b^w \, b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon) \, dH(\varepsilon) + b^w \, b_1 \left[a_1 - b_1 \, r_1 + d_1 \, r_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, b_1 \left[r_1 - c_1^r \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon) \, dH(\varepsilon)$$

$$+ \, b^w \, d_2 \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon) \, dH(\varepsilon) + \left[1 - b^w \, d_2 \right] \left[a_1 - b_1 \, r_1 + d_1 \, r_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, b^w \, b_1 \int_{\varepsilon_0}^{\varepsilon_{12}} q_1^{*1c}(\varepsilon) \, dH(\varepsilon) + b^w \, b_1 \left[a_1 - b_1 \, r_1 + d_1 \, r_2 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, b_1 \left[r_1 - c_1^r \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_1 \int_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon) \, dH(\varepsilon)$$

$$+ \, b^w \, d_2 \int_{\varepsilon_0}^{\varepsilon_{12}} q_1^{*1c}(\varepsilon) \, dH(\varepsilon) + \left[1 - b^w \, d_2 \right] \left[a_1 - b_1 \, r_1 + d_1 \, r_2 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, b^w \, b_1 \, K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b^w \, b_1 \left[a_1 - b_1 \, r_1 + d_1 \, r_2 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, b_1 \left[r_1 - c_1^r \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_1 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{2c}(\varepsilon) \, dH(\varepsilon)$$

$$+ \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \, d_2 \, K_1 + (1 - b^w \, d_2)(a_1 - b_1 \, r_1 + d_1 \, r_2) \right] \; = \; 0$$

$$\Rightarrow \quad b^w \left[\frac{2}{3}\, d_2 - b_1 \right] \int\limits_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon)\, dH(\varepsilon) - \; b^w \,[\, b_1 - d_2 \,] \int\limits_{\varepsilon_0}^{\varepsilon_{12}} q_1^{*1c}(\varepsilon)\, dH(\varepsilon)$$

$$+ \left[\frac{1}{3}\, b^w\, d_2 + b^w\, b_1 + 1 - b^w\, d_2 \right] [\, a_1 - b_1\, r_1 + d_1\, r_2 \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, b_1 \,[\, r_1 - c_1^r \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - \; b_1 \,[\, r_1 - c_1^r \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b_1 \,[\, r_1 - c_1^r \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \, b_1 \int\limits_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon)\, dH(\varepsilon) + b_1 \int\limits_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon)\, dH(\varepsilon) + b_1 \int\limits_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{2c}(\varepsilon)\, dH(\varepsilon)$$

$$+ \, [\, b^w\, b_1 + 1 - b^w\, d_2 \,]\, [\, a_1 - b_1\, r_1 + d_1\, r_2 \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \, [\, b^w\, b_1 + 1 - b^w\, d_2 \,]\, [\, a_1 - b_1\, r_1 + d_1\, r_2 \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \; b^w\, d_2\, K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w\, b_1\, K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \; = \; 0$$

$$\Rightarrow \; b^w \left[\frac{2}{3}\, d_2 - b_1 \right] \int\limits_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon)\, dH(\varepsilon) - \; b^w \,[\, b_1 - d_2 \,] \int\limits_{\varepsilon_0}^{\varepsilon_{12}} q_1^{*1c}(\varepsilon)\, dH(\varepsilon)$$

$$+ \left[b^w\, b_1 + 1 - \frac{2}{3}\, b^w\, d_2 \right] [\, a_1 - b_1\, r_1 + d_1\, r_2 \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - \; b_1 \,[\, r_1 - c_1^r \,]$$

$$+ \, b_1 \int\limits_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon)\, dH(\varepsilon) + b_1 \int\limits_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon)\, dH(\varepsilon) + b_1 \int\limits_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{2c}(\varepsilon)\, dH(\varepsilon)$$

$$+ \, [\, b^w\, b_1 + 1 - b^w\, d_2 \,]\, [\, a_1 - b_1\, r_1 + d_1\, r_2 \,] \left[\frac{\overline{\varepsilon} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \; b^w \,[\, b_1 - d_2 \,]\, K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \; = \; 0 \qquad (91)$$

where $q_1^{*0c}(\varepsilon)$, $q_1^{*1c}(\varepsilon)$, $w^{0c}(\varepsilon)$, $w^{1c}(\varepsilon)$, and $w^{2c}(\varepsilon)$ are as specified in (83) – (85), the integral terms are defined in (90), and where ε_0 and ε_{12} are as specified in (86).

We now characterize R2's choice of r_2 in the present case. (17) implies that when $\alpha_1^G = 1$ and $\alpha_2^G = 0$:

$$
\begin{aligned}
E\{w(\varepsilon)\} \;=\;& \frac{1}{3}\int_{\underline{\varepsilon}}^{\varepsilon_0} \left[\, b^w \left(a^L + Q_2^r \right) + \varepsilon + c_1 + c_2 \,\right] \, dH(\varepsilon) \\
&+ \frac{1}{2}\int_{\varepsilon_0}^{\varepsilon_{12}} \left[\, b^w \left(a^L + Q_2^r - K_2 \right) + c_1 + \varepsilon \,\right] \, dH(\varepsilon) \\
&+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) + \varepsilon \,\right] \, dH(\varepsilon)
\end{aligned}
$$

$$
\begin{aligned}
=\;& \left[\frac{1}{3}\, H(\varepsilon_0) + \frac{1}{2}\,(\, H(\varepsilon_{12}) - H(\varepsilon_0)\,) + 1 - H(\varepsilon_{12}) \right] b^w \left[\, a^L + Q_2^r \,\right] + \left[\, 1 - H(\varepsilon_{12}) \,\right] b^w \, Q_1^r \\
&+ \frac{1}{3}\, H(\varepsilon_0)\,[\, c_1 + c_2 \,] - \frac{1}{2}\,[\, H(\varepsilon_{12}) - H(\varepsilon_0) \,]\, b^w \, K_2 + \frac{1}{2}\,[\, H(\varepsilon_{12}) - H(\varepsilon_0) \,]\, c_1 \\
&- [\, 1 - H(\varepsilon_{12}) \,]\, b^w \,[\, K_1 + K_2 \,] + \frac{1}{3}\int_{\underline{\varepsilon}}^{\varepsilon_0} \varepsilon \; dH(\varepsilon) + \frac{1}{2}\int_{\varepsilon_0}^{\varepsilon_{12}} \varepsilon \; dH(\varepsilon) + \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \varepsilon \; dH(\varepsilon)
\end{aligned}
$$

$$
\begin{aligned}
=\;& \frac{1}{6}\,[\, 6 - 3\, H(\varepsilon_{12}) - H(\varepsilon_0) \,]\, b^w \left[\, a^L + Q_2^r \,\right] + [\, 1 - H(\varepsilon_{12}) \,]\, b^w \, Q_1^r \\
&+ \frac{1}{3}\, H(\varepsilon_0)\,[\, c_1 + c_2 \,] - \frac{1}{2}\,[\, H(\varepsilon_{12}) - H(\varepsilon_0) \,]\, b^w \, K_2 + \frac{1}{2}\,[\, H(\varepsilon_{12}) - H(\varepsilon_0) \,]\, c_1 \\
&- [\, 1 - H(\varepsilon_{12}) \,]\, b^w \,[\, K_1 + K_2 \,] + \frac{1}{3}\int_{\underline{\varepsilon}}^{\varepsilon_0} \varepsilon \; dH(\varepsilon) + \frac{1}{2}\int_{\varepsilon_0}^{\varepsilon_{12}} \varepsilon \; dH(\varepsilon) + \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \varepsilon \; dH(\varepsilon)\,. \qquad (92)
\end{aligned}
$$

(86) implies that in this case:

$$
\begin{aligned}
\frac{d\varepsilon_0}{dr_1} \;&=\; -\, b^w \, d_2 \,; \quad \frac{d\varepsilon_0}{dr_2} \;=\; b^w \, b_2 \,; \\
\frac{d\varepsilon_{12}}{dr_1} \;&=\; b^w \,[\, 2\, b_1 - d_2 \,]\,; \quad \frac{d\varepsilon_{12}}{dr_2} \;=\; b^w \,[\, b_2 - 2\, d_1 \,]\,. \qquad (93)
\end{aligned}
$$

When $H(\cdot)$ is the uniform distribution function:

$$\frac{1}{3}\int_{\underline{\varepsilon}}^{\varepsilon_0} \varepsilon \, dH(\varepsilon) + \frac{1}{2}\int_{\varepsilon_0}^{\varepsilon_{12}} \varepsilon \, dH(\varepsilon) + \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \varepsilon \, dH(\varepsilon)$$

$$= \frac{1}{6\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\left[\,(\varepsilon_0)^2 - (\underline{\varepsilon})^2\,\right] + \frac{1}{4\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\left[\,(\varepsilon_{12})^2 - (\varepsilon_0)^2\,\right] + \frac{1}{2\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\left[\,(\overline{\varepsilon})^2 - (\varepsilon_{12})^2\,\right]$$

$$= \frac{1}{12\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\left[\,6\,(\overline{\varepsilon})^2 - 3\,(\varepsilon_{12})^2 - (\varepsilon_0)^2 - 2\,(\underline{\varepsilon})^2\,\right];$$

$$6 - 3\,H(\varepsilon_{12}) - H(\varepsilon_0) \;=\; \frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\,[\,6\,\overline{\varepsilon} - 3\,\varepsilon_{12} - \varepsilon_0 - 2\,\underline{\varepsilon}\,];$$

$$3\,H(\varepsilon_{12}) - H(\varepsilon_0) \;=\; \frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\,[\,3\,\varepsilon_{12} - 3\,\underline{\varepsilon} - \varepsilon_0 + \underline{\varepsilon}\,] \;=\; \frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\,[\,3\,\varepsilon_{12} - \varepsilon_0 - 2\,\underline{\varepsilon}\,];$$

$$H(\varepsilon_0) \;=\; \frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}; \quad 1 - H(\varepsilon_{12}) \;=\; \frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\,[\,\overline{\varepsilon} - \underline{\varepsilon} - \varepsilon_{12} + \underline{\varepsilon}\,] \;=\; \frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\,. \tag{94}$$

Because $\frac{\partial Q_2^r}{\partial r_2} = -\,b_2$ and $\frac{\partial Q_1^r}{\partial r_1} = d_1$, (92) and (94) imply that when $H(\cdot)$ is the uniform distribution:

$$\begin{aligned}
\frac{\partial E\{w(Q)\}}{\partial r_2} \;=\; & -\frac{1}{6}\,[\,6 - 3\,H(\varepsilon_{12}) - H(\varepsilon_0)\,]\,b^w\,b_2 + [\,1 - H(\varepsilon_{12})\,]\,b^w\,d_1 \\
& - \frac{b^w}{6\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\left[\,a^L + a_2 - b_2\,r_2 + d_2\,r_1\,\right]\left[3\,\frac{d\varepsilon_{12}}{dr_2} + \frac{d\varepsilon_0}{dr_2}\right] \\
& - \frac{b^w}{\overline{\varepsilon} - \underline{\varepsilon}}\,[\,a_1 - b_1\,r_1 + d_1\,r_2\,]\,\frac{d\varepsilon_{12}}{dr_2} + \frac{c_1 + c_2}{3\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\,\frac{d\varepsilon_0}{dr_2} \\
& + \frac{c_1 - b^w\,K_2}{2\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2}\right] + \frac{b^w\,[\,K_1 + K_2\,]}{\overline{\varepsilon} - \underline{\varepsilon}}\,\frac{d\varepsilon_{12}}{dr_2} \\
& - \frac{1}{6\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\left[3\,\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_2} + \varepsilon_0\,\frac{d\varepsilon_0}{dr_2}\right].
\end{aligned} \tag{95}$$

(1) and (4) imply that in the present case, R2 seeks to maximize:

$$E\left\{\pi_2^R(r_1, r_2)\right\} = [\,r_2 - c_2^r - E\,\{\,w(\varepsilon)\,\}\,]\,Q_2^r\,. \tag{96}$$

Because $\frac{\partial Q_2^r}{\partial r_2} = -\,b_2$, (96) implies that in this case, R2's choice of r_2 is determined by:

$$\frac{\partial E\left\{\pi_2^R(r_1,r_2)\right\}}{\partial r_2} = [\,a_2 - b_2\,r_2 + d_2\,r_1\,]\left[1 - \frac{\partial E\{w(\varepsilon)\}}{\partial r_2}\right]$$

$$- \, b_2\,[\,r_2 - c_2^r - E\,\{\,w(Q)\,\}\,] \;=\; 0\,, \tag{97}$$

where $E\{w(\varepsilon)\}$ and $\frac{\partial E\{w(\varepsilon)\}}{\partial r_2}$ are as specified in (92) and (95), respectively. In addition, ε_0, ε_{12}, $\frac{d\varepsilon_0}{dr_2}$, and $\frac{d\varepsilon_{12}}{dr_2}$ are specified in (86) and (93).

Case 3B. G1 is capacity-constrained when $\varepsilon \in (\,\varepsilon_0, \varepsilon_{12}\,)$.

(6), (7), and (9) imply that in this case:

$$q_1^{*0c}(\varepsilon) \;=\; \frac{1}{3}\left[\,a^L + 3\,Q_1^r + Q_2^r\,\right] + \frac{1}{3\,b^w}\,[\,\varepsilon + c_2 - 2\,c_1\,]\,;$$

$$q_2^{*0c}(\varepsilon) \;=\; \frac{1}{3}\left[\,a^L + Q_2^r\,\right] + \frac{1}{3\,b^w}\,[\,\varepsilon + c_1 - 2\,c_2\,]\,;$$

$$w^{0c}(\varepsilon) \;=\; \frac{1}{3}\,b^w\left[\,a^L + Q_2^r\,\right] + \frac{1}{3}\,[\,\varepsilon + c_1 + c_2\,]\,. \tag{98}$$

(13) and (15) imply that in this case:

$$q_1^{*1c}(\varepsilon) \;=\; K_1\,; \quad q_2^{*1c}(\varepsilon) \;=\; \frac{1}{2}\left[\,a^L + Q_1^r + Q_2^r - K_1\,\right] + \frac{1}{2\,b^w}\,[\,\varepsilon - c_2\,]\,;$$

$$w^{1c}(\varepsilon) \;=\; \frac{1}{2}\,b^w\left[\,a^L + Q_1^r + Q_2^r - K_1\,\right] + \frac{1}{2}\,[\,c_2 + \varepsilon\,]\,. \tag{99}$$

(16) implies that in this case:

$$q_1^{*2c}(\varepsilon) \;=\; K_1\,; \quad q_2^{*2c}(\varepsilon) \;=\; K_2\,;$$

$$w^{2c}(\varepsilon) \;=\; b^w\left[\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right] + \varepsilon\,. \tag{100}$$

(23) and (25) imply that in this case:

$$\varepsilon_0 \;=\; 3\,b^w\,K_1 + 2\,c_1 - c_2 - b^w\left[\,a^L + 3\,Q_1^r + Q_2^r\,\right];$$

$$\varepsilon_{12} \;=\; 2\,b^w\,K_2 + b^w\,K_1 + c_2 - b^w\left[\,a^L + Q_1^r + Q_2^r\,\right]. \tag{101}$$

(98) – (100) imply that in this case:

$$\frac{dq_2^{*0c}}{dr_1} \;=\; \frac{1}{3}\,d_2, \quad \frac{dq_2^{*1c}}{dr_1} \;=\; -\,\frac{1}{2}\,[\,b_1 - d_2\,]\,, \text{ and } \quad \frac{dq_2^{*2c}}{dr_1} \;=\; 0\,. \tag{102}$$

Because $\frac{\partial Q_1^r}{\partial r_1} = -\, b_1$ and $\frac{\partial Q_2^r}{\partial r_1} = d_2$, (39), (41), (49), and (102) imply that R1's choice of r_1 is given by:

$$\int_{\underline{\varepsilon}}^{\varepsilon_0} \left[\frac{1}{3}\, d_2 \, \frac{\partial \Pi_1^{0c}(\varepsilon)}{\partial q_2^*} - b_1 \, \frac{\partial \Pi_1^{0c}(\varepsilon)}{\partial Q_1^r} + \frac{\partial \Pi_1^{0c}(\varepsilon)}{\partial r_1} \right] dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \left[-\, \frac{1}{2}\, (\, b_1 - d_2 \,)\, \frac{\partial \Pi_1^{1c}(\varepsilon)}{\partial q_2^*} - \, b_1 \, \frac{\partial \Pi_1^{1c}(\varepsilon)}{\partial Q_1^r} + \frac{\partial \Pi_1^{1c}(\varepsilon)}{\partial r_1} \right] dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left[-\, b_1 \, \frac{\partial \Pi_1^{2c}(\varepsilon)}{\partial Q_1^r} + \frac{\partial \Pi_1^{2c}(\varepsilon)}{\partial r_1} \right] dH(\varepsilon)$$

$$+ \left[\Pi_1^{0c}(\varepsilon_0) - \Pi_1^{1c}(\varepsilon_0) \right] h(\varepsilon_0)\, \frac{d\varepsilon_0}{dr_1} + \left[\Pi_1^{1c}(\varepsilon_{12}) - \Pi_1^{2c}(\varepsilon_{12}) \right] h(\varepsilon_{12})\, \frac{d\varepsilon_{12}}{dr_1} \;=\; 0 \, . \qquad (103)$$

Arguments that parallel those in (52) – (58) imply that in this case:

$$\Pi_1^{0c}(\varepsilon_0) \;=\; \Pi_1^{1c}(\varepsilon_0) \quad \text{and} \quad \Pi_1^{1c}(\varepsilon_{12}) \;=\; \Pi_1^{2c}(\varepsilon_{12}) \, . \qquad (104)$$

(98) – (100) imply:

$$\int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon)\, dH(\varepsilon) \;=\; \int_{\underline{\varepsilon}}^{\varepsilon_0} \left(\frac{1}{3} \left[a^L + 3\, Q_1^r + Q_2^r \right] + \frac{1}{3\, b^w} \left[\varepsilon + c_2 - 2\, c_1 \right] \right) dH(\varepsilon)$$

$$= \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\, (\, a_1 - b_1\, r_1 + d_1\, r_2 \,) + a_2 - b_2\, r_2 + d_2\, r_1 \right]$$

$$+ \frac{1}{3\, b^w} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[c_2 - 2\, c_1 \right] + \frac{1}{6\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right] ;$$

$$\int_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon)\, dH(\varepsilon) \;=\; \int_{\underline{\varepsilon}}^{\varepsilon_0} \left(\frac{1}{3}\, b^w \left[a^L + Q_2^r \right] + \frac{1}{3} \left[c_1 + c_2 \right] + \frac{1}{3}\, \varepsilon \right) dH(\varepsilon)$$

$$= \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + a_2 - b_2\, r_2 + d_2\, r_1 \right) + c_1 + c_2 \right]$$

$$+ \frac{1}{6} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right] ;$$

$$\int_{\varepsilon_0}^{\varepsilon_{12}} w^{*1c}(\varepsilon)\, dH(\varepsilon) = \int_{\varepsilon_0}^{\varepsilon_{12}} \left(\frac{1}{2}\, b^w \left[a^L + Q_1^r + Q_2^r - K_1 \right] + \frac{1}{2}\, c_2 + \frac{1}{2}\, \varepsilon \right) dH(\varepsilon)$$

$$= \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + a_1 - b_1\, r_1 + d_1\, r_2 + a_2 - b_2\, r_2 + d_2\, r_1 - K_1 \right) + c_2 \right]$$

$$+ \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right];$$

$$\int_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{*2c}(\varepsilon)\, dH(\varepsilon) = \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left(b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] + \varepsilon \right) dH(\varepsilon)$$

$$= b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + a_1 - b_1\, r_1 + d_1\, r_2 + a_2 - b_2\, r_2 + d_2\, r_1 - K_1 - K_2 \right]$$

$$+ \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right]. \tag{105}$$

(49), (103) and (104) imply that under partial vertical integration, if G1 is capacity-constrained when $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$, R1's choice of r_1 is determined by:

$$- \frac{1}{3}\, b^w\, d_2 \int_{\underline{\varepsilon}}^{\varepsilon_0} \left[q_1^{*0c}(\varepsilon) - (\, a_1 - b_1\, r_1 + d_1\, r_2 \,) \right] dH(\varepsilon)$$

$$- \, b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} \left\{ b^w \left[q_1^{*0c}(\varepsilon) - (\, a_1 - b_1\, r_1 + d_1\, r_2 \,) \right] + r_1 - c_1^r - w^{0c}(\varepsilon) \right\} dH(\varepsilon)$$

$$+ \int_{\underline{\varepsilon}}^{\varepsilon_0} \left[b^w\, d_2\, q_1^{*0c}(\varepsilon) + (\, 1 - b^w\, d_2 \,)\, (\, a_1 - b_1\, r_1 + d_1\, r_2 \,) \right] dH(\varepsilon)$$

$$+ \, b^w\, \frac{1}{2} \left[b_1 - d_2 \right] \left[K_1 - (\, a_1 - b_1\, r_1 + d_1\, r_2 \,) \right] \left[H(\varepsilon_{12}) - H(\varepsilon_0) \right]$$

$$- \, b_1 \int_{\varepsilon_0}^{\varepsilon_{12}} \left\{ b^w \left[K_1 - (\, a_1 - b_1\, r_1 + d_1\, r_2 \,) \right] + r_1 - c_1^r - w^{1c}(\varepsilon) \right\} dH(\varepsilon)$$

$$+ \left[b^w\, d_2\, K_1 + (\, 1 - b^w\, d_2 \,)\, (\, a_1 - b_1\, r_1 + d_1\, r_2 \,) \right] \left[H(\varepsilon_{12}) - H(\varepsilon_0) \right]$$

$$- \, b_1 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left\{ b^w \left[K_1 - (\, a_1 - b_1\, r_1 + d_1\, r_2 \,) \right] + r_1 - c_1^r - w^{2c}(\varepsilon) \right\} dH(\varepsilon)$$

$$+ \left[1 - H(\varepsilon_{12}) \right] \left[b^w d_2 K_1 + (1 - b^w d_2) (a_1 - b_1 r_1 + d_1 r_2) \right] \; = \; 0$$

$$\Rightarrow \quad - \frac{1}{3} b^w d_2 \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon) \, dH(\varepsilon) + \frac{1}{3} b^w d_2 \left[a_1 - b_1 r_1 + d_1 r_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b^w b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon) \, dH(\varepsilon) + b^w b_1 \left[a_1 - b_1 r_1 + d_1 r_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b_1 \left[r_1 - c_1^r \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon) \, dH(\varepsilon)$$

$$+ b^w d_2 \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon) \, dH(\varepsilon) + \left[1 - b^w d_2 \right] \left[a_1 - b_1 r_1 + d_1 r_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ b^w \frac{1}{2} \left[b_1 - d_2 \right] \left[K_1 - (a_1 - b_1 r_1 + d_1 r_2) \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b^w b_1 K_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b^w b_1 \left[a_1 - b_1 r_1 + d_1 r_2 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b_1 \left[r_1 - c_1^r \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_1 \int_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon) \, dH(\varepsilon)$$

$$+ \left[b^w d_2 K_1 + (1 - b^w d_2) (a_1 - b_1 r_1 + d_1 r_2) \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b^w b_1 K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b^w b_1 \left[a_1 - b_1 r_1 + d_1 r_2 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b_1 \left[r_1 - c_1^r \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_1 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{2c}(\varepsilon) \, dH(\varepsilon)$$

$$+ \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w d_2 K_1 + (1 - b^w d_2) (a_1 - b_1 r_1 + d_1 r_2) \right] \; = \; 0$$

$$\Rightarrow \quad b^w \left[\frac{2}{3} d_2 - b_1 \right] \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon) \, dH(\varepsilon)$$

$$+ \left[\frac{1}{3} b^w d_2 + b^w b_1 + 1 - b^w d_2 \right] \left[a_1 - b_1 r_1 + d_1 r_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$
\begin{aligned}
&+ b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon)\, dH(\varepsilon) + b_1 \int_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon)\, dH(\varepsilon) + b_1 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{2c}(\varepsilon)\, dH(\varepsilon) \\
&- b_1 [\, r_1 - c_1^r \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b_1 [\, r_1 - c_1^r \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b_1 [\, r_1 - c_1^r \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ b^w \frac{1}{2} [\, b_1 - d_2 \,] K_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w b_1 K_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ \left[b^w b_1 - b^w \frac{1}{2} (\, b_1 - d_2 \,) + 1 - b^w d_2 \right] [\, a_1 - b_1 r_1 + d_1 r_2 \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ b^w d_2 K_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w b_1 K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ [\, b^w b_1 + 1 - b^w d_2 \,] [\, a_1 - b_1 r_1 + d_1 r_2 \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b^w d_2 K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \; = \; 0
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow \quad & b^w \left[\frac{2}{3} d_2 - b_1 \right] \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon)\, dH(\varepsilon) \\
&+ \left[b^w b_1 + 1 - \frac{2}{3} b^w d_2 \right] [\, a_1 - b_1 r_1 + d_1 r_2 \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ b_1 \int_{\underline{\varepsilon}}^{\varepsilon_0} w^{0c}(\varepsilon)\, dH(\varepsilon) + b_1 \int_{\varepsilon_0}^{\varepsilon_{12}} w^{1c}(\varepsilon)\, dH(\varepsilon) + b_1 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} w^{2c}(\varepsilon)\, dH(\varepsilon) \\
&- b_1 \, [\, r_1 - c_1^r \,] - \frac{1}{2} b^w [\, b_1 + d_2 \,] K_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ \left[1 + \frac{1}{2} b^w b_1 - \frac{1}{2} b^w d_2 \right] [\, a_1 - b_1 r_1 + d_1 r_2 \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ b^w d_2 K_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w [\, b_1 - d_2 \,] K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ [\, b^w b_1 + 1 - b^w d_2 \,] [\, a_1 - b_1 r_1 + d_1 r_2 \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \; = \; 0 \qquad (106)
\end{aligned}
$$

where $q_1^{*0c}(\varepsilon)$, $w^{0c}(\varepsilon)$, $w^{1c}(\varepsilon)$, and $w^{2c}(\varepsilon)$ are as specified in (98) – (100), the integrals are defined in (105), and where ε_0 and ε_{12} are as specified in (101).

We now characterize R2's choice of r_2 in the present case. (22) implies that when $\alpha_1^G = 1$ and $\alpha_2^G = 0$:

$$
\begin{aligned}
E\{w(\varepsilon)\} \;=\; & \frac{1}{3}\int_{\underline{\varepsilon}}^{\varepsilon_0} \left[\, b^w \left(a^L + Q_2^r \right) + \varepsilon + c_1 + c_2 \,\right] \, dH(\varepsilon) \\
& + \frac{1}{2}\int_{\varepsilon_0}^{\varepsilon_{12}} \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 + \varepsilon \,\right] \, dH(\varepsilon) \\
& + \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) + \varepsilon \,\right] \, dH(\varepsilon)
\end{aligned}
$$

$$
\begin{aligned}
= \; & \left[\frac{1}{3} H(\varepsilon_0) + \frac{1}{2} \left(H(\varepsilon_{12}) - H(\varepsilon_0) \right) + 1 - H(\varepsilon_{12}) \right] b^w \left[\, a^L + Q_2^r \,\right] \\
& + \left[1 - H(\varepsilon_{12}) + \frac{1}{2} \left(H(\varepsilon_{12}) - H(\varepsilon_0) \right) \right] b^w \, Q_1^r + \frac{1}{3} H(\varepsilon_0) \left[\, c_1 + c_2 \,\right] \\
& - \frac{1}{2} \left[\, H(\varepsilon_{12}) - H(\varepsilon_0) \,\right] b^w \, K_1 + \frac{1}{2} \left[\, H(\varepsilon_{12}) - H(\varepsilon_0) \,\right] c_2 \\
& - \left[\, 1 - H(\varepsilon_{12}) \,\right] b^w \left[\, K_1 + K_2 \,\right] + \frac{1}{3} \int_{\underline{\varepsilon}}^{\varepsilon_0} \varepsilon \, dH(\varepsilon) + \frac{1}{2} \int_{\varepsilon_0}^{\varepsilon_{12}} \varepsilon \, dH(\varepsilon) + \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \varepsilon \, dH(\varepsilon)
\end{aligned}
$$

$$
\begin{aligned}
= \; & \frac{1}{6} \left[\, 6 - 3\, H(\varepsilon_{12}) - H(\varepsilon_0) \,\right] b^w \left[\, a^L + Q_2^r \,\right] + \left[1 - \frac{1}{2} H(\varepsilon_{12}) - \frac{1}{2} H(\varepsilon_0) \right] b^w \, Q_1^r \\
& + \frac{1}{3} H(\varepsilon_0) \left[\, c_1 + c_2 \,\right] - \frac{1}{2} \left[\, H(\varepsilon_{12}) - H(\varepsilon_0) \,\right] b^w \, K_1 + \frac{1}{2} \left[\, H(\varepsilon_{12}) - H(\varepsilon_0) \,\right] c_2 \\
& - \left[\, 1 - H(\varepsilon_{12}) \,\right] b^w \left[\, K_1 + K_2 \,\right] + \frac{1}{3} \int_{\underline{\varepsilon}}^{\varepsilon_0} \varepsilon \, dH(\varepsilon) + \frac{1}{2} \int_{\varepsilon_0}^{\varepsilon_{12}} \varepsilon \, dH(\varepsilon) + \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \varepsilon \, dH(\varepsilon) \, . \qquad (107)
\end{aligned}
$$

(101) implies that in this case:

$$
\begin{aligned}
\frac{d\varepsilon_0}{dr_1} \;&=\; b^w \left[\, 3\, b_1 - d_2 \,\right] ; \quad \frac{d\varepsilon_0}{dr_2} \;=\; b^w \left[\, b_2 - 3\, d_1 \,\right] ; \\
\frac{d\varepsilon_{12}}{dr_1} \;&=\; b^w \left[\, b_1 - d_2 \,\right] ; \quad \frac{d\varepsilon_{12}}{dr_2} \;=\; b^w \left[\, b_2 - d_1 \,\right] . \qquad (108)
\end{aligned}
$$

Because $\frac{\partial Q_2^r}{\partial r_2} = -\, b_2$ and $\frac{\partial Q_1^r}{\partial r_1} = d_1$, (94) and (107) imply that when $H(\cdot)$ is the uniform distribution:

$$\frac{\partial E\{w(Q)\}}{\partial r_2} = -\frac{1}{6}[6 - 3H(\varepsilon_{12}) - H(\varepsilon_0)]\, b^w b_2 + \left[1 - \frac{1}{2}H(\varepsilon_{12}) - \frac{1}{2}H(\varepsilon_0)\right] b^w d_1$$
$$- \frac{b^w}{6[\overline{\varepsilon} - \underline{\varepsilon}]}[a^L + a_2 - b_2 r_2 + d_2 r_1]\left[3\frac{d\varepsilon_{12}}{dr_2} + \frac{d\varepsilon_0}{dr_2}\right]$$
$$- \frac{b^w}{2[\overline{\varepsilon} - \underline{\varepsilon}]}[a_1 - b_1 r_1 + d_1 r_2]\left[\frac{d\varepsilon_{12}}{dr_2} + \frac{d\varepsilon_0}{dr_2}\right] + \frac{c_1 + c_2}{3[\overline{\varepsilon} - \underline{\varepsilon}]}\frac{d\varepsilon_0}{dr_2}$$
$$+ \frac{c_2 - b^w K_1}{2[\overline{\varepsilon} - \underline{\varepsilon}]}\left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2}\right] + \frac{b^w[K_1 + K_2]}{\overline{\varepsilon} - \underline{\varepsilon}}\frac{d\varepsilon_{12}}{dr_2}$$
$$- \frac{1}{6[\overline{\varepsilon} - \underline{\varepsilon}]}\left[3\varepsilon_{12}\frac{d\varepsilon_{12}}{dr_2} + \varepsilon_0\frac{d\varepsilon_0}{dr_2}\right]. \tag{109}$$

(96) and (97) imply that in the present case, R2's choice of r_2 is determined by:

$$\frac{\partial E\left\{\pi_2^R(r_1, r_2)\right\}}{\partial r_2} = [a_2 - b_2 r_2 + d_2 r_1]\left[1 - \frac{\partial E\{w(\varepsilon)\}}{\partial r_2}\right]$$
$$- b_2[r_2 - c_2^r - E\{w(Q)\}] = 0, \tag{110}$$

where $E\{w(\varepsilon)\}$ and $\frac{\partial E\{w(\varepsilon)\}}{\partial r_2}$ are as specified in (107) and (109), respectively. In addition, ε_0, ε_{12}, $\frac{d\varepsilon_0}{dr_2}$, and $\frac{d\varepsilon_{12}}{dr_2}$ are specified in (101) and (108).

Characterizing Capacity Investment Decisions

(6), (7), and (9) imply that for $\varepsilon \in [\,\underline{\varepsilon}, \varepsilon_0\,]$:

$$q_1^{*0c}(\varepsilon) \;=\; \frac{1}{3}\left[\,a^L + \left(1 + 2\,\alpha_1^G\right) Q_1^r + \left(1 - \alpha_2^G\right) Q_2^r\,\right] + \frac{1}{3\,b^w}\left[\,\varepsilon + c_2 - 2\,c_1\,\right];$$

$$q_2^{*0c}(\varepsilon) \;=\; \frac{1}{3}\left[\,a^L + \left(1 + 2\,\alpha_2^G\right) Q_2^r + \left(1 - \alpha_1^G\right) Q_1^r\,\right] + \frac{1}{3\,b^w}\left[\,\varepsilon + c_1 - 2\,c_2\,\right];$$

$$w^{0c}(\varepsilon) \;=\; \frac{1}{3}\,b^w\left[\,a^L + \left(1 - \alpha_1^G\right) Q_1^r + \left(1 - \alpha_2^G\right) Q_2^r\,\right] + \frac{1}{3}\left[\,\varepsilon + c_1 + c_2\,\right]. \qquad (111)$$

(10) and (12) imply that when only G2 is capacity-constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$q_1^{*1c}(\varepsilon) \;=\; \frac{1}{2}\left[\,a^L + Q_2^r + Q_1^r\left(1 + \alpha_1^G\right) - K_2\,\right] + \frac{1}{2\,b^w}\left[\,\varepsilon - c_1\,\right];$$

$$q_2^{*1c}(\varepsilon) \;=\; K_2\,;$$

$$w^{1c}(\varepsilon) \;=\; \frac{1}{2}\,b^w\left[\,a^L + Q_1^r\left(1 - \alpha_1^G\right) + Q_2^r - K_2\,\right] + \frac{1}{2}\left[\,\varepsilon + c_1\,\right]. \qquad (112)$$

Furthermore, (18) and (20) imply that in this case:

$$\varepsilon_0 \;=\; 3\,b^w\,K_2 + 2\,c_2 - c_1 \;-\; b^w\left[\,a^L + \left(1 + 2\,\alpha_2^G\right)\,Q_2^r + \left(1 - \alpha_1^G\right) Q_1^r\,\right];$$

$$\varepsilon_{12} \;=\; 2\,b^w\,K_1 + b^w\,K_2 + c_1 - b^w\left[\,a^L + Q_1^r\left(\,1 + \alpha_1^G\,\right) + Q_2^r\,\right]. \qquad (113)$$

(13) and (15) imply that when only G1 is capacity-constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$q_1^{*1c}(\varepsilon) \;=\; K_1\,;$$

$$q_2^{*1c}(\varepsilon) \;=\; \frac{1}{2}\left[\,a^L + Q_1^r + Q_2^r\left(1 + \alpha_2^G\right) - K_1\,\right] + \frac{1}{2\,b^w}\left[\,\varepsilon - c_2\,\right];$$

$$w^{1c}(\varepsilon) \;=\; \frac{1}{2}\,b^w\left[\,a^L + Q_1^r + Q_2^r\left(1 - \alpha_2^G\right) - K_1\,\right] + \frac{1}{2}\left[\,\varepsilon + c_2\,\right]. \qquad (114)$$

Furthermore, (23) and (25) imply that in this case:

$$\varepsilon_0 \;=\; 3\,b^w\,K_1 + 2\,c_1 - c_2 - b^w\left[\,a^L + \left(1 + 2\,\alpha_1^G\right)\,Q_1^r + \left(1 - \alpha_2^G\right) Q_2^r\,\right]$$

$$\varepsilon_{12} \;=\; 2\,b^w\,K_2 + b^w\,K_1 + c_2 - b^w\left[\,a^L + Q_1^r + Q_2^r\left(1 + \alpha_2^G\right)\,\right]. \qquad (115)$$

(16) implies that for $\varepsilon \in [\,\varepsilon_{12}, \overline{\varepsilon}\,]$:

$$q_1^{*2c}(\varepsilon) \;=\; K_1\,;\;\; q_2^{*2c}(\varepsilon) \;=\; K_2\,;\;\; w^{2c}(\varepsilon) \;=\; b^w\left[\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right] + \varepsilon\,. \qquad (116)$$

Case 1. Vertical Separation ($\alpha_1^R = \alpha_2^R = \alpha_1^G = \alpha_2^G = 0$).

(111) implies that in this case, for $\varepsilon \in [\underline{\varepsilon}, \varepsilon_0]$:

$$\begin{aligned} q_1^{*0c}(\varepsilon) &= \frac{1}{3}\left[a^L + Q_1^r + Q_2^r \right] + \frac{1}{3\,b^w}\left[\varepsilon + c_2 - 2\,c_1 \right] ; \\ q_2^{*0c}(\varepsilon) &= \frac{1}{3}\left[a^L + Q_1^r + Q_2^r \right] + \frac{1}{3\,b^w}\left[\varepsilon + c_1 - 2\,c_2 \right] ; \\ w^{0c}(\varepsilon) &= \frac{1}{3}\,b^w\left[a^L + Q_1^r + Q_2^r \right] + \frac{1}{3}\left[\varepsilon + c_1 + c_2 \right] . \end{aligned} \tag{117}$$

Furthermore, for $\varepsilon \in [\varepsilon_{12}, \overline{\varepsilon}]$, $q_1^{*2c}(\varepsilon)$, $q_2^{*2c}(\varepsilon)$, and $w^{2c}(\varepsilon)$ are as specified in (116).

Case 1A. G2 is capacity-constrained when $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$.

(113) implies that in this case:

$$\begin{aligned} \varepsilon_0 &= 3\,b^w K_2 + 2\,c_2 - c_1 - b^w\left[a^L + Q_1^r + Q_2^r \right] \\ \varepsilon_{12} &= 2\,b^w K_1 + b^w K_2 + c_1 - b^w\left[a^L + Q_1^r + Q_2^r \right] . \end{aligned} \tag{118}$$

(112) implies that for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$\begin{aligned} q_1^{*1c}(\varepsilon) &= \frac{1}{2}\left[a^L + Q_1^r + Q_2^r - K_2 \right] + \frac{1}{2\,b^w}\left[\varepsilon - c_1 \right] ; \\ q_2^{*1c}(\varepsilon) &= K_2 ; \\ w^{1c}(\varepsilon) &= \frac{b^w}{2}\left[a^L + Q_1^r + Q_2^r - K_2 \right] + \frac{1}{2}\left[c_1 + \varepsilon \right] . \end{aligned} \tag{119}$$

(28) and (29) imply that r_1^* and r_2^* are characterized in this case by:

$$\begin{aligned} J_1(r_1, r_2) &= \left[a_1 - b_1\,r_1 + d_1\,r_2 \right]\left[1 - \frac{\partial E\{ w(\varepsilon) \}}{\partial\, r_1} \right] - b_1\left[r_1 - c_1^r - E\{ w(\varepsilon) \} \right] = 0 ; \\ J_2(r_1, r_2) &= \left[a_2 - b_2\,r_2 + d_2\,r_1 \right]\left[1 - \frac{\partial E\{ w(\varepsilon) \}}{\partial\, r_2} \right] - b_2\left[r_2 - c_2^r - E\{ w(\varepsilon) \} \right] = 0 ; \end{aligned} \tag{120}$$

where $E\{w(\varepsilon)\}$, $\frac{\partial E w(\varepsilon)}{\partial r_1}$, and $\frac{\partial E w(\varepsilon)}{\partial r_2}$ are defined in (31), (32), and (33), respectively.

Case 1B. G1 is capacity-constrained when $\varepsilon \in (\, \varepsilon_0, \varepsilon_{12}\,)$.

(115) implies that in this case:

$$\varepsilon_0 \;=\; 3\, b^w\, K_1 + 2\, c_1 - c_2 - b^w \left[\, a^L + Q_1^r + Q_2^r \,\right];$$

$$\varepsilon_{12} \;=\; 2\, b^w\, K_2 + b^w\, K_1 + c_2 - b^w \left[\, a^L + Q_1^r + Q_2^r \,\right]. \tag{121}$$

(114) implies that for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$q_1^{*1c}(\varepsilon) \;=\; K_1\,;$$

$$q_2^{*1c}(\varepsilon) \;=\; \frac{1}{2}\left[\, a^L + Q_1^r + Q_2^r - K_1 \,\right] + \frac{1}{2\, b^w}\left[\, \varepsilon - c_2 \,\right];$$

$$w^{1c}(\varepsilon) \;=\; \frac{b^w}{2}\left[\, a^L + Q_1^r + Q_2^r - K_1 \,\right] + \frac{1}{2}\left[\, c_2 + \varepsilon \,\right]. \tag{122}$$

(3) implies that in this case, Gi chooses K_i to maximize:

$$E\,\{\, [\, w(\varepsilon) - c_i \,]\, q_i^*(\varepsilon)\,\} - k_i\, K_i$$

$$= \int_{\underline{\varepsilon}}^{\varepsilon_0} \left[\, w^{0c}(\varepsilon) - c_i \,\right] q_i^{*0c}(\varepsilon)\, dH(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_{12}} \left[\, w^{1c}(\varepsilon) - c_i \,\right] q_i^{*1c}(\varepsilon)\, dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left[\, w^{2c}(\varepsilon) - c_i \,\right] K_i\, dH(\varepsilon) - k_i\, K_i \tag{123}$$

(28) and (29) imply that r_1^* and r_2^* are characterized in this case by:

$$J_1(r_1, r_2) \;\equiv\; [\, a_1 - b_1\, r_1 + d_1\, r_2 \,] \left[1 - \frac{\partial E\,\{\, w(\varepsilon)\,\}}{\partial\, r_1} \right] - \; b_1\,[\, r_1 - c_1^r - E\,\{\, w(\varepsilon)\,\}\,] \;=\; 0\,;$$

$$J_2(r_1, r_2) \;\equiv\; [\, a_2 - b_2\, r_2 + d_2\, r_1 \,] \left[1 - \frac{\partial E\,\{\, w(\varepsilon)\,\}}{\partial\, r_2} \right] - \; b_2\,[\, r_2 - c_2^r - E\,\{\, w(\varepsilon)\,\}\,] \;=\; 0\,; \tag{124}$$

where $E\{w(\varepsilon)\}$, $\frac{\partial E w(\varepsilon)}{\partial r_1}$, and $\frac{\partial E w(\varepsilon)}{\partial r_2}$ are defined in (35), (36), and (37), respectively.

Case 1A. Vertical separation where G2 is capacity-constrained for $\varepsilon \in (\, \varepsilon_0, \varepsilon_{12}\,)$.

(116), (117), (119), and (123) imply that when $h(\varepsilon) = \frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}$:

$$E\{\, [\, w(\varepsilon) - c_1\,]\, q_1^*(\varepsilon)\,\} \;=$$

$$\frac{1}{9} \int_{\underline{\varepsilon}}^{\varepsilon_0} \{\, b^w \left[\, a^L + Q_1^r + Q_2^r \,\right] + c_2 - 2\, c_1 + \varepsilon \,\} \left\{ a^L + Q_1^r + Q_2^r + \frac{1}{b^w} [\, c_2 - 2\, c_1 + \varepsilon \,] \right\} dH(\varepsilon)$$

$$+ \frac{1}{4} \int_{\varepsilon_0}^{\varepsilon_{12}} \{\, b^w \left[\, a^L + Q_1^r + Q_2^r - K_2 \,\right] - c_1 + \varepsilon \,\} \left\{ a^L + Q_1^r + Q_2^r - K_2 + \frac{1}{b^w} [\, - c_1 + \varepsilon \,] \right\} dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \{\, b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] - c_1 + \varepsilon \,\} \; K_1 \; dH(\varepsilon)$$

$$= \frac{1}{9\, b^w} \int_{\underline{\varepsilon}}^{\varepsilon_0} \{\, b^w \left[\, a^L + Q_1^r + Q_2^r \,\right] + c_2 - 2\, c_1 + \varepsilon \,\}^2 \; dH(\varepsilon)$$

$$+ \frac{1}{4\, b^w} \int_{\varepsilon_0}^{\varepsilon_{12}} \{\, b^w \left[\, a^L + Q_1^r + Q_2^r - K_2 \,\right] - c_1 + \varepsilon \,\}^2 \; dH(\varepsilon)$$

$$+ K_1 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \{\, b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] - c_1 + \varepsilon \,\} \; dH(\varepsilon)$$

$$= \frac{1}{9\, b^w} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \{\, b^w \left[\, a^L + Q_1^r + Q_2^r \,\right] + c_2 - 2\, c_1 \,\}^2$$

$$+ \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \{\, b^w \left[\, a^L + Q_1^r + Q_2^r \,\right] + c_2 - 2\, c_1 \,\} \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right]$$

$$+ \frac{1}{27\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^3 - (\underline{\varepsilon})^3 \,\right]$$

$$+ \frac{1}{4\, b^w} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \{\, b^w \left[\, a^L + Q_1^r + Q_2^r - K_2 \,\right] - c_1 \,\}^2$$

$$+ \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \{\, b^w \left[\, a^L + Q_1^r + Q_2^r - K_2 \,\right] - c_1 \,\} \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right]$$

$$+ \frac{1}{12\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^3 - (\varepsilon_0)^3 \,\right]$$

$$+ \; K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - c_1 \right\}$$

$$+ \; \frac{K_1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right] \tag{125}$$

where ε_0 and ε_{12} are as specified in (118).

(116), (117), (119), and (123) also imply that when $h(\varepsilon) = \frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}$:

$$E \left\{ \left[w(\varepsilon) - c_2 \right] q_2^*(\varepsilon) \right\} \; =$$

$$\frac{1}{9} \int_{\underline{\varepsilon}}^{\varepsilon_0} \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_1 - 2\, c_2 + \varepsilon \right\} \left\{ a^L + Q_1^r + Q_2^r + \frac{1}{b^w} \left[c_1 - 2\, c_2 + \varepsilon \right] \right\} dH(\varepsilon)$$

$$+ \; \frac{1}{2} \int_{\varepsilon_0}^{\varepsilon_{12}} \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_2 \right] + c_1 - 2\, c_2 + \varepsilon \right\} K_2 \; dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - c_2 + \varepsilon \right\} \; K_2 \; dH(\varepsilon)$$

$$= \; \frac{1}{9\, b^w} \int_{\underline{\varepsilon}}^{\varepsilon_0} \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_1 - 2\, c_2 + \varepsilon \right\}^2 \; dH(\varepsilon)$$

$$+ \; \frac{K_2}{2} \int_{\varepsilon_0}^{\varepsilon_{12}} \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_2 \right] + c_1 - 2\, c_2 + \varepsilon \right\} dH(\varepsilon)$$

$$+ \; K_2 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - c_2 + \varepsilon \right\} dH(\varepsilon)$$

$$= \; \frac{1}{9\, b^w} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_1 - 2\, c_2 \right\}^2$$

$$+ \; \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_1 - 2\, c_2 \right\} \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]$$

$$+ \; \frac{1}{27\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^3 - (\underline{\varepsilon})^3 \right]$$

$$+ \; \frac{K_2}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_2 \right] + c_1 - 2\, c_2 \right\}$$

$$+ \frac{K_2}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$+ K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - c_2 \right\}$$

$$+ \frac{K_2}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right] \tag{126}$$

where ε_0 and ε_{12} are as specified in (118).

From (118):

$$\frac{d\varepsilon_0}{dK_1} = 0 ; \quad \frac{d\varepsilon_0}{dK_2} = 3\, b^w ; \quad \frac{d\varepsilon_0}{dr_1} = b^w [\, b_1 - d_2 \,] ; \quad \frac{d\varepsilon_0}{dr_2} = b^w [\, b_2 - d_1 \,] ;$$

$$\frac{d\varepsilon_{12}}{dK_1} = 2\, b^w ; \quad \frac{d\varepsilon_{12}}{dK_2} = b^w ; \quad \frac{d\varepsilon_{12}}{dr_1} = b^w [\, b_1 - d_2 \,] ; \quad \frac{d\varepsilon_{12}}{dr_2} = b^w [\, b_2 - d_1 \,] . \tag{127}$$

(127) implies:

$$\frac{d^2\varepsilon_0}{dr_1\, dz} = \frac{d^2\varepsilon_0}{dr_2\, dz} = \frac{d^2\varepsilon_{12}}{dr_1\, dz} = \frac{d^2\varepsilon_{12}}{dr_2\, dz} = 0 \text{ for } z \in \{ K_1, K_2, r_1, r_2 \} . \tag{128}$$

(31), (118), (127), and (128) imply:

$$\frac{\partial E\{w(\varepsilon)\}}{\partial K_1} = \frac{1}{3} \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dK_1}$$

$$+ \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \frac{d\varepsilon_0}{dK_1}$$

$$+ \frac{1}{2} \left[b^w \left(a^L + Q_1^r + Q_2^r - K_2 \right) + c_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1} \right]$$

$$+ \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dK_1} - \varepsilon_0 \frac{d\varepsilon_0}{dK_1} \right]$$

$$- b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dK_1}$$

$$- \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \frac{d\varepsilon_{12}}{dK_1} ;$$

$$\frac{\partial E\{w(\varepsilon)\}}{\partial K_2} = \frac{1}{3} \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dK_2}$$

$$+ \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_2} - \frac{1}{2}\,b^w\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+\ \frac{1}{2}\left[\,b^w\left(a^L + Q_1^r + Q_2^r - K_2\right) + c_1\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2}\right]$$

$$+ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2} - \varepsilon_0\,\frac{d\varepsilon_0}{dK_2}\right]$$

$$-\ b^w\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] - b^w\left[\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dK_2}$$

$$-\ \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2}\,. \qquad (129)$$

(32), (33), (127), and (128) imply:

$$\frac{\partial^2 E\{\,w(\varepsilon)\,\}}{\partial\, r_1 \partial K_1} = \frac{1}{3}\,b^w\,\left[\,d_2 - b_1\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_1}$$

$$+ \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_1}\,\frac{d\varepsilon_0}{dK_1} + \frac{1}{2}\,b^w\,\left[\,d_2 - b_1\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1}\right]$$

$$+ \frac{1}{2\left[\,\overline{\varepsilon}-\underline{\varepsilon}\right]}\left[\frac{d\varepsilon_{12}}{dK_1}\,\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dK_1}\,\frac{d\varepsilon_0}{dr_1}\right] - b^w\,\left[\,d_2 - b_1\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dK_1}$$

$$+ \left[\frac{b^w}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_1} - \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dK_1}\,\frac{d\varepsilon_{12}}{dr_1}\,;$$

$$\frac{\partial^2 E\{\,w(\varepsilon)\,\}}{\partial\, r_1 \partial K_2} = \frac{1}{3}\,b^w\,\left[\,d_2 - b_1\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_2}$$

$$+ \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_1}\,\frac{d\varepsilon_0}{dK_2} + \frac{1}{2}\,b^w\,\left[\,d_2 - b_1\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2}\right]$$

$$-\ \frac{b^w}{2\left[\overline{\varepsilon}-\underline{\varepsilon}\right]}\left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1}\right]$$

$$+ \frac{1}{2\left[\,\overline{\varepsilon}-\underline{\varepsilon}\right]}\left[\frac{d\varepsilon_{12}}{dK_2}\,\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dK_2}\,\frac{d\varepsilon_0}{dr_1}\right] - b^w\,\left[\,d_2 - b_1\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dK_2}$$

$$+ \left[\frac{b^w}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_1} - \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dK_2}\,\frac{d\varepsilon_{12}}{dr_1}\,;$$

$$\frac{\partial^2 E\{\,w(\varepsilon)\,\}}{\partial\, r_1^2} = \frac{1}{3}\,b^w\,\left[\,d_2 - b_1\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_1} + \frac{1}{3\left[\,\overline{\varepsilon}-\underline{\varepsilon}\right]}\,b^w\left[\,d_2 - b_1\,\right]\frac{d\varepsilon_0}{dr_1}$$

$$
\begin{aligned}
& + \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_0}{dr_1}\right]^2 + \frac{1}{2}\,b^w\,[\,d_2-b_1\,]\,\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_1}-\frac{d\varepsilon_0}{dr_1}\right] \\
& + \frac{b^w}{2\,[\,\overline{\varepsilon}-\underline{\varepsilon}\,]}\,[\,d_2-b_1\,]\,\left[\frac{d\varepsilon_{12}}{dr_1}-\frac{d\varepsilon_0}{dr_1}\right] \\
& + \frac{1}{2\,[\,\overline{\varepsilon}-\underline{\varepsilon}\,]}\left[\left(\frac{d\varepsilon_{12}}{dr_1}\right)^2-\left(\frac{d\varepsilon_0}{dr_1}\right)^2\right] - b^w\,[\,d_2-b_1\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_1} \\
& - \left[\frac{b^w}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\,[\,d_2-b_1\,]\,\frac{d\varepsilon_{12}}{dr_1} - \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_1}\right]^2\,;
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 E\,\{\,w(\varepsilon)\,\}}{\partial\,r_1\partial r_2} &= \frac{1}{3}\,b^w\,[\,d_2-b_1\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_2} + \frac{1}{3\,[\,\overline{\varepsilon}-\underline{\varepsilon}\,]}\,b^w\,[\,d_1-b_2\,]\,\frac{d\varepsilon_0}{dr_1} \\
& + \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_1}\,\frac{d\varepsilon_0}{dr_2} + \frac{1}{2}\,b^w\,[\,d_2-b_1\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_2}-\frac{d\varepsilon_0}{dr_2}\right] \\
& + \frac{b^w}{2\,[\,\overline{\varepsilon}-\underline{\varepsilon}\,]}\,[\,d_1-b_2\,]\left[\frac{d\varepsilon_{12}}{dr_1}-\frac{d\varepsilon_0}{dr_1}\right] \\
& + \frac{1}{2\,[\,\overline{\varepsilon}-\underline{\varepsilon}\,]}\left[\frac{d\varepsilon_{12}}{dr_1}\frac{d\varepsilon_{12}}{dr_2}-\frac{d\varepsilon_0}{dr_1}\,\frac{d\varepsilon_0}{dr_2}\right] - b^w\,[\,d_2-b_1\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_2} \\
& - \left[\frac{b^w}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,d_1-b_2\,]\,\frac{d\varepsilon_{12}}{dr_1} - \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_1}\,\frac{d\varepsilon_{12}}{dr_2}\,;
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 E\,\{\,w(\varepsilon)\,\}}{\partial\,r_2\partial K_1} &= \frac{1}{3}\,b^w\,[\,d_1-b_2\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_1} \\
& + \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_2}\,\frac{d\varepsilon_0}{dK_1} + \frac{1}{2}\,b^w\,[\,d_1-b_2\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_1}-\frac{d\varepsilon_0}{dK_1}\right] \\
& + \frac{1}{2\,[\,\overline{\varepsilon}-\underline{\varepsilon}\,]}\left[\frac{d\varepsilon_{12}}{dK_1}\,\frac{d\varepsilon_{12}}{dr_2}-\frac{d\varepsilon_0}{dK_1}\,\frac{d\varepsilon_0}{dr_2}\right] - b^w\,[\,d_1-b_2\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dK_1} \\
& + \left[\frac{b^w}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_2} - \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dK_1}\,\frac{d\varepsilon_{12}}{dr_2}\,;
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 E\,\{\,w(\varepsilon)\,\}}{\partial\,r_2\partial K_2} &= \frac{1}{3}\,b^w\,[\,d_1-b_2\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_2} \\
& + \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_2}\,\frac{d\varepsilon_0}{dK_2} + \frac{1}{2}\,b^w\,[\,d_1-b_2\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_2}-\frac{d\varepsilon_0}{dK_2}\right]
\end{aligned}
$$

$$
\begin{aligned}
& - \frac{b^w}{2\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]} \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right] \\
& + \frac{1}{2\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]} \left[\frac{d\varepsilon_{12}}{dK_2}\,\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dK_2}\,\frac{d\varepsilon_0}{dr_2} \right] - b^w\,[\,d_1 - b_2\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dK_2} \\
& + \left[\frac{b^w}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_2} - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dK_2}\,\frac{d\varepsilon_{12}}{dr_2}\,;
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 E\,\{\,w(\varepsilon)\,\}}{\partial\, r_2^2} = {} & \frac{1}{3}\, b^w\,[\,d_1 - b_2\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_2} + \frac{1}{3\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\, b^w\,[\,d_1 - b_2\,]\,\frac{d\varepsilon_0}{dr_2} \\
& + \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_0}{dr_2} \right]^2 + \frac{1}{2}\, b^w\,[\,d_1 - b_2\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right] \\
& + \frac{b^w}{2\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\,[\,d_1 - b_2\,] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right] \\
& + \frac{1}{2\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]} \left[\left(\frac{d\varepsilon_{12}}{dr_2} \right)^2 - \left(\frac{d\varepsilon_0}{dr_2} \right)^2 \right] - b^w\,[\,d_1 - b_2\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_2} \\
& - \left[\frac{b^w}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\,d_1 - b_2\,]\,\frac{d\varepsilon_{12}}{dr_2} - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dr_2} \right]^2 ;
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 E\,\{\,w(\varepsilon)\,\}}{\partial\, r_2 \partial r_1} = {} & \frac{1}{3}\, b^w\,[\,d_1 - b_2\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_1} + \frac{1}{3\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\, b^w\,[\,d_2 - b_1\,]\,\frac{d\varepsilon_0}{dr_2} \\
& + \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_1}\,\frac{d\varepsilon_0}{dr_2} + \frac{1}{2}\, b^w\,[\,d_1 - b_2\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1} \right] \\
& + \frac{b^w}{2\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\,[\,d_2 - b_1\,] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right] \\
& + \frac{1}{2\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]} \left[\frac{d\varepsilon_{12}}{dr_1}\,\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_1}\,\frac{d\varepsilon_0}{dr_2} \right] - b^w\,[\,d_1 - b_2\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_1} \\
& - \left[\frac{b^w}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\,d_2 - b_1\,]\,\frac{d\varepsilon_{12}}{dr_2} - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_1}\,\frac{d\varepsilon_{12}}{dr_2}\,. \qquad (130)
\end{aligned}
$$

From (120):

$$
\begin{aligned}
\frac{\partial J_1(\cdot)}{\partial r_1} &= -\,b_1 \left[1 - \frac{\partial E\{\,w(\varepsilon)\,\}}{\partial\, r_1} \right] - [\, a_1 - b_1\, r_1 + d_1\, r_2 \,]\; \frac{\partial^2 E\{\,w(\varepsilon)\,\}}{\partial\, r_1^2} - b_1 + b_1\, \frac{\partial E\{\,w(\varepsilon)\,\}}{\partial r_1}; \\
\frac{\partial J_1(\cdot)}{\partial r_2} &= d_1 \left[1 - \frac{\partial E\{\,w(\varepsilon)\,\}}{\partial\, r_1} \right] - [\, a_1 - b_1\, r_1 + d_1\, r_2 \,]\; \frac{\partial^2 E\{\,w(\varepsilon)\,\}}{\partial\, r_1 \partial r_2} + b_1\, \frac{\partial E\{\,w(\varepsilon)\,\}}{\partial r_2}; \\
\frac{\partial J_1(\cdot)}{\partial K_1} &= -\,[\, a_1 - b_1\, r_1 + d_1\, r_2 \,]\; \frac{\partial^2 E\{\,w(\varepsilon)\,\}}{\partial\, r_1 \partial K_1} + b_1\, \frac{\partial E\{\,w(\varepsilon)\,\}}{\partial K_1}; \\
\frac{\partial J_1(\cdot)}{\partial K_2} &= -\,[\, a_1 - b_1\, r_1 + d_1\, r_2 \,]\; \frac{\partial^2 E\{\,w(\varepsilon)\,\}}{\partial\, r_1 \partial K_2} + b_1\, \frac{\partial E\{\,w(\varepsilon)\,\}}{\partial K_2}; \\
\frac{\partial J_2(\cdot)}{\partial r_2} &= -\,b_2 \left[1 - \frac{\partial E\{\,w(\varepsilon)\,\}}{\partial\, r_2} \right] - [\, a_2 - b_2\, r_2 + d_2\, r_1 \,]\; \frac{\partial^2 E\{\,w(\varepsilon)\,\}}{\partial\, r_2^2} - b_2 + b_2\, \frac{\partial E\{\,w(\varepsilon)\,\}}{\partial r_2}; \\
\frac{\partial J_2(\cdot)}{\partial r_1} &= d_2 \left[1 - \frac{\partial E\{\,w(\varepsilon)\,\}}{\partial\, r_2} \right] - [\, a_2 - b_2\, r_2 + d_2\, r_1 \,]\; \frac{\partial^2 E\{\,w(\varepsilon)\,\}}{\partial\, r_2 \partial r_1} + b_2\, \frac{\partial E\{\,w(\varepsilon)\,\}}{\partial r_1}; \\
\frac{\partial J_2(\cdot)}{\partial K_2} &= -\;[\, a_2 - b_2\, r_2 + d_2\, r_1 \,]\; \frac{\partial^2 E\{\,w(\varepsilon)\,\}}{\partial\, r_2 \partial K_2} + b_2\, \frac{\partial E\{\,w(\varepsilon)\,\}}{\partial K_2}; \\
\frac{\partial J_2(\cdot)}{\partial K_1} &= -\;[\, a_2 - b_2\, r_2 + d_2\, r_1 \,]\; \frac{\partial^2 E\{\,w(\varepsilon)\,\}}{\partial\, r_2 \partial K_1} + b_2\, \frac{\partial E\{\,w(\varepsilon)\,\}}{\partial K_1}\,; \qquad (131)
\end{aligned}
$$

where $\frac{\partial E\{\,w(\varepsilon)\,\}}{\partial r_i}$ is defined in (32) and (33) for $i = 1, 2$, $\frac{\partial E\{\,w(\varepsilon)\,\}}{\partial K_i}$ is defined in (129) for $i = 1, 2$, and $\frac{\partial^2 E\{\,w(\varepsilon)\,\}}{\partial\, r_i \partial z}$ is defined in (130) for $i = 1, 2$ and for $z \in \{r_1, r_2, K_1, K_2\}$.

(125), (126), (127), and (128) imply:

$$
\begin{aligned}
\frac{\partial E\{\,[\,w(\varepsilon) - c_1\,]\, q_1^*(\varepsilon)\,\}}{\partial K_1} &= \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_2 - 2\, c_1 \right\}^2 \frac{d\varepsilon_0}{dK_1} \\
&+ \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_2 - 2\, c_1 \right\} \left[2\, \varepsilon_0\, \frac{d\varepsilon_0}{dK_1} \right] \\
&+ \frac{1}{27\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[3\, (\varepsilon_0)^2\, \frac{d\varepsilon_0}{dK_1} \right] \\
&+ \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_2 \right] - c_1 \right\}^2 \left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1} \right] \\
&+ \frac{1}{2\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_2 \right] - c_1 \right\} \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_1} - \,\varepsilon_0\, \frac{d\varepsilon_0}{dK_1} \right] \\
&+ \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2\, \frac{d\varepsilon_{12}}{dK_1} - \,(\varepsilon_0)^2\, \frac{d\varepsilon_0}{dK_1} \right]
\end{aligned}
$$

$$+ \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - c_1 \right\}$$

$$- b^w K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - c_1 \right\} \frac{d\varepsilon_{12}}{dK_1}$$

$$+ \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right] - K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \frac{d\varepsilon_{12}}{dK_1} ;$$

$$\frac{\partial E \{ [w(\varepsilon) - c_1] q_1^*(\varepsilon) \}}{\partial K_2} = \frac{1}{9 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_2 - 2 c_1 \right\}^2 \frac{d\varepsilon_0}{dK_2}$$

$$+ \frac{1}{9 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_2 - 2 c_1 \right\} \left[2 \varepsilon_0 \frac{d\varepsilon_0}{dK_2} \right]$$

$$+ \frac{1}{27 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[3 (\varepsilon_0)^2 \frac{d\varepsilon_0}{dK_2} \right]$$

$$+ \frac{1}{4 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_2 \right] - c_1 \right\}^2 \left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2} \right]$$

$$- \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_2 \right] - c_1 \right\}$$

$$+ \frac{1}{2 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_2 \right] - c_1 \right\} \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dK_2} - \varepsilon_0 \frac{d\varepsilon_0}{dK_2} \right]$$

$$- \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$+ \frac{1}{4 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 \frac{d\varepsilon_{12}}{dK_2} - (\varepsilon_0)^2 \frac{d\varepsilon_0}{dK_2} \right]$$

$$- b^w K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - c_1 \right\} \frac{d\varepsilon_{12}}{dK_2}$$

$$- K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \frac{d\varepsilon_{12}}{dK_2} ;$$

$$\frac{\partial E \{ [w(\varepsilon) - c_1] q_1^*(\varepsilon) \}}{\partial r_1} = \frac{1}{9 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_2 - 2 c_1 \right\}^2 \frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{2}{9} \left[d_2 - b_1 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_2 - 2 c_1 \right\}$$

$$+ \frac{1}{9\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_2 - 2\,c_1 \right\} \left[2\,\varepsilon_0 \, \frac{d\varepsilon_0}{dr_1} \right]$$

$$+ \frac{1}{9} \left[d_2 - b_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]$$

$$+ \frac{1}{9\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2 \, \frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{4\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_2 \right] - c_1 \right\}^2 \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1} \right]$$

$$+ \frac{1}{2} \left[d_2 - b_1 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_2 \right] - c_1 \right\}$$

$$+ \frac{1}{2\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_2 \right] - c_1 \right\} \left[\varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0 \frac{d\varepsilon_0}{dr_1} \right]$$

$$+ \frac{1}{4} \left[d_2 - b_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$+ \frac{1}{4\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 \, \frac{d\varepsilon_{12}}{dr_1} - (\varepsilon_0)^2 \, \frac{d\varepsilon_0}{dr_1} \right]$$

$$- \, K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - c_1 \right\} \frac{d\varepsilon_{12}}{dr_1}$$

$$+ \, b^w \left[d_2 - b_1 \right] K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_1} \, ;$$

$$\frac{\partial E \left\{ \left[w(\varepsilon) - c_1 \right] q_1^*(\varepsilon) \right\}}{\partial r_2} \; = \; \frac{1}{9\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_2 - 2\,c_1 \right\}^2 \frac{d\varepsilon_0}{dr_2}$$

$$+ \frac{2}{9} \left[d_1 - b_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_2 - 2\,c_1 \right\}$$

$$+ \frac{1}{9\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_2 - 2\,c_1 \right\} \left[2\,\varepsilon_0 \, \frac{d\varepsilon_0}{dr_2} \right]$$

$$+ \frac{1}{9} \left[d_1 - b_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]$$

$$+ \frac{1}{9\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2 \, \frac{d\varepsilon_0}{dr_2}$$

$$+ \frac{1}{4\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_2 \right] - c_1 \right\}^2 \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right]$$

$$+ \frac{1}{2}\,[\,d_1 - b_2\,]\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L + Q_1^r + Q_2^r - K_2\,\right] - c_1\,\right\}$$

$$+ \frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L + Q_1^r + Q_2^r - K_2\,\right] - c_1\,\right\}\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0\frac{d\varepsilon_0}{dr_2}\right]$$

$$+ \frac{1}{4}\,[\,d_1 - b_2\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2 - (\varepsilon_0)^2\,\right]$$

$$+ \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[(\varepsilon_{12})^2\,\frac{d\varepsilon_{12}}{dr_2} - (\varepsilon_0)^2\,\frac{d\varepsilon_0}{dr_2}\right]$$

$$- K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right] - c_1\,\right\}\frac{d\varepsilon_{12}}{dr_2}$$

$$+ b^w\,[\,d_1 - b_2\,]\,K_1\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] - K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_2}\,;$$

$$\frac{\partial E\left\{\,[\,w(\varepsilon) - c_2\,]\,q_2^*(\varepsilon)\,\right\}}{\partial K_1} = \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L + Q_1^r + Q_2^r\,\right] + c_1 - 2\,c_2\,\right\}^2\,\frac{d\varepsilon_0}{dK_1}$$

$$+ \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L + Q_1^r + Q_2^r\,\right] + c_1 - 2\,c_2\,\right\}\left[2\,\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right]$$

$$+ \frac{1}{27\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[3\,(\varepsilon_0)^2\,\frac{d\varepsilon_0}{dK_1}\right]$$

$$+ \frac{K_2}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L + Q_1^r + Q_2^r - K_2\,\right] + c_1 - 2\,c_2\,\right\}\left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1}\right]$$

$$+ \frac{K_2}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1} - \varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right]$$

$$- K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right] - c_2\,\right\}\frac{d\varepsilon_{12}}{dK_1}$$

$$- b^w\,K_2\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] - K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}\,;$$

$$\frac{\partial E\left\{\,[\,w(\varepsilon) - c_2\,]\,q_2^*(\varepsilon)\,\right\}}{\partial K_2} = \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L + Q_1^r + Q_2^r\,\right] + c_1 - 2\,c_2\,\right\}^2\,\frac{d\varepsilon_0}{dK_2}$$

$$+ \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L + Q_1^r + Q_2^r\,\right] + c_1 - 2\,c_2\,\right\}\left[2\,\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}\right]$$

$$+ \frac{1}{27\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[3\,(\varepsilon_0)^2\,\frac{d\varepsilon_0}{dK_2}\right]$$

$$
\begin{aligned}
&+ \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_2 \right] + c_1 - 2\, c_2 \right\} - \; b^w \, \frac{K_2}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ \frac{K_2}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_2 \right] + c_1 - 2\, c_2 \right\} \left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2} \right] \\
&+ \frac{K_2}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \, \frac{d\varepsilon_{12}}{dK_2} - \; \varepsilon_0 \, \frac{d\varepsilon_0}{dK_2} \right] + \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\left(\varepsilon_{12} \right)^2 - \left(\varepsilon_0 \right)^2 \right] \\
&+ \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - c_2 \right\} \\
&- \; K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - c_2 \right\} \frac{d\varepsilon_{12}}{dK_2} \\
&- \; b^w \, K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dK_2} + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\left(\overline{\varepsilon} \right)^2 - \left(\varepsilon_{12} \right)^2 \right];
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E \left\{ \left[w(\varepsilon) - c_2 \right] q_2^*(\varepsilon) \right\}}{\partial r_1} \;=\; & \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_1 - 2\, c_2 \right\}^2 \frac{d\varepsilon_0}{dr_1} \\
&+ \frac{2}{9} \left[d_2 - b_1 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_1 - 2\, c_2 \right\} \\
&+ \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_1 - 2\, c_2 \right\} \left[2\, \varepsilon_0 \, \frac{d\varepsilon_0}{dr_1} \right] \\
&+ \frac{1}{9} \left[d_2 - b_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\left(\varepsilon_0 \right)^2 - \left(\underline{\varepsilon} \right)^2 \right] \\
&+ \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left(\varepsilon_0 \right)^2 \frac{d\varepsilon_0}{dr_1} \\
&+ \frac{K_2}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_2 \right] + c_1 - 2\, c_2 \right\} \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1} \right] \\
&+ \frac{K_2}{2} \, b^w \left[d_2 - b_1 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{K_2}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0 \, \frac{d\varepsilon_0}{dr_1} \right] \\
&- \; K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - c_2 \right\} \frac{d\varepsilon_{12}}{dr_1} \\
&+ \; K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[d_2 - b_1 \right] - K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_1};
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E\left\{\,[\,w(\varepsilon)-c_2\,]\,q_2^*(\varepsilon)\,\right\}}{\partial r_2} \;=\; & \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r\right)+c_1-2\,c_2\,\right]^2\frac{d\varepsilon_0}{dr_2} \\
& + \frac{2}{9}\,[\,d_1-b_2\,]\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r\right)+c_1-2\,c_2\,\right] \\
& + \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r\,\right]+c_1-2\,c_2\,\right\}\left[2\,\varepsilon_0\,\frac{d\varepsilon_0}{dr_2}\right] \\
& + \frac{1}{9}\,[\,d_1-b_2\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right] \\
& + \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right](\varepsilon_0)^2\,\frac{d\varepsilon_0}{dr_2} \\
& + \frac{K_2}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r-K_2\,\right]+c_1-2\,c_2\,\right\}\left[\frac{d\varepsilon_{12}}{dr_2}-\frac{d\varepsilon_0}{dr_2}\right] \\
& + \frac{K_2}{2}\,b^w\,[\,d_1-b_2\,]\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]+\frac{K_2}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_2}-\varepsilon_0\,\frac{d\varepsilon_0}{dr_2}\right] \\
& - K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]-c_2\,\right\}\frac{d\varepsilon_{12}}{dr_2} \\
& + K_2\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]b^w\,[\,d_1-b_2\,]-K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_2}\,. \qquad (132)
\end{aligned}
$$

Case 1B. Vertical separation where G1 is capacity-constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$.

(116), (117), (122), and (123) imply that when $h(\varepsilon) = \frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}$:

$$E\{[w(\varepsilon) - c_1] q_1^*(\varepsilon)\} =$$

$$\frac{1}{9} \int_{\underline{\varepsilon}}^{\varepsilon_0} \{ b^w [a^L + Q_1^r + Q_2^r] + c_2 - 2c_1 + \varepsilon \} \left\{ a^L + Q_1^r + Q_2^r + \frac{1}{b^w} [c_2 - 2c_1 + \varepsilon] \right\} dH(\varepsilon)$$

$$+ \frac{1}{2} \int_{\varepsilon_0}^{\varepsilon_{12}} \{ b^w [a^L + Q_1^r + Q_2^r - K_1] + c_2 - 2c_1 + \varepsilon \} K_1 \, dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \{ b^w [a^L + Q_1^r + Q_2^r - K_1 - K_2] - c_1 + \varepsilon \} K_1 \, dH(\varepsilon)$$

$$= \frac{1}{9 b^w} \int_{\underline{\varepsilon}}^{\varepsilon_0} \{ b^w [a^L + Q_1^r + Q_2^r] + c_2 - 2c_1 + \varepsilon \}^2 \, dH(\varepsilon)$$

$$+ \frac{K_1}{2} \int_{\varepsilon_0}^{\varepsilon_{12}} \{ b^w [a^L + Q_1^r + Q_2^r - K_1] + c_2 - 2c_1 + \varepsilon \} \, dH(\varepsilon)$$

$$+ K_1 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \{ b^w [a^L + Q_1^r + Q_2^r - K_1 - K_2] - c_1 + \varepsilon \} \, dH(\varepsilon)$$

$$= \frac{1}{9 b^w} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \{ b^w [a^L + Q_1^r + Q_2^r] + c_2 - 2c_1 \}^2$$

$$+ \frac{1}{9 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \{ b^w [a^L + Q_1^r + Q_2^r] + c_2 - 2c_1 \} [(\varepsilon_0)^2 - (\underline{\varepsilon})^2]$$

$$+ \frac{1}{27 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [(\varepsilon_0)^3 - (\underline{\varepsilon})^3]$$

$$+ \frac{K_1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \{ b^w [a^L + Q_1^r + Q_2^r - K_1] + c_2 - 2c_1 \}$$

$$+ \frac{K_1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [(\varepsilon_{12})^2 - (\varepsilon_0)^2]$$

$$+ K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \{ b^w [a^L + Q_1^r + Q_2^r - K_1 - K_2] - c_1 \}$$

$$+ \frac{K_1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right] \qquad (133)$$

where ε_0 and ε_{12} are as specified in (121).

(116), (117), (122), and (123) also imply that when $h(\varepsilon) = \frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}$:

$$E\{ [w(\varepsilon) - c_2] q_2^*(\varepsilon) \} \ =$$

$$\frac{1}{9} \int_{\underline{\varepsilon}}^{\varepsilon_0} \{ b^w [a^L + Q_1^r + Q_2^r] + c_1 - 2 c_2 + \varepsilon \} \left\{ a^L + Q_1^r + Q_2^r + \frac{1}{b^w} [c_1 - 2 c_2 + \varepsilon] \right\} dH(\varepsilon)$$

$$+ \frac{1}{4} \int_{\varepsilon_0}^{\varepsilon_{12}} \{ b^w [a^L + Q_1^r + Q_2^r - K_1] - c_2 + \varepsilon \} \left\{ a^L + Q_1^r + Q_2^r - K_1 + \frac{1}{b^w} [\varepsilon - c_2] \right\} dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \{ b^w [a^L + Q_1^r + Q_2^r - K_1 - K_2] - c_2 + \varepsilon \} \ K_2 \ dH(\varepsilon)$$

$$= \frac{1}{9 b^w} \int_{\underline{\varepsilon}}^{\varepsilon_0} \{ b^w [a^L + Q_1^r + Q_2^r] + c_1 - 2 c_2 + \varepsilon \}^2 \ dH(\varepsilon)$$

$$+ \frac{1}{4 b^w} \int_{\varepsilon_0}^{\varepsilon_{12}} \{ b^w [a^L + Q_1^r + Q_2^r - K_1] - c_2 + \varepsilon \}^2 \ dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \{ b^w [a^L + Q_1^r + Q_2^r - K_1 - K_2] - c_2 + \varepsilon \} \ K_2 \ dH(\varepsilon)$$

$$= \frac{1}{9 b^w} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \{ b^w [a^L + Q_1^r + Q_2^r] + c_1 - 2 c_2 \}^2$$

$$+ \frac{1}{9 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \{ b^w [a^L + Q_1^r + Q_2^r] + c_1 - 2 c_2 \} \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]$$

$$+ \frac{1}{27 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^3 - (\underline{\varepsilon})^3 \right]$$

$$+ \frac{1}{4 b^w} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \{ b^w [a^L + Q_1^r + Q_2^r - K_1] - c_2 \}^2$$

$$+ \frac{1}{4 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \{ b^w [a^L + Q_1^r + Q_2^r - K_1] - c_2 \} \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$+ \frac{1}{12\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^3-(\varepsilon_0)^3\,\right]$$

$$+ K_2\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]-c_2\,\right\}$$

$$+ \frac{K_2}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2-(\varepsilon_{12})^2\,\right] \tag{134}$$

where ε_0 and ε_{12} are as specified in (121).

From (121):

$$\frac{d\varepsilon_0}{dK_1} = 3\,b^w\,; \quad \frac{d\varepsilon_0}{dK_2} = 0\,; \quad \frac{d\varepsilon_0}{dr_1} = b^w\,[\,b_1-d_2\,]\,; \quad \frac{d\varepsilon_0}{dr_2} = b^w\,[\,b_2-d_1\,]\,;$$

$$\frac{d\varepsilon_{12}}{dK_1} = b^w\,; \quad \frac{d\varepsilon_{12}}{dK_2} = 2\,b^w\,; \quad \frac{d\varepsilon_{12}}{dr_1} = b^w\,[\,b_1-d_2\,]\,; \quad \frac{d\varepsilon_{12}}{dr_2} = b^w\,[\,b_2-d_1\,]\,. \tag{135}$$

(135) implies:

$$\frac{d^2\varepsilon_0}{dr_1\,dz} = \frac{d^2\varepsilon_0}{dr_2\,dz} = \frac{d^2\varepsilon_{12}}{dr_1\,dz} = \frac{d^2\varepsilon_{12}}{dr_2\,dz} = 0 \;\text{ for }\; z \in \{\,K_1, K_2, r_1, r_2\,\}\,. \tag{136}$$

(35), (121), (135), and (136) imply:

$$\frac{\partial E\{w(\varepsilon)\}}{\partial K_1} = \frac{1}{3}\left[\,b^w\left(\,a^L+Q_1^r+Q_2^r\,\right)+c_1+c_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_1}$$

$$+ \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1} - \frac{1}{2}\,b^w\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+ \frac{1}{2}\left[\,b^w\left(\,a^L+Q_1^r+Q_2^r-K_1\,\right)+c_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_1}-\frac{d\varepsilon_0}{dK_1}\right]$$

$$+ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right]$$

$$- b^w\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] - b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dK_1}$$

$$- \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}\,;$$

$$\frac{\partial E\{w(\varepsilon)\}}{\partial K_2} = \frac{1}{3}\left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dK_2}$$

$$+ \frac{1}{3}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \frac{d\varepsilon_0}{dK_2}$$

$$+ \frac{1}{2}\left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2} \right]$$

$$+ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dK_2} - \varepsilon_0 \frac{d\varepsilon_0}{dK_2} \right]$$

$$- b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dK_2}$$

$$- \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \frac{d\varepsilon_{12}}{dK_2} . \tag{137}$$

(36), (37), (135), and (136) imply:

$$\frac{\partial^2 E\{w(\varepsilon)\}}{\partial r_1 \partial K_1} = \frac{1}{3} b^w \left[d_2 - b_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dK_1} + \frac{1}{3}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_1} \frac{d\varepsilon_0}{dK_1}$$

$$+ \frac{1}{2} b^w \left[d_2 - b_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1} \right] - \frac{b^w}{2\left[\overline{\varepsilon} - \underline{\varepsilon}\right]} \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1} \right]$$

$$+ \frac{1}{2\left[\overline{\varepsilon} - \underline{\varepsilon}\right]} \left[\frac{d\varepsilon_{12}}{dK_1} \frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dK_1} \frac{d\varepsilon_0}{dr_1} \right] - b^w \left[d_2 - b_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dK_1}$$

$$+ \left[\frac{b^w}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_1} - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dK_1} \frac{d\varepsilon_{12}}{dr_1} ;$$

$$\frac{\partial^2 E\{w(\varepsilon)\}}{\partial r_1 \partial K_2} = \frac{1}{3} b^w \left[d_2 - b_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dK_2}$$

$$+ \frac{1}{3}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_1} \frac{d\varepsilon_0}{dK_2} + \frac{1}{2} b^w \left[d_2 - b_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2} \right]$$

$$+ \frac{1}{2\left[\overline{\varepsilon} - \underline{\varepsilon}\right]} \left[\frac{d\varepsilon_{12}}{dK_2} \frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dK_2} \frac{d\varepsilon_0}{dr_1} \right] - b^w \left[d_2 - b_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dK_2}$$

$$+ \left[\frac{b^w}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_1} - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dK_2} \frac{d\varepsilon_{12}}{dr_1} ;$$

$$\frac{\partial^2 E\{w(\varepsilon)\}}{\partial r_1^2} = \frac{1}{3}\, b^w\, [\, d_2 - b_1\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right] \frac{d\varepsilon_0}{dr_1} + \frac{1}{3\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\, b^w\, [\, d_2 - b_1\,]\, \frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{3}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\frac{d\varepsilon_0}{dr_1}\right]^2 + \frac{1}{2}\, b^w\, [\, d_2 - b_1\,]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1}\right]$$

$$+ \frac{b^w}{2\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\,[\, d_2 - b_1\,]\left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1}\right]$$

$$+ \frac{1}{2\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\left[\left(\frac{d\varepsilon_{12}}{dr_1}\right)^2 - \left(\frac{d\varepsilon_0}{dr_1}\right)^2\right] - b^w\, [\, d_2 - b_1\,]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_1}$$

$$- \left[\frac{b^w}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\, d_2 - b_1\,]\,\frac{d\varepsilon_{12}}{dr_1} - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_1}\right]^2 ;$$

$$\frac{\partial^2 E\{w(\varepsilon)\}}{\partial r_1 \partial r_2} = \frac{1}{3}\, b^w\, [\, d_2 - b_1\,]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_2} + \frac{1}{3\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\, b^w\, [\, d_1 - b_2\,]\,\frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{3}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_1}\,\frac{d\varepsilon_0}{dr_2} + \frac{1}{2}\, b^w\, [\, d_2 - b_1\,]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2}\right]$$

$$+ \frac{b^w}{2\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\,[\, d_1 - b_2\,]\left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1}\right]$$

$$+ \frac{1}{2\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\left[\frac{d\varepsilon_{12}}{dr_1}\,\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_1}\,\frac{d\varepsilon_0}{dr_2}\right] - b^w\, [\, d_2 - b_1\,]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_2}$$

$$- \left[\frac{b^w}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\, d_1 - b_2\,]\,\frac{d\varepsilon_{12}}{dr_1} - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_1}\,\frac{d\varepsilon_{12}}{dr_2} ;$$

$$\frac{\partial^2 E\{w(\varepsilon)\}}{\partial r_2 \partial K_1} = \frac{1}{3}\, b^w\, [\, d_1 - b_2\,]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_1} + \frac{1}{3}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_2}\,\frac{d\varepsilon_0}{dK_1}$$

$$+ \frac{1}{2}\, b^w\, [\, d_1 - b_2\,]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1}\right] - \frac{b^w}{2\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2}\right]$$

$$+ \frac{1}{2\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\left[\frac{d\varepsilon_{12}}{dK_1}\,\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dK_1}\,\frac{d\varepsilon_0}{dr_2}\right] - b^w\, [\, d_1 - b_2\,]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dK_1}$$

$$+ \left[\frac{b^w}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_2} - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dK_1}\,\frac{d\varepsilon_{12}}{dr_2} ;$$

$$
\begin{aligned}
\frac{\partial^2 E\{w(\varepsilon)\}}{\partial\, r_2 \partial K_2} \;=\;& \frac{1}{3}\, b^w\, [\, d_1 - b_2\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dK_2} \\
&+ \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_2} \frac{d\varepsilon_0}{dK_2} + \frac{1}{2}\, b^w\, [\, d_1 - b_2\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2} \right] \\
&+ \frac{1}{2\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]} \left[\frac{d\varepsilon_{12}}{dK_2} \frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dK_2} \frac{d\varepsilon_0}{dr_2} \right] - b^w\, [\, d_1 - b_2\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dK_2} \\
&+ \left[\frac{b^w}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_2} - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dK_2} \frac{d\varepsilon_{12}}{dr_2}\,;
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 E\{w(\varepsilon)\}}{\partial\, r_2^2} \;=\;& \frac{1}{3}\, b^w\, [\, d_1 - b_2\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_2} + \frac{1}{3\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\, b^w\, [\, d_1 - b_2\,] \frac{d\varepsilon_0}{dr_2} \\
&+ \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_0}{dr_2} \right]^2 + \frac{1}{2}\, b^w\, [\, d_1 - b_2\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right] \\
&+ \frac{b^w}{2\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]} [\, d_1 - b_2\,] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right] \\
&+ \frac{1}{2\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]} \left[\left(\frac{d\varepsilon_{12}}{dr_2} \right)^2 - \left(\frac{d\varepsilon_0}{dr_2} \right)^2 \right] - b^w\, [\, d_1 - b_2\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_2} \\
&- \left[\frac{b^w}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, d_1 - b_2\,] \frac{d\varepsilon_{12}}{dr_2} - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dr_2} \right]^2 ;
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 E\{w(\varepsilon)\}}{\partial\, r_2 \partial r_1} \;=\;& \frac{1}{3}\, b^w\, [\, d_1 - b_2\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_1} + \frac{1}{3\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]}\, b^w\, [\, d_2 - b_1\,] \frac{d\varepsilon_0}{dr_2} \\
&+ \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_1} \frac{d\varepsilon_0}{dr_2} + \frac{1}{2}\, b^w\, [\, d_1 - b_2\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1} \right] \\
&+ \frac{b^w}{2\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]} [\, d_2 - b_1\,] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right] \\
&+ \frac{1}{2\,[\,\overline{\varepsilon} - \underline{\varepsilon}\,]} \left[\frac{d\varepsilon_{12}}{dr_1} \frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_1} \frac{d\varepsilon_0}{dr_2} \right] - b^w\, [\, d_1 - b_2\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_1} \\
&- \left[\frac{b^w}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, d_2 - b_1\,] \frac{d\varepsilon_{12}}{dr_2} - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_1} \frac{d\varepsilon_{12}}{dr_2}\,. \qquad (138)
\end{aligned}
$$

The expressions for $\frac{\partial J_i(\cdot)}{\partial z}$ for $i \in \{1, 2\}$ and $z \in \{r_1, r_2, K_1, K_2\}$ are as specified in (131), where $\frac{\partial E\{w(\varepsilon)\}}{\partial r_i}$ is defined in (36) and (37) , $\frac{\partial E\{w(\varepsilon)\}}{\partial K_i}$ is defined in (137), and $\frac{\partial^2 E\{w(\varepsilon)\}}{\partial r_i \partial z}$ is defined in (138).

(133), (134), (135), and (136) imply:

$$
\begin{aligned}
\frac{\partial E\{\,[\,w(\varepsilon) - c_1\,]\,q_1^*(\varepsilon)\,\}}{\partial K_1} \;=\; & \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L + Q_1^r + Q_2^r\,\right] + c_2 - 2\,c_1\,\right\}^2 \frac{d\varepsilon_0}{dK_1} \\
& + \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L + Q_1^r + Q_2^r\,\right] + c_2 - 2\,c_1\,\right\}\left[2\,\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right] \\
& + \frac{1}{27\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[3\,(\varepsilon_0)^2\,\frac{d\varepsilon_0}{dK_1}\right] - \frac{1}{2}\,K_1\,b^w\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right] \\
& + \frac{1}{2}\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L + Q_1^r + Q_2^r - K_1\,\right] + c_2 - 2\,c_1\,\right\} \\
& + \frac{K_1}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L + Q_1^r + Q_2^r - K_1\,\right] + c_2 - 2\,c_1\,\right\}\left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1}\right] \\
& + \frac{1}{4}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2 - (\varepsilon_0)^2\,\right] \\
& + \frac{K_1}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1} - \varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right] \\
& + \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right] - c_1\,\right\} \\
& - b^w\,K_1\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] \\
& - K_1\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right] - c_1\,\right\}\frac{d\varepsilon_{12}}{dK_1} \\
& + \frac{1}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2 - (\varepsilon_{12})^2\,\right] - K_1\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}\,;
\end{aligned}
$$

$$\frac{\partial E\left\{\,[\,w(\varepsilon)-c_1\,]\,q_1^*(\varepsilon)\,\right\}}{\partial K_2} \;=\; \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r\,\right]+c_2-2\,c_1\,\right\}^2\,\frac{d\varepsilon_0}{dK_2}$$

$$+\;\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r\,\right]+c_2-2\,c_1\,\right\}\left[2\,\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}\right]$$

$$+\;\frac{1}{27\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[3\,(\varepsilon_0)^2\,\frac{d\varepsilon_0}{dK_2}\right]$$

$$+\;\frac{K_1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r-K_1\,\right]+c_2-2\,c_1\,\right\}\left[\frac{d\varepsilon_{12}}{dK_2}-\frac{d\varepsilon_0}{dK_2}\right]$$

$$+\;\frac{K_1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2}-\,\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}\right]$$

$$-\;b^w\;K_1\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]-\;K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]-c_1\,\right\}\frac{d\varepsilon_{12}}{dK_2}$$

$$-\;K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2}\,;$$

$$\frac{\partial E\left\{\,[\,w(\varepsilon)-c_1\,]\,q_1^*(\varepsilon)\,\right\}}{\partial r_1} \;=\; \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r\,\right]+c_2-2\,c_1\,\right\}^2\,\frac{d\varepsilon_0}{dr_1}$$

$$+\;\frac{2}{9}\,[\,d_2-b_1\,]\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r\,\right]+c_2-2\,c_1\,\right\}$$

$$+\;\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r\,\right]+c_2-2\,c_1\,\right\}\left[2\,\varepsilon_0\,\frac{d\varepsilon_0}{dr_1}\right]$$

$$+\;\frac{1}{9}\,[\,d_2-b_1\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right]$$

$$+\;\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\,(\varepsilon_0)^2\,\frac{d\varepsilon_0}{dr_1}$$

$$+\;\frac{K_1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r-K_1\,\right]+c_2-2\,c_1\,\right\}\left[\frac{d\varepsilon_{12}}{dr_1}-\frac{d\varepsilon_0}{dr_1}\right]$$

$$+\;\frac{1}{2}\,b^w\,K_1\,[\,d_2-b_1\,]\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+\;\frac{K_1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_1}-\varepsilon_0\,\frac{d\varepsilon_0}{dr_1}\right]$$

$$-\;K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]-c_1\,\right\}\frac{d\varepsilon_{12}}{dr_1}$$

$$+\, b^w\,[\,d_2-b_1\,]\;K_1\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]-K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_1}\,;$$

$$\frac{\partial E\,\{\,[\,w(\varepsilon)-c_1\,]\,q_1^*(\varepsilon)\,\}}{\partial r_2} \;=\; \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r\,\right]+c_2-2\,c_1\,\right\}^2\,\frac{d\varepsilon_0}{dr_2}$$

$$+\,\frac{2}{9}\,[\,d_1-b_2\,]\,\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r\,\right]+c_2-2\,c_1\,\right\}$$

$$+\,\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r\,\right]+c_2-2\,c_1\,\right\}\left[\,2\,\varepsilon_0\,\frac{d\varepsilon_0}{dr_2}\right]$$

$$+\,\frac{1}{9}\,[\,d_1-b_2\,]\,\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right]$$

$$+\,\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right](\varepsilon_0)^2\,\frac{d\varepsilon_0}{dr_2}$$

$$+\,\frac{K_1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r-K_1\,\right]+c_2-2\,c_1\,\right\}\left[\frac{d\varepsilon_{12}}{dr_2}-\frac{d\varepsilon_0}{dr_2}\right]$$

$$+\,\frac{K_1}{2}\,b^w\,[\,d_1-b_2\,]\,\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]+\frac{K_1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_2}-\varepsilon_0\frac{d\varepsilon_0}{dr_2}\right]$$

$$-\,K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]-c_1\,\right\}\frac{d\varepsilon_{12}}{dr_2}$$

$$+\,b^w\,K_1\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,d_1-b_2\,]-\,K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_2}\,;$$

$$\frac{\partial E\left\{\left[w(\varepsilon)-c_2\right]q_2^*(\varepsilon)\right\}}{\partial K_1} = \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{b^w\left[a^L+Q_1^r+Q_2^r\right]+c_1-2\,c_2\right\}^2\frac{d\varepsilon_0}{dK_1}$$

$$+\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{b^w\left[a^L+Q_1^r+Q_2^r\right]+c_1-2\,c_2\right\}\left[2\,\varepsilon_0\frac{d\varepsilon_0}{dK_1}\right]$$

$$+\frac{1}{27\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[3\,(\varepsilon_0)^2\frac{d\varepsilon_0}{dK_1}\right]$$

$$-\frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{b^w\left[a^L+Q_1^r+Q_2^r-K_1\right]-c_2\right\}$$

$$+\frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{b^w\left[a^L+Q_1^r+Q_2^r-K_1\right]-c_2\right\}^2\left[\frac{d\varepsilon_{12}}{dK_1}-\frac{d\varepsilon_0}{dK_1}\right]$$

$$-\frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[(\varepsilon_{12})^2-(\varepsilon_0)^2\right]$$

$$+\frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{b^w\left[a^L+Q_1^r+Q_2^r-K_1\right]-c_2\right\}\left[\varepsilon_{12}\frac{d\varepsilon_{12}}{dK_1}-\varepsilon_0\frac{d\varepsilon_0}{dK_1}\right]$$

$$+\frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[(\varepsilon_{12})^2\frac{d\varepsilon_{12}}{dK_1}-(\varepsilon_0)^2\frac{d\varepsilon_0}{dK_1}\right]-\;b^w\,K_2\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$-\;K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{b^w\left[a^L+Q_1^r+Q_2^r-K_1-K_2\right]-\;c_2\right\}\frac{d\varepsilon_{12}}{dK_1}$$

$$-\;K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\frac{d\varepsilon_{12}}{dK_1}\,;$$

$$\frac{\partial E\left\{\left[w(\varepsilon)-c_2\right]q_2^*(\varepsilon)\right\}}{\partial K_2} = \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{b^w\left[a^L+Q_1^r+Q_2^r\right]+c_1-2\,c_2\right\}^2\frac{d\varepsilon_0}{dK_2}$$

$$+\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{b^w\left[a^L+Q_1^r+Q_2^r\right]+c_1-2\,c_2\right\}\left[2\,\varepsilon_0\frac{d\varepsilon_0}{dK_2}\right]$$

$$+\frac{1}{27\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[3\,(\varepsilon_0)^2\frac{d\varepsilon_0}{dK_2}\right]$$

$$+\frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{b^w\left[a^L+Q_1^r+Q_2^r-K_1\right]-c_2\right\}^2\left[\frac{d\varepsilon_{12}}{dK_2}-\frac{d\varepsilon_0}{dK_2}\right]$$

$$+\frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{b^w\left[a^L+Q_1^r+Q_2^r-K_1\right]-c_2\right\}\left[\varepsilon_{12}\frac{d\varepsilon_{12}}{dK_2}-\,\varepsilon_0\frac{d\varepsilon_0}{dK_2}\right]$$

$$+\ \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[(\varepsilon_{12})^2\ \frac{d\varepsilon_{12}}{dK_2}-\ (\varepsilon_0)^2\ \frac{d\varepsilon_0}{dK_2}\right]$$

$$+\ \left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]-c_2\,\right\}$$

$$-\ b^w\,K_2\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]-\ K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]-c_2\,\right\}\frac{d\varepsilon_{12}}{dK_2}$$

$$-\ K_2\ \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2}+\frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2-(\varepsilon_{12})^2\,\right];$$

$$\frac{\partial E\,\{\,[\,w(\varepsilon)-c_2\,]\,q_2^*(\varepsilon)\,\}}{\partial r_1}\ =\ \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r\,\right]+c_1-2\,c_2\,\right\}^2\ \frac{d\varepsilon_0}{dr_1}$$

$$+\ \frac{2}{9}\,[\,d_2-b_1\,]\ \left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r\,\right]+c_1-2\,c_2\,\right\}$$

$$+\ \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r\,\right]+c_1-2\,c_2\,\right\}\left[2\,\varepsilon_0\,\frac{d\varepsilon_0}{dr_1}\right]$$

$$+\ \frac{1}{9}\ [\,d_2-b_1\,]\ \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right]$$

$$+\ \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right](\varepsilon_0)^2\ \frac{d\varepsilon_0}{dr_1}$$

$$+\ \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\ \left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r-K_1\,\right]-c_2\,\right\}^2\left[\frac{d\varepsilon_{12}}{dr_1}-\frac{d\varepsilon_0}{dr_1}\right]$$

$$+\ \frac{1}{2}\,[\,d_2-b_1\,]\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\ \left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r-K_1\,\right]-c_2\,\right\}$$

$$+\ \frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\ \left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r-K_1\,\right]-c_2\,\right\}\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_1}-\varepsilon_0\,\frac{d\varepsilon_0}{dr_1}\right]$$

$$+\ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,d_2-b_1\,]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right]$$

$$+\ \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[(\varepsilon_{12})^2\ \frac{d\varepsilon_{12}}{dr_1}-(\varepsilon_0)^2\ \frac{d\varepsilon_0}{dr_1}\right]$$

$$+\ K_2\,b^w\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,d_2-b_1\,]$$

$$- \; K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - c_2 \right\} \frac{d\varepsilon_{12}}{dr_1}$$

$$- \; K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_1} \, ;$$

$$\frac{\partial E \left\{ \left[w(\varepsilon) - c_2 \right] q_2^*(\varepsilon) \right\}}{\partial r_2} \; = \; \frac{1}{9 \, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_1 - 2 \, c_2 \right\}^2 \frac{d\varepsilon_0}{dr_2}$$

$$+ \; \frac{2}{9} \left[d_1 - b_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_1 - 2 \, c_2 \right\}$$

$$+ \; \frac{1}{9 \, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r \right] + c_1 - 2 \, c_2 \right\} \left[2 \, \varepsilon_0 \, \frac{d\varepsilon_0}{dr_2} \right]$$

$$+ \; \frac{1}{9} \left[d_1 - b_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\left(\varepsilon_0 \right)^2 - \left(\underline{\varepsilon} \right)^2 \right]$$

$$+ \; \frac{1}{9 \, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left(\varepsilon_0 \right)^2 \frac{d\varepsilon_0}{dr_2}$$

$$+ \; \frac{1}{4 \, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 \right] - c_2 \right\}^2 \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right]$$

$$+ \; \frac{1}{2} \left[d_1 - b_2 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 \right] - c_2 \right\}$$

$$+ \; \frac{1}{2 \, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 \right] - c_2 \right\} \left[\varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0 \, \frac{d\varepsilon_0}{dr_2} \right]$$

$$+ \; \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[d_1 - b_2 \right] \left[\left(\varepsilon_{12} \right)^2 - \left(\varepsilon_0 \right)^2 \right]$$

$$+ \; \frac{1}{4 \, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\left(\varepsilon_{12} \right)^2 \frac{d\varepsilon_{12}}{dr_2} - \left(\varepsilon_0 \right)^2 \frac{d\varepsilon_0}{dr_2} \right]$$

$$+ \; K_2 \, b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[d_1 - b_2 \right]$$

$$- \; K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - c_2 \right\} \frac{d\varepsilon_{12}}{dr_2}$$

$$- \; K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_2} \, . \tag{139}$$

Case 2. Full Vertical Integration ($\alpha_1^R = \alpha_2^R = \alpha_1^G = \alpha_2^G = 1$).

(111) implies that in this case, for $\varepsilon \in [\,\underline{\varepsilon}, \varepsilon_0\,]$:

$$\begin{aligned}
q_1^{*0c}(\varepsilon) &= \frac{1}{3}\left[\,a^L + 3\,Q_1^r\,\right] + \frac{1}{3\,b^w}\,[\,\varepsilon + c_2 - 2\,c_1\,]\,; \\
q_2^{*0c}(\varepsilon) &= \frac{1}{3}\left[\,a^L + 3\,Q_2^r\,\right] + \frac{1}{3\,b^w}\,[\,\varepsilon + c_1 - 2\,c_2\,]\,; \\
w^{0c}(\varepsilon) &= \frac{1}{3}\left[\,b^w\,a^L + c_1 + c_2 + \varepsilon\,\right]. \qquad (140)
\end{aligned}$$

(112) and (113) imply that if G2 is capacity-constrained for $\varepsilon \in (\,\varepsilon_0, \varepsilon_{12}\,)$:

$$\begin{aligned}
\varepsilon_0 &= 3\,b^w\,K_2 + 2\,c_2 - c_1 - \;b^w\left[\,a^L + 3\,Q_2^r\,\right]; \\
\varepsilon_{12} &= 2\,b^w\,K_1 + b^w\,K_2 + c_1 - \;b^w\left[\,a^L + 2\,Q_1^r + Q_2^r\,\right]; \\
q_1^{*1c}(\varepsilon) &= \frac{1}{2}\left[\,a^L + 2\,Q_1^r + Q_2^r - K_2\,\right] + \frac{1}{2\,b^w}\,[\,\varepsilon - c_1\,]\,; \\
q_2^{*1c}(\varepsilon) &= K_2\,; \\
w^{1c}(\varepsilon) &= \frac{b^w}{2}\left[\,a^L + Q_2^r - K_2\,\right] + \frac{1}{2}\,[\,c_1 + \varepsilon\,]\,. \qquad (141)
\end{aligned}$$

(114) and (115) imply that if G1 is capacity-constrained for $\varepsilon \in (\,\varepsilon_0, \varepsilon_{12}\,)$:

$$\begin{aligned}
\varepsilon_0 &= 3\,b^w\,K_1 + 2\,c_1 - c_2 - b^w\left[\,a^L + 3\,Q_1^r\,\right]; \\
\varepsilon_{12} &= 2\,b^w\,K_2 + b^w\,K_1 + c_2 - b^w\left[\,a^L + Q_1^r + 2\,Q_2^r\,\right]; \\
q_1^{*1c}(\varepsilon) &= K_1\,; \\
q_2^{*1c}(\varepsilon) &= \frac{1}{2}\left[\,a^L + Q_1^r + 2\,Q_2^r - K_1\,\right] + \frac{1}{2\,b^w}\,[\,\varepsilon - c_2\,]\,; \\
w^{1c}(\varepsilon) &= \frac{b^w}{2}\left[\,a^L + Q_1^r - K_1\,\right] + \frac{1}{2}\,[\,c_2 + \varepsilon\,]\,. \qquad (142)
\end{aligned}$$

(116) implies that for $\varepsilon \in [\,\varepsilon_{12}, \overline{\varepsilon}\,]$:

$$q_1^{*2c}(\varepsilon) = K_1\,; \quad q_2^{*2c}(\varepsilon) = K_2\,; \quad w^{2c}(\varepsilon) = b^w\left[\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right] + \varepsilon\,. \qquad (143)$$

Under full vertical integration, Gi chooses K_i to maximize:

$$E\{\,[\,w(\cdot) - c_i\,]\,q_i + [\,r_i - w(\cdot) - c_i^r\,]\,Q_i^r\,\} - -k_i\,K_i$$

$$= \int_{\underline{\varepsilon}}^{\varepsilon_0} [\,w^{0c}(\varepsilon) - c_i\,]\,q_i^{*0c}(\varepsilon)\,dH(\varepsilon) + \int_{\underline{\varepsilon}}^{\varepsilon_0} [\,r_i - w^{0c}(\varepsilon) - c_i^r\,]\,Q_i^r\,dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} [\,w^{1c}(\varepsilon) - c_i\,]\,q_i^{*1c}(\varepsilon)\,dH(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_{12}} [\,r_i - w^{1c}(\varepsilon) - c_i^r\,]\,Q_i^r\,dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} [\,w^{2c}(\varepsilon) - c_i\,]\,K_i\,dH(\varepsilon) + \int_{\varepsilon_{12}}^{\overline{\varepsilon}} [\,r_i - w^{2c}(\varepsilon) - c_i^r\,]\,Q_i^r\,dH(\varepsilon) - k_i\,K_i\,, \qquad (144)$$

where, from (1), for $i, j \in \{1, 2\}$ $(j \neq i)$:

$$Q_i^r \;=\; a_i - b_i\,r_i + d_i\,r_j\,. \qquad (145)$$

Case 2A. Full vertical integration where G2 is capacity-constrained for $\varepsilon \in (\, \varepsilon_0, \varepsilon_{12}\,))$

(140), (141), and (143) imply that when $h(\varepsilon) = \frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}$:

$$E\,\{\,[\,w(\varepsilon) - c_1\,]\,q_1^*(\varepsilon)\,\} \;=$$

$$\frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0} \left[\,b^w\,a^L + c_2 - 2\,c_1 + \varepsilon\,\right] \left[a^L + 3\,Q_1^r + \frac{1}{b^w}\,(\,c_2 - 2\,c_1 + \varepsilon\,)\right] dH(\varepsilon)$$

$$+\;\frac{1}{4}\int_{\varepsilon_0}^{\varepsilon_{12}} \left\{\,b^w\left[\,a^L + Q_2^r - K_2\,\right] - c_1 + \varepsilon\,\right\} \left\{a^L + 2\,Q_1^r + Q_2^r - K_2 + \frac{1}{b^w}\,[\,-\,c_1 + \varepsilon\,]\right\} dH(\varepsilon)$$

$$+\int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left\{\,b^w\left[\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right] - c_1 + \varepsilon\,\right\}\,K_1\;dH(\varepsilon)$$

$$= \;\frac{1}{9}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,b^w\,a^L + c_2 - 2\,c_1\,\right]\left[a^L + 3\,Q_1^r + \frac{1}{b^w}\,(\,c_2 - 2\,c_1\,)\right]$$

$$+\;\frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0}\left[\,b^w\,a^L + c_2 - 2\,c_1\,\right]\frac{1}{b^w}\;\varepsilon\;dH(\varepsilon)$$

$$+\;\frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0}\left[a^L + 3\,Q_1^r + \frac{1}{b^w}\,(\,c_2 - 2\,c_1)\right]\varepsilon\;dH(\varepsilon) + \frac{1}{9\,b^w}\int_{\underline{\varepsilon}}^{\varepsilon_0}\varepsilon^2\,dH(\varepsilon)$$

$$+\;\frac{1}{4}\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L + Q_2^r - K_2\,\right) - c_1\,\right]\left[\,a^L + 2\,Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w}\,\right]$$

$$+\;\frac{1}{4}\int_{\varepsilon_0}^{\varepsilon_{12}}\left[\,b^w\left(\,a^L + Q_2^r - K_2\,\right) - c_1\,\right]\frac{1}{b^w}\;\varepsilon\;dH(\varepsilon)$$

$$+\;\frac{1}{4}\int_{\varepsilon_0}^{\varepsilon_{12}}\left[\,a^L + 2\,Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w}\,\right]\varepsilon\;dH(\varepsilon) + \;\frac{1}{4\,b^w}\int_{\varepsilon_0}^{\varepsilon_{12}}\varepsilon^2\,dH(\varepsilon)$$

$$+\;K_1\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right) - c_1\,\right]$$

$$+\;\frac{K_1}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2 - (\varepsilon_{12})^2\,\right]$$

$$
\begin{aligned}
&= \frac{1}{9}\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\,a^L+c_2-2\,c_1\,\right]\left[a^L+3\,Q_1^r+\frac{1}{b^w}\,(\,c_2-2\,c_1\,)\right] \\
&\quad+\frac{1}{18\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\,a^L+c_2-2\,c_1\,\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right] \\
&\quad+\frac{1}{18}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+3\,Q_1^r+\frac{1}{b^w}\,(\,c_2-2\,c_1)\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right] \\
&\quad+\frac{1}{27\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^3-(\underline{\varepsilon})^3\,\right] \\
&\quad+\frac{1}{4}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r-K_2\,\right)-c_1\,\right]\left[a^L+2\,Q_1^r+Q_2^r-K_2-\frac{c_1}{b^w}\right] \\
&\quad+\frac{1}{8\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r-K_2\,\right)-c_1\,\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right] \\
&\quad+\frac{1}{8}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+2\,Q_1^r+Q_2^r-K_2-\frac{c_1}{b^w}\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right] \\
&\quad+\frac{1}{12\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^3-(\varepsilon_0)^3\,\right] \\
&\quad+K_1\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right)-c_1\,\right] \\
&\quad+\frac{K_1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2-(\varepsilon_{12})^2\,\right] \qquad (146)
\end{aligned}
$$

where ε_0 and ε_{12} are as specified in (141).

Furthermore, from (60):

$$
\begin{aligned}
&E\{\,[\,r_1-w(\cdot)-c_1^r\,]\,Q_1^r\,\} \;=\; [\,r_1-c_1^r\,]\,Q_1^r-Q_1^r\,E\{\,w(\cdot)\,\} \\
&= [\,r_1-c_1^r\,]\,Q_1^r \\
&\quad-Q_1^r\left\{\frac{1}{3}\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\,a^L+c_1+c_2\,\right]+\frac{1}{6}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right]\right. \\
&\qquad+\frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r-K_2\,\right)+c_1\,\right]+\frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right] \\
&\qquad\left.+\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]+\frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2-(\varepsilon_{12})^2\,\right]\right\}. \qquad (147)
\end{aligned}
$$

When it chooses K_1, G1 seeks to maximize the sum of (146) and (147).

(140), (141), and (143) also imply that when $h(\varepsilon) = \frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}$:

$$E\{[w(\varepsilon) - c_2]\, q_2^*(\varepsilon)\} \;=$$

$$\frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0} [\, b^w a^L + c_1 - 2\,c_2 + \varepsilon\,] \left[a^L + 3\,Q_2^r + \frac{1}{b^w}(\,c_1 - 2\,c_2 + \varepsilon\,)\right] dH(\varepsilon)$$

$$+ \frac{1}{2}\int_{\varepsilon_0}^{\varepsilon_{12}} \left\{ b^w \left[a^L + Q_2^r - K_2 \right] + c_1 - 2\,c_2 + \varepsilon \right\} K_2 \; dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - c_2 + \varepsilon \right\} K_2 \; dH(\varepsilon)$$

$$= \frac{1}{9}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b^w a^L + c_1 - 2\,c_2\,] \left[a^L + 3\,Q_2^r + \frac{1}{b^w}(\,c_1 - 2\,c_2\,)\right]$$

$$+ \frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0} [\, b^w a^L + c_1 - 2\,c_2\,] \frac{1}{b^w}\, \varepsilon \; dH(\varepsilon)$$

$$+ \frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0} \left[a^L + 3\,Q_2^r + \frac{1}{b^w}(\,c_1 - 2\,c_2)\right] \varepsilon \; dH(\varepsilon) + \frac{1}{9\,b^w}\int_{\underline{\varepsilon}}^{\varepsilon_0} \varepsilon^2 \, dH(\varepsilon)$$

$$+ \frac{K_2}{2}\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 - 2\,c_2 \right]$$

$$+ \frac{K_2}{4}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$+ K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_2 \right]$$

$$+ \frac{K_2}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right]$$

$$= \frac{1}{9}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b^w a^L + c_1 - 2\,c_2\,] \left[a^L + 3\,Q_2^r + \frac{1}{b^w}(\,c_1 - 2\,c_2\,)\right]$$

$$+ \frac{1}{18\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b^w a^L + c_1 - 2\,c_2\,] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]$$

$$+ \frac{1}{18}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\,Q_2^r + \frac{1}{b^w}(\,c_1 - 2\,c_2)\right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]$$

$$+\ \frac{1}{27\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^3-(\underline{\varepsilon})^3\,\right]$$

$$+\ \frac{K_2}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_2^r-K_2\right)+c_1-2\,c_2\,\right]$$

$$+\ \frac{K_2}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right]$$

$$+\ K_2\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1-K_2\right)-c_2\,\right]$$

$$+\ \frac{K_2}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2-(\varepsilon_{12})^2\,\right] \tag{148}$$

where ε_0 and ε_{12} are as specified in (141).

Furthermore, from (60):

$$E\{\,[\,r_2-w(\cdot)-c_2^r\,]\,Q_2^r\,\} \;=\; [\,r_2-c_2^r\,]\,Q_2^r-Q_2^r\,E\{\,w(\cdot)\,\}$$

$$=\;[\,r_2-c_2^r\,]\,Q_2^r$$

$$-\ Q_2^r\left\{\frac{1}{3}\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\,a^L+c_1+c_2\,\right]+\frac{1}{6}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right]\right.$$

$$+\ \frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_2^r-K_2\right)+c_1\,\right]+\frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right]$$

$$\left.+\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]+\frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2-(\varepsilon_{12})^2\,\right]\right\}. \tag{149}$$

When it chooses K_2, G2 seeks to maximize the sum of (148) and (149).

From (141):

$$\frac{d\varepsilon_0}{dK_1}\;=\;0\,;\quad\frac{d\varepsilon_0}{dK_2}\;=\;3\,b^w\,;\quad\frac{d\varepsilon_0}{dr_1}\;=\;-\,3\,b^w\,d_2\,;\quad\frac{d\varepsilon_0}{dr_2}\;=\;3\,b^w\,b_2\,;$$

$$\frac{d\varepsilon_{12}}{dK_1}\;=\;2\,b^w\,;\quad\frac{d\varepsilon_{12}}{dK_2}\;=\;b^w\,;\quad\frac{d\varepsilon_{12}}{dr_1}\;=\;b^w\,[\,2\,b_1-d_2\,]\,;\quad\frac{d\varepsilon_{12}}{dr_2}\;=\;b^w\,[\,b_2-2\,d_1\,]\,. \tag{150}$$

(150) implies:

$$\frac{d^2\varepsilon_0}{dr_1\,dz}\;=\;\frac{d^2\varepsilon_0}{dr_2\,dz}\;=\;\frac{d^2\varepsilon_{12}}{dr_1\,dz}\;=\;\frac{d^2\varepsilon_{12}}{dr_2\,dz}\;=\;0\ \text{ for }\ z\in\{\,K_1,K_2,r_1,r_2\,\}\,. \tag{151}$$

Furthermore, from (60):

$$E\{w(\varepsilon)\} = \frac{1}{3}\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\,a^L+c_1+c_2\,\right]+\frac{1}{6}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right]$$
$$+\ \frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_2^r-K_2\right)+c_1\,\right]+\frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right]$$
$$+\ \left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]+\frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2-(\varepsilon_{12})^2\,\right]. \quad (152)$$

(150), (151), (152) imply:

$$\frac{\partial E\{w(\varepsilon)\}}{\partial K_1} = \frac{1}{3}\left[\,b^w\,a^L+c_1+c_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_1}+\frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}$$
$$+\ \frac{1}{2}\left[\,b^w\left(a^L+Q_2^r-K_2\right)+c_1\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_1}-\frac{d\varepsilon_0}{dK_1}\right]$$
$$+\ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right]$$
$$-\ b^w\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]-b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dK_1}$$
$$-\ \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}\,;$$

$$\frac{\partial E\{w(\varepsilon)\}}{\partial r_1} = \frac{1}{3}\left[\,b^w\,a^L+c_1+c_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_1}+\frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_1}$$
$$+\ \frac{1}{2}\left[\,b^w\left(a^L+Q_2^r-K_2\right)+c_1\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_1}-\frac{d\varepsilon_0}{dr_1}\right]$$
$$+\ \frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]b^w\,d_2+\frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_1}-\varepsilon_0\,\frac{d\varepsilon_0}{dr_1}\right]$$
$$+\ b^w\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,d_2-b_1\,\right]-b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_1}$$
$$-\ \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_1}\,;$$

$$\frac{\partial E\{w(\varepsilon)\}}{\partial K_2} = \frac{1}{3}\left[\,b^w\,a^L+c_1+c_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_2}$$

$$+ \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \frac{d\varepsilon_0}{dK_2} - \frac{1}{2} b^w \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \frac{1}{2} \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2} \right]$$

$$+ \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dK_2} - \varepsilon_0 \frac{d\varepsilon_0}{dK_2} \right]$$

$$- b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dK_2}$$

$$- \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \frac{d\varepsilon_{12}}{dK_2} ;$$

$$\frac{\partial E\{w(\varepsilon)\}}{\partial r_2} = \frac{1}{3} \left[b^w a^L + c_1 + c_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_2} + \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \frac{d\varepsilon_0}{dr_2}$$

$$+ \frac{1}{2} \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right]$$

$$- \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w b_2 + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0 \frac{d\varepsilon_0}{dr_2} \right]$$

$$+ b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[d_1 - b_2 \right] - b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_2}$$

$$- \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \frac{d\varepsilon_{12}}{dr_2} . \tag{153}$$

Define $E\{\Pi_i^v\} \equiv E\{[w(\varepsilon) - c_i] q_i^*(\varepsilon)\} + E\{[r_i - w(\cdot) - c_i^r] Q_i^r\}$ for $i \in \{1, 2\}$. (154)

(146), (147), and (154) imply that in the present case:

$$\frac{\partial E\{\Pi_1^v\}}{\partial K_1} = \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w a^L + c_2 - 2 c_1 \right] \left[a^L + 3 Q_1^r + \frac{1}{b^w} (c_2 - 2 c_1) \right] \frac{d\varepsilon_0}{dK_1}$$

$$+ \frac{1}{9 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w a^L + c_2 - 2 c_1 \right] \varepsilon_0 \frac{d\varepsilon_0}{dK_1}$$

$$+ \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3 Q_1^r + \frac{1}{b^w} (c_2 - 2 c_1) \right] \varepsilon_0 \frac{d\varepsilon_0}{dK_1} + \frac{1}{9 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2 \frac{d\varepsilon_0}{dK_1}$$

$$+ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, b^w \left(\, a^L + Q_2^r - K_2 \,\right) - c_1 \,\right]\left[\, a^L + 2\, Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w}\,\right]\left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1}\right]$$

$$+ \frac{1}{4\, b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, b^w \left(\, a^L + Q_2^r - K_2 \,\right) - c_1 \,\right]\left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_1} - \varepsilon_0\, \frac{d\varepsilon_0}{dK_1}\right]$$

$$+ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, a^L + 2\, Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w}\,\right]\left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_1} - \varepsilon_0\, \frac{d\varepsilon_0}{dK_1}\right]$$

$$+ \frac{1}{4\, b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[(\varepsilon_{12})^2\, \frac{d\varepsilon_{12}}{dK_1} - (\varepsilon_0)^2\, \frac{d\varepsilon_0}{dK_1}\right]$$

$$+ \left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, b^w \left(\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right) - c_1 \,\right] - K_1\, b^w \left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$- K_1 \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, b^w \left(\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right) - c_1 \,\right]\frac{d\varepsilon_{12}}{dK_1}$$

$$+ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \,\right] - K_1 \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_1}$$

$$- \frac{1}{3}\, Q_1^r \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, b^w\, a^L + c_1 + c_2 \,\right]\frac{d\varepsilon_0}{dK_1} - \frac{1}{3}\, Q_1^r \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\, \frac{d\varepsilon_0}{dK_1}$$

$$- \frac{1}{2}\, Q_1^r \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, b^w \left(\, a^L + Q_2^r - K_2 \,\right) + c_1 \,\right]\left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1}\right]$$

$$- \frac{1}{2}\, Q_1^r \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_1} - \varepsilon_0\, \frac{d\varepsilon_0}{dK_1}\right] + b^w\, Q_1^r \left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+ Q_1^r \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right]\frac{d\varepsilon_{12}}{dK_1} + Q_1^r \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_1}\, ; \tag{155}$$

$$\frac{\partial E\left\{\Pi_1^v\right\}}{\partial K_2} = \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, b^w\, a^L + c_2 - 2\, c_1 \,\right]\left[\, a^L + 3\, Q_1^r + \frac{1}{b^w}\left(\, c_2 - 2\, c_1 \right)\right]\frac{d\varepsilon_0}{dK_2}$$

$$+ \frac{1}{9\, b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, b^w\, a^L + c_2 - 2\, c_1 \,\right]\varepsilon_0\, \frac{d\varepsilon_0}{dK_2}$$

$$+ \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, a^L + 3\, Q_1^r + \frac{1}{b^w}\left(\, c_2 - 2\, c_1 \right)\right]\varepsilon_0\, \frac{d\varepsilon_0}{dK_2} + \frac{1}{9\, b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right](\varepsilon_0)^2\, \frac{d\varepsilon_0}{dK_2}$$

$$+ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, b^w \left(\, a^L + Q_2^r - K_2 \,\right) - c_1 \,\right]\left[\, a^L + 2\, Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w}\,\right]\left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2}\right]$$

$$- \frac{1}{4} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r - K_2 \right) - c_1 \right]$$

$$- \frac{1}{4} b^w \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 2\, Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w} \right] - \frac{1}{8} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$+ \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r - K_2 \right) - c_1 \right] \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dK_2} - \varepsilon_0 \frac{d\varepsilon_0}{dK_2} \right]$$

$$- \frac{1}{8} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$+ \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 2\, Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w} \right] \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dK_2} - \varepsilon_0 \frac{d\varepsilon_0}{dK_2} \right]$$

$$+ \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 \frac{d\varepsilon_{12}}{dK_2} - (\varepsilon_0)^2 \frac{d\varepsilon_0}{dK_2} \right] - K_1\, b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_1 \right] \frac{d\varepsilon_{12}}{dK_2} - K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \frac{d\varepsilon_{12}}{dK_2}$$

$$- \frac{1}{3} Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w\, a^L + c_1 + c_2 \right] \frac{d\varepsilon_0}{dK_2} - \frac{1}{3} Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \frac{d\varepsilon_0}{dK_2}$$

$$- \frac{1}{2} Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right] \left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2} \right] + \frac{1}{2} b^w\, Q_1^r \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \frac{1}{2} Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dK_2} - \varepsilon_0 \frac{d\varepsilon_0}{dK_2} \right] + b^w\, Q_1^r \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \frac{d\varepsilon_{12}}{dK_2} + Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \frac{d\varepsilon_{12}}{dK_2}\,; \qquad (156)$$

$$\frac{\partial E\left\{\Pi_1^v\right\}}{\partial r_1} = - \frac{1}{3} b_1 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w\, a^L + c_2 - 2\, c_1 \right]$$

$$+ \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w\, a^L + c_2 - 2\, c_1 \right] \left[a^L + 3\, Q_1^r + \frac{1}{b^w} \left(c_2 - 2\, c_1 \right) \right] \frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w\, a^L + c_2 - 2\, c_1 \right] \varepsilon_0 \frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\, Q_1^r + \frac{1}{b^w} \left(c_2 - 2\, c_1 \right) \right] \varepsilon_0 \frac{d\varepsilon_0}{dr_1}$$

$$- \frac{1}{6}\, b_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right] + \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2 \, \frac{d\varepsilon_0}{dr_1}$$

$$+ \; \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r - K_2 \right) - c_1 \right] \left[a^L + 2\, Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w} \right] \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1} \right]$$

$$- \frac{1}{4} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r - K_2 \right) - c_1 \right] \left[2\, b_1 - d_2 \right]$$

$$+ \frac{1}{4}\, b^w\, d_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 2\, Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w} \right]$$

$$+ \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r - K_2 \right) - c_1 \right] \left[\varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0 \, \frac{d\varepsilon_0}{dr_1} \right]$$

$$+ \frac{1}{8}\, d_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$+ \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 2\, Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w} \right] \left[\varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0 \, \frac{d\varepsilon_0}{dr_1} \right]$$

$$- \frac{1}{8} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[2\, b_1 - d_2 \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$+ \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 \, \frac{d\varepsilon_{12}}{dr_1} - (\varepsilon_0)^2 \, \frac{d\varepsilon_0}{dr_1} \right] - \; K_1\, b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b_1 - d_2 \right]$$

$$- \; K_1 \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_1} - \; K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_1}$$

$$+ Q_1^r - b_1 \left[r_1 - c_1^r \right] - \frac{1}{3}\, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w\, a^L + c_1 + c_2 \right] \frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{3}\, b_1 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w\, a^L + c_1 + c_2 \right] - \frac{1}{3}\, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \, \frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{6}\, b_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]$$

$$- \frac{1}{2}\, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right] \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1} \right]$$

$$- \frac{1}{2}\, b^w\, d_2\, Q_1^r \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{2}\, b_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right]$$

$$- \frac{1}{2}\, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0 \, \frac{d\varepsilon_0}{dr_1} \right] + \frac{1}{4}\, b_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$+\; b^w\, Q_1^r \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b_1 - d_2 \,] + \; b^w\, b_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right]$$

$$+\, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \frac{d\varepsilon_{12}}{dr_1}$$

$$+\; \frac{1}{2}\, b_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \,\right] + \; Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12}\, \frac{d\varepsilon_{12}}{dr_1}\,; \qquad (157)$$

$$\frac{\partial E\,\{\Pi_1^v\}}{\partial r_2} = \; \frac{1}{3}\, d_1 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_2 - 2\, c_1 \,\right]$$

$$+\; \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_2 - 2\, c_1 \,\right] \left[a^L + 3\, Q_1^r + \frac{1}{b^w}\, (\, c_2 - 2\, c_1 \,) \right] \frac{d\varepsilon_0}{dr_2}$$

$$+\; \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_2 - 2\, c_1 \,\right] \varepsilon_0\, \frac{d\varepsilon_0}{dr_2}$$

$$+\; \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\, Q_1^r + \frac{1}{b^w}\, (\, c_2 - 2\, c_1) \right] \varepsilon_0\, \frac{d\varepsilon_0}{dr_2}$$

$$+\; \frac{1}{6}\, d_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right] + \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2\, \frac{d\varepsilon_0}{dr_2}$$

$$+\; \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r - K_2 \right) - c_1 \,\right] \left[a^L + 2\, Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w} \right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right]$$

$$-\; \frac{1}{4} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r - K_2 \right) - c_1 \,\right] [\, b_2 - 2\, d_1 \,]$$

$$-\; \frac{1}{4}\, b^w\, b_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 2\, Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w} \right]$$

$$+\; \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r - K_2 \right) - c_1 \,\right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0\, \frac{d\varepsilon_0}{dr_2} \right]$$

$$-\; \frac{1}{8}\, b_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right]$$

$$+\; \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 2\, Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w} \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0\, \frac{d\varepsilon_0}{dr_2} \right]$$

$$-\; \frac{1}{8} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b_2 - 2\, d_1 \,] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right]$$

$$+\ \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[(\varepsilon_{12})^2\,\frac{d\varepsilon_{12}}{dr_2}-(\varepsilon_0)^2\,\frac{d\varepsilon_0}{dr_2}\right]-\ K_1\,b^w\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b_2-d_1\,]$$

$$-\ \ K_1\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1-K_2\right)-c_1\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_2}-\ K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_2}$$

$$+\ d_1\,[\,r_1-c_1^r\,]-\frac{1}{3}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b^w\,a^L+c_1+c_2\,]\,\frac{d\varepsilon_0}{dr_2}$$

$$-\ \frac{1}{3}\,d_1\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b^w\,a^L+c_1+c_2\,]-\frac{1}{3}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_2}$$

$$-\ \frac{1}{6}\,d_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,]$$

$$-\ \frac{1}{2}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b^w\left(a^L+Q_2^r-K_2\right)+c_1\,]\left[\frac{d\varepsilon_{12}}{dr_2}-\frac{d\varepsilon_0}{dr_2}\right]$$

$$+\ \frac{1}{2}\,b_2\,b^w\,Q_1^r\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]-\frac{1}{2}\,d_1\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b^w\left(a^L+Q_2^r-K_2\right)+c_1\,]$$

$$-\ \frac{1}{2}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_2}-\varepsilon_0\,\frac{d\varepsilon_0}{dr_2}\right]-\ \frac{1}{4}\,d_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,]$$

$$+\ \ b^w\,Q_1^r\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b_2-d_1\,]-\ b^w\,d_1\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,a^L+Q_1^r+Q_2^r-K_1-K_2\,]$$

$$+\ Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]b^w\,[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,]\,\frac{d\varepsilon_{12}}{dr_2}$$

$$-\ \ \frac{1}{2}\,d_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,(\overline{\varepsilon})^2-(\varepsilon_{12})^2\,]+\ Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_2}\,. \qquad (158)$$

where ε_0 and ε_{12} are as specified in (141).

(148), (149), and (154) imply that in the present case:

$$\frac{\partial E\,\{\Pi_2^v\}}{\partial K_1}\ =\ \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b^w\,a^L+c_1-2\,c_2\,]\left[a^L+3\,Q_2^r+\frac{1}{b^w}\,(\,c_1-2\,c_2)\right]\frac{d\varepsilon_0}{dK_1}$$

$$+\ \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b^w\,a^L+c_1-2\,c_2\,]\,\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}$$

$$+\ \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+3\,Q_2^r+\frac{1}{b^w}\,(\,c_1-2\,c_2)\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}+\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right](\varepsilon_0)^2\,\frac{d\varepsilon_0}{dK_1}$$

$$+\ \frac{K_2}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r-K_2\,\right)+c_1-2\,c_2\,\right]\left[\frac{d\varepsilon_{12}}{dK_1}-\frac{d\varepsilon_0}{dK_1}\right]$$

$$+\ \frac{K_2}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right]-\ K_2\,b^w\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$-\ K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right)-c_2\,\right]\frac{d\varepsilon_{12}}{dK_1}-\ K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}$$

$$-\ \frac{1}{3}\,Q_2^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\,a^L+c_1+c_2\,\right]\frac{d\varepsilon_0}{dK_1}-\frac{1}{3}\,Q_2^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}$$

$$-\ \frac{1}{2}\,Q_2^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r-K_2\,\right)+c_1\,\right]\left[\frac{d\varepsilon_{12}}{dK_1}-\frac{d\varepsilon_0}{dK_1}\right]$$

$$-\ \frac{1}{2}\,Q_2^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right]+\ b^w\,Q_2^r\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+\ Q_2^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]\frac{d\varepsilon_{12}}{dK_1}+\ Q_2^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}\,; \tag{159}$$

$$\frac{\partial E\left\{\Pi_2^v\right\}}{\partial K_2}\ =\ \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\,a^L+c_1-2\,c_2\,\right]\left[a^L+3\,Q_2^r+\frac{1}{b^w}\left(\,c_1-2\,c_2\right)\right]\frac{d\varepsilon_0}{dK_2}$$

$$+\ \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\,a^L+c_1-2\,c_2\,\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}$$

$$+\ \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+3\,Q_2^r+\frac{1}{b^w}\left(\,c_1-2\,c_2\right)\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}+\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left(\varepsilon_0\right)^2\,\frac{d\varepsilon_0}{dK_2}$$

$$+\ \frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r-K_2\,\right)+c_1-2\,c_2\,\right]-\frac{1}{2}\,b^w\,K_2\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+\ \frac{K_2}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r-K_2\,\right)+c_1-2\,c_2\,\right]\left[\frac{d\varepsilon_{12}}{dK_2}-\frac{d\varepsilon_0}{dK_2}\right]$$

$$+\ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right]+\ \frac{K_2}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}\right]$$

$$+\ \left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right)-c_2\,\right]-\ K_2\,b^w\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$-\ K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right)-c_2\,\right]\frac{d\varepsilon_{12}}{dK_2}$$

$$+\ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2-(\varepsilon_{12})^2\,\right]-\ K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\ \frac{d\varepsilon_{12}}{dK_2}$$

$$-\ \frac{1}{3}\,Q_2^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\,a^L+c_1+c_2\,\right]\frac{d\varepsilon_0}{dK_2}-\frac{1}{3}\,Q_2^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\ \frac{d\varepsilon_0}{dK_2}$$

$$-\ \frac{1}{2}\,Q_2^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_2^r-K_2\right)+c_1\,\right]\left[\frac{d\varepsilon_{12}}{dK_2}-\frac{d\varepsilon_0}{dK_2}\right]+\frac{1}{2}\,b^w\,Q_2^r\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$-\ \frac{1}{2}\,Q_2^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\ \frac{d\varepsilon_{12}}{dK_2}-\varepsilon_0\ \frac{d\varepsilon_0}{dK_2}\right]+\ b^w\,Q_2^r\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+\ Q_2^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]\frac{d\varepsilon_{12}}{dK_2}+\ Q_2^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\ \frac{d\varepsilon_{12}}{dK_2}\,; \tag{160}$$

$$\frac{\partial E\left\{\Pi_2^v\right\}}{\partial r_1}=\ \frac{1}{3}\,d_2\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\,a^L+c_1-2\,c_2\,\right]$$

$$+\ \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\,a^L+c_1-2\,c_2\,\right]\left[a^L+3\,Q_2^r+\frac{1}{b^w}\left(\,c_1-2\,c_2\,\right)\right]\frac{d\varepsilon_0}{dr_1}$$

$$+\ \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\,a^L+c_1-2\,c_2\,\right]\varepsilon_0\ \frac{d\varepsilon_0}{dr_1}$$

$$+\ \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+3\,Q_2^r+\frac{1}{b^w}\left(\,c_1-2\,c_2\,\right)\right]\varepsilon_0\ \frac{d\varepsilon_0}{dr_1}$$

$$+\ \frac{1}{6}\,d_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right]+\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right](\varepsilon_0)^2\ \frac{d\varepsilon_0}{dr_1}$$

$$+\ \frac{K_2}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_2^r-K_2\right)+c_1-2\,c_2\,\right]\left[\frac{d\varepsilon_{12}}{dr_1}-\frac{d\varepsilon_0}{dr_1}\right]$$

$$+\ \frac{K_2}{2}\,b^w\,d_2\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]+\ \frac{K_2}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\ \frac{d\varepsilon_{12}}{dr_1}-\varepsilon_0\ \frac{d\varepsilon_0}{dr_1}\right]$$

$$-\ K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1-K_2\right)-c_2\,\right]\frac{d\varepsilon_{12}}{dr_1}$$

$$-\ K_2\,b^w\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b_1-d_2\,\right]-\ K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\ \frac{d\varepsilon_{12}}{dr_1}$$

$$+\ d_2\left[\,r_2-c_2^r\,\right]-\frac{1}{3}\,Q_2^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\,a^L+c_1+c_2\,\right]\frac{d\varepsilon_0}{dr_1}$$

$$
\begin{aligned}
& - \frac{1}{3}\, d_2 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_1 + c_2 \,\right] - \frac{1}{3}\, Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0\, \frac{d\varepsilon_0}{dr_1} \\
& - \frac{1}{6}\, d_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right] \\
& - \frac{1}{2}\, Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \,\right] \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1} \right] \\
& - \frac{1}{2}\, b^w\, d_2\, Q_2^r \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - \frac{1}{2}\, d_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \,\right] \\
& - \frac{1}{2}\, Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0\, \frac{d\varepsilon_0}{dr_1} \right] - \frac{1}{4}\, d_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right] \\
& + b^w\, Q_2^r \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b_1 - d_2 \,\right] - b^w\, d_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \\
& + Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \frac{d\varepsilon_{12}}{dr_1} \\
& - \frac{1}{2}\, d_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \,\right] + Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12}\, \frac{d\varepsilon_{12}}{dr_1}\, ;
\end{aligned} \tag{161}
$$

$$
\begin{aligned}
\frac{\partial E\left\{\Pi_2^v\right\}}{\partial r_2} = & -\frac{1}{3}\, b_2 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_1 - 2\, c_2 \,\right] \\
& + \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_1 - 2\, c_2 \,\right] \left[a^L + 3\, Q_2^r + \frac{1}{b^w} \left(\, c_1 - 2\, c_2 \,\right) \right] \frac{d\varepsilon_0}{dr_2} \\
& + \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_1 - 2\, c_2 \,\right] \varepsilon_0\, \frac{d\varepsilon_0}{dr_2} \\
& + \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\, Q_2^r + \frac{1}{b^w} \left(\, c_1 - 2\, c_2 \,\right) \right] \varepsilon_0\, \frac{d\varepsilon_0}{dr_2} \\
& - \frac{1}{6}\, b_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right] + \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2\, \frac{d\varepsilon_0}{dr_2} \\
& + \frac{K_2}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r - K_2 \right) + c_1 - 2\, c_2 \,\right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right] \\
& - \frac{K_2}{2}\, b^w\, b_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{K_2}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0\, \frac{d\varepsilon_0}{dr_2} \right] \\
& - K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_2 \,\right] \frac{d\varepsilon_{12}}{dr_2} \\
& - K_2\, b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b_2 - d_1 \,\right] - K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12}\, \frac{d\varepsilon_{12}}{dr_2} \\
& + Q_2^r - b_2 \left[\, r_2 - c_2^r \,\right] - \frac{1}{3}\, Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_1 + c_2 \,\right] \frac{d\varepsilon_0}{dr_2} \\
& + \frac{1}{3}\, b_2 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_1 + c_2 \,\right] - \frac{1}{3}\, Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0\, \frac{d\varepsilon_0}{dr_2} \\
& + \frac{1}{6}\, b_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right] \\
& - \frac{1}{2}\, Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \,\right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right] \\
& + \frac{1}{2}\, b^w\, b_2\, Q_2^r \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{2}\, b_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \,\right] \\
& - \frac{1}{2}\, Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0\, \frac{d\varepsilon_0}{dr_2} \right] + \frac{1}{4}\, b_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right]
\end{aligned}
$$

$$+ \; b^w \, Q_2^r \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b_2 - d_1 \,] + \, b^w \, b_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right]$$

$$+ \, Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \frac{d\varepsilon_{12}}{dr_2}$$

$$+ \; \frac{1}{2} \, b_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \,\right] + \, Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_2} \, , \qquad (162)$$

where ε_0 and ε_{12} are as specified in (141).

Further Characterization of the Optimal Retail Prices

(60) implies that when G2 is capacity-constrained for $\varepsilon \in (\, \varepsilon_0, \varepsilon_{12} \,)$:

$$E\left\{q_1^{*0c}\right\} \;\; \equiv \;\; \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon)\, dH(\varepsilon) \;\; = \;\; \frac{1}{6\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right]$$

$$+ \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ \frac{1}{3} \left[\, a^L + 3\,(a_1 - b_1\, r_1 + d_1\, r_2) \,\right] + \frac{1}{3\, b^w} \left[\, c_2 - 2\, c_1 \,\right] \right\};$$

$$E\left\{q_1^{*1c}\right\} \;\; \equiv \;\; \int_{\varepsilon_0}^{\varepsilon_{12}} q_1^{*1c}(\varepsilon)\, dH(\varepsilon) \;\; = \;\; \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right]$$

$$+ \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ \frac{1}{2} \left[\, a^L + 2\,(a_1 - b_1\, r_1 + d_1\, r_2) + (a_2 - b_2\, r_2 + d_2\, r_1) - K_2 \,\right] - \frac{1}{2\, b^w}\, c_1 \right\};$$

$$E\left\{q_2^{*0c}\right\} \;\; \equiv \;\; \int_{\underline{\varepsilon}}^{\varepsilon_0} q_2^{*0c}(\varepsilon)\, dH(\varepsilon) \;\; = \;\; \frac{1}{6\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right]$$

$$+ \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ \frac{1}{3} \left[\, a^L + 3\,(a_2 - b_2\, r_2 + d_2\, r_1) \,\right] + \frac{1}{3\, b^w} \left[\, c_1 - 2\, c_2 \,\right] \right\} \qquad (163)$$

where ε_0 and ε_{12} are as specified in (141).

(163) implies that in this case:

$$\frac{\partial E\left\{q_1^{*0c}\right\}}{\partial r_1} \;\; = \;\; - \, b_1 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{3\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \, \frac{d\varepsilon_0}{dr_1}$$

$$+ \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ \frac{1}{3} \left[\, a^L + 3\,(a_1 - b_1\, r_1 + d_1\, r_2) \,\right] + \frac{1}{3\, b^w} \left[\, c_2 - 2\, c_1 \,\right] \right\} \frac{d\varepsilon_0}{dr_1}\, ;$$

$$\frac{\partial E\left\{q_1^{*0c}\right\}}{\partial r_2} = d_1\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]+\frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_2}$$
$$+\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\frac{1}{3}\left[\,a^L+3\left(a_1-b_1\,r_1+d_1\,r_2\right)\right]+\frac{1}{3\,b^w}\left[\,c_2-2\,c_1\,\right]\right\}\frac{d\varepsilon_0}{dr_2}\,;$$

$$\frac{\partial E\left\{q_1^{*0c}\right\}}{\partial K_1} = \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}$$
$$+\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\frac{1}{3}\left[\,a^L+3\left(a_1-b_1\,r_1+d_1\,r_2\right)\right]+\frac{1}{3\,b^w}\left[\,c_2-2\,c_1\,\right]\right\}\frac{d\varepsilon_0}{dK_1}\,;$$

$$\frac{\partial E\left\{q_1^{*0c}\right\}}{\partial K_2} = \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}$$
$$+\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\frac{1}{3}\left[\,a^L+3\left(a_1-b_1\,r_1+d_1\,r_2\right)\right]+\frac{1}{3\,b^w}\left[\,c_2-2\,c_1\,\right]\right\}\frac{d\varepsilon_0}{dK_2}\,;$$

$$\frac{\partial E\left\{q_1^{*1c}\right\}}{\partial r_1} = -\frac{1}{2}\left[\,2\,b_1-d_2\,\right]\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]+\frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_1}-\varepsilon_0\,\frac{d\varepsilon_0}{dr_1}\right]$$
$$+\left\{\frac{1}{2}\left[\,a^L+2\left(a_1-b_1\,r_1+d_1\,r_2\right)+\left(a_2-b_2\,r_2+d_2\,r_1\right)-K_2\,\right]-\frac{1}{2\,b^w}\,c_1\right\}$$
$$\cdot\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_1}-\frac{d\varepsilon_0}{dr_1}\right];$$

$$\frac{\partial E\left\{q_1^{*1c}\right\}}{\partial r_2} = -\frac{1}{2}\left[\,b_2-2\,d_1\,\right]\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]+\frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_2}-\varepsilon_0\,\frac{d\varepsilon_0}{dr_2}\right]$$
$$+\left\{\frac{1}{2}\left[\,a^L+2\left(a_1-b_1\,r_1+d_1\,r_2\right)+\left(a_2-b_2\,r_2+d_2\,r_1\right)-K_2\,\right]-\frac{1}{2\,b^w}\,c_1\right\}$$
$$\cdot\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_2}-\frac{d\varepsilon_0}{dr_2}\right];$$

$$\frac{\partial E\left\{q_1^{*1c}\right\}}{\partial K_1} = \frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right]$$
$$+\left\{\frac{1}{2}\left[\,a^L+2\left(a_1-b_1\,r_1+d_1\,r_2\right)+\left(a_2-b_2\,r_2+d_2\,r_1\right)-K_2\,\right]-\frac{1}{2\,b^w}\,c_1\right\}$$
$$\cdot\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_1}-\frac{d\varepsilon_0}{dK_1}\right];$$

$$\frac{\partial E\left\{q_1^{*1c}\right\}}{\partial K_2} = -\frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right] + \frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2} - \varepsilon_0\,\frac{d\varepsilon_0}{dK_2}\right]$$

$$+ \left\{\frac{1}{2}\left[\,a^L + 2\,(a_1 - b_1\,r_1 + d_1\,r_2) + (a_2 - b_2\,r_2 + d_2\,r_1) - K_2\,\right] - \frac{1}{2\,b^w}\,c_1\right\}$$

$$\cdot\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2}\right];$$

$$\frac{\partial E\left\{q_2^{*0c}\right\}}{\partial r_1} = d_2\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] + \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_1}$$

$$+ \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\frac{1}{3}\left[\,a^L + 3\,(a_2 - b_2\,r_2 + d_2\,r_1)\,\right] + \frac{1}{3\,b^w}\left[\,c_1 - 2\,c_2\,\right]\right\}\frac{d\varepsilon_0}{dr_1};$$

$$\frac{\partial E\left\{q_2^{*0c}\right\}}{\partial r_2} = -\,b_2\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] + \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_2}$$

$$+ \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\frac{1}{3}\left[\,a^L + 3\,(a_2 - b_2\,r_2 + d_2\,r_1)\,\right] + \frac{1}{3\,b^w}\left[\,c_1 - 2\,c_2\,\right]\right\}\frac{d\varepsilon_0}{dr_2};$$

$$\frac{\partial E\left\{q_2^{*0c}\right\}}{\partial K_1} = \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}$$

$$+ \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\frac{1}{3}\left[\,a^L + 3\,(a_2 - b_2\,r_2 + d_2\,r_1)\,\right] + \frac{1}{3\,b^w}\left[\,c_1 - 2\,c_2\,\right]\right\}\frac{d\varepsilon_0}{dK_1};$$

$$\frac{\partial E\left\{q_2^{*0c}\right\}}{\partial K_2} = \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}$$

$$+ \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left\{\frac{1}{3}\left[\,a^L + 3\,(a_2 - b_2\,r_2 + d_2\,r_1)\,\right] + \frac{1}{3\,b^w}\left[\,c_1 - 2\,c_2\,\right]\right\}\frac{d\varepsilon_0}{dK_2}. \quad (164)$$

(61) and (69) imply that when G2 is capacity-constrained for $\varepsilon \in (\,\varepsilon_0, \varepsilon_{12}\,)$:

$$J_1(r_1, r_2) \equiv \left[\,1 + b^w\,b_1\,\right]\left[\,a_1 - b_1\,r_1 + d_1\,r_2\,\right]\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+ \left[\,1 - b^w\,d_2 + b^w\,b_1\right]\left[\,a_1 - b_1\,r_1 + d_1\,r_2\,\right]\left[\frac{\overline{\varepsilon}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$-\,b^w\,b_1\,E\left\{q_1^{*0c}\right\} - b^w\left[\,b_1 - d_2\,\right]E\left\{q_1^{*1c}\right\} + b_1\,E\left\{w\left(\cdot\right)\right\}$$

$$-\,b_1\left[\,r_1 - c_1^r\,\right] - \left[\,b_1 - d_2\,\right]b^w\,K_1\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] = 0\,;$$

$$
\begin{aligned}
J_2(r_1, r_2) \;\equiv\; & [\,1 + b^w\, b_2\,]\,[\,a_2 - b_2\, r_2 + d_2\, r_1\,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w\, b_2\, E\left\{ q_2^{*0c} \right\} \\
& + \left[1 + \frac{1}{2}\, b^w\, b_2 \right] [\,a_2 - b_2\, r_2 + d_2\, r_1\,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b_2\, E\{ w(\cdot) \} \\
& - \; b_2\,[\,r_2 - c_2^r\,] - \frac{1}{2}\, b^w\, b_2\, K_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w\,[\,b_2 - d_1\,]\, K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
& + [\,1 - b^w\, d_1 + b^w\, b_2\,]\,[\,a_2 - b_2\, r_2 + d_2\, r_1\,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \;=\; 0\,, \qquad (165)
\end{aligned}
$$

where $E\{w(\cdot)\}$ is specified in (152), $E\{q_1^{*0c}\}$, $E\{q_1^{*1c}\}$, and $E\{q_2^{*0c}\}$ are specified in (163), and ε_0 and ε_{12} are specified in (141).

For $v \in \{\, r_1,\, r_2,\, K_1,\, K_2 \}$, define:

$$
\begin{aligned}
M_1^A(v) \;\equiv\; & [\,1 + b^w\, b_1\,]\,[\,a_1 - b_1\, r_1 + d_1\, r_2\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dv} \\
& - [\,1 - b^w\, d_2 + b^w\, b_1]\,[\,a_1 - b_1\, r_1 + d_1\, r_2\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dv} \\
& - b^w\, b_1\, \frac{\partial E\{q_1^{*0c}\}}{\partial v} - b^w\,[\,b_1 - d_2\,]\, \frac{\partial E\{q_1^{*1c}\}}{\partial v} + b_1\, \frac{\partial E\{w(\cdot)\}}{\partial v} \\
& + [\,b_1 - d_2\,]\, b^w\, K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dv}\,;
\end{aligned}
$$

$$
\begin{aligned}
M_2^A(v) \;\equiv\; & [\,1 + b^w\, b_2\,]\,[\,a_2 - b_2\, r_2 + d_2\, r_1\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dv} \\
& + \left[1 + \frac{1}{2}\, b^w\, b_2 \right] [\,a_2 - b_2\, r_2 + d_2\, r_1\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dv} - \frac{d\varepsilon_0}{dv} \right] \\
& - \; \frac{1}{2}\, b^w\, b_2\, K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dv} - \frac{d\varepsilon_0}{dv} \right] + b^w\,[\,b_2 - d_1\,]\, K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dv} \\
& - [\,1 - b^w\, d_1 + b^w\, b_2\,]\,[\,a_2 - b_2\, r_2 + d_2\, r_1\,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dv} \\
& - \; b^w\, b_2\, \frac{\partial E\{q_2^{*0c}\}}{\partial v} + b_2\, \frac{\partial E\{w(\cdot)\}}{\partial v}\,. \qquad (166)
\end{aligned}
$$

From (165) and (166):

$$\frac{\partial J_1(\cdot)}{\partial r_1} = -b_1 \left[1 + b^w b_1 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b_1 \left[1 - b^w d_2 + b^w b_1 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b_1 + M_1^A(r_1) ;$$

$$\frac{\partial J_1(\cdot)}{\partial r_2} = d_1 \left[1 + b^w b_1 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + d_1 \left[1 - b^w d_2 + b^w b_1 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + M_1^A(r_2) ;$$

$$\frac{\partial J_1(\cdot)}{\partial K_1} = -\left[b_1 - d_2 \right] b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + M_1^A(K_1) ;$$

$$\frac{\partial J_1(\cdot)}{\partial K_2} = M_1^A(K_2) ;$$

$$\frac{\partial J_2(\cdot)}{\partial r_1} = d_2 \left[1 + b^w b_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + d_2 \left[1 + \frac{1}{2} b^w b_2 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$
$$+ d_2 \left[1 - b^w d_1 + b^w b_2 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + M_2^A(r_1) ;$$

$$\frac{\partial J_2(\cdot)}{\partial r_2} = -b_2 \left[1 + b^w b_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b_2 \left[1 + \frac{1}{2} b^w b_2 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$
$$- b_2 - b_2 \left[1 - b^w d_1 + b^w b_2 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + M_2^A(r_2) ;$$

$$\frac{\partial J_2(\cdot)}{\partial K_1} = M_2^A(K_1) ;$$

$$\frac{\partial J_2(\cdot)}{\partial K_2} = -\frac{1}{2} b^w b_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w \left[b_2 - d_1 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + M_2^A(K_2) . \tag{167}$$

Case 2B. Full vertical integration where G1 is capacity-constrained for $\varepsilon \in (\, \varepsilon_0, \varepsilon_{12}\,))$

(140), (142), and (143) imply that when $h(\varepsilon) = \frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}$:

$$E\{\, [\, w(\varepsilon) - c_1\,]\, q_1^*(\varepsilon)\,\} \;=$$

$$\frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0} [\, b^w a^L + c_2 - 2\,c_1 + \varepsilon\,] \left[a^L + 3\,Q_1^r + \frac{1}{b^w}(\, c_2 - 2\,c_1 + \varepsilon\,)\right] dH(\varepsilon)$$

$$+ \frac{1}{2}\int_{\varepsilon_0}^{\varepsilon_{12}} \left\{\, b^w \left[\, a^L + Q_1^r - K_1\,\right] + c_2 - 2\,c_1 + \varepsilon\,\right\} K_1\, dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left\{\, b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right] - c_1 + \varepsilon\,\right\} K_1\, dH(\varepsilon)$$

$$= \frac{1}{9}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] [\, b^w a^L + c_2 - 2\,c_1\,] \left[a^L + 3\,Q_1^r + \frac{1}{b^w}(\, c_2 - 2\,c_1\,)\right]$$

$$+ \frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0} [\, b^w a^L + c_2 - 2\,c_1\,]\, \frac{1}{b^w}\, \varepsilon\, dH(\varepsilon)$$

$$+ \frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0} \left[a^L + 3\,Q_1^r + \frac{1}{b^w}(\, c_2 - 2\,c_1)\right] \varepsilon\, dH(\varepsilon) + \frac{1}{9\,b^w}\int_{\underline{\varepsilon}}^{\varepsilon_0} \varepsilon^2\, dH(\varepsilon)$$

$$+ \frac{K_1}{2}\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right] \left[\, b^w \left(\, a^L + Q_1^r - K_1\,\right) + c_2 - 2\,c_1\,\right] + \frac{K_1}{4}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2\,\right]$$

$$+ K_1\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] \left\{\, b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right] - c_1\,\right\}$$

$$+ \frac{K_1}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right] \left[\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2\,\right]$$

$$= \frac{1}{9}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] [\, b^w a^L + c_2 - 2\,c_1\,] \left[a^L + 3\,Q_1^r + \frac{1}{b^w}(\, c_2 - 2\,c_1\,)\right]$$

$$+ \frac{1}{18\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right] [\, b^w a^L + c_2 - 2\,c_1\,] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2\,\right]$$

$$+ \frac{1}{18}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right] \left[a^L + 3\,Q_1^r + \frac{1}{b^w}(\, c_2 - 2\,c_1)\right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2\,\right]$$

$$+ \frac{1}{27\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^3 - (\underline{\varepsilon})^3 \right]$$

$$+ \frac{K_1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r - K_1 \right) + c_2 - 2\, c_1 \right] + \frac{K_1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$+ K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_1 \right]$$

$$+ \frac{K_1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right] \tag{168}$$

where ε_0 and ε_{12} are as specified in (142).

Furthermore, from (77):

$$E\,\{ [\, r_1 - w(\cdot) - c_1^r \,]\, Q_1^r \} \;=\; [\, r_1 - c_1^r \,]\, Q_1^r - Q_1^r\, E\,\{ w(\cdot) \}$$

$$= [\, r_1 - c_1^r \,]\, Q_1^r$$

$$- Q_1^r \left\{ \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w\, a^L + c_1 + c_2 \right] + \frac{1}{6} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right] \right.$$

$$+ \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r - K_1 \right) + c_2 \right] + \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$\left. + \; b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right] \right\}. \tag{169}$$

When it chooses K_1, G1 seeks to maximize the sum of (168) and (169).

(140), (142), and (143) also imply that when $h(\varepsilon) = \frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}$:

$$E\,\{ [\, w(\varepsilon) - c_2 \,]\, q_2^*(\varepsilon) \} \;=$$

$$\frac{1}{9} \int_{\underline{\varepsilon}}^{\varepsilon_0} \left[b^w\, a^L + c_1 - 2\, c_2 + \varepsilon \right] \left[a^L + 3\, Q_2^r + \frac{1}{b^w} \left(c_1 - 2\, c_2 + \varepsilon \right) \right] dH(\varepsilon)$$

$$+ \frac{1}{4} \int_{\varepsilon_0}^{\varepsilon_{12}} \left\{ b^w \left[a^L + Q_1^r - K_1 \right] - c_2 + \varepsilon \right\} \left\{ a^L + Q_1^r + 2\, Q_2^r - K_1 + \frac{1}{b^w} \left[\varepsilon - c_2 \right] \right\} dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left\{ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - c_2 + \varepsilon \right\} K_2\; dH(\varepsilon)$$

$$= \frac{1}{9}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, b^w a^L + c_1 - 2\,c_2 \,\right]\left[a^L + 3\,Q_2^r + \frac{1}{b^w}\left(c_1 - 2\,c_2 \right)\right]$$

$$+ \frac{1}{18\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, b^w a^L + c_1 - 2\,c_2 \,\right]\left[\,(\varepsilon_0)^2 - (\underline{\varepsilon})^2\,\right]$$

$$+ \frac{1}{18}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[a^L + 3\,Q_2^r + \frac{1}{b^w}\left(c_1 - 2\,c_2\right)\right]\left[\,(\varepsilon_0)^2 - (\underline{\varepsilon})^2\,\right]$$

$$+ \frac{1}{27\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^3 - (\underline{\varepsilon})^3\,\right]$$

$$+ \frac{1}{4}\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, b^w \left(a^L + Q_1^r - K_1\right) - c_2 \,\right]\left[a^L + Q_1^r + 2\,Q_2^r - K_1 - \frac{c_2}{b^w}\right]$$

$$+ \frac{1}{8\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, b^w \left(a^L + Q_1^r - K_1\right) - c_2 \,\right]\left[\,(\varepsilon_{12})^2 - (\varepsilon_0)^2\,\right]$$

$$+ \frac{1}{8}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[a^L + Q_1^r + 2\,Q_2^r - K_1 - \frac{c_2}{b^w}\right]\left[\,(\varepsilon_{12})^2 - (\varepsilon_0)^2\,\right]$$

$$+ \frac{1}{12\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^3 - (\varepsilon_0)^3\,\right]$$

$$+ K_2\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2\right) - c_2 \,\right]$$

$$+ \frac{K_2}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2 - (\varepsilon_{12})^2\,\right] \qquad (170)$$

where ε_0 and ε_{12} are as specified in (142).

Furthermore, from (77):

$$E\left\{\left[\, r_2 - w(\cdot) - c_2^r \,\right] Q_2^r \right\} \;=\; \left[\, r_2 - c_2^r \,\right] Q_2^r - Q_2^r\, E\left\{ w(\cdot) \right\}$$

$$= \left[\, r_2 - c_2^r \,\right] Q_2^r$$

$$- Q_2^r \left\{ \frac{1}{3}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, b^w a^L + c_1 + c_2 \,\right] + \frac{1}{6}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2 - (\underline{\varepsilon})^2\,\right] \right.$$

$$+ \frac{1}{2}\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, b^w \left(a^L + Q_1^r - K_1 \right) + c_2 \,\right] + \frac{1}{4}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2 - (\varepsilon_0)^2\,\right]$$

$$\left. + \; b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] + \frac{1}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2 - (\varepsilon_{12})^2\,\right] \right\}. \qquad (171)$$

When it chooses K_2, G2 seeks to maximize the sum of (170) and (171).

From (142)

$$\varepsilon_0 \;=\; 3\, b^w\, K_1 + 2\, c_1 - c_2 - b^w \left[\, a^L + 3\, Q_1^r \,\right] \quad \text{and}$$

$$\varepsilon_{12} \;=\; 2\, b^w\, K_2 + b^w\, K_1 + c_2 - b^w \left[\, a^L + Q_1^r + 2\, Q_2^r \,\right]$$

$$\Rightarrow \quad \frac{d\varepsilon_0}{dK_1} \;=\; 3\, b^w\, ; \quad \frac{d\varepsilon_0}{dK_2} \;=\; 0\, ; \quad \frac{d\varepsilon_0}{dr_1} \;=\; 3\, b^w\, b_1\, ; \quad \frac{d\varepsilon_0}{dr_2} \;=\; -\, 3\, b^w\, d_1\, ;$$

$$\frac{d\varepsilon_{12}}{dK_1} \;=\; b^w\, ; \quad \frac{d\varepsilon_{12}}{dK_2} \;=\; 2\, b^w\, ; \quad \frac{d\varepsilon_{12}}{dr_1} \;=\; b^w\, [\, b_1 - 2\, d_2\,]\, ; \quad \frac{d\varepsilon_{12}}{dr_2} \;=\; b^w\, [\, 2\, b_2 - d_1\,]\, . \tag{172}$$

(172) implies:

$$\frac{d^2\varepsilon_0}{dr_1\, dz} \;=\; \frac{d^2\varepsilon_0}{dr_2\, dz} \;=\; \frac{d^2\varepsilon_{12}}{dr_1\, dz} \;=\; \frac{d^2\varepsilon_{12}}{dr_2\, dz} \;=\; 0 \quad \text{for} \;\; z \in \{\, K_1, K_2, r_1, r_2\,\}\, . \tag{173}$$

Furthermore, from (77):

$$\begin{aligned} E\,\{w(\varepsilon)\} \;=\;& \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_1 + c_2 \,\right] + \frac{1}{6} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right] \\ &+ \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r - K_1 \right) + c_2 \,\right] + \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right] \\ &+ \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \,\right]. \end{aligned} \tag{174}$$

(172), (173), (174) imply:

$$\begin{aligned} \frac{\partial E\,\{w(\varepsilon)\}}{\partial K_1} \;=\;& \frac{1}{3} \left[\, b^w\, a^L + c_1 + c_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dK_1} + \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0\, \frac{d\varepsilon_0}{dK_1} \\ &+ \frac{1}{2} \left[\, b^w \left(a^L + Q_1^r - K_1 \right) + c_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1} \right] \\ &- \frac{1}{2}\, b^w \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_1} - \varepsilon_0\, \frac{d\varepsilon_0}{dK_1} \right] \\ &- b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dK_1} \\ &- \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_1}\, ; \end{aligned}$$

$$
\begin{aligned}
\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial r_1} \;=\;& \frac{1}{3}\left[\, b^w\, a^L + c_1 + c_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_1} + \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \, \frac{d\varepsilon_0}{dr_1} \\
&+ \frac{1}{2}\left[\, b^w \left(a^L + Q_1^r - K_1 \right) + c_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1} \right] \\
&- \frac{1}{2}\, b^w\, b_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0 \, \frac{d\varepsilon_0}{dr_1} \right] \\
&- \; b^w \left[\, b_1 - d_2 \,\right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - \; b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_1} \\
&- \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_1} \,;
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial K_2} \;=\;& \frac{1}{3}\left[\, b^w\, a^L + c_1 + c_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dK_2} + \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \, \frac{d\varepsilon_0}{dK_2} \\
&+ \; \frac{1}{2}\left[\, b^w \left(a^L + Q_1^r - K_1 \right) + c_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2} \right] \\
&+ \; \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_2} - \varepsilon_0 \, \frac{d\varepsilon_0}{dK_2} \right] \\
&- \; b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - \; b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dK_2} \\
&- \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dK_2} \,;
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial r_2} \;=\;& \frac{1}{3}\left[\, b^w\, a^L + c_1 + c_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_2} + \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \, \frac{d\varepsilon_0}{dr_2} \\
&+ \frac{1}{2}\left[\, b^w \left(a^L + Q_1^r - K_1 \right) + c_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right] \\
&+ \frac{1}{2}\, b^w\, d_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0 \, \frac{d\varepsilon_0}{dr_2} \right] \\
&- \; b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b_2 - d_1 \,\right] - \; b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_2} \\
&- \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_2} \,. \qquad (175)
\end{aligned}
$$

(154), (168), and (169) imply that in the present case:

$$
\begin{aligned}
\frac{\partial E\{\Pi_1^v\}}{\partial K_1} = \; & \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, b^w a^L + c_2 - 2\,c_1 \,\right]\left[a^L + 3\,Q_1^r + \frac{1}{b^w}\,(\,c_2 - 2\,c_1)\right]\frac{d\varepsilon_0}{dK_1} \\
& + \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, b^w a^L + c_2 - 2\,c_1 \,\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1} \\
& + \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L + 3\,Q_1^r + \frac{1}{b^w}\,(\,c_2 - 2\,c_1)\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1} + \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right](\varepsilon_0)^2\,\frac{d\varepsilon_0}{dK_1} \\
& + \frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, b^w\left(a^L + Q_1^r - K_1\right) + c_2 - 2\,c_1 \,\right] - \frac{1}{2}\,b^w K_1\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \\
& + \frac{K_1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, b^w\left(a^L + Q_1^r - K_1\right) + c_2 - 2\,c_1 \,\right]\left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1}\right] \\
& + \frac{K_1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1} - \varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right] + \frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2 - (\varepsilon_0)^2\,\right] \\
& + \left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, b^w\left(a^L + Q_1^r + Q_2^r - K_1 - K_2\right) - c_1 \,\right] - K_1\,b^w\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \\
& - K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, b^w\left(a^L + Q_1^r + Q_2^r - K_1 - K_2\right) - c_1 \,\right]\frac{d\varepsilon_{12}}{dK_1} \\
& + \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2 - (\varepsilon_{12})^2\,\right] - K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1} \\
& - \frac{1}{3}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, b^w a^L + c_1 + c_2 \,\right]\frac{d\varepsilon_0}{dK_1} - \frac{1}{3}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1} \\
& + \frac{1}{2}\,b^w Q_1^r\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right] - \frac{1}{2}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, b^w\left(a^L + Q_1^r - K_1\right) + c_2 \,\right]\left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1}\right] \\
& - \frac{1}{2}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1} - \varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right] + b^w Q_1^r\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \\
& + Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] b^w\left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right]\frac{d\varepsilon_{12}}{dK_1} + Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}\,; \qquad (176)
\end{aligned}
$$

$$
\frac{\partial E\{\Pi_1^v\}}{\partial K_2} = \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\, b^w a^L + c_2 - 2\,c_1 \,\right]\left[a^L + 3\,Q_1^r + \frac{1}{b^w}\,(\,c_2 - 2\,c_1)\right]\frac{d\varepsilon_0}{dK_2}
$$

$$+ \frac{1}{9\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_2 - 2\, c_1 \,\right] \varepsilon_0\, \frac{d\varepsilon_0}{dK_2}$$

$$+ \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\, Q_1^r + \frac{1}{b^w} \left(c_2 - 2\, c_1 \right) \right] \varepsilon_0\, \frac{d\varepsilon_0}{dK_2} + \frac{1}{9\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2\, \frac{d\varepsilon_0}{dK_2}$$

$$+ \frac{1}{2}\, K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r - K_1 \right) + c_2 - 2\, c_1 \,\right] \left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2} \right]$$

$$+ \frac{1}{2}\, K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_2} - \varepsilon_0\, \frac{d\varepsilon_0}{dK_2} \right]$$

$$-\, K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_1 \,\right] \frac{d\varepsilon_{12}}{dK_2}$$

$$-\, K_1\, b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] -\, K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_2}$$

$$- \frac{1}{3}\, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_1 + c_2 \,\right] \frac{d\varepsilon_0}{dK_2} - \frac{1}{3}\, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0\, \frac{d\varepsilon_0}{dK_2}$$

$$- \frac{1}{2}\, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r - K_1 \right) + c_2 \,\right] \left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2} \right]$$

$$- \frac{1}{2}\, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_2} - \varepsilon_0\, \frac{d\varepsilon_0}{dK_2} \right] +\, b^w\, Q_1^r \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+\, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \frac{d\varepsilon_{12}}{dK_2} +\, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_2}\,; \qquad (177)$$

$$\frac{\partial E\left\{\Pi_1^v\right\}}{\partial r_1} = -\frac{1}{3}\, b_1 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_2 - 2\, c_1 \,\right]$$

$$+ \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_2 - 2\, c_1 \,\right] \left[a^L + 3\, Q_1^r + \frac{1}{b^w} \left(c_2 - 2\, c_1 \right) \right] \frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{9\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_2 - 2\, c_1 \,\right] \varepsilon_0\, \frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\, Q_1^r + \frac{1}{b^w} \left(c_2 - 2\, c_1 \right) \right] \varepsilon_0\, \frac{d\varepsilon_0}{dr_1}$$

$$- \frac{1}{6}\, b_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right] + \frac{1}{9\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2\, \frac{d\varepsilon_0}{dr_1}$$

$$
\begin{aligned}
&+\ \frac{1}{2}\,K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r-K_1\right)+c_2-2\,c_1\,\right]\left[\frac{d\varepsilon_{12}}{dr_1}-\frac{d\varepsilon_0}{dr_1}\right]\\
&-\frac{1}{2}\,K_1\,b^w\,b_1\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]+\frac{1}{2}\,K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_1}-\varepsilon_0\,\frac{d\varepsilon_0}{dr_1}\right]\\
&-\ K_1\,b^w\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b_1-d_2\,\right]\\
&-\ K_1\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1-K_2\right)-c_1\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_1}-\ K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_1}\\
&+\ Q_1^r-b_1\left[\,r_1-c_1^r\,\right]-\frac{1}{3}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\,a^L+c_1+c_2\,\right]\frac{d\varepsilon_0}{dr_1}\\
&+\ \frac{1}{3}\,b_1\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\,a^L+c_1+c_2\,\right]-\frac{1}{3}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_1}\\
&+\ \frac{1}{6}\,b_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right]\\
&-\frac{1}{2}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r-K_1\right)+c_2\,\right]\left[\frac{d\varepsilon_{12}}{dr_1}-\frac{d\varepsilon_0}{dr_1}\right]\\
&+\frac{1}{2}\,b^w\,b_1\,Q_1^r\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]+\frac{1}{2}\,b_1\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r-K_1\right)+c_2\,\right]\\
&-\ \frac{1}{2}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_1}-\varepsilon_0\,\frac{d\varepsilon_0}{dr_1}\right]+\ \frac{1}{4}\,b_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right]\\
&+\ b^w\,Q_1^r\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b_1-d_2\,\right]+\ b^w\,b_1\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]\\
&+\ Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]\frac{d\varepsilon_{12}}{dr_1}\\
&+\ \frac{1}{2}\,b_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2-(\varepsilon_{12})^2\,\right]+\ Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_1}\,;
\end{aligned}
\tag{178}
$$

$$
\begin{aligned}
\frac{\partial E\left\{\Pi_1^v\right\}}{\partial r_2} = {} & \frac{1}{3}\, d_1 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_2 - 2\, c_1 \,\right] \\
& + \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_2 - 2\, c_1 \,\right] \left[a^L + 3\, Q_1^r + \frac{1}{b^w}\,(\, c_2 - 2\, c_1\,) \right] \frac{d\varepsilon_0}{dr_2} \\
& + \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_2 - 2\, c_1 \,\right] \varepsilon_0\, \frac{d\varepsilon_0}{dr_2} \\
& + \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\, Q_1^r + \frac{1}{b^w}\,(\, c_2 - 2\, c_1) \right] \varepsilon_0\, \frac{d\varepsilon_0}{dr_2} \\
& + \frac{1}{6}\, d_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right] + \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2\, \frac{d\varepsilon_0}{dr_2} \\
& + \frac{1}{2}\, K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r - K_1 \right) + c_2 - 2\, c_1 \,\right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right] \\
& + \frac{1}{2}\, K_1\, b^w\, d_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
& + \frac{1}{2}\, K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0\, \frac{d\varepsilon_0}{dr_2} \right] -\, K_1\, b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b_2 - d_1 \,\right] \\
& -\, K_1 \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_1 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_2} -\, K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12}\, \frac{d\varepsilon_{12}}{dr_2} \\
& + d_1 \left[\, r_1 - c_1^r \,\right] - \frac{1}{3}\, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_1 + c_2 \,\right] \frac{d\varepsilon_0}{dr_2} \\
& - \frac{1}{3}\, d_1 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_1 + c_2 \,\right] - \frac{1}{3}\, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0\, \frac{d\varepsilon_0}{dr_2} \\
& - \frac{1}{6}\, d_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right] \\
& - \frac{1}{2}\, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r - K_1 \right) + c_2 \,\right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right] \\
& - \frac{1}{2}\, d_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r - K_1 \right) + c_2 \,\right] - \frac{1}{2}\, Q_1^r\, b^w\, d_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
& - \frac{1}{2}\, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0\, \frac{d\varepsilon_0}{dr_2} \right] -\, \frac{1}{4}\, d_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right]
\end{aligned}
$$

$$+ \, b^w \, Q_1^r \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b_2 - d_1 \,] - \, b^w \, d_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right]$$

$$+ \, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \frac{d\varepsilon_{12}}{dr_2}$$

$$- \, \frac{1}{2} \, d_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \,\right] + \, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_2} \tag{179}$$

where ε_0 and ε_{12} are as specified in (172).

(170), (171), and (154) imply that in the present case:

$$\frac{\partial E\{\Pi_2^v\}}{\partial K_1} = \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \, a^L + c_1 - 2\, c_2 \,\right] \left[a^L + 3\, Q_2^r + \frac{1}{b^w} \left(\, c_1 - 2\, c_2 \right) \right] \frac{d\varepsilon_0}{dK_1}$$

$$+ \, \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \, a^L + c_1 - 2\, c_2 \,\right] \varepsilon_0 \, \frac{d\varepsilon_0}{dK_1}$$

$$+ \, \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\, Q_2^r + \frac{1}{b^w} \left(\, c_1 - 2\, c_2 \right) \right] \varepsilon_0 \, \frac{d\varepsilon_0}{dK_1} + \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2 \, \frac{d\varepsilon_0}{dK_1}$$

$$- \, \frac{1}{4} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(\, a^L + Q_1^r - K_1 \,\right) - c_2 \,\right]$$

$$- \, \frac{1}{4} \, b^w \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, a^L + Q_1^r + 2\, Q_2^r - K_1 - \frac{c_2}{b^w} \,\right]$$

$$+ \, \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(\, a^L + Q_1^r - K_1 \,\right) - c_2 \,\right] \left[\, a^L + Q_1^r + 2\, Q_2^r - K_1 - \frac{c_2}{b^w} \,\right] \left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1} \right]$$

$$- \, \frac{1}{8} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right]$$

$$+ \, \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(\, a^L + Q_1^r - K_1 \,\right) - c_2 \,\right] \left[\varepsilon_{12} \, \frac{d\varepsilon_{12}}{dK_1} - \varepsilon_0 \, \frac{d\varepsilon_0}{dK_1} \right]$$

$$- \, \frac{1}{8} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right]$$

$$+ \, \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, a^L + Q_1^r + 2\, Q_2^r - K_1 - \frac{c_2}{b^w} \,\right] \left[\varepsilon_{12} \, \frac{d\varepsilon_{12}}{dK_1} - \varepsilon_0 \, \frac{d\varepsilon_0}{dK_1} \right]$$

$$+ \, \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 \, \frac{d\varepsilon_{12}}{dK_1} - (\varepsilon_0)^2 \, \frac{d\varepsilon_0}{dK_1} \right] - \, K_2 \, b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$
\begin{aligned}
&- K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_2 \right] \frac{d\varepsilon_{12}}{dK_1} - K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \frac{d\varepsilon_{12}}{dK_1} \\
&- \frac{1}{3} Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w a^L + c_1 + c_2 \right] \frac{d\varepsilon_0}{dK_1} - \frac{1}{3} Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \frac{d\varepsilon_0}{dK_1} \\
&- \frac{1}{2} Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r - K_1 \right) + c_2 \right] \left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1} \right] + \frac{1}{2} b^w Q_2^r \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&- \frac{1}{2} Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dK_1} - \varepsilon_0 \frac{d\varepsilon_0}{dK_1} \right] + b^w Q_2^r \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \frac{d\varepsilon_{12}}{dK_1} + Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \frac{d\varepsilon_{12}}{dK_1} ; \qquad (180)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E\left\{\Pi_2^v\right\}}{\partial K_2} = &\frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w a^L + c_1 - 2 c_2 \right] \left[a^L + 3 Q_2^r + \frac{1}{b^w} \left(c_1 - 2 c_2 \right) \right] \frac{d\varepsilon_0}{dK_2} \\
&+ \frac{1}{9 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w a^L + c_1 - 2 c_2 \right] \varepsilon_0 \frac{d\varepsilon_0}{dK_2} \\
&+ \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3 Q_2^r + \frac{1}{b^w} \left(c_1 - 2 c_2 \right) \right] \varepsilon_0 \frac{d\varepsilon_0}{dK_2} + \frac{1}{9 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left(\varepsilon_0 \right)^2 \frac{d\varepsilon_0}{dK_2} \\
&+ \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r - K_1 \right) - c_2 \right] \left[a^L + Q_1^r + 2 Q_2^r - K_1 - \frac{c_2}{b^w} \right] \left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2} \right] \\
&+ \frac{1}{4 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r - K_1 \right) - c_2 \right] \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dK_2} - \varepsilon_0 \frac{d\varepsilon_0}{dK_2} \right] \\
&+ \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + 2 Q_2^r - K_1 - \frac{c_2}{b^w} \right] \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dK_2} - \varepsilon_0 \frac{d\varepsilon_0}{dK_2} \right] \\
&+ \frac{1}{4 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\left(\varepsilon_{12} \right)^2 \frac{d\varepsilon_{12}}{dK_2} - \left(\varepsilon_0 \right)^2 \frac{d\varepsilon_0}{dK_2} \right] - K_2 b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_2 \right] \\
&- K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_2 \right] \frac{d\varepsilon_{12}}{dK_2} \\
&+ \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\left(\overline{\varepsilon} \right)^2 - \left(\varepsilon_{12} \right)^2 \right] - K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \frac{d\varepsilon_{12}}{dK_2}
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{1}{3}\, Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_1 + c_2 \,\right] \frac{d\varepsilon_0}{dK_2} - \frac{1}{3}\, Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0\, \frac{d\varepsilon_0}{dK_2} \\
&- \frac{1}{2}\, Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r - K_1 \right) + c_2 \,\right] \left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2} \right] \\
&- \frac{1}{2}\, Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_2} - \varepsilon_0\, \frac{d\varepsilon_0}{dK_2} \right] + b^w\, Q_2^r \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \frac{d\varepsilon_{12}}{dK_2} + Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_2}\,; \qquad (181)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E\left\{\Pi_2^v\right\}}{\partial r_1} = \;& \frac{1}{3}\, d_2 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_1 - 2\, c_2 \,\right] \\
&+ \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_1 - 2\, c_2 \,\right] \left[a^L + 3\, Q_2^r + \frac{1}{b^w} \left(\, c_1 - 2\, c_2 \,\right) \right] \frac{d\varepsilon_0}{dr_1} \\
&+ \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_1 - 2\, c_2 \,\right] \varepsilon_0\, \frac{d\varepsilon_0}{dr_1} \\
&+ \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\, Q_2^r + \frac{1}{b^w} \left(\, c_1 - 2\, c_2 \,\right) \right] \varepsilon_0\, \frac{d\varepsilon_0}{dr_1} \\
&+ \frac{1}{6}\, d_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right] + \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2\, \frac{d\varepsilon_0}{dr_1} \\
&+ \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r - K_1 \right) - c_2 \,\right] \left[a^L + Q_1^r + 2\, Q_2^r - K_1 - \frac{c_2}{b^w} \right] \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1} \right] \\
&- \frac{1}{4}\, b^w\, b_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + 2\, Q_2^r - K_1 - \frac{c_2}{b^w} \right] \\
&- \frac{1}{4} \left[\, b_1 - 2\, d_2 \,\right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r - K_1 \right) - c_2 \,\right] \\
&- \frac{1}{8}\, b_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right] \\
&+ \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r - K_1 \right) - c_2 \,\right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0\, \frac{d\varepsilon_0}{dr_1} \right] \\
&+ \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + 2\, Q_2^r - K_1 - \frac{c_2}{b^w} \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0\, \frac{d\varepsilon_0}{dr_1} \right] \\
&- \frac{1}{8} \left[\, b_1 - 2\, d_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right]
\end{aligned}
$$

$$+ \frac{1}{4\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 \, \frac{d\varepsilon_{12}}{dr_1} - (\varepsilon_0)^2 \, \frac{d\varepsilon_0}{dr_1} \right]$$

$$- \, K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \, \right) - c_2 \, \right] \frac{d\varepsilon_{12}}{dr_1}$$

$$- \, K_2 \, b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b_1 - d_2 \,] - \, K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_1}$$

$$+ \, d_2 \, [\, r_2 - c_2^r \,] - \frac{1}{3} \, Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \, a^L + c_1 + c_2 \, \right] \frac{d\varepsilon_0}{dr_1}$$

$$- \, \frac{1}{3} \, d_2 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \, a^L + c_1 + c_2 \, \right] - \frac{1}{3} \, Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \, \frac{d\varepsilon_0}{dr_1}$$

$$- \, \frac{1}{6} \, d_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \, \right]$$

$$- \, \frac{1}{2} \, Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(\, a^L + Q_1^r - K_1 \, \right) + c_2 \, \right] \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1} \right]$$

$$+ \, \frac{1}{2} \, b^w \, b_1 \, Q_2^r \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - \frac{1}{2} \, d_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(\, a^L + Q_1^r - K_1 \, \right) + c_2 \, \right]$$

$$- \, \frac{1}{2} \, Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0 \, \frac{d\varepsilon_0}{dr_1} \right] - \, \frac{1}{4} \, d_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \, \right]$$

$$+ \, b^w \, Q_2^r \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b_1 - d_2 \,] - \, b^w \, d_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \, \right]$$

$$+ \, Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \, \right] \frac{d\varepsilon_{12}}{dr_1}$$

$$- \, \frac{1}{2} \, d_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \, \right] + \, Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_1} \, ; \tag{182}$$

$$\frac{\partial E \, \{ \Pi_2^v \}}{\partial r_2} = \, - \frac{1}{3} \, b_2 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \, a^L + c_1 - 2 \, c_2 \, \right]$$

$$+ \, \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \, a^L + c_1 - 2 \, c_2 \, \right] \left[a^L + 3 \, Q_2^r + \frac{1}{b^w} \left(\, c_1 - 2 \, c_2 \, \right) \right] \frac{d\varepsilon_0}{dr_2}$$

$$+ \, \frac{1}{9 \, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \, a^L + c_1 - 2 \, c_2 \, \right] \varepsilon_0 \, \frac{d\varepsilon_0}{dr_2}$$

$$+\ \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+3\,Q_2^r+\frac{1}{b^w}\,(\,c_1-2\,c_2\,)\right]\varepsilon_0\ \frac{d\varepsilon_0}{dr_2}$$

$$-\ \frac{1}{6}\,b_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right]+\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right](\varepsilon_0)^2\ \frac{d\varepsilon_0}{dr_2}$$

$$+\ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r-K_1\,\right)-c_2\,\right]\left[\,a^L+Q_1^r+2\,Q_2^r-K_1-\frac{c_2}{b^w}\,\right]\left[\frac{d\varepsilon_{12}}{dr_2}-\frac{d\varepsilon_0}{dr_2}\right]$$

$$+\ \frac{1}{4}\,b^w\,d_1\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,a^L+Q_1^r+2\,Q_2^r-K_1-\frac{c_2}{b^w}\,\right]$$

$$-\ \frac{1}{4}\left[\,2\,b_2-d_1\,\right]\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r-K_1\,\right)-c_2\,\right]$$

$$+\ \frac{1}{8}\,d_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right]$$

$$+\ \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r-K_1\,\right)-c_2\,\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_2}-\varepsilon_0\,\frac{d\varepsilon_0}{dr_2}\right]$$

$$+\ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,a^L+Q_1^r+2\,Q_2^r-K_1-\frac{c_2}{b^w}\,\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_2}-\varepsilon_0\,\frac{d\varepsilon_0}{dr_2}\right]$$

$$-\ \frac{1}{8}\left[\,2\,b_2-d_1\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right]$$

$$+\ \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[(\varepsilon_{12})^2\ \frac{d\varepsilon_{12}}{dr_2}-(\varepsilon_0)^2\ \frac{d\varepsilon_0}{dr_2}\right]$$

$$-\ K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right)-c_2\,\right]\frac{d\varepsilon_{12}}{dr_2}$$

$$-\ K_2\,b^w\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b_2-d_1\,\right]-\ K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\ \frac{d\varepsilon_{12}}{dr_2}$$

$$+\ Q_2^r-b_2\left[\,r_2-c_2^r\,\right]-\frac{1}{3}\,Q_2^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\,a^L+c_1+c_2\,\right]\frac{d\varepsilon_0}{dr_2}$$

$$+\ \frac{1}{3}\,b_2\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\,a^L+c_1+c_2\,\right]-\frac{1}{3}\,Q_2^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\ \frac{d\varepsilon_0}{dr_2}$$

$$+\ \frac{1}{6}\,b_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right]$$

$$-\ \frac{1}{2}\,Q_2^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r-K_1\,\right)+c_2\,\right]\left[\frac{d\varepsilon_{12}}{dr_2}-\frac{d\varepsilon_0}{dr_2}\right]$$

$$- \frac{1}{2} b^w d_1 Q_2^r \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{2} b_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r - K_1 \right) + c_2 \right]$$

$$- \frac{1}{2} Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0 \frac{d\varepsilon_0}{dr_2} \right] + \frac{1}{4} b_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$+ b^w Q_2^r \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [b_2 - d_1] + b^w b_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right]$$

$$+ Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \frac{d\varepsilon_{12}}{dr_2}$$

$$+ \frac{1}{2} b_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right] + Q_2^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \frac{d\varepsilon_{12}}{dr_2} \tag{183}$$

where ε_0 and ε_{12} are as specified in (172).

Additional Characterization of the Optimal Retail Prices

(77) implies that when G1 is capacity-constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$E\left\{q_1^{*0c}\right\} \equiv \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon)\, dH(\varepsilon) = \frac{1}{6\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]$$

$$+ \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\,(a_1 - b_1 r_1 + d_1 r_2) + \frac{1}{b^w}(c_2 - 2\, c_1) \right];$$

$$E\left\{q_2^{*0c}\right\} \equiv \int_{\underline{\varepsilon}}^{\varepsilon_0} q_2^{*0c}(\varepsilon)\, dH(\varepsilon) = \frac{1}{6\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]$$

$$+ \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\,(a_2 - b_2 r_2 + d_2 r_1) + \frac{1}{b^w}(c_1 - 2\, c_2) \right]$$

$$E\left\{q_2^{*1c}\right\} \equiv \int_{\varepsilon_0}^{\varepsilon_{12}} q_2^{*1c}(\varepsilon)\, dH(\varepsilon) = \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$+ \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + a_1 - b_1 r_1 + d_1 r_2 + 2\,(a_2 - b_2 r_2 + d_2 r_1) - K_1 - \frac{c_2}{b^w} \right] \tag{184}$$

where ε_0 and ε_{12} are as specified in (142). (184) implies:

$$\frac{\partial E\left\{q_1^{*0c}\right\}}{\partial r_1} = -b_1 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{3\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \frac{d\varepsilon_0}{dr_1}$$

$$+ \; \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+3\,(a_1-b_1\,r_1+d_1\,r_2)+\frac{1}{b^w}\,(c_2-2\,c_1)\right]\frac{d\varepsilon_0}{dr_1}\,;$$

$$\frac{\partial E\left\{q_1^{*0c}\right\}}{\partial r_2} \;=\; d_1\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]+\frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_2}$$

$$+ \; \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+3\,(a_1-b_1\,r_1+d_1\,r_2)+\frac{1}{b^w}\,(c_2-2\,c_1)\right]\frac{d\varepsilon_0}{dr_2}\,;$$

$$\frac{\partial E\left\{q_1^{*0c}\right\}}{\partial K_1} \;=\; \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}$$

$$+ \; \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+3\,(a_1-b_1\,r_1+d_1\,r_2)+\frac{1}{b^w}\,(c_2-2\,c_1)\right]\frac{d\varepsilon_0}{dK_1}\,;$$

$$\frac{\partial E\left\{q_1^{*0c}\right\}}{\partial K_2} \;=\; \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}$$

$$+ \; \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+3\,(a_1-b_1\,r_1+d_1\,r_2)+\frac{1}{b^w}\,(c_2-2\,c_1)\right]\frac{d\varepsilon_0}{dK_2}\,;$$

$$\frac{\partial E\left\{q_2^{*0c}\right\}}{\partial r_1} \;=\; d_2\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]+\frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_1}$$

$$+ \; \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+3\,(a_2-b_2\,r_2+d_2\,r_1)+\frac{1}{b^w}\,(\,c_1-2\,c_2\,)\right]\frac{d\varepsilon_0}{dr_1}\,;$$

$$\frac{\partial E\left\{q_2^{*0c}\right\}}{\partial r_2} \;=\; -\,b_2\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]+\frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_2}$$

$$+ \; \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+3\,(a_2-b_2\,r_2+d_2\,r_1)+\frac{1}{b^w}\,(\,c_1-2\,c_2\,)\right]\frac{d\varepsilon_0}{dr_2}\,;$$

$$\frac{\partial E\left\{q_2^{*0c}\right\}}{\partial K_1} \;=\; \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}$$

$$+ \; \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+3\,(a_2-b_2\,r_2+d_2\,r_1)+\frac{1}{b^w}\,(\,c_1-2\,c_2\,)\right]\frac{d\varepsilon_0}{dK_1}\,;$$

$$\frac{\partial E\left\{q_2^{*0c}\right\}}{\partial K_2} \;=\; \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}$$

$$+ \; \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+3\,(a_2-b_2\,r_2+d_2\,r_1)+\frac{1}{b^w}\,(\,c_1-2\,c_2\,)\right]\frac{d\varepsilon_0}{dK_2}\,;$$

$$\frac{\partial E\left\{q_2^{*1c}\right\}}{\partial r_1} = -\frac{1}{2}\left[b_1 - 2\,d_2\right]\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right] + \frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0\,\frac{d\varepsilon_0}{dr_1}\right]$$
$$+\ \frac{1}{2}\left[a^L + a_1 - b_1\,r_1 + d_1\,r_2 + 2\left(a_2 - b_2\,r_2 + d_2\,r_1\right) - K_1 - \frac{c_2}{b^w}\right]$$
$$\cdot\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1}\right];$$

$$\frac{\partial E\left\{q_2^{*1c}\right\}}{\partial r_2} = -\frac{1}{2}\left[2\,b_2 - d_1\right]\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right] + \frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0\,\frac{d\varepsilon_0}{dr_2}\right]$$
$$+\ \frac{1}{2}\left[a^L + a_1 - b_1\,r_1 + d_1\,r_2 + 2\left(a_2 - b_2\,r_2 + d_2\,r_1\right) - K_1 - \frac{c_2}{b^w}\right]$$
$$\cdot\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2}\right];$$

$$\frac{\partial E\left\{q_2^{*1c}\right\}}{\partial K_1} = -\frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right] + \frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1} - \varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right]$$
$$+\ \frac{1}{2}\left[a^L + a_1 - b_1\,r_1 + d_1\,r_2 + 2\left(a_2 - b_2\,r_2 + d_2\,r_1\right) - K_1 - \frac{c_2}{b^w}\right]$$
$$\cdot\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1}\right];$$

$$\frac{\partial E\left\{q_2^{*1c}\right\}}{\partial K_2} = \frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2} - \varepsilon_0\,\frac{d\varepsilon_0}{dK_2}\right]$$
$$+\ \frac{1}{2}\left[a^L + a_1 - b_1\,r_1 + d_1\,r_2 + 2\left(a_2 - b_2\,r_2 + d_2\,r_1\right) - K_1 - \frac{c_2}{b^w}\right]$$
$$\cdot\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2}\right]. \tag{185}$$

(78) and (82) imply that when G1 is capacity-constrained for $\varepsilon \in (\,\varepsilon_0, \varepsilon_{12}\,)$:

$$J_1(r_1, r_2) \equiv \left[1 + b^w\,b_1\right]\left[a_1 - b_1\,r_1 + d_1\,r_2\right]\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] - b^w\,b_1\,E\left\{q_1^{*0c}\right\} + b_1\,E\left\{w(\cdot)\right\}$$
$$-\ b_1\left[r_1 - c_1^r\right] - \frac{1}{2}\,b^w\,b_1\,K_1\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$
$$+\left[1 + \frac{1}{2}\,b^w\,b_1\right]\left[a_1 - b_1\,r_1 + d_1\,r_2\right]\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$- \; b^w \, [\, b_1 - d_2 \,] \; K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + [\, b^w \, b_1 + 1 - b^w \, d_2 \,] \, [\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,] \; \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \; = \; 0;$$

$$J_2(r_1, r_2) \;\equiv\; [\, 1 + b^w \, b_2 \,] \, [\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - \; b^w \, b_2 \, E \left\{ q_2^{*0c} \right\} + b_2 \, E \left\{ w(\cdot) \right\}$$

$$- \; b_2 \, [\, r_2 - c_2^r \,] - \; b^w \, [\, b_2 - d_1 \,] \, E \left\{ q_2^{*1c} \right\}$$

$$+ \; [\, b^w \, b_2 + 1 - b^w \, d_1 \,] \, [\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w \, [\, b_2 - d_1 \,] \, K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \; [\, b^w \, b_2 + 1 - b^w \, d_1 \,] \, [\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \; = \; 0 \tag{186}$$

where: (i) $E\{w(\cdot)\}$ is specified in (174); (ii) $E\{q_1^{*0c}\}$, $E\{q_2^{*0c}\}$, and $E\{q_2^{*1c}\}$ are specified in (184); and (iii) ε_0 and ε_{12} are as specified in (172).

For $v \in \{\, r_1, \, r_2, \, K_1, \, K_2 \}$, define:

$$M_1^B(v) \;\equiv\; [\, 1 + b^w \, b_1 \,] \, [\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dv} - \; b^w \, b_1 \, \frac{\partial E \left\{ q_1^{*0c} \right\}}{\partial v}$$

$$+ \; b_1 \, \frac{\partial E \left\{ w \left(\cdot \right) \right\}}{\partial v} - \frac{1}{2} \, b^w \, b_1 \, K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dv} - \frac{d\varepsilon_0}{dv} \right]$$

$$+ \; \left[1 + \frac{1}{2} \, b^w \, b_1 \right] [\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dv} - \frac{d\varepsilon_0}{dv} \right]$$

$$+ \; b^w \, K_1 \, [\, b_1 - d_2 \,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dv}$$

$$- \; [\, b^w \, b_1 + 1 - b^w \, d_2 \,] \, [\, a_1 - b_1 \, r_1 + d_1 \, r_2 \,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dv} \, ;$$

$$M_2^B(v) \;\equiv\; [\, 1 + b^w \, b_2 \,] \, [\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dv}$$

$$- \; b^w \, b_2 \, \frac{\partial E \left\{ q_2^{*0c} \right\}}{\partial v} + b_2 \, \frac{\partial E \left\{ w \left(\cdot \right) \right\}}{\partial v} - \; b^w \, [\, b_2 - d_1 \,] \; \frac{\partial E \left\{ q_2^{*1c} \right\}}{\partial v}$$

$$+ \; [\, b^w \, b_2 + 1 - b^w \, d_1 \,] \, [\, a_2 - b_2 \, r_2 + d_2 \, r_1 \,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dv} - \frac{d\varepsilon_0}{dv} \right]$$

$$+ \; b^w \, [\, b_2 - d_1 \,] \, K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dv}$$

$$- \left[b^w b_2 + 1 - b^w d_1 \right] \left[a_2 - b_2 r_2 + d_2 r_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dv}. \tag{187}$$

(186) and (187) imply:

$$\begin{aligned}
\frac{\partial J_1(\cdot)}{\partial r_1} &= -b_1 \left[1 + b^w b_1 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b_1 - b_1 \left[1 + \frac{1}{2} b^w b_1 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&\quad - b_1 \left[b^w b_1 + 1 - b^w d_2 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + M_1^B(r_1); \\
\frac{\partial J_1(\cdot)}{\partial r_2} &= d_1 \left[1 + b^w b_1 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + d_1 \left[1 + \frac{1}{2} b^w b_1 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&\quad + d_1 \left[b^w b_1 + 1 - b^w d_2 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + M_1^B(r_2); \\
\frac{\partial J_1(\cdot)}{\partial K_1} &= -\frac{1}{2} b^w b_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w \left[b_1 - d_2 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + M_1^B(K_1); \\
\frac{\partial J_1(\cdot)}{\partial K_2} &= M_1^B(K_2); \\
\frac{\partial J_2(\cdot)}{\partial r_1} &= d_2 \left[1 + b^w b_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + d_2 \left[b^w b_2 + 1 - b^w d_1 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&\quad + d_2 \left[b^w b_2 + 1 - b^w d_1 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + M_2^B(r_1); \\
\frac{\partial J_2(\cdot)}{\partial r_2} &= -b_2 \left[1 + b^w b_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b_2 \left[1 + b^w b_2 - b^w d_1 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&\quad - b_2 - b_2 \left[1 + b^w b_2 - b^w d_1 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + M_2^B(r_2); \\
\frac{\partial J_2(\cdot)}{\partial K_1} &= M_2^B(K_1); \\
\frac{\partial J_2(\cdot)}{\partial K_2} &= -b^w \left[b_2 - d_1 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + M_2^B(K_2).
\end{aligned} \tag{188}$$

Case 3. Partial Vertical Integration ($\alpha_1^R = \alpha_1^G = 1$, $\alpha_2^R = \alpha_2^G = 0$).

(111) implies that in this case, for $\varepsilon \in [\, \underline{\varepsilon}, \varepsilon_0 \,]$:

$$q_1^{*0c}(\varepsilon) \;=\; \frac{1}{3}\left[\, a^L + 3\, Q_1^r + Q_2^r \,\right] + \frac{1}{3\, b^w}\left[\, \varepsilon + c_2 - 2\, c_1 \,\right] ;$$

$$q_2^{*0c}(\varepsilon) \;=\; \frac{1}{3}\left[\, a^L + Q_2^r \,\right] + \frac{1}{3\, b^w}\left[\, \varepsilon + c_1 - 2\, c_2 \,\right] ;$$

$$w^{0c}(\varepsilon) \;=\; \frac{1}{3}\, b^w \left[\, a^L + Q_2^r \,\right] + \frac{1}{3}\left[\, \varepsilon + c_1 + c_2 \,\right] . \tag{189}$$

(112) and (113) imply that if G2 is capacity-constrained for $\varepsilon \in (\, \varepsilon_0, \varepsilon_{12} \,)$:

$$\varepsilon_0 \;=\; 3\, b^w\, K_2 + 2\, c_2 - c_1 - b^w \left[\, a^L + Q_2^r \,\right] ;$$

$$\varepsilon_{12} \;=\; 2\, b^w\, K_1 + b^w\, K_2 + c_1 - b^w \left[\, a^L + 2\, Q_1^r + Q_2^r \,\right] ;$$

$$q_1^{*1c}(\varepsilon) \;=\; \frac{1}{2}\left[\, a^L + 2\, Q_1^r + Q_2^r - K_2 \,\right] + \frac{1}{2\, b^w}\left[\, \varepsilon - c_1 \,\right] ;$$

$$q_2^{*1c}(\varepsilon) \;=\; K_2 \,;$$

$$w^{1c}(\varepsilon) \;=\; \frac{b^w}{2}\left[\, a^L + Q_2^r - K_2 \,\right] + \frac{1}{2}\left[\, c_1 + \varepsilon \,\right] . \tag{190}$$

(114) and (115) imply that if G1 is capacity-constrained for $\varepsilon \in (\, \varepsilon_0, \varepsilon_{12} \,)$:

$$\varepsilon_0 \;=\; 3\, b^w\, K_1 + 2\, c_1 - c_2 - b^w \left[\, a^L + 3\, Q_1^r + Q_2^r \,\right] ;$$

$$\varepsilon_{12} \;=\; 2\, b^w\, K_2 + b^w\, K_1 + c_2 - b^w \left[\, a^L + Q_1^r + Q_2^r \,\right] ;$$

$$q_1^{*1c}(\varepsilon) \;=\; K_1 \,;$$

$$q_2^{*1c}(\varepsilon) \;=\; \frac{1}{2}\left[\, a^L + Q_1^r + Q_2^r - K_1 \,\right] + \frac{1}{2\, b^w}\left[\, \varepsilon - c_2 \,\right] ;$$

$$w^{1c}(\varepsilon) \;=\; \frac{b^w}{2}\left[\, a^L + Q_1^r + Q_2^r - K_1 \,\right] + \frac{1}{2}\left[\, c_2 + \varepsilon \,\right] . \tag{191}$$

(116) implies that for $\varepsilon \in [\, \varepsilon_{12}, \overline{\varepsilon} \,]$:

$$q_1^{*2c}(\varepsilon) \;=\; K_1 \,; \quad q_2^{*2c}(\varepsilon) \;=\; K_2 \,; \quad w^{2c}(\varepsilon) \;=\; b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] + \varepsilon \,. \tag{192}$$

Under partial vertical integration, G1 chooses K_1 to maximize:

$$
\begin{aligned}
& E\,\{\,[\,w(\cdot) - c_1\,]\,q_1 + [\,r_1 - w(\cdot) - c_1^r\,]\,Q_1^r\,\} - k_1\,K_1 \\
& = \int_{\underline{\varepsilon}}^{\varepsilon_0} \left[\,w^{0c}(\varepsilon) - c_1\,\right] q_1^{*0c}(\varepsilon)\,dH(\varepsilon) + \int_{\underline{\varepsilon}}^{\varepsilon_0} \left[\,r_1 - w^{0c}(\varepsilon) - c_1^r\,\right] Q_1^r\,dH(\varepsilon) \\
& \quad + \int_{\varepsilon_0}^{\varepsilon_{12}} \left[\,w^{1c}(\varepsilon) - c_1\,\right] q_1^{*1c}(\varepsilon)\,dH(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_{12}} \left[\,r_1 - w^{1c}(\varepsilon) - c_1^r\,\right] Q_1^r\,dH(\varepsilon) \\
& \quad + \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left[\,w^{2c}(\varepsilon) - c_1\,\right] K_1\,dH(\varepsilon) + \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left[\,r_1 - w^{2c}(\varepsilon) - c_1^r\,\right] Q_1^r\,dH(\varepsilon) - k_1\,K_1\,, \qquad (193)
\end{aligned}
$$

where, from (1):

$$
Q_1^r \;=\; a_1 - b_1\,r_1\, + d_1\,r_2\,. \qquad (194)
$$

(3) implies that under partial vertical integration, G2 chooses K_2 to maximize:

$$
\begin{aligned}
& E\,\{\,[\,w(\varepsilon) - c_2\,]\,q_2^*(\varepsilon)\,\} - k_2\,K_2 \\
& = \int_{\underline{\varepsilon}}^{\varepsilon_0} \left[\,w^{0c}(\varepsilon) - c_2\,\right] q_2^{*0c}(\varepsilon)\,dH(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_{12}} \left[\,w^{1c}(\varepsilon) - c_2\,\right] q_2^{*1c}(\varepsilon)\,dH(\varepsilon) \\
& \quad + \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left[\,w^{2c}(\varepsilon) - c_2\,\right] K_2\,dH(\varepsilon) - k_2\,K_2\,. \qquad (195)
\end{aligned}
$$

Case 3A. Partial vertical integration where G2 is capacity-constrained for $\varepsilon \in (\, \varepsilon_0, \varepsilon_{12} \,)$.

(189), (190), and (192) imply that in this case:

$$E\{w(\cdot)\} = \frac{1}{3}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, b^w(a^L + Q_2^r) + c_1 + c_2 \,\right] + \frac{1}{6}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right]$$

$$+ \frac{1}{2}\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, b^w(a^L + Q_2^r - K_2) + c_1 \,\right] + \frac{1}{4}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right]$$

$$+ \; b^w\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] + \; \frac{1}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \,\right]. \quad (196)$$

(189), (190), and (192) also imply:(189), (190), and (192) imply that in this case:

$$E\{\, [\, w(\varepsilon) - c_1 \,]\, q_1^*(\varepsilon) \,\} \;\; =$$

$$\frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0} \{\, b^w\left[\, a^L + Q_2^r \,\right] + c_2 - 2\,c_1 + \varepsilon \,\} \left\{ a^L + 3\,Q_1^r + Q_2^r + \frac{1}{b^w}\left[\, c_2 - 2\,c_1 + \varepsilon \,\right] \right\} dH(\varepsilon)$$

$$+ \; \frac{1}{4}\int_{\varepsilon_0}^{\varepsilon_{12}} \{\, b^w\left[\, a^L + Q_2^r - K_2 \,\right] - c_1 + \varepsilon \,\} \left\{ a^L + 2\,Q_1^r + Q_2^r - K_2 + \frac{1}{b^w}\left[\, - c_1 + \varepsilon \,\right] \right\} dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \{\, b^w\left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] - c_1 + \varepsilon \,\}\; K_1 \; dH(\varepsilon)$$

$$= \frac{1}{9}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, b^w\left(\, a^L + Q_2^r \,\right) + c_2 - 2\,c_1 \,\right]\left[\, a^L + 3\,Q_1^r + Q_2^r + \frac{1}{b^w}\left(\, c_2 - 2\,c_1 \,\right)\right]$$

$$+ \; \frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0} \left[\, b^w\left(\, a^L + Q_2^r \,\right) + c_2 - 2\,c_1 \,\right] \frac{1}{b^w}\, \varepsilon \; dH(\varepsilon)$$

$$+ \; \frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0} \left[\, a^L + 3\,Q_1^r + Q_2^r + \frac{1}{b^w}\left(\, c_2 - 2\,c_1 \,\right)\right] \varepsilon \; dH(\varepsilon) + \frac{1}{9\,b^w}\int_{\underline{\varepsilon}}^{\varepsilon_0} \varepsilon^2 \, dH(\varepsilon)$$

$$+ \; \frac{1}{4}\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, b^w\left(\, a^L + Q_2^r - K_2 \,\right) - c_1 \,\right]\left[\, a^L + 2\,Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w}\,\right]$$

$$+ \; \frac{1}{4}\int_{\varepsilon_0}^{\varepsilon_{12}} \left[\, b^w\left(\, a^L + Q_2^r - K_2 \,\right) - c_1 \,\right] \frac{1}{b^w}\, \varepsilon \; dH(\varepsilon)$$

$$
\begin{aligned}
&+\ \frac{1}{4}\int_{\varepsilon_0}^{\varepsilon_{12}}\left[\,a^L+2\,Q_1^r+Q_2^r-K_2-\frac{c_1}{b^w}\,\right]\varepsilon\;dH(\varepsilon)+\frac{1}{4\,b^w}\int_{\varepsilon_0}^{\varepsilon_{12}}\varepsilon^2\,dH(\varepsilon)\\
&+\ K_1\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1-K_2\right)-c_1\,\right]\\
&+\ \frac{K_1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2-(\varepsilon_{12})^2\,\right]\\
=&\ \ \frac{1}{9}\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_2^r\right)+c_2-2\,c_1\,\right]\left[a^L+3\,Q_1^r+Q_2^r+\frac{1}{b^w}\left(\,c_2-2\,c_1\,\right)\right]\\
&+\ \frac{1}{18\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_2^r\right)+c_2-2\,c_1\,\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right]\\
&+\ \frac{1}{18}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+3\,Q_1^r+Q_2^r+\frac{1}{b^w}\left(\,c_2-2\,c_1\,\right)\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right]\\
&+\ \frac{1}{27\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^3-(\underline{\varepsilon})^3\,\right]\\
&+\ \frac{1}{4}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_2^r-K_2\right)-c_1\,\right]\left[\,a^L+2\,Q_1^r+Q_2^r-K_2-\frac{c_1}{b^w}\,\right]\\
&+\ \frac{1}{8\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_2^r-K_2\right)-c_1\,\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right]\\
&+\ \frac{1}{8}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,a^L+2\,Q_1^r+Q_2^r-K_2-\frac{c_1}{b^w}\,\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right]\\
&+\ \frac{1}{12\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^3-(\varepsilon_0)^3\,\right]\\
&+\ K_1\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1-K_2\right)-c_1\,\right]\\
&+\ \frac{K_1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2-(\varepsilon_{12})^2\,\right]
\end{aligned}
\tag{197}
$$

where ε_0 and ε_{12} are as specified in (190).

Furthermore, from (196):

$$E\{\,[\,r_1 - w(\cdot) - c_1^r\,]\,Q_1^r\,\} \;=\; [\,r_1 - c_1^r\,]\,Q_1^r - Q_1^r\,E\{\,w(\cdot)\,\}$$

$$= \;[\,r_1 - c_1^r\,]\,Q_1^r$$

$$- \,Q_1^r \left\{ \frac{1}{3}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\,b^w\left(a^L + Q_2^r\right) + c_1 + c_2\,] + \frac{1}{6}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\,(\varepsilon_0)^2 - (\underline{\varepsilon})^2\,]\right.$$

$$+ \,\frac{1}{2}\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\,b^w\left(a^L + Q_2^r - K_2\right) + c_1\,] + \frac{1}{4}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\,(\varepsilon_{12})^2 - (\varepsilon_0)^2\,]$$

$$\left. + \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] b^w\,[\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,] + \frac{1}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\,(\overline{\varepsilon})^2 - (\varepsilon_{12})^2\,]\right\}. \quad (198)$$

When it chooses K_1, G1 seeks to maximize the sum of (197) and (198).

(189), (190), and (192) also imply that when G2 is capacity-constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$E\{\,[\,w(\varepsilon) - c_2\,]\,q_2^*(\varepsilon)\,\} \;=$$

$$\frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0} \{\,b^w\,[\,a^L + Q_2^r\,] + c_1 - 2\,c_2 + \varepsilon\,\}\left\{a^L + Q_2^r + \frac{1}{b^w}\,[\,c_1 - 2\,c_2 + \varepsilon\,]\right\} dH(\varepsilon)$$

$$+ \,\frac{1}{2}\int_{\varepsilon_0}^{\varepsilon_{12}} \{\,b^w\,[\,a^L + Q_2^r - K_2\,] + c_1 - 2\,c_2 + \varepsilon\,\}\,K_2\;dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \{\,b^w\,[\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,] - c_2 + \varepsilon\,\}\;K_2\;dH(\varepsilon)$$

$$= \;\frac{1}{9}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\,b^w\left(a^L + Q_2^r\right) + c_1 - 2\,c_2\,]\left[a^L + Q_2^r + \frac{1}{b^w}\,(\,c_1 - 2\,c_2\,)\right]$$

$$+ \,\frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0} [\,b^w\left(a^L + Q_2^r\right) + c_1 - 2\,c_2\,]\,\frac{1}{b^w}\;\varepsilon\;dH(\varepsilon)$$

$$+ \,\frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0} \left[a^L + Q_2^r + \frac{1}{b^w}\,(\,c_1 - 2\,c_2\,)\right]\varepsilon\;dH(\varepsilon) + \frac{1}{9\,b^w}\int_{\underline{\varepsilon}}^{\varepsilon_0} \varepsilon^2\,dH(\varepsilon)$$

$$+ \,\frac{K_2}{2}\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\,b^w\left(a^L + Q_2^r - K_2\right) + c_1 - 2\,c_2\,] + \frac{K_2}{2}\int_{\varepsilon_0}^{\varepsilon_{12}} \varepsilon\;dH(\varepsilon)$$

$$+ K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_2 \right]$$

$$+ \frac{K_2}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right]$$

$$= \frac{1}{9} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_1 - 2\, c_2 \right] \left[a^L + Q_2^r + \frac{1}{b^w} \left(c_1 - 2\, c_2 \right) \right]$$

$$+ \frac{1}{18\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_1 - 2\, c_2 \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]$$

$$+ \frac{1}{18} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_2^r + \frac{1}{b^w} \left(c_1 - 2\, c_2 \right) \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]$$

$$+ \frac{1}{27\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^3 - (\underline{\varepsilon})^3 \right]$$

$$+ \frac{K_2}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 - 2\, c_2 \right]$$

$$+ \frac{K_2}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$+ K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_2 \right]$$

$$+ \frac{K_2}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right] \tag{199}$$

where ε_0 and ε_{12} are as specified in (190).

Recall that in this case, G1 seeks to maximize:

$$E\{\Pi_1^p\} \equiv E\{ [\, w(\varepsilon) - c_1 \,]\, q_1^*(\varepsilon) \} + E\{ [\, r_1 - w(\cdot) - c_1^r \,]\, Q_1^r \}\,. \tag{200}$$

(197), (198), and (200) imply that in the present case:

$$\frac{\partial E\{\Pi_1^p\}}{\partial K_1} = \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_2 - 2\, c_1 \right] \left[a^L + 3\, Q_1^r + Q_2^r + \frac{1}{b^w} \left(c_2 - 2\, c_1 \right) \right] \frac{d\varepsilon_0}{dK_1}$$

$$+ \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_2 - 2\, c_1 \right] \varepsilon_0 \, \frac{d\varepsilon_0}{dK_1}$$

$$+ \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\, Q_1^r + Q_2^r + \frac{1}{b^w} \left(c_2 - 2\, c_1 \right) \right] \varepsilon_0 \, \frac{d\varepsilon_0}{dK_1} + \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2 \, \frac{d\varepsilon_0}{dK_1}$$

$$+\ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r-K_2\,\right)-c_1\,\right]\left[a^L+2\,Q_1^r+Q_2^r-K_2-\frac{c_1}{b^w}\right]\left[\frac{d\varepsilon_{12}}{dK_1}-\frac{d\varepsilon_0}{dK_1}\right]$$

$$+\ \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r-K_2\,\right)-c_1\,\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right]$$

$$+\ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+2\,Q_1^r+Q_2^r-K_2-\frac{c_1}{b^w}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right]$$

$$+\ \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[(\varepsilon_{12})^2\,\frac{d\varepsilon_{12}}{dK_1}-(\varepsilon_0)^2\,\frac{d\varepsilon_0}{dK_1}\right]$$

$$+\ \left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right)-c_1\,\right]-\ K_1\,b^w\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$-\ K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right)-c_1\,\right]\frac{d\varepsilon_{12}}{dK_1}$$

$$+\ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2-(\varepsilon_{12})^2\,\right]-\ K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}$$

$$-\ \frac{1}{3}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r\,\right)+c_1+c_2\,\right]\frac{d\varepsilon_0}{dK_1}-\frac{1}{3}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}$$

$$-\ \frac{1}{2}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r-K_2\,\right)+c_1\,\right]\left[\frac{d\varepsilon_{12}}{dK_1}-\frac{d\varepsilon_0}{dK_1}\right]$$

$$-\ \frac{1}{2}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right]+\ b^w\,Q_1^r\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+\ Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]\frac{d\varepsilon_{12}}{dK_1}+\ Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}\,; \qquad (201)$$

$$\frac{\partial E\left\{\Pi_1^p\right\}}{\partial K_2}\ =\ \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r\,\right)+c_2-2\,c_1\,\right]\left[a^L+3\,Q_1^r+Q_2^r+\frac{1}{b^w}\left(\,c_2-2\,c_1\,\right)\right]\frac{d\varepsilon_0}{dK_2}$$

$$+\ \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r\,\right)+c_2-2\,c_1\,\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}$$

$$+\ \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+3\,Q_1^r+Q_2^r+\frac{1}{b^w}\left(\,c_2-2\,c_1\,\right)\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}+\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right](\varepsilon_0)^2\,\frac{d\varepsilon_0}{dK_2}$$

$$+\ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r-K_2\,\right)-c_1\,\right]\left[a^L+2\,Q_1^r+Q_2^r-K_2-\frac{c_1}{b^w}\right]\left[\frac{d\varepsilon_{12}}{dK_2}-\frac{d\varepsilon_0}{dK_2}\right]$$

$$-\ \frac{1}{4}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r-K_2\,\right)-c_1\,\right]$$

$$- \frac{1}{4}\, b^w \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 2\, Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w} \right] - \frac{1}{8} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \, \right]$$

$$+ \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r - K_2 \right) - c_1 \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_2} - \varepsilon_0\, \frac{d\varepsilon_0}{dK_2} \right]$$

$$- \frac{1}{8} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \, \right]$$

$$+ \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 2\, Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w} \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_2} - \varepsilon_0\, \frac{d\varepsilon_0}{dK_2} \right]$$

$$+ \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2\, \frac{d\varepsilon_{12}}{dK_2} - (\varepsilon_0)^2\, \frac{d\varepsilon_0}{dK_2} \right] - \, K_1\, b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_1 \right] \frac{d\varepsilon_{12}}{dK_2} - \, K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_2}$$

$$- \frac{1}{3}\, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \right] \frac{d\varepsilon_0}{dK_2} - \frac{1}{3}\, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0\, \frac{d\varepsilon_0}{dK_2}$$

$$- \frac{1}{2}\, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right] \left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2} \right] + \frac{1}{2}\, b^w\, Q_1^r \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \frac{1}{2}\, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_2} - \varepsilon_0\, \frac{d\varepsilon_0}{dK_2} \right] + \, b^w\, Q_1^r \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \frac{d\varepsilon_{12}}{dK_2} + \, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_2}\, ; \qquad (202)$$

$$\frac{\partial E\, \{\Pi_1^p\}}{\partial r_1} = \frac{1}{9}\, b^w\, d_2 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\, Q_1^r + Q_2^r + \frac{1}{b^w} \left(\, c_2 - 2\, c_1 \, \right) \right]$$

$$- \frac{1}{9} \left[\, 3\, b_1 - d_2 \, \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r \right) + c_2 - 2\, c_1 \right]$$

$$+ \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r \right) + c_2 - 2\, c_1 \right] \left[a^L + 3\, Q_1^r + Q_2^r + \frac{1}{b^w} \left(\, c_2 - 2\, c_1 \, \right) \right] \frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r \right) + c_2 - 2\, c_1 \right] \varepsilon_0\, \frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{18}\, d_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \, \right]$$

$$+ \frac{1}{9}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[a^L + 3\,Q_1^r + Q_2^r + \frac{1}{b^w}\,(\,c_2 - 2\,c_1)\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_1}$$

$$- \frac{1}{18}\,[\,3\,b_1 - d_2\,]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2 - (\underline{\varepsilon})^2\,\right] + \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right](\varepsilon_0)^2\,\frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,b^w\left(a^L + Q_2^r - K_2\right) - c_1\,\right]\left[a^L + 2\,Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w}\right]\left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1}\right]$$

$$- \frac{1}{4}\,[\,2\,b_1 - d_2\,]\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,b^w\left(a^L + Q_2^r - K_2\right) - c_1\,\right]$$

$$+ \frac{1}{4}\,b^w\,d_2\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[a^L + 2\,Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w}\right]$$

$$+ \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,b^w\left(a^L + Q_2^r - K_2\right) - c_1\,\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0\,\frac{d\varepsilon_0}{dr_1}\right]$$

$$+ \frac{1}{8}\,d_2\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2 - (\varepsilon_0)^2\,\right]$$

$$+ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[a^L + 2\,Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0\,\frac{d\varepsilon_0}{dr_1}\right]$$

$$- \frac{1}{8}\,[\,2\,b_1 - d_2\,]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2 - (\varepsilon_0)^2\,\right]$$

$$+ \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[(\varepsilon_{12})^2\,\frac{d\varepsilon_{12}}{dr_1} - (\varepsilon_0)^2\,\frac{d\varepsilon_0}{dr_1}\right] - \;K_1\,b^w\,[\,b_1 - d_2\,]\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]$$

$$- \;K_1\left[\,b^w\left(a^L + Q_1^r + Q_2^r - K_1 - K_2\right) - c_1\,\right]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_1} - \;K_1\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_1}$$

$$+ \,Q_1^r - b_1\,[\,r_1 - c_1^r\,] - \frac{1}{3}\,Q_1^r\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,b^w\left(a^L + Q_2^r\right) + c_1 + c_2\,\right]\frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{3}\,b_1\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,b^w\left(a^L + Q_2^r\right) + c_1 + c_2\,\right] - \frac{1}{3}\,b^w\,d_2\,Q_1^r\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]$$

$$- \frac{1}{3}\,Q_1^r\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_1} + \frac{1}{6}\,b_1\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2 - (\underline{\varepsilon})^2\,\right]$$

$$- \frac{1}{2}\,Q_1^r\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,b^w\left(a^L + Q_2^r - K_2\right) + c_1\,\right]\left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1}\right]$$

$$- \frac{1}{2}\,b^w\,d_2\,Q_1^r\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right] + \frac{1}{2}\,b_1\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,b^w\left(a^L + Q_2^r - K_2\right) + c_1\,\right]$$

$$- \frac{1}{2} Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0 \frac{d\varepsilon_0}{dr_1} \right] + \frac{1}{4} b_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$+ b^w Q_1^r \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [b_1 - d_2] + b^w b_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right]$$

$$+ Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \frac{d\varepsilon_{12}}{dr_1}$$

$$+ \frac{1}{2} b_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right] + Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \frac{d\varepsilon_{12}}{dr_1} ; \qquad (203)$$

$$\frac{\partial E \{\Pi_1^p\}}{\partial r_2} = - \frac{1}{9} b^w b_2 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3 Q_1^r + Q_2^r + \frac{1}{b^w} (c_2 - 2 c_1) \right]$$

$$+ \frac{1}{9} [3 d_1 - b_2] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_2 - 2 c_1 \right]$$

$$+ \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_2 - 2 c_1 \right] \left[a^L + 3 Q_1^r + Q_2^r + \frac{1}{b^w} (c_2 - 2 c_1) \right] \frac{d\varepsilon_0}{dr_2}$$

$$+ \frac{1}{9 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_2 - 2 c_1 \right] \varepsilon_0 \frac{d\varepsilon_0}{dr_2} - \frac{1}{18} b_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]$$

$$+ \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3 Q_1^r + Q_2^r + \frac{1}{b^w} (c_2 - 2 c_1) \right] \varepsilon_0 \frac{d\varepsilon_0}{dr_2}$$

$$+ \frac{1}{18} [3 d_1 - b_2] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right] + \frac{1}{9 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2 \frac{d\varepsilon_0}{dr_2}$$

$$+ \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r - K_2 \right) - c_1 \right] \left[a^L + 2 Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w} \right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right]$$

$$- \frac{1}{4} [b_2 - 2 d_1] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r - K_2 \right) - c_1 \right]$$

$$- \frac{1}{4} b^w b_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 2 Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w} \right]$$

$$+ \frac{1}{4 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r - K_2 \right) - c_1 \right] \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0 \frac{d\varepsilon_0}{dr_2} \right]$$

$$- \frac{1}{8} b_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$
\begin{aligned}
&+ \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 2\, Q_1^r + Q_2^r - K_2 - \frac{c_1}{b^w} \right] \left[\varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0 \, \frac{d\varepsilon_0}{dr_2} \right] \\
&- \frac{1}{8} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b_2 - 2\, d_1 \,] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \, \right] \\
&+ \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 \, \frac{d\varepsilon_{12}}{dr_2} - (\varepsilon_0)^2 \, \frac{d\varepsilon_0}{dr_2} \right] - \; K_1 \, b^w \, [\, b_2 - d_1 \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&- \; K_1 \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_1 \, \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_2} - \; K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_2} \\
&+ d_1 \, [\, r_1 - c_1^r \,] - \frac{1}{3} \, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \, \right] \frac{d\varepsilon_0}{dr_2} \\
&- \frac{1}{3} \, d_1 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \, \right] + \frac{1}{3} \, b^w \, b_2 \, Q_1^r \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&- \frac{1}{3} \, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \, \frac{d\varepsilon_0}{dr_2} - \frac{1}{6} \, d_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \, \right] \\
&- \frac{1}{2} \, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \, \right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right] \\
&+ \frac{1}{2} \, b_2 \, b^w \, Q_1^r \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - \frac{1}{2} \, d_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \, \right] \\
&- \frac{1}{2} \, Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0 \, \frac{d\varepsilon_0}{dr_2} \right] - \frac{1}{4} \, d_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \, \right] \\
&+ \; b^w \, Q_1^r \, [\, b_2 - d_1 \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - \; b^w \, d_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \, \right] \\
&+ Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \, \right] \frac{d\varepsilon_{12}}{dr_2} \\
&- \; \frac{1}{2} \, d_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \, \right] + \; Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_2} \, . \qquad (204)
\end{aligned}
$$

where ε_0 and ε_{12} are as specified in (190).

Recall that in this case, G2 seeks to maximize

$$
E\,\{\Pi_2^p\} \;\; \equiv \;\; E\,\{\, [\, w(\varepsilon) - c_2 \,] \, q_2^*(\varepsilon) \,\} \, . \qquad (205)
$$

(199) and (205) imply:

$$\frac{\partial E\{\Pi_2^p\}}{\partial K_1} = \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[b^w\left(a^L+Q_2^r\right)+c_1-2\,c_2\right]\left[a^L+Q_2^r+\frac{1}{b^w}\left(c_1-2\,c_2\right)\right]\frac{d\varepsilon_0}{dK_1}$$

$$+\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[b^w\left(a^L+Q_2^r\right)+c_1-2\,c_2\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}$$

$$+\frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+Q_2^r+\frac{1}{b^w}\left(c_1-2\,c_2\right)\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}$$

$$+\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right](\varepsilon_0)^2\,\frac{d\varepsilon_0}{dK_1}$$

$$+\frac{K_2}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[b^w\left(a^L+Q_2^r-K_2\right)+c_1-2\,c_2\right]\left[\frac{d\varepsilon_{12}}{dK_1}-\frac{d\varepsilon_0}{dK_1}\right]$$

$$+\frac{K_2}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right]$$

$$-\,K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[b^w\left(a^L+Q_1^r+Q_2^r-K_1-K_2\right)-\,c_2\right]\frac{d\varepsilon_{12}}{dK_1}$$

$$-\,b^w\,K_2\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]-K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}\,;$$

$$\frac{\partial E\{\Pi_2^p\}}{\partial K_2} = \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[b^w\left(a^L+Q_2^r\right)+c_1-2\,c_2\right]\left[a^L+Q_2^r+\frac{1}{b^w}\left(c_1-2\,c_2\right)\right]\frac{d\varepsilon_0}{dK_2}$$

$$+\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[b^w\left(a^L+Q_2^r\right)+c_1-2\,c_2\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}$$

$$+\frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+Q_2^r+\frac{1}{b^w}\left(c_1-2\,c_2\right)\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}$$

$$+\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right](\varepsilon_0)^2\,\frac{d\varepsilon_0}{dK_2}$$

$$+\frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[b^w\left(a^L+Q_2^r-K_2\right)+c_1-2\,c_2\right]-\frac{1}{2}\,K_2\,b^w\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+\frac{K_2}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[b^w\left(a^L+Q_2^r-K_2\right)+c_1-2\,c_2\right]\left[\frac{d\varepsilon_{12}}{dK_2}-\frac{d\varepsilon_0}{dK_2}\right]$$

$$+\frac{K_2}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}\right]+\frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[(\varepsilon_{12})^2-(\varepsilon_0)^2\right]$$

$$+ \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_2 \right]$$

$$- K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_2 \right] \frac{d\varepsilon_{12}}{dK_2}$$

$$- b^w K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \frac{d\varepsilon_{12}}{dK_2} + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right];$$

$$\frac{\partial E\{\Pi_2^p\}}{\partial r_1} = \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_1 - 2\, c_2 \right] \left[a^L + Q_2^r + \frac{1}{b^w} (c_1 - 2\, c_2) \right] \frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{9} b^w d_2 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_2^r + \frac{1}{b^w} (c_1 - 2\, c_2) \right]$$

$$+ \frac{1}{9} d_2 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_1 - 2\, c_2 \right]$$

$$+ \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_1 - 2\, c_2 \right] \varepsilon_0 \frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{18} d_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]$$

$$+ \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_2^r + \frac{1}{b^w} (c_1 - 2\, c_2) \right] \varepsilon_0 \frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{18} d_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right] + \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2 \frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{K_2}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 - 2\, c_2 \right] \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1} \right]$$

$$+ \frac{1}{2} K_2\, b^w d_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{K_2}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0 \frac{d\varepsilon_0}{dr_1} \right]$$

$$- K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_2 \right] \frac{d\varepsilon_{12}}{dr_1}$$

$$- K_2\, b^w [b_1 - d_2] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \frac{d\varepsilon_{12}}{dr_1};$$

$$\frac{\partial E\{\Pi_2^p\}}{\partial r_2} = \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_1 - 2\, c_2 \right] \left[a^L + Q_2^r + \frac{1}{b^w} (c_1 - 2\, c_2) \right] \frac{d\varepsilon_0}{dr_2}$$

$$
\begin{aligned}
& - \frac{1}{9} b^w b_2 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_2^r + \frac{1}{b^w} (c_1 - 2 c_2) \right] \\
& - \frac{1}{9} b_2 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_1 - 2 c_2 \right] \\
& + \frac{1}{9 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_1 - 2 c_2 \right] \varepsilon_0 \frac{d\varepsilon_0}{dr_2} \\
& - \frac{1}{18} b_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right] \\
& + \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_2^r + \frac{1}{b^w} (c_1 - 2 c_2) \right] \varepsilon_0 \frac{d\varepsilon_0}{dr_2} \\
& - \frac{1}{18} b_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right] + \frac{1}{9 b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2 \frac{d\varepsilon_0}{dr_2} \\
& + \frac{K_2}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 - 2 c_2 \right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right] \\
& - \frac{1}{2} K_2 b^w b_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{K_2}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0 \frac{d\varepsilon_0}{dr_2} \right] \\
& - K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_2 \right] \frac{d\varepsilon_{12}}{dr_2} \\
& - K_2 b^w [b_2 - d_1] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \frac{d\varepsilon_{12}}{dr_2} . \qquad (206)
\end{aligned}
$$

(190) implies that when G2 is capacity-constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$ under partial vertical integration:

$$
\begin{aligned}
\varepsilon_0 &= 3 b^w K_2 + 2 c_2 - c_1 - b^w \left[a^L + Q_2^r \right] ; \\
\varepsilon_{12} &= 2 b^w K_1 + b^w K_2 + c_1 - b^w \left[a^L + 2 Q_1^r + Q_2^r \right] \\
\Rightarrow \frac{d\varepsilon_0}{dK_1} &= 0 ; \ \frac{d\varepsilon_0}{dK_2} = 3 b^w ; \ \frac{d\varepsilon_0}{dr_1} = - b^w d_2 ; \ \frac{d\varepsilon_0}{dr_2} = b^w b_2 ; \\
\frac{d\varepsilon_{12}}{dK_1} &= 2 b^w ; \ \frac{d\varepsilon_{12}}{dK_2} = b^w ; \ \frac{d\varepsilon_{12}}{dr_1} = b^w [2 b_1 - d_2] ; \ \frac{d\varepsilon_{12}}{dr_2} = b^w [b_2 - 2 d_1] . \qquad (207)
\end{aligned}
$$

(207) implies:

$$
\frac{d^2\varepsilon_0}{dr_1 \, dz} = \frac{d^2\varepsilon_0}{dr_2 \, dz} = \frac{d^2\varepsilon_{12}}{dr_1 \, dz} = \frac{d^2\varepsilon_{12}}{dr_2 \, dz} = 0 \ \text{ for } \ z \in \{ K_1, K_2, r_1, r_2 \} . \qquad (208)
$$

(196), (207), and (208) imply:

$$\begin{aligned}
\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial r_1} &= \frac{1}{3}\left[\, b^w(a^L + Q_2^r) + c_1 + c_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_1} + \frac{1}{3}\, b^w\, d_2 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ \frac{1}{3}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0\, \frac{d\varepsilon_0}{dr_1} + \frac{1}{2}\left[\, b^w(a^L + Q_2^r - K_2) + c_1 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1} \right] \\
&+ \frac{1}{2}\, b^w\, d_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0\, \frac{d\varepsilon_0}{dr_1} \right] \\
&- b^w \left[\, b_1 - d_2 \,\right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_1} \\
&- \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12}\, \frac{d\varepsilon_{12}}{dr_1}\,;
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial r_2} &= \frac{1}{3}\left[\, b^w(a^L + Q_2^r) + c_1 + c_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_2} - \frac{1}{3}\, b^w\, b_2 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ \frac{1}{3}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0\, \frac{d\varepsilon_0}{dr_2} + \frac{1}{2}\left[\, b^w(a^L + Q_2^r - K_2) + c_1 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right] \\
&- \frac{1}{2}\, b^w\, b_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0\, \frac{d\varepsilon_0}{dr_2} \right] \\
&- b^w \left[\, b_2 - d_1 \,\right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_2} \\
&- \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12}\, \frac{d\varepsilon_{12}}{dr_2}\,;
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial K_1} &= \frac{1}{3}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w(a^L + Q_2^r) + c_1 + c_2 \,\right] \frac{d\varepsilon_0}{dK_1} \\
&+ \frac{1}{3}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0\, \frac{d\varepsilon_0}{dK_1} + \frac{1}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w(a^L + Q_2^r - K_2) + c_1 \,\right] \left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1} \right] \\
&+ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_1} - \varepsilon_0\, \frac{d\varepsilon_0}{dK_1} \right] - b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&- b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \frac{d\varepsilon_{12}}{dK_1} - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_1}\,;
\end{aligned}$$

$$\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial K_2} = \frac{1}{3}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w(a^L + Q_2^r) + c_1 + c_2 \,\right] \frac{d\varepsilon_0}{dK_2}$$

$$
\begin{aligned}
&+ \ \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}+\frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w(a^L+Q_2^r-K_2)+c_1\,\right]\left[\frac{d\varepsilon_{12}}{dK_2}-\frac{d\varepsilon_0}{dK_2}\right] \\
&- \ \frac{1}{2}\,b^w\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]+\frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}\right] \\
&- \ b^w\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]-\,b^w\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]\frac{d\varepsilon_{12}}{dK_2} \\
&- \ \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2}\,. \qquad (209)
\end{aligned}
$$

(208) and (209) imply:

$$
\begin{aligned}
\frac{\partial}{\partial K_1}\left(\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial r_1}\right) \ &= \ \frac{1}{3}\,b^w\,d_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_1}+\frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_1}\,\frac{d\varepsilon_0}{dr_1} \\
&+ \frac{1}{2}\,b^w\,d_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_1}-\frac{d\varepsilon_0}{dK_1}\right]+\frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_1}\,\frac{d\varepsilon_{12}}{dr_1}-\frac{d\varepsilon_0}{dK_1}\,\frac{d\varepsilon_0}{dr_1}\right] \\
&+ b^w\left[\,b_1-d_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dK_1}+\,b^w\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_1}-\,\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dK_1}\,\frac{d\varepsilon_{12}}{dr_1}\,;
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial}{\partial K_2}\left(\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial r_1}\right) \ &= \ \frac{1}{3}\,b^w\,d_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_2}+\frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_2}\,\frac{d\varepsilon_0}{dr_1} \\
&- \ \frac{1}{2}\,b^w\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_1}-\frac{d\varepsilon_0}{dr_1}\right]+\frac{1}{2}\,b^w\,d_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_2}-\frac{d\varepsilon_0}{dK_2}\right] \\
&+ \ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_2}\,\frac{d\varepsilon_{12}}{dr_1}-\frac{d\varepsilon_0}{dK_2}\,\frac{d\varepsilon_0}{dr_1}\right]+b^w\left[\,b_1-d_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dK_2} \\
&+ b^w\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_1}-\,\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dK_2}\,\frac{d\varepsilon_{12}}{dr_1}\,;
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial}{\partial r_1}\left(\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial r_1}\right) \ &= \ \frac{2}{3}\,b^w\,d_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_1}+\frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_0}{dr_1}\right]^2 \\
&+ \ \frac{1}{2}\,b^w\,d_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_1}-\frac{d\varepsilon_0}{dr_1}\right]+\frac{1}{2}\,b^w\,d_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_1}-\frac{d\varepsilon_0}{dr_1}\right] \\
&+ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\left(\frac{d\varepsilon_{12}}{dr_1}\right)^2-\left(\frac{d\varepsilon_0}{dr_1}\right)^2\right]
\end{aligned}
$$

$$+ 2\, b^w \,[\, b_1 - d_2 \,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_1} - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dr_1} \right]^2 ;$$

$$\begin{aligned}
\frac{\partial}{\partial r_2} \left(\frac{\partial E\left\{ w(\varepsilon) \right\}}{\partial r_1} \right) \;=\; & - \frac{1}{3}\, b^w\, b_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_1} + \frac{1}{3}\, b^w\, d_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_2} \\
& + \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_2}\, \frac{d\varepsilon_0}{dr_1} - \frac{1}{2}\, b^w\, b_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1} \right] \\
& + \frac{1}{2}\, b^w\, d_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right] + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dr_2}\, \frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_2}\, \frac{d\varepsilon_0}{dr_1} \right] \\
& + b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b_1 - d_2 \,] \frac{d\varepsilon_{12}}{dr_2} + b^w \,[\, b_2 - d_1 \,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_1} \\
& - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_2}\, \frac{d\varepsilon_{12}}{dr_1} ;
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial K_1} \left(\frac{\partial E\left\{ w(\varepsilon) \right\}}{\partial r_2} \right) \;=\; & - \frac{1}{3}\, b^w\, b_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dK_1} + \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dK_1}\, \frac{d\varepsilon_0}{dr_2} \\
& - \frac{1}{2}\, b^w\, b_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1} \right] + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dK_1}\, \frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dK_1}\, \frac{d\varepsilon_0}{dr_2} \right] \\
& + b^w \,[\, b_2 - d_1 \,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dK_1} + b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_2} \\
& - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dK_1}\, \frac{d\varepsilon_{12}}{dr_2} ;
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial K_2} \left(\frac{\partial E\left\{ w(\varepsilon) \right\}}{\partial r_2} \right) \;=\; & - \frac{1}{3}\, b^w\, b_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dK_2} + \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dK_2}\, \frac{d\varepsilon_0}{dr_2} \\
& - \frac{1}{2}\, b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right] - \frac{1}{2}\, b^w\, b_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2} \right] \\
& + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dK_2}\, \frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dK_2}\, \frac{d\varepsilon_0}{dr_2} \right] \\
& + b^w \,[\, b_2 - d_1 \,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dK_2} + b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_2} \\
& - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dK_2}\, \frac{d\varepsilon_{12}}{dr_2} ;
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial}{\partial r_1}\left(\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial r_2}\right) \;=\;& \frac{1}{3}\, b^w\, d_2 \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_0}{dr_2} - \frac{1}{3}\, b^w\, b_2 \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_0}{dr_1} \\
&+\; \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_0}{dr_1}\,\frac{d\varepsilon_0}{dr_2} + \frac{1}{2}\, b^w\, d_2 \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2}\right] \\
&-\; \frac{1}{2}\, b^w\, b_2 \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1}\right] \\
&+\; \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \left[\frac{d\varepsilon_{12}}{dr_1}\,\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_1}\,\frac{d\varepsilon_0}{dr_2}\right] \\
&+\; b^w\left[\,b_2 - d_1\,\right] \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_{12}}{dr_1} + b^w\left[\,b_1 - d_2\,\right] \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_{12}}{dr_2} \\
&-\; \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_{12}}{dr_1}\,\frac{d\varepsilon_{12}}{dr_2}\,;
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial}{\partial r_2}\left(\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial r_2}\right) \;=\;& -\frac{2}{3}\, b^w\, b_2 \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_0}{dr_2} + \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \left[\frac{d\varepsilon_0}{dr_2}\right]^2 \\
&-\; \frac{1}{2}\, b^w\, b_2 \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2}\right] - \frac{1}{2}\, b^w\, d_2 \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2}\right] \\
&+\; \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \left[\left(\frac{d\varepsilon_{12}}{dr_2}\right)^2 - \left(\frac{d\varepsilon_0}{dr_2}\right)^2\right] + b^w\left[\,b_2 - d_1\,\right] \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_{12}}{dr_2} \\
&+\; b^w\left[\,b_2 - d_1\,\right] \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_{12}}{dr_2} - \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \left[\frac{d\varepsilon_{12}}{dr_2}\right]^2 . \qquad (210)
\end{aligned}
$$

(189) and (190) imply that when G2 is capacity-constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$E\left\{q_1^{*0c}\right\} \equiv \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon)\, dH(\varepsilon) = \frac{1}{3}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, a^L + 3\, Q_1^r + Q_2^r \,\right]$$

$$+ \frac{1}{3\, b^w}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, c_2 - 2\, c_1 \,\right] + \frac{1}{6\, b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right];$$

$$E\left\{q_2^{*0c}\right\} \equiv \int_{\underline{\varepsilon}}^{\varepsilon_0} q_2^{*0c}(\varepsilon)\, dH(\varepsilon) = \frac{1}{3}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, a^L + Q_2^r \,\right] + \frac{1}{3\, b^w}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, c_1 - 2\, c_2 \,\right]$$

$$+ \frac{1}{6\, b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right];$$

$$E\left\{q_1^{*1c}\right\} \equiv \int_{\varepsilon_0}^{\varepsilon_{12}} q_1^{*1c}(\varepsilon)\, dH(\varepsilon) = \frac{1}{4\, b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right] - \frac{1}{2\, b^w}\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right] c_1$$

$$+ \frac{1}{2}\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, a^L + 2\, Q_1^r + Q_2^r - K_2 \,\right]. \tag{211}$$

(211) implies that in this case:

$$\frac{\partial E\left\{q_1^{*0c}\right\}}{\partial r_1} = -\frac{1}{3}\left[\, 3\, b_1 - d_2 \,\right]\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] + \frac{1}{3}\left[\, a^L + 3\, Q_1^r + Q_2^r \,\right]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{3\, b^w}\left[\, c_2 - 2\, c_1 \,\right]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_1} + \frac{1}{3\, b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_1};$$

$$\frac{\partial E\left\{q_1^{*0c}\right\}}{\partial r_2} = \frac{1}{3}\left[\, 3\, d_1 - b_2 \,\right]\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] + \frac{1}{3}\left[\, a^L + 3\, Q_1^r + Q_2^r \,\right]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_2}$$

$$+ \frac{1}{3\, b^w}\left[\, c_2 - 2\, c_1 \,\right]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_2} + \frac{1}{3\, b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_2};$$

$$\frac{\partial E\left\{q_1^{*0c}\right\}}{\partial K_1} = \frac{1}{3}\left[\, a^L + 3\, Q_1^r + Q_2^r \,\right]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_1}$$

$$+ \frac{1}{3\, b^w}\left[\, c_2 - 2\, c_1 \,\right]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_1} + \frac{1}{3\, b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1};$$

$$
\begin{aligned}
\frac{\partial E\left\{q_1^{*0c}\right\}}{\partial K_2} &= \frac{1}{3}\left[\, a^L + 3\,Q_1^r + Q_2^r\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_2} \\
&+ \frac{1}{3\,b^w}\left[\, c_2 - 2\,c_1\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_2} + \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}\,;
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E\left\{q_2^{*0c}\right\}}{\partial r_1} &= \frac{1}{3}\,d_2\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] + \frac{1}{3}\left[\, a^L + Q_2^r\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_1} \\
&+ \frac{1}{3\,b^w}\left[\, c_1 - 2\,c_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_1} + \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_1}\,;
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E\left\{q_2^{*0c}\right\}}{\partial r_2} &= -\frac{1}{3}\,b_2\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] + \frac{1}{3}\left[\, a^L + Q_2^r\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_2} \\
&+ \frac{1}{3\,b^w}\left[\, c_1 - 2\,c_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_2} + \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_2}\,;
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E\left\{q_2^{*0c}\right\}}{\partial K_1} &= \frac{1}{3}\left[\, a^L + Q_2^r\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_1} \\
&+ \frac{1}{3\,b^w}\left[\, c_1 - 2\,c_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_1} + \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\,;
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E\left\{q_2^{*0c}\right\}}{\partial K_2} &= \frac{1}{3}\left[\, a^L + Q_2^r\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_2} \\
&+ \frac{1}{3\,b^w}\left[\, c_1 - 2\,c_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_2} + \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}\,;
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E\left\{q_1^{*1c}\right\}}{\partial r_1} &= -\frac{1}{2}\left[\, 2\,b_1 - d_2\,\right]\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right] + \frac{1}{2}\left[\, a^L + 2\,Q_1^r + Q_2^r - K_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1}\right] \\
&+ \frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0\,\frac{d\varepsilon_0}{dr_1}\right] - \frac{c_1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1}\right];
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E\left\{q_1^{*1c}\right\}}{\partial r_2} &= \frac{1}{2}\left[\, 2\,d_1 - b_2\,\right]\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right] + \frac{1}{2}\left[\, a^L + 2\,Q_1^r + Q_2^r - K_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2}\right] \\
&+ \frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0\,\frac{d\varepsilon_0}{dr_2}\right] - \frac{c_1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2}\right];
\end{aligned}
$$

$$
\frac{\partial E\left\{q_1^{*1c}\right\}}{\partial K_1} = \frac{1}{2}\left[\, a^L + 2\,Q_1^r + Q_2^r - K_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1}\right]
$$

$$+\ \frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right]-\frac{c_1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_1}-\frac{d\varepsilon_0}{dK_1}\right];$$

$$\frac{\partial E\left\{q_1^{*1c}\right\}}{\partial K_2} = -\frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]+\frac{1}{2}\left[\,a^L+2\,Q_1^r+Q_2^r-K_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_2}-\frac{d\varepsilon_0}{dK_2}\right]$$

$$+\ \frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}\right]-\frac{c_1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_2}-\frac{d\varepsilon_0}{dK_2}\right]. \quad (212)$$

(91) and (97) imply that when G2 is capacity-constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$J_1(r_1,r_2) = b^w\left[\frac{2}{3}\,d_2-b_1\right]E\left\{q_1^{*0c}\right\}-\ b^w\left[\,b_1-d_2\,\right]E\left\{q_1^{*1c}\right\}$$

$$+\left[1+b^w\,b_1-\frac{2}{3}\,b^w\,d_2\right]\left[\,a_1-b_1\,r_1+d_1\,r_2\,\right]\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]-\ b_1\left[\,r_1-c_1^r\,\right]$$

$$+\ b_1\,E\left\{w(\cdot)\right\}+\left[\,1+b^w\,b_1-b^w\,d_2\,\right]\left[\,a_1-b_1\,r_1+d_1\,r_2\,\right]\left[\frac{\overline{\varepsilon}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$-\ b^w\left[\,b_1-d_2\,\right]K_1\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] = 0\,;$$

$$J_2(r_1,r_2) = \left[\,a_2-b_2\,r_2+d_2\,r_1\,\right]\left[1-\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial r_2}\right]$$

$$-\ b_2\left[\,r_2-c_2^r-E\left\{\,w(Q)\,\right\}\,\right] = 0\,, \quad (213)$$

where $E\{w(\cdot)\}$ is specified in (196), $E\{q_1^{*0c}\}$ and $E\{q_1^{*1c}\}$ are specified in (211), and ε_0 and ε_{12} are specified in (207).

For $v \in \{\,r_1,\ r_2,\ K_1,\ K_2\}$, define:

$$\widetilde{M}_1^A(v) \equiv b^w\left[\frac{2}{3}\,d_2-b_1\right]\frac{\partial E\left\{q_1^{*0c}\right\}}{\partial v}-b^w\left[\,b_1-d_2\,\right]\frac{\partial E\left\{q_1^{*1c}\right\}}{\partial v}$$

$$+\left[1+b^w\,b_1-\frac{2}{3}\,b^w\,d_2\right]\left[\,a_1-b_1\,r_1+d_1\,r_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dv}+\ b_1\,\frac{\partial E\left\{w\left(\cdot\right)\right\}}{\partial v}$$

$$-\left[\,1+b^w\,b_1-b^w\,d_2\,\right]\left[\,a_1-b_1\,r_1+d_1\,r_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dv}+\ b^w\,K_1\left[\,b_1-d_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dv}\,;$$

$$\widetilde{M}_2^A(v) \equiv -\left[\,a_2-b_2\,r_2+d_2\,r_1\,\right]\frac{\partial}{\partial v}\left(\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial r_2}\right)+\ b_2\,\frac{\partial E\left\{w\left(\cdot\right)\right\}}{\partial v}\,. \quad (214)$$

From (213) and (214):

$$\frac{\partial J_1(\cdot)}{\partial r_1} = -\,b_1\left[1 + b^w\,b_1 - \frac{2}{3}\,b^w\,d_2\right]\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] - b_1$$

$$-\,b_1\,[\,1 + b^w\,b_1 - b^w\,d_2\,]\left[\frac{\overline{\varepsilon} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right] + \widetilde{M}_1^A(r_1)\,;$$

$$\frac{\partial J_1(\cdot)}{\partial r_2} = d_1\left[1 + b^w\,b_1 - \frac{2}{3}\,b^w\,d_2\right]\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]$$

$$+\,d_1\,[\,1 + b^w\,b_1 - b^w\,d_2\,]\left[\frac{\overline{\varepsilon} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right] + \widetilde{M}_1^A(r_2)\,;$$

$$\frac{\partial J_1(\cdot)}{\partial K_1} = -\,b^w\,[\,b_1 - d_2\,]\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] + \widetilde{M}_1^A(K_1)\,; \qquad \frac{\partial J_1(\cdot)}{\partial K_2} = \widetilde{M}_1^A(K_2)\,;$$

$$\frac{\partial J_2(\cdot)}{\partial r_1} = d_2\left[1 - \frac{\partial E\,\{w(\varepsilon)\}}{\partial r_2}\right] + \widetilde{M}_2^A(r_1)\,;$$

$$\frac{\partial J_2(\cdot)}{\partial r_2} = -\,b_2\left[1 - \frac{\partial E\,\{w(\varepsilon)\}}{\partial r_2}\right] - b_2 + \widetilde{M}_2^A(r_2)\,;$$

$$\frac{\partial J_2(\cdot)}{\partial K_1} = \widetilde{M}_2^A(K_1)\,; \qquad \frac{\partial J_2(\cdot)}{\partial K_2} = \widetilde{M}_2^A(K_2)\,. \tag{215}$$

Case 3B. Partial vertical integration where G1 is capacity-constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$.

(189), (191), and (192) imply that in this case:

$$
\begin{aligned}
E\{w(\cdot)\} \;=\; & \frac{1}{3}\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w(a^L+Q_2^r)+c_1+c_2\,\right]+\frac{1}{6}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right] \\
& +\frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w(a^L+Q_1^r+Q_2^r-K_1)+c_2\,\right]+\frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right] \\
& +\; b^w\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]+\;\frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2-(\varepsilon_{12})^2\,\right]. \qquad (216)
\end{aligned}
$$

(189), (191), and (192) also imply that in this case:

$$
\begin{aligned}
& E\{\,[\,w(\varepsilon)-c_1\,]\,q_1^*(\varepsilon)\,\} \;= \\
& \frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0}\left\{\,b^w\left[\,a^L+Q_2^r\,\right]+c_2-2\,c_1+\varepsilon\,\right\}\left\{\,a^L+3\,Q_1^r+Q_2^r+\frac{1}{b^w}\left[\,c_2-2\,c_1+\varepsilon\,\right]\right\}dH(\varepsilon) \\
& \quad +\frac{1}{2}\int_{\varepsilon_0}^{\varepsilon_{12}}\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r-K_1\,\right]+c_2-2\,c_1+\varepsilon\,\right\}K_1\,dH(\varepsilon) \\
& \quad +\int_{\varepsilon_{12}}^{\overline{\varepsilon}}\left\{\,b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]-c_1+\varepsilon\,\right\}\,K_1\,dH(\varepsilon) \\
& \quad =\;\frac{1}{9}\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r\,\right)+c_2-2\,c_1\,\right]\left[\,a^L+3\,Q_1^r+Q_2^r+\frac{1}{b^w}\,(\,c_2-2\,c_1\,)\right] \\
& \qquad +\frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0}\left[\,b^w\left(\,a^L+Q_2^r\,\right)+c_2-2\,c_1\,\right]\frac{1}{b^w}\;\varepsilon\;dH(\varepsilon) \\
& \qquad +\frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0}\left[\,a^L+3\,Q_1^r+Q_2^r+\frac{1}{b^w}\,(\,c_2-2\,c_1\,)\right]\varepsilon\;dH(\varepsilon)+\frac{1}{9\,b^w}\int_{\underline{\varepsilon}}^{\varepsilon_0}\varepsilon^2\,dH(\varepsilon) \\
& \qquad +\;\frac{K_1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r+Q_2^r-K_1\,\right)+c_2-2\,c_1\,\right]+\frac{K_1}{2}\int_{\varepsilon_0}^{\varepsilon_{12}}\varepsilon\;dH(\varepsilon) \\
& \qquad +\,K_1\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right)-c_1\,\right]
\end{aligned}
$$

$$+ \frac{K_1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right]$$

$$= \frac{1}{9} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_2 - 2\, c_1 \right] \left[a^L + 3\, Q_1^r + Q_2^r + \frac{1}{b^w} (c_2 - 2\, c_1) \right]$$

$$+ \frac{1}{18\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_2 - 2\, c_1 \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]$$

$$+ \frac{1}{18} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\, Q_1^r + Q_2^r + \frac{1}{b^w} (c_2 - 2\, c_1) \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]$$

$$+ \frac{1}{27\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^3 - (\underline{\varepsilon})^3 \right]$$

$$+ \frac{K_1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 - 2\, c_1 \right]$$

$$+ \frac{K_1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$+ K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_1 \right]$$

$$+ \frac{K_1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right] \qquad (217)$$

where ε_0 and ε_{12} are as specified in (191).

Furthermore, from (216):

$$E \{ [r_1 - w(\cdot) - c_1^r] \, Q_1^r \} = [r_1 - c_1^r] \, Q_1^r - Q_1^r \, E \{ w(\cdot) \}$$

$$= [r_1 - c_1^r] \, Q_1^r$$

$$- Q_1^r \left\{ \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \right] + \frac{1}{6} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right] \right.$$

$$+ \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \right] + \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$\left. + \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right] \right\}. \qquad (218)$$

When it chooses K_1, G1 seeks to maximize the sum of (217) and (218).

(189), (191), and (192) also imply that in this case:

$$E\{\,[\,w(\varepsilon) - c_2\,]\,q_2^*(\varepsilon)\,\} \;=$$

$$\frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0}\left\{\,b^w\left[\,a^L + Q_2^r\,\right] + c_1 - 2\,c_2 + \varepsilon\,\right\}\left\{a^L + Q_2^r + \frac{1}{b^w}\,[\,c_1 - 2\,c_2 + \varepsilon\,]\right\}dH(\varepsilon)$$

$$+\,\frac{1}{4}\int_{\varepsilon_0}^{\varepsilon_{12}}\left\{\,b^w\left[\,a^L + Q_1^r + Q_2^r - K_1\,\right] - c_2 + \varepsilon\,\right\}\left\{a^L + Q_1^r + Q_2^r - K_1 + \frac{1}{b^w}\,[\,-\,c_2 + \varepsilon\,]\right\}dH(\varepsilon)$$

$$+\int_{\varepsilon_{12}}^{\overline{\varepsilon}}\left\{\,b^w\left[\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right] - c_2 + \varepsilon\,\right\}\,K_2\;dH(\varepsilon)$$

$$= \;\frac{1}{9}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L + Q_2^r\,\right) + c_1 - 2\,c_2\,\right]\left[\,a^L + Q_2^r + \frac{1}{b^w}\,(\,c_1 - 2\,c_2\,)\right]$$

$$+\,\frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0}\left\{\,b^w\left[\,a^L + Q_2^r\,\right] + c_1 - 2\,c_2\,\right\}\frac{1}{b^w}\;\varepsilon\;dH(\varepsilon)$$

$$+\,\frac{1}{9}\int_{\underline{\varepsilon}}^{\varepsilon_0}\left[\,a^L + Q_2^r + \frac{1}{b^w}\,(\,c_1 - 2\,c_2\,)\right]\varepsilon\;dH(\varepsilon) + \frac{1}{9\,b^w}\int_{\underline{\varepsilon}}^{\varepsilon_0}\varepsilon^2\,dH(\varepsilon)$$

$$+\;\frac{1}{4}\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L + Q_1^r + Q_2^r - K_1\,\right) - c_2\,\right]\left[\,a^L + Q_1^r + Q_2^r - K_1 - \frac{c_2}{b^w}\,\right]$$

$$+\,\frac{1}{4}\int_{\varepsilon_0}^{\varepsilon_{12}}\left[\,b^w\left(\,a^L + Q_1^r + Q_2^r - K_1\,\right) - c_2\,\right]\frac{1}{b^w}\;\varepsilon\;dH(\varepsilon)$$

$$+\,\frac{1}{4}\int_{\varepsilon_0}^{\varepsilon_{12}}\left[\,a^L + Q_1^r + Q_2^r - K_1 - \frac{c_2}{b^w}\,\right]\varepsilon\;dH(\varepsilon) + \;\frac{1}{4\,b^w}\int_{\varepsilon_0}^{\varepsilon_{12}}\varepsilon^2\,dH(\varepsilon)$$

$$+\,K_2\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right) - c_2\,\right]$$

$$+\,\frac{K_2}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2 - (\varepsilon_{12})^2\,\right]$$

$$= \frac{1}{9}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, b^w \left(a^L + Q_2^r \right) + c_1 - 2\,c_2 \,\right]\left[a^L + Q_2^r + \frac{1}{b^w}\left(c_1 - 2\,c_2 \right)\right]$$

$$+ \frac{1}{18\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, b^w \left(a^L + Q_2^r \right) + c_1 - 2\,c_2 \,\right]\left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right]$$

$$+ \frac{1}{18}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[a^L + Q_2^r + \frac{1}{b^w}\left(c_1 - 2\,c_2 \right)\right]\left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right]$$

$$+ \frac{1}{27\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, (\varepsilon_0)^3 - (\underline{\varepsilon})^3 \,\right]$$

$$+ \frac{1}{4}\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) - c_2 \,\right]\left[a^L + Q_1^r + Q_2^r - K_1 - \frac{c_2}{b^w}\right]$$

$$+ \frac{1}{8\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) - c_2 \,\right]\left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right]$$

$$+ \frac{1}{8}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[a^L + Q_1^r + Q_2^r - K_1 - \frac{c_2}{b^w}\right]\left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right]$$

$$+ \frac{1}{12\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, (\varepsilon_{12})^3 - (\varepsilon_0)^3 \,\right]$$

$$+ K_2\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_2 \,\right]$$

$$+ \frac{K_2}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \,\right] \tag{219}$$

where ε_0 and ε_{12} are as specified in (191).

Recall that in this case, G1 seeks to maximize:

$$E\{\Pi_1^p\} \;\equiv\; E\{\,[\, w(\varepsilon) - c_1 \,]\, q_1^*(\varepsilon)\,\} + E\{\,[\, r_1 - w(\cdot) - c_1^r \,]\, Q_1^r\,\}\,.$$

Therefore, (217) and (218) imply that in the present case:

$$\frac{\partial E\{\Pi_1^p\}}{\partial K_1} = \frac{1}{9}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, b^w \left(a^L + Q_2^r \right) + c_2 - 2\,c_1 \,\right]\left[a^L + 3\,Q_1^r + Q_2^r + \frac{1}{b^w}\left(c_2 - 2\,c_1 \right)\right]\frac{d\varepsilon_0}{dK_1}$$

$$+ \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\, b^w \left(a^L + Q_2^r \right) + c_2 - 2\,c_1 \,\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}$$

$$+ \frac{1}{9}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[a^L + 3\,Q_1^r + Q_2^r + \frac{1}{b^w}\left(c_2 - 2\,c_1 \right)\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1} + \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right](\varepsilon_0)^2\,\frac{d\varepsilon_0}{dK_1}$$

$$
\begin{aligned}
&+\frac{1}{2}\,K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r+Q_2^r-K_1\,\right)+c_2-2\,c_1\,\right]\left[\frac{d\varepsilon_{12}}{dK_1}-\frac{d\varepsilon_0}{dK_1}\right]\\
&+\frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r+Q_2^r-K_1\,\right)+c_2-2\,c_1\,\right]-\frac{1}{2}\,K_1\,b^w\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\\
&+\frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right]+\frac{1}{2}\,K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right]\\
&+\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right)-c_1\,\right]-\,K_1\,b^w\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\\
&-\,K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right)-c_1\,\right]\frac{d\varepsilon_{12}}{dK_1}\\
&+\frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2-(\varepsilon_{12})^2\,\right]-\,K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}\\
&-\frac{1}{3}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r\,\right)+c_1+c_2\,\right]\frac{d\varepsilon_0}{dK_1}-\frac{1}{3}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\\
&-\frac{1}{2}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r+Q_2^r-K_1\,\right)+c_2\,\right]\left[\frac{d\varepsilon_{12}}{dK_1}-\frac{d\varepsilon_0}{dK_1}\right]+\frac{1}{2}\,Q_1^r\,b^w\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\\
&-\frac{1}{2}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right]+\,b^w\,Q_1^r\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\\
&+\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]\frac{d\varepsilon_{12}}{dK_1}+\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}\,; \qquad (220)
\end{aligned}
$$

$$\frac{\partial E\left\{\Pi_1^p\right\}}{\partial K_2} \;=\; \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_2^r\right)+c_2-2\,c_1\,\right]\left[a^L+3\,Q_1^r+Q_2^r+\frac{1}{b^w}\left(\,c_2-2\,c_1\,\right)\right]\frac{d\varepsilon_0}{dK_2}$$

$$+\;\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_2^r\right)+c_2-2\,c_1\,\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}$$

$$+\;\frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+3\,Q_1^r+Q_2^r+\frac{1}{b^w}\left(\,c_2-2\,c_1\,\right)\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}+\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right](\varepsilon_0)^2\,\frac{d\varepsilon_0}{dK_2}$$

$$+\;\frac{1}{2}\,K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1\right)+c_2-2\,c_1\,\right]\left[\frac{d\varepsilon_{12}}{dK_2}-\frac{d\varepsilon_0}{dK_2}\right]$$

$$+\;\frac{1}{2}\,K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}\right]-\;K_1\,b^w\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$-\;K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1-K_2\right)-c_1\,\right]\frac{d\varepsilon_{12}}{dK_2}-\;K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2}$$

$$-\;\frac{1}{3}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_2^r\right)+c_1+c_2\,\right]\frac{d\varepsilon_0}{dK_2}-\frac{1}{3}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}$$

$$-\;\frac{1}{2}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1\right)+c_2\,\right]\left[\frac{d\varepsilon_{12}}{dK_2}-\frac{d\varepsilon_0}{dK_2}\right]$$

$$-\;\frac{1}{2}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}\right]+\;b^w\,Q_1^r\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+\;Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]\frac{d\varepsilon_{12}}{dK_2}+\;Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2}\,; \qquad (221)$$

$$\frac{\partial E\left\{\Pi_1^p\right\}}{\partial r_1} = \;\frac{1}{9}\,b^w\,d_2\,\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+3\,Q_1^r+Q_2^r+\frac{1}{b^w}\left(\,c_2-2\,c_1\,\right)\right]$$

$$-\;\frac{1}{9}\left[\,3\,b_1-d_2\,\right]\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_2^r\right)+c_2-2\,c_1\,\right]$$

$$+\;\frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_2^r\right)+c_2-2\,c_1\,\right]\left[a^L+3\,Q_1^r+Q_2^r+\frac{1}{b^w}\left(\,c_2-2\,c_1\,\right)\right]\frac{d\varepsilon_0}{dr_1}$$

$$+\;\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_2^r\right)+c_2-2\,c_1\,\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_1}$$

$$+\;\frac{1}{18}\,d_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right]$$

$$
\begin{aligned}
&+ \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L + 3\,Q_1^r + Q_2^r + \frac{1}{b^w}\,(\,c_2 - 2\,c_1)\right]\varepsilon_0\;\frac{d\varepsilon_0}{dr_1} \\
&- \frac{1}{18}\,[\,3\,b_1 - d_2\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2 - (\underline{\varepsilon})^2\,\right] + \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right](\varepsilon_0)^2\;\frac{d\varepsilon_0}{dr_1} \\
&+ \frac{1}{2}\,K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L + Q_1^r + Q_2^r - K_1\,\right) + c_2 - 2\,c_1\,\right]\left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1}\right] \\
&- \frac{1}{2}\,K_1\,b^w\,[\,b_1 - d_2\,]\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right] + \;\frac{1}{2}\,K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\;\frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0\;\frac{d\varepsilon_0}{dr_1}\right] \\
&- \;K_1\,b^w\,[\,b_1 - d_2\,]\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \\
&- \;K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right) - c_1\,\right]\frac{d\varepsilon_{12}}{dr_1} - \;K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\;\frac{d\varepsilon_{12}}{dr_1} \\
&+ \;Q_1^r - b_1\,[\,r_1 - c_1^r\,] - \frac{1}{3}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L + Q_2^r\,\right) + c_1 + c_2\,\right]\frac{d\varepsilon_0}{dr_1} \\
&+ \;\frac{1}{3}\,b_1\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L + Q_2^r\,\right) + c_1 + c_2\,\right] - \frac{1}{3}\,b^w\,d_2\,Q_1^r\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \\
&- \;\frac{1}{3}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\;\frac{d\varepsilon_0}{dr_1} + \frac{1}{6}\,b_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2 - (\underline{\varepsilon})^2\,\right] \\
&- \frac{1}{2}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L + Q_1^r + Q_2^r - K_1\,\right) + c_2\,\right]\left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1}\right] \\
&+ \;\frac{1}{2}\,Q_1^r\,b^w\,[\,b_1 - d_2\,]\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right] + \frac{1}{2}\,b_1\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L + Q_1^r + Q_2^r - K_1\,\right) + c_2\,\right] \\
&- \;\frac{1}{2}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\;\frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0\;\frac{d\varepsilon_0}{dr_1}\right] + \;\frac{1}{4}\,b_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2 - (\varepsilon_0)^2\,\right] \\
&+ \;b^w\,Q_1^r\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b_1 - d_2\,] + \;b^w\,b_1\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right] \\
&+ \;Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]b^w\left[\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right]\frac{d\varepsilon_{12}}{dr_1} \\
&+ \;\frac{1}{2}\,b_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2 - (\varepsilon_{12})^2\,\right] + \;Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\;\frac{d\varepsilon_{12}}{dr_1}\,; \qquad (222)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E\left\{\Pi_1^p\right\}}{\partial r_2} = & -\frac{1}{9}\, b^w\, b_2 \left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+3\,Q_1^r+Q_2^r+\frac{1}{b^w}\left(\,c_2-2\,c_1\,\right)\right] \\
& + \frac{1}{9}\left[\,3\,d_1-b_2\,\right]\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r\right)+c_2-2\,c_1\,\right] \\
& + \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r\right)+c_2-2\,c_1\,\right]\left[a^L+3\,Q_1^r+Q_2^r+\frac{1}{b^w}\left(\,c_2-2\,c_1\,\right)\right]\frac{d\varepsilon_0}{dr_2} \\
& + \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r\right)+c_2-2\,c_1\,\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_2}-\frac{1}{18}\,b_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right] \\
& + \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+3\,Q_1^r+Q_2^r+\frac{1}{b^w}\left(\,c_2-2\,c_1\,\right)\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_2} \\
& + \frac{1}{18}\left[\,3\,d_1-b_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right]+\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right](\varepsilon_0)^2\,\frac{d\varepsilon_0}{dr_2} \\
& + \frac{1}{2}\,K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r+Q_2^r-K_1\right)+c_2-2\,c_1\,\right]\left[\frac{d\varepsilon_{12}}{dr_2}-\frac{d\varepsilon_0}{dr_2}\right] \\
& - \frac{1}{2}\,K_1\,b^w\left[\,b_2-d_1\,\right]\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]+\frac{1}{2}\,K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_2}-\varepsilon_0\,\frac{d\varepsilon_0}{dr_2}\right] \\
& - K_1\,b^w\left[\,b_2-d_1\,\right]\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \\
& - K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r+Q_2^r-K_1-K_2\right)-c_1\,\right]\frac{d\varepsilon_{12}}{dr_2}-K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_2} \\
& + d_1\left[\,r_1-c_1^r\,\right]-\frac{1}{3}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r\right)+c_1+c_2\,\right]\frac{d\varepsilon_0}{dr_2} \\
& - \frac{1}{3}\,d_1\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r\right)+c_1+c_2\,\right]+\frac{1}{3}\,b^w\,b_2\,Q_1^r\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \\
& - \frac{1}{3}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_2}-\frac{1}{6}\,d_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right] \\
& - \frac{1}{2}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r+Q_2^r-K_1\right)+c_2\,\right]\left[\frac{d\varepsilon_{12}}{dr_2}-\frac{d\varepsilon_0}{dr_2}\right] \\
& + \frac{1}{2}b^w\,Q_1^r\left[\,b_2-d_1\,\right]\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]-\frac{1}{2}\,d_1\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_1^r+Q_2^r-K_1\right)+c_2\,\right] \\
& - \frac{1}{2}\,Q_1^r\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_2}-\varepsilon_0\,\frac{d\varepsilon_0}{dr_2}\right]-\frac{1}{4}\,d_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right]
\end{aligned}
$$

$$+ \; b^w\, Q_1^r\,[\, b_2 - d_1\,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - \; b^w\, d_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, a^L + Q_1^r + Q_2^r - K_1 - K_2\,]$$

$$+ \; Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w\, [\, a^L + Q_1^r + Q_2^r - K_1 - K_2\,]\, \frac{d\varepsilon_{12}}{dr_2}$$

$$- \; \frac{1}{2}\, d_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2\,] + \; Q_1^r \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12}\, \frac{d\varepsilon_{12}}{dr_2}\,. \tag{223}$$

where ε_0 and ε_{12} are as specified in (191).

Recall that in this case, G2 seeks to maximize

$$E\,\{\Pi_2^p\} \;\; \equiv \;\; E\,\{\,[\, w(\varepsilon) - c_2\,]\, q_2^*(\varepsilon)\,\}\,.$$

Therefore, (219) implies:

$$\frac{\partial E\,\{\Pi_2^v\}}{\partial K_1} \;=\; \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r \right) + c_1 - 2\, c_2\,\right] \left[a^L + Q_2^r + \frac{1}{b^w}\,(\, c_1 - 2\, c_2) \right] \frac{d\varepsilon_0}{dK_1}$$

$$+ \; \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r \right) + c_1 - 2\, c_2\,\right] \varepsilon_0\, \frac{d\varepsilon_0}{dK_1}$$

$$+ \; \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_2^r + \frac{1}{b^w}\,(\, c_1 - 2\, c_2) \right] \varepsilon_0\, \frac{d\varepsilon_0}{dK_1} + \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2\, \frac{d\varepsilon_0}{dK_1}$$

$$- \; \frac{1}{4} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) - c_2\,\right]$$

$$- \; \frac{1}{4}\, b^w \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, a^L + Q_1^r + Q_2^r - K_1 - \frac{c_2}{b^w} \right]$$

$$+ \; \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) - c_2\,\right] \left[\, a^L + Q_1^r + Q_2^r - K_1 - \frac{c_2}{b^w} \right] \left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1} \right]$$

$$- \; \frac{1}{8} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, (\varepsilon_{12})^2 - (\varepsilon_0)^2\,]$$

$$+ \; \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) - c_2\,\right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_1} - \varepsilon_0\, \frac{d\varepsilon_0}{dK_1} \right]$$

$$- \; \frac{1}{8} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, (\varepsilon_{12})^2 - (\varepsilon_0)^2\,]$$

$$+ \; \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, a^L + Q_1^r + Q_2^r - K_1 - \frac{c_2}{b^w} \right] \left[\varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_1} - \varepsilon_0\, \frac{d\varepsilon_0}{dK_1} \right]$$

$$+\ \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[(\varepsilon_{12})^2\ \frac{d\varepsilon_{12}}{dK_1}-(\varepsilon_0)^2\ \frac{d\varepsilon_0}{dK_1}\right]-\ K_2\,b^w\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$-\ K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1-K_2\right)-c_2\,\right]\frac{d\varepsilon_{12}}{dK_1}-\ K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\ \frac{d\varepsilon_{12}}{dK_1}\,;\quad (224)$$

$$\frac{\partial E\left\{\Pi_2^v\right\}}{\partial K_2}\ =\ \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_2^r\right)+c_1-2\,c_2\,\right]\left[a^L+Q_2^r+\frac{1}{b^w}\left(\,c_1-2\,c_2\right)\right]\frac{d\varepsilon_0}{dK_2}$$

$$+\ \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_2^r\right)+c_1-2\,c_2\,\right]\varepsilon_0\ \frac{d\varepsilon_0}{dK_2}$$

$$+\ \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+Q_2^r+\frac{1}{b^w}\left(\,c_1-2\,c_2\right)\right]\varepsilon_0\ \frac{d\varepsilon_0}{dK_2}+\frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right](\varepsilon_0)^2\ \frac{d\varepsilon_0}{dK_2}$$

$$+\ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1\right)-c_2\,\right]\left[\,a^L+Q_1^r+Q_2^r-K_1-\frac{c_2}{b^w}\,\right]\left[\frac{d\varepsilon_{12}}{dK_2}-\frac{d\varepsilon_0}{dK_2}\right]$$

$$+\ \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1\right)-c_2\,\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}\right]$$

$$+\ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,a^L+Q_1^r+Q_2^r-K_1-\frac{c_2}{b^w}\,\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}\right]$$

$$+\ \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[(\varepsilon_{12})^2\ \frac{d\varepsilon_{12}}{dK_2}-(\varepsilon_0)^2\ \frac{d\varepsilon_0}{dK_2}\right]-\ K_2\,b^w\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+\ \left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1-K_2\right)-c_2\,\right]$$

$$-\ K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1-K_2\right)-c_2\,\right]\frac{d\varepsilon_{12}}{dK_2}$$

$$+\ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2-(\varepsilon_{12})^2\,\right]-\ K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\ \frac{d\varepsilon_{12}}{dK_2}\,;\qquad (225)$$

$$\frac{\partial E\left\{\Pi_2^v\right\}}{\partial r_1}=\ \frac{1}{9}\,d_2\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_2^r\right)+c_1-2\,c_2\,\right]$$

$$+\ \frac{1}{9}\,b^w d_2\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+Q_2^r+\frac{1}{b^w}\left(\,c_1-2\,c_2\,\right)\right]$$

$$+\ \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_2^r\right)+c_1-2\,c_2\,\right]\left[a^L+Q_2^r+\frac{1}{b^w}\left(\,c_1-2\,c_2\,\right)\right]\frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{9\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r \right) + c_1 - 2\,c_2 \,\right] \varepsilon_0 \, \frac{d\varepsilon_0}{dr_1} + \frac{1}{18}\, d_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right]$$

$$+ \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_2^r + \frac{1}{b^w} \left(c_1 - 2\,c_2 \right) \right] \varepsilon_0 \, \frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{18}\, d_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right] + \frac{1}{9\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2 \, \frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) - c_2 \,\right] \left[a^L + Q_1^r + Q_2^r - K_1 - \frac{c_2}{b^w} \right] \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1} \right]$$

$$- \frac{1}{4}\, b^w \left[\, b_1 - d_2 \,\right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 - \frac{c_2}{b^w} \right]$$

$$- \frac{1}{4} \left[\, b_1 - d_2 \,\right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) - c_2 \,\right]$$

$$- \frac{1}{8} \left[\, b_1 - d_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right]$$

$$+ \frac{1}{4\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) - c_2 \,\right] \left[\varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0 \, \frac{d\varepsilon_0}{dr_1} \right]$$

$$+ \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 - \frac{c_2}{b^w} \right] \left[\varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0 \, \frac{d\varepsilon_0}{dr_1} \right]$$

$$- \frac{1}{8} \left[\, b_1 - d_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right]$$

$$+ \frac{1}{4\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 \, \frac{d\varepsilon_{12}}{dr_1} - (\varepsilon_0)^2 \, \frac{d\varepsilon_0}{dr_1} \right]$$

$$- K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - c_2 \,\right] \frac{d\varepsilon_{12}}{dr_1}$$

$$- K_2\, b^w \left[\, b_1 - d_2 \,\right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_1} \,; \tag{226}$$

$$\frac{\partial E\left\{\Pi_2^v\right\}}{\partial r_2} = - \frac{1}{9}\, b_2 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r \right) + c_1 - 2\,c_2 \,\right]$$

$$- \frac{1}{9}\, b^w b_2 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_2^r + \frac{1}{b^w} \left(c_1 - 2\,c_2 \right) \right]$$

$$
\begin{aligned}
&+\frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^{w}\left(\,a^{L}+Q_{2}^{r}\right)+c_{1}-2\,c_{2}\,\right]\left[a^{L}+Q_{2}^{r}+\frac{1}{b^{w}}\left(\,c_{1}-2\,c_{2}\,\right)\right]\frac{d\varepsilon_{0}}{dr_{2}}\\
&+\frac{1}{9\,b^{w}}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^{w}\left(\,a^{L}+Q_{2}^{r}\right)+c_{1}-2\,c_{2}\,\right]\varepsilon_{0}\,\frac{d\varepsilon_{0}}{dr_{2}}-\frac{1}{18}\,b_{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{0})^{2}-(\underline{\varepsilon})^{2}\,\right]\\
&+\frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^{L}+Q_{2}^{r}+\frac{1}{b^{w}}\left(\,c_{1}-2\,c_{2}\,\right)\right]\varepsilon_{0}\,\frac{d\varepsilon_{0}}{dr_{2}}\\
&-\frac{1}{18}\,b_{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{0})^{2}-(\underline{\varepsilon})^{2}\,\right]+\frac{1}{9\,b^{w}}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right](\varepsilon_{0})^{2}\,\frac{d\varepsilon_{0}}{dr_{2}}\\
&+\frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^{w}\left(\,a^{L}+Q_{1}^{r}+Q_{2}^{r}-K_{1}\,\right)-c_{2}\,\right]\left[a^{L}+Q_{1}^{r}+Q_{2}^{r}-K_{1}-\frac{c_{2}}{b^{w}}\right]\left[\frac{d\varepsilon_{12}}{dr_{2}}-\frac{d\varepsilon_{0}}{dr_{2}}\right]\\
&-\frac{1}{4}\,b^{w}\left[\,b_{2}-d_{1}\right]\left[\frac{\varepsilon_{12}-\varepsilon_{0}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,a^{L}+Q_{1}^{r}+Q_{2}^{r}-K_{1}-\frac{c_{2}}{b^{w}}\,\right]\\
&-\frac{1}{4}\left[\,b_{2}-d_{1}\,\right]\left[\frac{\varepsilon_{12}-\varepsilon_{0}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^{w}\left(\,a^{L}+Q_{1}^{r}+Q_{2}^{r}-K_{1}\,\right)-c_{2}\,\right]\\
&-\frac{1}{8}\left[\,b_{2}-d_{1}\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^{2}-(\varepsilon_{0})^{2}\,\right]\\
&+\frac{1}{4\,b^{w}}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^{w}\left(\,a^{L}+Q_{1}^{r}+Q_{2}^{r}-K_{1}\,\right)-c_{2}\,\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_{2}}-\varepsilon_{0}\,\frac{d\varepsilon_{0}}{dr_{2}}\right]\\
&+\frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,a^{L}+Q_{1}^{r}+Q_{2}^{r}-K_{1}-\frac{c_{2}}{b^{w}}\,\right]\left[\,\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_{2}}-\varepsilon_{0}\,\frac{d\varepsilon_{0}}{dr_{2}}\,\right]\\
&-\frac{1}{8}\left[\,b_{2}-d_{1}\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^{2}-(\varepsilon_{0})^{2}\,\right]\\
&+\frac{1}{4\,b^{w}}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[(\varepsilon_{12})^{2}\,\frac{d\varepsilon_{12}}{dr_{2}}-(\varepsilon_{0})^{2}\,\frac{d\varepsilon_{0}}{dr_{2}}\right]\\
&-K_{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^{w}\left(\,a^{L}+Q_{1}^{r}+Q_{2}^{r}-K_{1}-K_{2}\,\right)-c_{2}\,\right]\frac{d\varepsilon_{12}}{dr_{2}}\\
&-K_{2}\,b^{w}\left[\,b_{2}-d_{1}\,\right]\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]-K_{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_{2}} \qquad (227)
\end{aligned}
$$

where ε_0 and ε_{12} are as specified in (191).

(191) implies that when G1 is capacity-constrained for $\varepsilon \in (\, \varepsilon_0, \varepsilon_{12}\,)$ under partial vertical integration:

$$\varepsilon_0 \;=\; 3\, b^w K_1 + 2\, c_1 - c_2 - b^w \left[\, a^L + 3\, Q_1^r + Q_2^r \,\right];$$

$$\varepsilon_{12} \;=\; 2\, b^w K_2 + b^w K_1 + c_2 - b^w \left[\, a^L + Q_1^r + Q_2^r \,\right]$$

$$\Rightarrow \;\; \frac{d\varepsilon_0}{dK_1} \;=\; 3\, b^w ; \;\; \frac{d\varepsilon_0}{dK_2} \;=\; 0 ; \;\; \frac{d\varepsilon_0}{dr_1} \;=\; b^w [\, 3\, b_1 - d_2 \,] ; \;\; \frac{d\varepsilon_0}{dr_2} \;=\; b^w [\, b_2 - 3\, d_1 \,] ;$$

$$\frac{d\varepsilon_{12}}{dK_1} \;=\; b^w ; \;\; \frac{d\varepsilon_{12}}{dK_2} \;=\; 2\, b^w ; \;\; \frac{d\varepsilon_{12}}{dr_1} \;=\; b^w [\, b_1 - d_2 \,] ; \;\; \frac{d\varepsilon_{12}}{dr_2} \;=\; b^w [\, b_2 - d_1 \,] . \qquad (228)$$

(228) implies:

$$\frac{d^2 \varepsilon_0}{dr_1\, dz} \;=\; \frac{d^2 \varepsilon_0}{dr_2\, dz} \;=\; \frac{d^2 \varepsilon_{12}}{dr_1\, dz} \;=\; \frac{d^2 \varepsilon_{12}}{dr_2\, dz} \;=\; 0 \;\; \text{for} \;\; z \in \{\, K_1, K_2, r_1, r_2 \,\} . \qquad (229)$$

(216), (228), and (229) imply:

$$\begin{aligned}
\frac{\partial E\{w(\varepsilon)\}}{\partial r_1} \;=\;& \frac{1}{3} \left[\, b^w (a^L + Q_2^r) + c_1 + c_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_1} + \frac{1}{3}\, b^w d_2 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \frac{d\varepsilon_0}{dr_1} + \frac{1}{2} \left[\, b^w (a^L + Q_1^r + Q_2^r - K_1) + c_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1} \right] \\
&- \frac{1}{2}\, b^w [\, b_1 - d_2 \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dr_1} - \varepsilon_0 \frac{d\varepsilon_0}{dr_1} \right] \\
&- b^w [\, b_1 - d_2 \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_1} \\
&- \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \frac{d\varepsilon_{12}}{dr_1} ;
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E\{w(\varepsilon)\}}{\partial r_2} \;=\;& \frac{1}{3} \left[\, b^w (a^L + Q_2^r) + c_1 + c_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dr_2} - \frac{1}{3}\, b^w b_2 \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&+ \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \frac{d\varepsilon_0}{dr_2} + \frac{1}{2} \left[\, b^w (a^L + Q_1^r + Q_2^r - K_1) + c_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2} \right] \\
&- \frac{1}{2}\, b^w [\, b_2 - d_1 \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dr_2} - \varepsilon_0 \frac{d\varepsilon_0}{dr_2} \right] \\
&- b^w [\, b_2 - d_1 \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_2}
\end{aligned}$$

$$- \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dr_2} \, ;$$

$$\frac{\partial E\{w(\varepsilon)\}}{\partial K_1} = \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w (a^L + Q_2^r) + c_1 + c_2 \right] \frac{d\varepsilon_0}{dK_1}$$

$$+ \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \, \frac{d\varepsilon_0}{dK_1} + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w (a^L + Q_1^r + Q_2^r - K_1) + c_2 \right] \left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1} \right]$$

$$- \frac{1}{2} b^w \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dK_1} - \varepsilon_0 \frac{d\varepsilon_0}{dK_1} \right] - b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \frac{d\varepsilon_{12}}{dK_1} - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dK_1} \, ;$$

$$\frac{\partial E\{w(\varepsilon)\}}{\partial K_2} = \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w (a^L + Q_2^r) + c_1 + c_2 \right] \frac{d\varepsilon_0}{dK_2}$$

$$+ \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \, \frac{d\varepsilon_0}{dK_2} + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w (a^L + Q_1^r + Q_2^r - K_1) + c_2 \right] \left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2} \right]$$

$$+ \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \frac{d\varepsilon_{12}}{dK_2} - \varepsilon_0 \frac{d\varepsilon_0}{dK_2} \right] - b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \frac{d\varepsilon_{12}}{dK_2}$$

$$- \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dK_2} \, . \tag{230}$$

(229) and (230) imply:

$$\frac{\partial}{\partial K_1} \left(\frac{\partial E\{w(\varepsilon)\}}{\partial r_1} \right) = \frac{1}{3} b^w d_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dK_1} + \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dK_1} \, \frac{d\varepsilon_0}{dr_1}$$

$$- \frac{1}{2} b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1} \right]$$

$$- \frac{1}{2} b^w \left[b_1 - d_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1} \right] + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dK_1} \frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dK_1} \frac{d\varepsilon_0}{dr_1} \right]$$

$$+ b^w \left[b_1 - d_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dK_1} + b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dr_1} - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dK_1} \, \frac{d\varepsilon_{12}}{dr_1} \, ;$$

$$\frac{\partial}{\partial K_2}\left(\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial r_1}\right) = \frac{1}{3}\, b^w\, d_2 \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_0}{dK_2} + \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_0}{dK_2}\,\frac{d\varepsilon_0}{dr_1}$$

$$- \frac{1}{2}\, b^w \left[\, b_1 - d_2\,\right] \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2}\right]$$

$$+ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \left[\frac{d\varepsilon_{12}}{dK_2}\,\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dK_2}\,\frac{d\varepsilon_0}{dr_1}\right] + b^w \left[\, b_1 - d_2\,\right] \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_{12}}{dK_2}$$

$$+ b^w \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_{12}}{dr_1} - \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_{12}}{dK_2}\,\frac{d\varepsilon_{12}}{dr_1}\,;$$

$$\frac{\partial}{\partial r_1}\left(\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial r_1}\right) = \frac{2}{3}\, b^w\, d_2 \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_0}{dr_1} + \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \left[\frac{d\varepsilon_0}{dr_1}\right]^2$$

$$- b^w \left[\, b_1 - d_2\,\right] \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1}\right]$$

$$+ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \left[\left(\frac{d\varepsilon_{12}}{dr_1}\right)^2 - \left(\frac{d\varepsilon_0}{dr_1}\right)^2\right]$$

$$+ 2\, b^w \left[\, b_1 - d_2\,\right] \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_{12}}{dr_1} - \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \left[\frac{d\varepsilon_{12}}{dr_1}\right]^2;$$

$$\frac{\partial}{\partial r_2}\left(\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial r_1}\right) = -\frac{1}{3}\, b^w\, b_2 \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_0}{dr_1} + \frac{1}{3}\, b^w\, d_2 \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_0}{dr_2}$$

$$+ \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_0}{dr_2}\,\frac{d\varepsilon_0}{dr_1} - \frac{1}{2}\, b^w \left[\, b_2 - d_1\,\right] \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1}\right]$$

$$- \frac{1}{2}\, b^w \left[\, b_1 - d_2\,\right] \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2}\right] + \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \left[\frac{d\varepsilon_{12}}{dr_2}\,\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_2}\,\frac{d\varepsilon_0}{dr_1}\right]$$

$$+ b^w \left[\, b_1 - d_2\,\right] \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_{12}}{dr_2} + b^w \left[\, b_2 - d_1\,\right] \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_{12}}{dr_1}$$

$$- \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_{12}}{dr_2}\,\frac{d\varepsilon_{12}}{dr_1}\,;$$

$$\frac{\partial}{\partial K_1}\left(\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial r_2}\right) = -\frac{1}{3}\, b^w\, b_2 \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_0}{dK_1} + \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \frac{d\varepsilon_0}{dK_1}\,\frac{d\varepsilon_0}{dr_2}$$

$$- \frac{1}{2}\, b^w \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \left[\frac{d\varepsilon_{12}}{dr_2} - \frac{d\varepsilon_0}{dr_2}\right] - \frac{1}{2}\, b^w \left[\, b_2 - d_1\,\right] \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1}\right]$$

$$+\ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_1}\,\frac{d\varepsilon_{12}}{dr_2}-\frac{d\varepsilon_0}{dK_1}\,\frac{d\varepsilon_0}{dr_2}\right]$$

$$+\, b^w\,[\,b_2-d_1\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dK_1}+\,b^w\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_2}$$

$$-\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dK_1}\,\frac{d\varepsilon_{12}}{dr_2}\,;$$

$$\frac{\partial}{\partial K_2}\left(\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial r_2}\right)\ =\ -\,\frac{1}{3}\,b^w\,b_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_2}+\frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_2}\,\frac{d\varepsilon_0}{dr_2}$$

$$-\,\frac{1}{2}\,b^w\,[\,b_2-d_1\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_2}-\frac{d\varepsilon_0}{dK_2}\right]$$

$$+\,\frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_2}\,\frac{d\varepsilon_{12}}{dr_2}-\frac{d\varepsilon_0}{dK_2}\,\frac{d\varepsilon_0}{dr_2}\right]$$

$$+\, b^w\,[\,b_2-d_1\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dK_2}+\,b^w\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_2}$$

$$-\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dK_2}\,\frac{d\varepsilon_{12}}{dr_2}\,;$$

$$\frac{\partial}{\partial r_1}\left(\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial r_2}\right)\ =\ \frac{1}{3}\,b^w\,d_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_2}-\frac{1}{3}\,b^w\,b_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_1}$$

$$+\ \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_1}\,\frac{d\varepsilon_0}{dr_2}-\frac{1}{2}\,b^w\,[\,b_1-d_2\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_2}-\frac{d\varepsilon_0}{dr_2}\right]$$

$$-\,\frac{1}{2}\,b^w\,[\,b_2-d_1\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_1}-\frac{d\varepsilon_0}{dr_1}\right]$$

$$+\,\frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_1}\,\frac{d\varepsilon_{12}}{dr_2}-\frac{d\varepsilon_0}{dr_1}\,\frac{d\varepsilon_0}{dr_2}\right]$$

$$+\, b^w\,[\,b_2-d_1\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_1}+\,b^w\,[\,b_1-d_2\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_2}$$

$$-\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_1}\,\frac{d\varepsilon_{12}}{dr_2}\,;$$

$$\frac{\partial}{\partial r_2}\left(\frac{\partial E\{w(\varepsilon)\}}{\partial r_2}\right) = -\frac{2}{3}\,b^w\,b_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_2}+\frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_0}{dr_2}\right]^2$$

$$-\,b^w\,[\,b_2-d_1\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_2}-\frac{d\varepsilon_0}{dr_2}\right]$$

$$+\,\frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\left(\frac{d\varepsilon_{12}}{dr_2}\right)^2-\left(\frac{d\varepsilon_0}{dr_2}\right)^2\right]+\,b^w\,[\,b_2-d_1\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_2}$$

$$+\,b^w\,[\,b_2-d_1\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dr_2}-\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_2}\right]^2. \tag{231}$$

<u>Further Characterization of the Optimal Retail Prices</u>

(189) and (190) imply that when G1 is capacity-constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$E\left\{q_1^{*0c}\right\} \equiv \int_{\underline{\varepsilon}}^{\varepsilon_0} q_1^{*0c}(\varepsilon)\,dH(\varepsilon) = \frac{1}{3}\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,a^L+3\,Q_1^r+Q_2^r\,\right]$$

$$+\,\frac{1}{3\,b^w}\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,c_2-2\,c_1\,]+\frac{1}{6\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right];$$

$$E\left\{q_2^{*0c}\right\} \equiv \int_{\underline{\varepsilon}}^{\varepsilon_0} q_2^{*0c}(\varepsilon)\,dH(\varepsilon) = \frac{1}{3}\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,a^L+Q_2^r\,\right]+\frac{1}{3\,b^w}\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,c_1-2\,c_2\,]$$

$$+\,\frac{1}{6\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right];$$

$$E\left\{q_2^{*1c}\right\} \equiv \int_{\varepsilon_0}^{\varepsilon_{12}} q_2^{*1c}(\varepsilon)\,dH(\varepsilon) = \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right]-\frac{1}{2\,b^w}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]c_2$$

$$+\,\frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,a^L+Q_1^r+Q_2^r-K_1\,\right]. \tag{232}$$

(232) implies that in this case:

$$\frac{\partial E\left\{q_1^{*0c}\right\}}{\partial r_1} = -\frac{1}{3}\,[\,3\,b_1-d_2\,]\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]+\frac{1}{3}\left[\,a^L+3\,Q_1^r+Q_2^r\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_1}$$

$$+\,\frac{1}{3\,b^w}\,[\,c_2-2\,c_1\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_1}+\frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_1};$$

$$\frac{\partial E\{q_1^{*0c}\}}{\partial r_2} = \frac{1}{3}[3\,d_1 - b_2]\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] + \frac{1}{3}[a^L + 3\,Q_1^r + Q_2^r]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_2}$$

$$+ \frac{1}{3\,b^w}[c_2 - 2\,c_1]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_2} + \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_2};$$

$$\frac{\partial E\{q_1^{*0c}\}}{\partial K_1} = \frac{1}{3}[a^L + 3\,Q_1^r + Q_2^r]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_1}$$

$$+ \frac{1}{3\,b^w}[c_2 - 2\,c_1]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_1} + \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1};$$

$$\frac{\partial E\{q_1^{*0c}\}}{\partial K_2} = \frac{1}{3}[a^L + 3\,Q_1^r + Q_2^r]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_2}$$

$$+ \frac{1}{3\,b^w}[c_2 - 2\,c_1]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_2} + \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_2};$$

$$\frac{\partial E\{q_2^{*0c}\}}{\partial r_1} = \frac{1}{3}\,d_2\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] + \frac{1}{3}[a^L + Q_2^r]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_1}$$

$$+ \frac{1}{3\,b^w}[c_1 - 2\,c_2]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_1} + \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_1};$$

$$\frac{\partial E\{q_2^{*0c}\}}{\partial r_2} = -\frac{1}{3}\,b_2\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] + \frac{1}{3}[a^L + Q_2^r]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_2}$$

$$+ \frac{1}{3\,b^w}[c_1 - 2\,c_2]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dr_2} + \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dr_2};$$

$$\frac{\partial E\{q_2^{*0c}\}}{\partial K_1} = \frac{1}{3}[a^L + Q_2^r]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_1}$$

$$+ \frac{1}{3\,b^w}[c_1 - 2\,c_2]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_1} + \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_1};$$

$$\frac{\partial E\{q_2^{*0c}\}}{\partial K_2} = \frac{1}{3}[a^L + Q_2^r]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_2}$$

$$+ \frac{1}{3\,b^w}[c_1 - 2\,c_2]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_2} + \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\varepsilon_0\,\frac{d\varepsilon_0}{dK_2};$$

$$\frac{\partial E\{q_2^{*1c}\}}{\partial r_1} = -\frac{1}{2}[b_1 - d_2]\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right] + \frac{1}{2}[a^L + Q_1^r + Q_2^r - K_1]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_1} - \frac{d\varepsilon_0}{dr_1}\right]$$

$$+\ \frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_1}-\varepsilon_0\,\frac{d\varepsilon_0}{dr_1}\right]-\frac{c_2}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_1}-\frac{d\varepsilon_0}{dr_1}\right];$$

$$\frac{\partial E\left\{q_2^{*1c}\right\}}{\partial r_2} = \frac{1}{2}\left[\,d_1-b_2\,\right]\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]+\frac{1}{2}\left[\,a^L+Q_1^r+Q_2^r-K_1\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_2}-\frac{d\varepsilon_0}{dr_2}\right]$$

$$+\ \frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dr_2}-\varepsilon_0\,\frac{d\varepsilon_0}{dr_2}\right]-\frac{c_2}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dr_2}-\frac{d\varepsilon_0}{dr_2}\right];$$

$$\frac{\partial E\left\{q_2^{*1c}\right\}}{\partial K_1} = -\frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]+\frac{1}{2}\left[\,a^L+Q_1^r+Q_2^r-K_1\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_1}-\frac{d\varepsilon_0}{dK_1}\right]$$

$$+\ \frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right]-\frac{c_2}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_1}-\frac{d\varepsilon_0}{dK_1}\right];$$

$$\frac{\partial E\left\{q_2^{*1c}\right\}}{\partial K_2} = \frac{1}{2}\left[\,a^L+Q_1^r+Q_2^r-K_1\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_2}-\frac{d\varepsilon_0}{dK_2}\right]$$

$$+\ \frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2}-\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}\right]-\frac{c_2}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{d\varepsilon_{12}}{dK_2}-\frac{d\varepsilon_0}{dK_2}\right]. \quad (233)$$

(106) and (110) imply that r_1^* and r_2^* are characterized in this case by:

$$J_1(r_1,r_2) = b^w\left[\frac{2}{3}\,d_2-b_1\right]E\left\{q_1^{*0c}\right\}$$

$$+\left[1+b^w\,b_1-\frac{2}{3}\,b^w\,d_2\right]\left[\,a_1-b_1\,r_1+d_1\,r_2\,\right]\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+\ b_1\,E\left\{w(\cdot)\right\}-b_1\left[\,r_1-c_1^r\,\right]-\frac{1}{2}\,b^w\left[\,b_1+d_2\,\right]K_1\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+\left[1+\frac{1}{2}\,b^w\,b_1-\frac{1}{2}\,b^w\,d_2\right]\left[\,a_1-b_1\,r_1+d_1\,r_2\,\right]\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+\,b^w\,d_2\,K_1\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]-\ b^w\left[\,b_1-d_2\,\right]K_1\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+\ \left[\,1+b^w\,b_1-b^w\,d_2\,\right]\left[\,a_1-b_1\,r_1+d_1\,r_2\,\right]\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] = 0\,;$$

$$J_2(r_1,r_2) = \left[\,a_2-b_2\,r_2+d_2\,r_1\,\right]\left[1-\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial r_2}\right]-\ b_2\left[\,r_2-c_2^r-E\left\{\,w(Q)\,\right\}\,\right] = 0\,, \quad (234)$$

where $E\left\{w(\varepsilon)\right\}$ and $\frac{\partial E\{w(\varepsilon)\}}{\partial r_2}$ are specified in (216) and (230), and ε_0, and ε_{12} are specified in (228).

For $v \in \{ r_1,\, r_2,\, K_1,\, K_2 \}$, define:

$$\widetilde{M}_1^B(v) \;\equiv\; b^w \left[\frac{2}{3}\, d_2 - b_1 \right] \frac{\partial E\,\{q_1^{*0c}\}}{\partial v} + \; b_1\, \frac{\partial E\,\{w\,(\cdot)\}}{\partial v}$$

$$+ \left[1 + b^w\, b_1 - \frac{2}{3}\, b^w\, d_2 \right] [\, a_1 - b_1\, r_1 + d_1\, r_2 \,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_0}{dv}$$

$$- \frac{1}{2}\, b^w\, [\, b_1 + d_2 \,]\, K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dv} - \frac{d\varepsilon_0}{dv} \right]$$

$$+ \left[1 + \frac{1}{2}\, b^w\, b_1 - \frac{1}{2}\, b^w\, d_2 \right] [\, a_1 - b_1\, r_1 + d_1\, r_2 \,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dv} - \frac{d\varepsilon_0}{dv} \right]$$

$$+ \; b^w\, K_1\, d_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{d\varepsilon_{12}}{dv} - \frac{d\varepsilon_0}{dv} \right] + b^w\, K_1\, [\, b_1 - d_2 \,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dv}$$

$$- \; [\, 1 + b^w\, b_1 - b^w\, d_2 \,]\, [\, a_1 - b_1\, r_1 + d_1\, r_2 \,] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{d\varepsilon_{12}}{dv}$$

$$\widetilde{M}_2^B(v) \;\equiv\; - \, [\, a_2 - b_2\, r_2 + d_2\, r_1 \,]\, \frac{\partial}{\partial v} \left(\frac{\partial E\,\{w(\varepsilon)\}}{\partial r_2} \right) + \; b_2\, \frac{\partial E\,\{w\,(\cdot)\}}{\partial v}\,. \tag{235}$$

From (234) and (235):

$$\frac{\partial J_1(\cdot)}{\partial r_1} \;=\; - \, b_1 \left[1 + b^w\, b_1 - \frac{2}{3}\, b^w\, d_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b_1 \left[1 + \frac{1}{2}\, b^w\, b_1 - \frac{1}{2}\, b^w\, d_2 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \; b_1 - b_1\, [\, 1 + b^w\, b_1 - b^w\, d_2 \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \widetilde{M}_1^B(r_1)\,;$$

$$\frac{\partial J_1(\cdot)}{\partial r_2} \;=\; d_1 \left[1 + b^w\, b_1 - \frac{2}{3}\, b^w\, d_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + d_1 \left[1 + \frac{1}{2}\, b^w\, b_1 - \frac{1}{2}\, b^w\, d_2 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \; d_1\, [\, 1 + b^w\, b_1 - b^w\, d_2 \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \widetilde{M}_1^B(r_2)\,;$$

$$\frac{\partial J_1(\cdot)}{\partial K_1} \;=\; - \, \frac{1}{2}\, b^w\, [\, b_1 + d_2 \,] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b^w\, d_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \; b^w\, [\, b_1 - d_2 \,] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \widetilde{M}_1^B(K_1)\,;$$

$$\frac{\partial J_1(\cdot)}{\partial K_2} = \widetilde{M}_1^B(K_2)\,;$$

$$\frac{\partial J_2(\cdot)}{\partial r_1} = d_2\left[1-\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial r_2}\right]+\widetilde{M}_2^B(r_1)\,;$$

$$\frac{\partial J_2(\cdot)}{\partial r_2} = -\,b_2\left[1-\frac{\partial E\left\{w(\varepsilon)\right\}}{\partial r_2}\right]-b_2+\widetilde{M}_2^B(r_2)\,;$$

$$\frac{\partial J_2(\cdot)}{\partial K_1} = \widetilde{M}_2^B(K_1)\,; \qquad \frac{\partial J_2(\cdot)}{\partial K_2} = \widetilde{M}_2^B(K_2)\,. \tag{236}$$

II. Additional Numerical Solutions.

Tables T1 – T65 below record the equilibrium outcomes that arise when the parameter or parameters identified in the title of the table are changed from their values in the baseline setting. All other parameter values are as specified in the baseline setting.

Parameter	Value		Parameter	Value		Parameter	Value
$c_1 = c_2$	20		$a_1 = a_2$	2,100		a^I	7,500
$k_1 = k_2$	35		$b_1 = b_2$	8		b^I	35
$c_1^r = c_2^r$	1		$d_1 = d_2$	3		$\underline{\varepsilon}$	0
						$\overline{\varepsilon}$	700

Table 1. Parameter Values in the Baseline Setting.

Observe that in all of the settings considered below: (i) retail prices are lowest under VI and highest under VS; (ii) industry capacity and expected wholesale output are highest under VI and lowest under VS; (iii) the combined expected profit of G1 and R1 is higher under PVI than under VS; (iv) G2's equilibrium investment and expected output are lower under PVI than under VS; (v) the combined expected profit of G2 and R2 ($E\left\{\pi_2^G + \pi_2^R\right\}$) is lower under PVI than under VS; (vi) $E\left\{\pi_2^G + \pi_2^R\right\}$ is higher under VI than under PVI; (vii) expected consumer surplus is highest under VI and lowest under VS; and (viii) expected total welfare is highest under VI and lowest under VS.

IIA. Additional Unilateral 20% Increases (Not Considered in the Text).

	VS	PVI	VI
r_1	314	244	217
r_2	337	322	239
$E\{w\}$	233	228	225
K_1	8,574	9,317	8,948
K_2	8,574	8,260	9,113
$K_1 + K_2$	17,147	17,576	18,062
$E\{q_1\}$	6,477	7,318	7,027
$E\{q_2\}$	6,477	6,223	7,200
$E\{q_1\} + E\{q_2\}$	12,953	13,541	14,227
$E\left\{\pi_1^G\right\}$	1,247,411	1,364,998	1,290,314
$E\left\{\pi_1^R\right\}$	47,840	16,635	-8,965
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,295,250	1,381,634	1,281,348
$E\left\{\pi_2^G\right\}$	1,247,411	1,175,880	1,319,421
$E\left\{\pi_2^R\right\}$	78,878	62,670	17,396
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,326,289	1,238,549	1,336,816
$E\{CS\}$	2,194,191	2,287,944	2,390,117
$E\{CS + \pi\}$	4,815,730	4,908,127	5,008,282

Table T1. Outcomes when $a_2 = 2,520$**.**

	VS	PVI	VI
r_1	300	232	205
r_2	273	260	180
$E\{w\}$	228	224	224
K_1	8,438	9,143	8,899
K_2	8,438	8,147	8,894
$K_1 + K_2$	16,876	17,290	17,793
$E\{q_1\}$	6,328	7,118	6,954
$E\{q_2\}$	6,328	6,101	6,949
$E\{q_1\} + E\{q_2\}$	12,656	13,219	13,903
$E\left\{\pi_1^G\right\}$	1,190,825	1,301,265	1,272,067
$E\left\{\pi_1^R\right\}$	36,601	6,585	-19,394
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,227,426	1,307,850	1,252,672
$E\left\{\pi_2^G\right\}$	1,190,825	1,129,337	1,271,192
$E\left\{\pi_2^R\right\}$	16,462	10,350	-44,730
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,207,287	1,139,687	1,226,463
$E\{CS\}$	2,215,750	2,302,034	2,343,088
$E\{CS + \pi\}$	4,650,463	4,749,570	4,822,223

Table T2. Outcomes when $b_2 = 9.6$**.**

	VS	PVI	VI
r_1	310	246	217
r_2	321	304	221
$E\{w\}$	232	227	225
K_1	8,518	9,214	8,912
K_2	8,518	8,212	8,946
$K_1 + K_2$	17,036	17,427	17,858
$E\{q_1\}$	6,429	7,211	6,982
$E\{q_2\}$	6,429	6,180	7,046
$E\{q_1\} + E\{q_2\}$	12,859	13,392	14,028
$E\left\{\pi_1^G\right\}$	1,232,420	1,338,169	1,283,434
$E\left\{\pi_1^R\right\}$	44,753	18,573	-9,235
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,277,174	1,356,741	1,274,199
$E\left\{\pi_2^G\right\}$	1,232,420	1,161,127	1,293,531
$E\left\{\pi_2^R\right\}$	57,083	41,932	-4,690
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,289,504	1,203,058	1,288,842
$E\{CS\}$	2,194,329	2,283,163	2,360,792
$E\{CS + \pi\}$	4,761,006	4,842,963	4,923,833

Table T3. Outcomes when $d_2 = 3.6$**.**

	VS	PVI	VI
r_1	308	239	214
r_2	308	293	215
$E\{w\}$	232	228	225
K_1	8,621	9,269	9,032
K_2	8,236	7,966	8,652
$K_1 + K_2$	16,857	17,235	17,683
$E\{q_1\}$	6,434	7,224	7,024
$E\{q_2\}$	6,319	6,085	6,890
$E\{q_1\} + E\{q_2\}$	12,752	13,310	13,913
$E\left\{\pi_1^G\right\}$	1,235,856	1,348,468	1,300,196
$E\left\{\pi_1^R\right\}$	42,072	11,220	-12,955
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,277,928	1,359,688	1,287,241
$E\left\{\pi_2^G\right\}$	1,152,564	1,091,963	1,210,072
$E\left\{\pi_2^R\right\}$	42,072	30,388	-11,510
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,194,635	1,122,351	1,198,562
$E\{CS\}$	2,184,188	2,268,179	2,336,144
$E\{CS + \pi\}$	4,656,751	4,750,219	4,821,947

Table T4. Outcomes when $k_2 = 42$**.**

	VS	PVI	VI
r_1	308	239	212
r_2	308	293	214
$E\{w\}$	232	228	226
K_1	8,537	9,254	8,968
K_2	8,389	8,087	8,812
$K_1 + K_2$	16,927	17,341	17,780
$E\{q_1\}$	6,441	7,254	7,040
$E\{q_2\}$	6,299	6,058	6,884
$E\{q_1\} + E\{q_2\}$	12,740	13,312	13,924
$E\left\{\pi_1^G\right\}$	1,235,279	1,350,054	1,299,535
$E\left\{\pi_1^R\right\}$	41,927	10,720	-14,987
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,277,205	1,360,773	1,284,548
$E\left\{\pi_2^G\right\}$	1,184,779	1,119,608	1,245,326
$E\left\{\pi_2^R\right\}$	41,927	30,292	-13,226
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,226,705	1,149,900	1,232,100
$E\{CS\}$	2,184,629	2,273,680	2,341,018
$E\{CS + \pi\}$	4,688,540	4,784,353	4,857,665

Table T5. Outcomes when $c_2 = 24$**.**

	VS	PVI	VI
r_1	308	239	212
r_2	308	293	212
$E\{w\}$	231	227	224
K_1	8,489	9,208	8,919
K_2	8,489	8,187	8,918
$K_1 + K_2$	16,977	17,394	17,837
$E\{q_1\}$	6,395	7,209	6,991
$E\{q_2\}$	6,395	6,154	6,990
$E\{q_1\} + E\{q_2\}$	12,791	13,363	13,982
$E\left\{\pi_1^G\right\}$	1,219,097	1,332,605	1,281,900
$E\left\{\pi_1^R\right\}$	42,517	11,834	-14,006
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,261,613	1,344,438	1,267,894
$E\left\{\pi_2^G\right\}$	1,219,097	1,152,324	1,281,743
$E\left\{\pi_2^R\right\}$	42,382	30,793	-14,114
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,261,479	1,183,117	1,267,630
$E\{CS\}$	2,199,770	2,289,669	2,358,267
$E\{CS + \pi\}$	4,722,862	4,817,224	4,893,791

Table T6. Outcomes when $c_2^r = 1.2$**.**

IIB. Unilateral 20% Reductions.

	VS	PVI	VI
r_1	300	236	211
r_2	300	286	211
$E\{w\}$	218	213	210
K_1	7,998	8,708	8,414
K_2	7,998	7,698	8,414
$K_1 + K_2$	15,996	16,407	16,828
$E\{q_1\}$	5,917	6,723	6,492
$E\{q_2\}$	5,917	5,669	6,492
$E\{q_1\} + E\{q_2\}$	11,834	12,391	12,985
$E\left\{\pi_1^G\right\}$	1,056,920	1,157,680	1,105,958
$E\left\{\pi_1^R\right\}$	48,695	23,133	-203
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,105,615	1,180,813	1,105,755
$E\left\{\pi_2^G\right\}$	1,056,920	992,161	1,105,958
$E\left\{\pi_2^R\right\}$	48,695	37,388	-203
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,105,615	1,029,549	1,105,755
$E\{CS\}$	1,874,319	1,961,835	2,031,236
$E\{CS + \pi\}$	4,085,549	4,172,196	4,242,746

Table T7. Outcomes when $a^I = 6,000$**.**

	VS	PVI	VI
r_1	319	242	213
r_2	319	302	213
$E\{w\}$	249	244	242
K_1	7,396	8,134	7,855
K_2	7,396	7,095	7,855
$K_1 + K_2$	14,793	15,230	15,710
$E\{q_1\}$	5,673	6,503	6,302
$E\{q_2\}$	5,673	5,444	6,302
$E\{q_1\} + E\{q_2\}$	11,345	11,947	12,604
$E\left\{\pi_1^G\right\}$	1,172,816	1,304,543	1,255,491
$E\left\{\pi_1^R\right\}$	34,924	-2,781	-31,213
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,207,740	1,301,762	1,224,279
$E\left\{\pi_2^G\right\}$	1,172,816	1,104,750	1,255,491
$E\left\{\pi_2^R\right\}$	34,924	23,442	-31,213
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,207,740	1,128,193	1,224,279
$E\{CS\}$	2,116,221	2,208,684	2,275,373
$E\{CS + \pi\}$	4,531,701	4,638,638	4,723,930

Table T8. Outcomes when $b^I = 28$**.**

	VS	PVI	VI
r_1	294	234	211
r_2	294	281	211
$E\{w\}$	209	204	201
K_1	7,144	7,849	7,552
K_2	7,144	6,844	7,552
$K_1 + K_2$	14,288	14,693	15,104
$E\{q_1\}$	5,622	6,419	6,177
$E\{q_2\}$	5,622	5,367	6,177
$E\{q_1\} + E\{q_2\}$	11,245	11,786	12,353
$E\left\{\pi_1^G\right\}$	917,922	1,010,886	958,837
$E\left\{\pi_1^R\right\}$	53,094	30,579	9,058
$E\left\{\pi_1^G + \pi_1^R\right\}$	971,016	1,041,465	967,894
$E\left\{\pi_2^G\right\}$	917,922	854,905	958,837
$E\left\{\pi_2^R\right\}$	53,094	41,964	9,058
$E\left\{\pi_2^G + \pi_2^R\right\}$	971,016	896,869	967,894
$E\{CS\}$	1,600,986	1,686,014	1,754,991
$E\{CS + \pi\}$	3,543,019	3,624,349	3,690,780

Table T9. Outcomes when $\overline{\varepsilon} = 560$**.**

	VS	PVI	VI
r_1	278	211	184
r_2	301	287	207
$E\{w\}$	229	226	224
K_1	8,404	8,996	8,712
K_2	8,404	8,164	8,889
$K_1 + K_2$	16,808	17,160	17,600
$E\{q_1\}$	6,315	6,985	6,777
$E\{q_2\}$	6,315	6,139	6,954
$E\{q_1\} + E\{q_2\}$	12,629	13,124	13,732
$E\left\{\pi_1^G\right\}$	1,191,267	1,291,448	1,244,390
$E\left\{\pi_1^R\right\}$	17,271	-13,703	-33,554
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,208,538	1,277,746	1,210,835
$E\left\{\pi_2^G\right\}$	1,191,267	1,146,848	1,274,294
$E\left\{\pi_2^R\right\}$	37,462	26,162	-18,622
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,228,729	1,173,010	1,255,673
$E\{CS\}$	2,210,830	2,266,704	2,330,645
$E\{CS + \pi\}$	4,648,097	4,717,460	4,797,152

Table T10. Outcomes when $a_1 = 1,680$**.**

	VS	PVI	VI
r_1	301	233	207
r_2	278	263	184
$E\{w\}$	229	225	224
K_1	8,404	9,099	8,889
K_2	8,404	8,114	8,712
$K_1 + K_2$	16,808	17,213	17,600
$E\{q_1\}$	6,315	7,100	6,954
$E\{q_2\}$	6,315	6,085	6,777
$E\{q_1\} + E\{q_2\}$	12,629	13,185	13,732
$E\left\{\pi_1^G\right\}$	1,191,267	1,300,743	1,274,294
$E\left\{\pi_1^R\right\}$	37,462	7,308	-18,622
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,228,729	1,308,051	1,255,673
$E\left\{\pi_2^G\right\}$	1,191,267	1,129,187	1,244,390
$E\left\{\pi_2^R\right\}$	17,271	10,231	-33,554
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,208,538	1,139,418	1,210,835
$E\{CS\}$	2,210,830	2,296,717	2,330,645
$E\{CS + \pi\}$	4,648,097	4,744,186	4,797,152

Table T11. Outcomes when $a_2 = 1,680$**.**

	VS	PVI	VI
r_1	359	295	261
r_2	319	304	222
$E\{w\}$	234	227	225
K_1	8,548	9,255	8,946
K_2	8,548	8,229	8,939
$K_1 + K_2$	17,096	17,484	17,885
$E\{q_1\}$	6,474	7,280	7,037
$E\{q_2\}$	6,474	6,185	7,044
$E\{q_1\} + E\{q_2\}$	12,948	13,465	14,081
$E\left\{\pi_1^G\right\}$	1,252,381	1,351,197	1,295,232
$E\left\{\pi_1^R\right\}$	94,247	74,716	38,273
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,346,629	1,425,913	1,333,505
$E\left\{\pi_2^G\right\}$	1,252,381	1,163,749	1,296,041
$E\left\{\pi_2^R\right\}$	52,652	41,735	-4,663
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,305,033	1,205,483	1,291,378
$E\{CS\}$	2,193,915	2,310,884	2,382,854
$E\{CS + \pi\}$	4,845,577	4,942,280	5,007,737

Table T12. Outcomes when $b_1 = 6.4$**.**

	VS	PVI	VI
r_1	319	249	222
r_2	359	341	261
$E\{w\}$	234	229	225
K_1	8,548	9,290	8,939
K_2	8,548	8,231	8,946
$K_1 + K_2$	17,096	17,520	17,885
$E\{q_1\}$	6,474	7,323	7,044
$E\{q_2\}$	6,474	6,211	7,037
$E\{q_1\} + E\{q_2\}$	12,948	13,535	14,081
$E\{\pi_1^G\}$	1,252,381	1,370,010	1,296,041
$E\{\pi_1^R\}$	52,652	21,094	-4,663
$E\{\pi_1^G + \pi_1^R\}$	1,305,033	1,391,104	1,291,378
$E\{\pi_2^G\}$	1,252,381	1,177,748	1,295,232
$E\{\pi_2^R\}$	94,247	73,651	38,273
$E\{\pi_2^G + \pi_2^R\}$	1,346,629	1,251,400	1,333,505
$E\{CS\}$	2,193,915	2,288,913	2,382,854
$E\{CS + \pi\}$	4,845,577	4,931,417	5,007,737

Table T13. Outcomes when $b_2 = 6.4$**.**

	VS	PVI	VI
r_1	295	227	203
r_2	305	290	207
$E\{w\}$	230	226	224
K_1	8,460	9,144	8,859
K_2	8,460	8,172	8,936
$K_1 + K_2$	16,920	17,316	17,795
$E\{q_1\}$	6,362	7,124	6,926
$E\{q_2\}$	6,362	6,146	7,003
$E\{q_1\} + E\{q_2\}$	12,725	13,270	13,929
$E\{\pi_1^G\}$	1,206,235	1,315,917	1,269,701
$E\{\pi_1^R\}$	30,250	-295	-21,477
$E\{\pi_1^G + \pi_1^R\}$	1,236,485	1,315,621	1,248,224
$E\{\pi_2^G\}$	1,206,235	1,148,449	1,282,692
$E\{\pi_2^R\}$	40,326	28,902	-18,625
$E\{\pi_2^G + \pi_2^R\}$	1,246,561	1,177,352	1,264,066
$E\{CS\}$	2,206,079	2,282,034	2,353,234
$E\{CS + \pi\}$	4,689,125	4,775,007	4,865,524

Table T14. Outcomes when $d_1 = 2.4$**.**

	VS	PVI	VI
r_1	305	231	207
r_2	295	282	203
$E\{w\}$	230	226	224
K_1	8,460	9,202	8,936
K_2	8,460	8,162	8,859
$K_1 + K_2$	16,920	17,364	17,795
$E\{q_1\}$	6,362	7,208	7,003
$E\{q_2\}$	6,362	6,129	6,926
$E\{q_1\} + E\{q_2\}$	12,725	13,337	13,929
$E\{\pi_1^G\}$	1,206,235	1,327,871	1,282,692
$E\{\pi_1^R\}$	40,326	4,839	-18,625
$E\{\pi_1^G + \pi_1^R\}$	1,246,561	1,332,711	1,264,066
$E\{\pi_2^G\}$	1,206,235	1,144,284	1,269,701
$E\{\pi_2^R\}$	30,250	21,990	-21,477
$E\{\pi_2^G + \pi_2^R\}$	1,236,485	1,166,274	1,248,224
$E\{CS\}$	2,206,079	2,296,465	2,353,234
$E\{CS + \pi\}$	4,689,125	4,795,450	4,865,524

Table T15. Outcomes when $d_2 = 2.4$.

	VS	PVI	VI
r_1	306	236	209
r_2	306	291	210
$E\{w\}$	228	224	221
K_1	8,906	9,575	9,333
K_2	8,511	8,223	8,924
$K_1 + K_2$	17,417	17,799	18,257
$E\{q_1\}$	6,512	7,328	7,127
$E\{q_2\}$	6,406	6,167	6,996
$E\{q_1\} + E\{q_2\}$	12,918	13,494	14,124
$E\{\pi_1^G\}$	1,278,635	1,395,712	1,345,241
$E\{\pi_1^R\}$	43,870	11,935	-13,972
$E\{\pi_1^G + \pi_1^R\}$	1,322,505	1,407,646	1,331,269
$E\{\pi_2^G\}$	1,194,750	1,130,036	1,252,893
$E\{\pi_2^R\}$	43,870	31,818	-12,469
$E\{\pi_2^G + \pi_2^R\}$	1,238,620	1,161,853	1,240,424
$E\{CS\}$	2,251,194	2,341,230	2,414,902
$E\{CS + \pi\}$	4,812,319	4,910,730	4,986,594

Table T16. Outcomes when $k_1 = 28$.

	VS	PVI	VI
r_1	306	238	210
r_2	306	291	209
$E\{w\}$	228	223	221
K_1	8,511	9,225	8,924
K_2	8,906	8,594	9,333
$K_1 + K_2$	17,417	17,819	18,257
$E\{q_1\}$	6,406	7,218	6,996
$E\{q_2\}$	6,512	6,271	7,127
$E\{q_1\} + E\{q_2\}$	12,918	13,489	14,124
$E\left\{\pi_1^G\right\}$	1,194,750	1,304,555	1,252,893
$E\left\{\pi_1^R\right\}$	43,870	14,334	-12,469
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,238,620	1,318,889	1,240,424
$E\left\{\pi_2^G\right\}$	1,278,635	1,209,995	1,345,241
$E\left\{\pi_2^R\right\}$	43,870	32,338	-13,972
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,322,505	1,242,333	1,331,269
$E\{CS\}$	2,251,193	2,342,509	2,414,902
$E\{CS + \pi\}$	4,812,319	4,903,730	4,986,594

Table T17. Outcomes when $k_2 = 28$.

	VS	PVI	VI
r_1	307	237	210
r_2	307	292	211
$E\{w\}$	230	225	223
K_1	8,591	9,316	9,026
K_2	8,443	8,137	8,870
$K_1 + K_2$	17,034	17,453	17,896
$E\{q_1\}$	6,493	7,315	7,099
$E\{q_2\}$	6,351	6,107	6,942
$E\{q_1\} + E\{q_2\}$	12,844	13,423	14,041
$E\left\{\pi_1^G\right\}$	1,253,893	1,370,730	1,319,008
$E\left\{\pi_1^R\right\}$	43,081	11,375	-14,838
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,296,974	1,382,105	1,304,171
$E\left\{\pi_2^G\right\}$	1,202,980	1,136,364	1,264,355
$E\left\{\pi_2^R\right\}$	43,081	31,194	-13,063
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,246,062	1,167,559	1,251,292
$E\{CS\}$	2,215,659	2,306,708	2,375,708
$E\{CS + \pi\}$	4,758,695	4,856,372	4,931,171

Table T18. Outcomes when $c_1 = 16$.

	VS	PVI	VI
r_1	307	238	211
r_2	307	292	210
$E\{w\}$	230	225	223
K_1	8,443	9,161	8,870
K_2	8,591	8,286	9,026
$K_1 + K_2$	17,034	17,448	17,896
$E\{q_1\}$	6,351	7,164	6,942
$E\{q_2\}$	6,493	6,251	7,099
$E\{q_1\} + E\{q_2\}$	12,844	13,414	14,041
$E\left\{\pi_1^G\right\}$	1,202,980	1,315,401	1,264,355
$E\left\{\pi_1^R\right\}$	43,081	12,894	-13,063
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,246,062	1,328,295	1,251,292
$E\left\{\pi_2^G\right\}$	1,253,893	1,185,720	1,319,008
$E\left\{\pi_2^R\right\}$	43,081	31,486	-14,838
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,296,974	1,217,206	1,304,171
$E\{CS\}$	2,215,659	2,305,611	2,375,708
$E\{CS + \pi\}$	4,758,695	4,851,112	4,931,171

Table T19. Outcomes when $c_2 = 16$**.**

	VS	PVI	VI
r_1	307	239	212
r_2	307	293	212
$E\{w\}$	231	227	224
K_1	8,489	9,208	8,920
K_2	8,489	8,187	8,919
$K_1 + K_2$	16,978	17,395	17,839
$E\{q_1\}$	6,396	7,210	6,992
$E\{q_2\}$	6,396	6,154	6,991
$E\{q_1\} + E\{q_2\}$	12,792	13,364	13,983
$E\left\{\pi_1^G\right\}$	1,219,230	1,332,779	1,282,009
$E\left\{\pi_1^R\right\}$	42,602	11,919	-13,934
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,261,833	1,344,699	1,268,075
$E\left\{\pi_2^G\right\}$	1,219,230	1,152,393	1,281,852
$E\left\{\pi_2^R\right\}$	42,468	30,867	-14,042
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,261,698	1,183,260	1,267,810
$E\{CS\}$	2,199,731	2,289,720	2,358,407
$E\{CS + \pi\}$	4,723,262	4,817,679	4,894,293

Table T20. Outcomes when $c_1^r = 0.8$**.**

	VS	PVI	VI
r_1	307	239	212
r_2	307	293	212
$E\{w\}$	231	227	224
K_1	8,489	9,208	8,919
K_2	8,489	8,187	8,920
$K_1 + K_2$	16,978	17,395	17,839
$E\{q_1\}$	6,396	7,209	6,991
$E\{q_2\}$	6,396	6,154	6,992
$E\{q_1\} + E\{q_2\}$	12,792	13,363	13,983
$E\left\{\pi_1^G\right\}$	1,219,230	1,332,731	1,281,852
$E\left\{\pi_1^R\right\}$	42,468	11,779	-14,042
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,261,698	1,344,511	1,267,810
$E\left\{\pi_2^G\right\}$	1,219,230	1,152,486	1,282,009
$E\left\{\pi_2^R\right\}$	42,602	30,979	-13,934
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,261,833	1,183,465	1,268,075
$E\{CS\}$	2,199,731	2,289,561	2,358,407
$E\{CS + \pi\}$	4,723,262	4,817,537	4,894,293

Table T21. Outcomes when $c_2^r = 0.8$.

IIC. Unilateral 50% Increases.

	VS	PVI	VI
r_1	327	246	214
r_2	327	310	214
$E\{w\}$	264	261	260
K_1	9,714	10,454	10,183
K_2	9,714	9,407	10,183
$K_1 + K_2$	19,429	19,861	20,366
$E\{q_1\}$	7,591	8,424	8,239
$E\{q_2\}$	7,591	7,367	8,239
$E\{q_1\} + E\{q_2\}$	15,182	15,791	16,477
$E\left\{\pi_1^G\right\}$	1,680,344	1,828,238	1,783,236
$E\left\{\pi_1^R\right\}$	28,825	-16,223	-48,140
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,709,169	1,812,015	1,735,096
$E\left\{\pi_2^G\right\}$	1,680,344	1,611,123	1,783,236
$E\left\{\pi_2^R\right\}$	28,825	17,334	-48,140
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,709,169	1,628,457	1,735,096
$E\{CS\}$	3,148,767	3,240,555	3,302,956
$E\{CS + \pi\}$	6,567,104	6,681,027	6,773,147

Table T22. Outcomes when $a^I = 11,250$.

	VS	PVI	VI
r_1	292	233	210
r_2	292	279	210
$E\{w\}$	206	203	201
K_1	11,191	11,886	11,586
K_2	11,191	10,890	11,586
$K_1 + K_2$	22,383	22,776	23,172
$E\{q_1\}$	8,163	8,953	8,714
$E\{q_2\}$	8,163	7,905	8,714
$E\{q_1\} + E\{q_2\}$	16,326	16,859	17,428
$E\left\{\pi_1^G\right\}$	1,379,309	1,469,695	1,418,726
$E\left\{\pi_1^R\right\}$	54,106	31,527	9,074
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,433,415	1,501,222	1,427,800
$E\left\{\pi_2^G\right\}$	1,379,309	1,315,864	1,418,726
$E\left\{\pi_2^R\right\}$	54,106	42,643	9,074
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,433,415	1,358,507	1,427,800
$E\{CS\}$	2,519,647	2,604,603	2,673,819
$E\{CS + \pi\}$	5,386,476	5,464,332	5,529,419

Table T23. Outcomes when $b^I = 52.5$.

	VS	PVI	VI
r_1	340	252	214
r_2	340	322	214
$E\{w\}$	286	283	282
K_1	11,951	12,701	12,439
K_2	11,951	11,647	12,439
$K_1 + K_2$	23,902	24,349	24,878
$E\{q_1\}$	8,330	9,180	9,026
$E\{q_2\}$	8,330	8,126	9,026
$E\{q_1\} + E\{q_2\}$	16,660	17,307	18,053
$E\left\{\pi_1^G\right\}$	2,174,484	2,345,564	2,308,745
$E\left\{\pi_1^R\right\}$	21,223	-34,103	-70,916
$E\left\{\pi_1^G + \pi_1^R\right\}$	2,195,707	2,311,461	2,237,829
$E\left\{\pi_2^G\right\}$	2,174,484	2,105,437	2,308,745
$E\left\{\pi_2^R\right\}$	21,223	10,619	-70,916
$E\left\{\pi_2^G + \pi_2^R\right\}$	2,195,707	2,116,056	2,237,829
$E\{CS\}$	4,164,724	4,256,929	4,310,648
$E\{CS + \pi\}$	8,556,138	8,684,446	8,786,306

Table T24. Outcomes when $\bar{\varepsilon} = 1,050$.

	VS	PVI	VI
r_1	381	308	281
r_2	324	307	225
$E\{w\}$	236	228	225
K_1	8,701	9,736	9,385
K_2	8,701	8,244	9,031
$K_1 + K_2$	17,402	17,981	18,415
$E\{q_1\}$	6,598	7,768	7,506
$E\{q_2\}$	6,598	6,194	7,094
$E\{q_1\} + E\{q_2\}$	13,196	13,962	14,600
$E\left\{\pi_1^G\right\}$	1,290,439	1,436,503	1,372,838
$E\left\{\pi_1^R\right\}$	154,392	126,949	86,993
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,444,832	1,563,452	1,459,830
$E\left\{\pi_2^G\right\}$	1,290,439	1,166,420	1,304,362
$E\left\{\pi_2^R\right\}$	56,455	44,408	-510
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,346,894	1,210,828	1,303,852
$E\{CS\}$	2,196,204	2,373,261	2,451,470
$E\{CS + \pi\}$	4,987,930	5,147,541	5,215,152

Table T25. Outcomes when $a_1 = 3,150$.

	VS	PVI	VI
r_1	324	253	225
r_2	381	366	281
$E\{w\}$	236	231	225
K_1	8,701	9,480	9,031
K_2	8,701	8,370	9,385
$K_1 + K_2$	17,402	17,849	18,415
$E\{q_1\}$	6,598	7,482	7,094
$E\{q_2\}$	6,598	6,327	7,506
$E\{q_1\} + E\{q_2\}$	13,196	13,808	14,600
$E\left\{\pi_1^G\right\}$	1,290,439	1,414,254	1,304,362
$E\left\{\pi_1^R\right\}$	56,455	24,498	-510
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,346,894	1,438,752	1,303,852
$E\left\{\pi_2^G\right\}$	1,290,439	1,211,573	1,372,838
$E\left\{\pi_2^R\right\}$	154,392	131,214	86,993
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,444,832	1,342,787	1,459,830
$E\{CS\}$	2,196,204	2,295,622	2,451,470
$E\{CS + \pi\}$	4,987,930	5,077,161	5,215,152

Table T26. Outcomes when $a_2 = 3,150$**.**

	VS	PVI	VI
r_1	238	165	148
r_2	291	278	199
$E\{w\}$	225	225	223
K_1	8,370	9,088	8,846
K_2	8,370	8,141	8,878
$K_1 + K_2$	16,739	17,228	17,724
$E\{q_1\}$	6,238	7,064	6,885
$E\{q_2\}$	6,238	6,117	6,917
$E\{q_1\} + E\{q_2\}$	12,475	13,181	13,802
$E\left\{\pi_1^G\right\}$	1,153,937	1,299,918	1,257,257
$E\left\{\pi_1^R\right\}$	1,398	-57,749	-70,270
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,155,334	1,242,169	1,186,987
$E\left\{\pi_2^G\right\}$	1,153,937	1,138,442	1,262,567
$E\left\{\pi_2^R\right\}$	31,441	19,212	-24,155
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,185,378	1,157,654	1,238,412
$E\{CS\}$	2,247,457	2,264,601	2,328,167
$E\{CS + \pi\}$	4,588,168	4,664,424	4,753,566

Table T27. Outcomes when $b_1 = 12$**.**

	VS	PVI	VI
r_1	291	225	199
r_2	238	227	148
$E\{w\}$	225	221	223
K_1	8,370	9,062	8,878
K_2	8,370	8,089	8,846
$K_1 + K_2$	16,739	17,151	17,724
$E\{q_1\}$	6,238	7,005	6,917
$E\{q_2\}$	6,238	6,025	6,885
$E\{q_1\} + E\{q_2\}$	12,475	13,031	13,802
$E\left\{\pi_1^G\right\}$	1,153,937	1,260,880	1,262,567
$E\left\{\pi_1^R\right\}$	31,441	2,261	-24,155
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,185,377	1,263,142	1,238,412
$E\left\{\pi_2^G\right\}$	1,153,937	1,097,804	1,257,257
$E\left\{\pi_2^R\right\}$	1,398	227	-70,270
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,155,334	1,098,031	1,186,987
$E\{CS\}$	2,247,457	2,329,968	2,328,167
$E\{CS + \pi\}$	4,588,168	4,691,140	4,753,566

Table T28. Outcomes when $b_2 = 12$.

	VS	PVI	VI
r_1	341	269	237
r_2	315	299	225
$E\{w\}$	234	228	225
K_1	8,563	9,355	8,939
K_2	8,563	8,232	8,954
$K_1 + K_2$	17,125	17,587	17,894
$E\{q_1\}$	6,481	7,420	7,115
$E\{q_2\}$	6,481	6,178	6,985
$E\{q_1\} + E\{q_2\}$	12,962	13,598	14,100
$E\left\{\pi_1^G\right\}$	1,252,944	1,375,400	1,307,575
$E\left\{\pi_1^R\right\}$	83,635	52,125	13,393
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,336,579	1,427,525	1,320,967
$E\left\{\pi_2^G\right\}$	1,252,944	1,163,771	1,289,300
$E\left\{\pi_2^R\right\}$	48,330	36,280	-1,509
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,301,274	1,200,051	1,287,791
$E\{CS\}$	2,188,015	2,312,295	2,366,226
$E\{CS + \pi\}$	4,825,867	4,939,871	4,974,984

Table T29. Outcomes when $d_1 = 4.5$.

	VS	PVI	VI
r_1	315	258	225
r_2	341	323	237
$E\{w\}$	234	228	225
K_1	8,563	9,225	8,954
K_2	8,563	8,252	8,939
$K_1 + K_2$	17,125	17,477	17,894
$E\{q_1\}$	6,481	7,218	6,985
$E\{q_2\}$	6,481	6,221	7,115
$E\{q_1\} + E\{q_2\}$	12,962	13,440	14,100
$E\left\{\pi_1^G\right\}$	1,252,944	1,347,814	1,289,300
$E\left\{\pi_1^R\right\}$	48,330	28,358	-1,509
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,301,274	1,376,173	1,287,791
$E\left\{\pi_2^G\right\}$	1,252,944	1,175,391	1,307,575
$E\left\{\pi_2^R\right\}$	83,635	63,207	13,393
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,336,579	1,238,598	1,320,967
$E\{CS\}$	2,188,015	2,274,471	2,366,226
$E\{CS + \pi\}$	4,825,867	4,889,242	4,974,984

Table T30. Outcomes when $d_2 = 4.5$**.**

	VS	PVI	VI
r_1	309	246	220
r_2	309	295	217
$E\{w\}$	233	229	227
K_1	7,863	8,523	8,238
K_2	8,816	8,524	9,236
$K_1 + K_2$	16,678	17,047	17,473
$E\{q_1\}$	6,190	6,942	6,717
$E\{q_2\}$	6,496	6,268	7,092
$E\{q_1\} + E\{q_2\}$	12,687	13,210	13,809
$E\left\{\pi_1^G\right\}$	1,056,637	1,151,824	1,104,286
$E\left\{\pi_1^R\right\}$	41,369	16,038	-7,795
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,098,006	1,167,862	1,096,492
$E\left\{\pi_2^G\right\}$	1,264,201	1,201,872	1,332,543
$E\left\{\pi_2^R\right\}$	41,369	30,873	-11,563
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,305,569	1,232,744	1,320,980
$E\{CS\}$	2,158,382	2,237,658	2,303,018
$E\{CS + \pi\}$	4,561,957	4,638,264	4,720,489

Table T31. Outcomes when $k_1 = 52.5$**.**

	VS	PVI	VI
r_1	309	241	217
r_2	309	295	220
$E\{w\}$	233	230	227
K_1	8,816	9,461	9,236
K_2	7,863	7,587	8,238
$K_1 + K_2$	16,678	17,048	17,473
$E\{q_1\}$	6,496	7,285	7,092
$E\{q_2\}$	6,190	5,953	6,717
$E\{q_1\} + E\{q_2\}$	12,687	13,239	13,809
$E\left\{\pi_1^G\right\}$	1,264,201	1,378,776	1,332,543
$E\left\{\pi_1^R\right\}$	41,369	10,522	-11,563
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,305,569	1,389,298	1,320,980
$E\left\{\pi_2^G\right\}$	1,056,637	998,507	1,104,286
$E\left\{\pi_2^R\right\}$	41,369	29,751	-7,795
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,098,006	1,028,258	1,096,492
$E\{CS\}$	2,158,382	2,240,303	2,303,018
$E\{CS + \pi\}$	4,561,957	4,657,859	4,720,489

Table T32. Outcomes when $k_2 = 52.5$**.**

	VS	PVI	VI
r_1	309	243	217
r_2	309	295	213
$E\{w\}$	234	230	228
K_1	8,240	8,937	8,651
K_2	8,610	8,310	9,042
$K_1 + K_2$	16,850	17,247	17,692
$E\{q_1\}$	6,153	6,942	6,723
$E\{q_2\}$	6,509	6,272	7,114
$E\{q_1\} + E\{q_2\}$	12,662	13,214	13,836
$E\left\{\pi_1^G\right\}$	1,134,195	1,240,020	1,191,577
$E\left\{\pi_1^R\right\}$	41,085	12,831	-12,060
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,175,280	1,252,851	1,179,516
$E\left\{\pi_2^G\right\}$	1,259,673	1,193,055	1,326,271
$E\left\{\pi_2^R\right\}$	41,085	30,121	-16,441
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,300,758	1,223,175	1,309,830
$E\{CS\}$	2,162,049	2,247,209	2,315,193
$E\{CS + \pi\}$	4,638,087	4,723,236	4,804,539

Table T33. Outcomes when $c_1 = 30$**.**

	VS	PVI	VI
r_1	309	239	213
r_2	309	294	217
$E\{w\}$	234	230	228
K_1	8,610	9,324	9,042
K_2	8,240	7,938	8,651
$K_1 + K_2$	16,850	17,262	17,692
$E\{q_1\}$	6,509	7,322	7,114
$E\{q_2\}$	6,153	5,914	6,723
$E\{q_1\} + E\{q_2\}$	12,662	13,236	13,836
$E\left\{\pi_1^G\right\}$	1,259,673	1,376,354	1,326,271
$E\left\{\pi_1^R\right\}$	41,085	9,090	-16,441
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,300,758	1,385,444	1,309,830
$E\left\{\pi_2^G\right\}$	1,134,195	1,071,384	1,191,577
$E\left\{\pi_2^R\right\}$	41,085	29,411	-12,060
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,175,280	1,100,796	1,179,516
$E\{CS\}$	2,162,049	2,249,895	2,315,193
$E\{CS + \pi\}$	4,638,087	4,736,135	4,804,539

Table T34. Outcomes when $c_2 = 30$.

	VS	PVI	VI
r_1	308	239	212
r_2	308	293	212
$E\{w\}$	231	227	224
K_1	8,488	9,206	8,917
K_2	8,488	8,187	8,919
$K_1 + K_2$	16,977	17,393	17,837
$E\{q_1\}$	6,395	7,207	6,989
$E\{q_2\}$	6,395	6,154	6,992
$E\{q_1\} + E\{q_2\}$	12,790	13,361	13,981
$E\left\{\pi_1^G\right\}$	1,218,996	1,332,389	1,281,546
$E\left\{\pi_1^R\right\}$	42,218	11,525	-14,248
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,261,214	1,343,914	1,267,298
$E\left\{\pi_2^G\right\}$	1,218,996	1,152,435	1,281,938
$E\left\{\pi_2^R\right\}$	42,553	30,933	-13,979
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,261,549	1,183,368	1,267,960
$E\{CS\}$	2,199,800	2,289,353	2,358,152
$E\{CS + \pi\}$	4,722,564	4,816,635	4,893,410

Table T35. Outcomes when $c_1^r = 1.5$.

	VS	PVI	VI
r_1	308	239	212
r_2	308	293	212
$E\{w\}$	231	227	224
K_1	8,488	9,208	8,919
K_2	8,488	8,186	8,917
$K_1 + K_2$	16,977	17,394	17,837
$E\{q_1\}$	6,395	7,209	6,992
$E\{q_2\}$	6,395	6,154	6,989
$E\{q_1\} + E\{q_2\}$	12,790	13,362	13,981
$E\left\{\pi_1^G\right\}$	1,218,996	1,332,510	1,281,938
$E\left\{\pi_1^R\right\}$	42,553	11,874	-13,979
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,261,549	1,344,384	1,267,960
$E\left\{\pi_2^G\right\}$	1,218,996	1,152,203	1,281,546
$E\left\{\pi_2^R\right\}$	42,218	30,654	-14,248
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,261,214	1,182,857	1,267,298
$E\{CS\}$	2,199,800	2,289,749	2,358,152
$E\{CS + \pi\}$	4,722,564	4,816,990	4,893,410

Table T36. Outcomes when $c_2^r = 1.5$.

IID. Unilateral 50% Reductions.

	VS	PVI	VI
r_1	288	231	210
r_2	288	275	210
$E\{w\}$	198	193	189
K_1	7,262	7,959	7,657
K_2	7,262	6,965	7,657
$K_1 + K_2$	14,523	14,924	15,313
$E\{q_1\}$	5,199	5,993	5,745
$E\{q_2\}$	5,199	4,941	5,745
$E\{q_1\} + E\{q_2\}$	10,398	10,934	11,489
$E\left\{\pi_1^G\right\}$	837,362	920,091	868,467
$E\left\{\pi_1^R\right\}$	58,785	40,246	20,724
$E\left\{\pi_1^G + \pi_1^R\right\}$	896,148	960,337	889,191
$E\left\{\pi_2^G\right\}$	837,362	776,682	868,467
$E\left\{\pi_2^R\right\}$	58,785	48,296	20,724
$E\left\{\pi_2^G + \pi_2^R\right\}$	896,148	824,978	889,191
$E\{CS\}$	1,444,267	1,526,565	1,594,912
$E\{CS + \pi\}$	3,236,562	3,311,880	3,373,294

Table T37. Outcomes when $a^I = 3,750$**.**

	VS	PVI	VI
r_1	350	252	215
r_2	350	329	215
$E\{w\}$	300	294	294
K_1	5,715	6,506	6,268
K_2	5,715	5,440	6,268
$K_1 + K_2$	11,431	11,946	12,537
$E\{q_1\}$	4,539	5,418	5,268
$E\{q_2\}$	4,539	4,352	5,268
$E\{q_1\} + E\{q_2\}$	9,078	9,770	10,536
$E\left\{\pi_1^G\right\}$	1,153,000	1,341,557	1,306,650
$E\left\{\pi_1^R\right\}$	17,303	-45,530	-82,439
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,170,303	1,296,027	1,224,211
$E\left\{\pi_2^G\right\}$	1,153,000	1,086,446	1,306,650
$E\left\{\pi_2^R\right\}$	17,303	7,632	-82,439
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,170,303	1,094,077	1,224,211
$E\{CS\}$	2,141,698	2,239,155	2,290,995
$E\{CS + \pi\}$	4,482,305	4,629,259	4,739,417

Table T38. Outcomes when $b^I = 17.5$**.**

	VS	PVI	VI
r_1	275	226	209
r_2	275	264	209
$E\{w\}$	176	170	166
K_1	5,201	5,886	5,578
K_2	5,201	4,903	5,578
$K_1 + K_2$	10,403	10,789	11,156
$E\{q_1\}$	4,466	5,231	4,954
$E\{q_2\}$	4,466	4,190	4,954
$E\{q_1\} + E\{q_2\}$	8,931	9,421	9,907
$E\left\{\pi_1^G\right\}$	552,016	616,944	567,600
$E\left\{\pi_1^R\right\}$	71,371	59,276	44,147
$E\left\{\pi_1^G + \pi_1^R\right\}$	623,387	676,220	611,747
$E\left\{\pi_2^G\right\}$	552,016	497,701	567,600
$E\left\{\pi_2^R\right\}$	71,371	61,819	44,147
$E\left\{\pi_2^G + \pi_2^R\right\}$	623,387	559,520	611,747
$E\{CS\}$	903,002	976,298	1,039,392
$E\{CS + \pi\}$	2,149,775	2,212,038	2,262,886

Table T39. Outcomes when $\bar{\varepsilon} = 350$.

	VS	PVI	VI
r_1	234	170	143
r_2	291	279	199
$E\{w\}$	226	225	224
K_1	8,277	8,672	8,401
K_2	8,277	8,144	8,843
$K_1 + K_2$	16,554	16,816	17,244
$E\{q_1\}$	6,193	6,647	6,457
$E\{q_2\}$	6,193	6,120	6,899
$E\{q_1\} + E\{q_2\}$	12,386	12,766	13,356
$E\left\{\pi_1^G\right\}$	1,150,080	1,228,872	1,188,301
$E\left\{\pi_1^R\right\}$	374	-29,856	-40,721
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,150,455	1,199,017	1,147,580
$E\left\{\pi_2^G\right\}$	1,150,080	1,139,008	1,262,952
$E\left\{\pi_2^R\right\}$	30,510	19,846	-24,753
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,180,590	1,158,854	1,238,200
$E\{CS\}$	2,237,799	2,244,679	2,300,442
$E\{CS + \pi\}$	4,568,843	4,602,549	4,686,221

Table T40. Outcomes when $a_1 = 1,050$.

	VS	PVI	VI
r_1	291	224	199
r_2	234	219	143
$E\{w\}$	226	222	224
K_1	8,277	8,936	8,843
K_2	8,277	8,004	8,401
$K_1 + K_2$	16,554	16,940	17,244
$E\{q_1\}$	6,193	6,936	6,899
$E\{q_2\}$	6,193	5,982	6,457
$E\{q_1\} + E\{q_2\}$	12,386	12,918	13,356
$E\left\{\pi_1^G\right\}$	1,150,080	1,253,616	1,262,952
$E\left\{\pi_1^R\right\}$	30,510	1,181	-24,753
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,180,590	1,254,797	1,238,200
$E\left\{\pi_2^G\right\}$	1,150,080	1,094,842	1,188,301
$E\left\{\pi_2^R\right\}$	374	115	-40,721
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,150,455	1,094,958	1,147,580
$E\{CS\}$	2,237,799	2,317,552	2,300,442
$E\{CS + \pi\}$	4,568,843	4,667,306	4,686,221

Table T41. Outcomes when $a_2 = 1,050$.

	VS	PVI	VI
r_1	517	471	415
r_2	353	340	253
$E\{w\}$	240	230	227
K_1	8,678	9,329	9,037
K_2	8,678	8,359	8,963
$K_1 + K_2$	17,355	17,688	18,000
$E\{q_1\}$	6,639	7,435	7,138
$E\{q_2\}$	6,639	6,281	7,193
$E\{q_1\} + E\{q_2\}$	13,277	13,716	14,330
$E\left\{\pi_1^G\right\}$	1,322,187	1,398,774	1,328,838
$E\left\{\pi_1^R\right\}$	300,955	296,154	224,723
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,623,142	1,694,928	1,553,561
$E\left\{\pi_2^G\right\}$	1,322,187	1,200,514	1,334,539
$E\left\{\pi_2^R\right\}$	92,877	86,121	33,643
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,415,064	1,286,636	1,368,182
$E\{CS\}$	2,244,753	2,386,910	2,471,915
$E\{CS + \pi\}$	5,282,959	5,368,474	5,393,658

Table T42. Outcomes when $b_1 = 4$.

	VS	PVI	VI
r_1	353	278	253
r_2	517	488	415
$E\{w\}$	240	233	227
K_1	8,678	9,493	9,135
K_2	8,678	8,316	8,831
$K_1 + K_2$	17,355	17,809	17,966
$E\{q_1\}$	6,639	7,602	7,253
$E\{q_2\}$	6,639	6,312	7,068
$E\{q_1\} + E\{q_2\}$	13,277	13,913	14,321
$E\left\{\pi_1^G\right\}$	1,322,187	1,450,564	1,351,358
$E\left\{\pi_1^R\right\}$	92,877	58,688	32,612
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,415,064	1,509,252	1,383,971
$E\left\{\pi_2^G\right\}$	1,322,187	1,223,547	1,315,888
$E\left\{\pi_2^R\right\}$	300,955	249,219	223,994
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,623,142	1,472,765	1,539,882
$E\{CS\}$	2,244,753	2,353,770	2,466,832
$E\{CS + \pi\}$	5,282,959	5,335,788	5,390,685

Table T43. Outcomes when $b_2 = 4$.

	VS	PVI	VI
r_1	276	210	191
r_2	301	287	201
$E\{w\}$	228	226	224
K_1	8,417	9,032	8,768
K_2	8,417	8,181	8,955
$K_1 + K_2$	16,835	17,213	17,723
$E\{q_1\}$	6,314	6,994	6,830
$E\{q_2\}$	6,314	6,143	7,018
$E\{q_1\} + E\{q_2\}$	12,627	13,137	13,849
$E\left\{\pi_1^G\right\}$	1,187,438	1,289,084	1,252,746
$E\left\{\pi_1^R\right\}$	16,041	-14,224	-30,065
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,203,479	1,274,860	1,222,681
$E\left\{\pi_2^G\right\}$	1,187,438	1,143,889	1,284,509
$E\left\{\pi_2^R\right\}$	37,246	26,178	-25,524
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,224,684	1,170,067	1,258,985
$E\{CS\}$	2,217,159	2,274,127	2,345,404
$E\{CS + \pi\}$	4,645,322	4,719,054	4,827,070

Table T44. Outcomes when $d_1 = 1.5$.

	VS	PVI	VI
r_1	301	221	201
r_2	276	267	191
$E\{w\}$	228	225	224
K_1	8,417	9,196	8,955
K_2	8,417	8,125	8,768
$K_1 + K_2$	16,835	17,321	17,723
$E\{q_1\}$	6,314	7,209	7,018
$E\{q_2\}$	6,314	6,094	6,830
$E\{q_1\} + E\{q_2\}$	12,627	13,304	13,849
$E\left\{\pi_1^G\right\}$	1,187,438	1,321,927	1,284,509
$E\left\{\pi_1^R\right\}$	37,246	-6,001	-25,524
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,224,684	1,315,926	1,258,985
$E\left\{\pi_2^G\right\}$	1,187,438	1,133,174	1,252,746
$E\left\{\pi_2^R\right\}$	16,041	12,088	-30,065
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,203,479	1,145,262	1,222,681
$E\{CS\}$	2,217,159	2,307,281	2,345,404
$E\{CS + \pi\}$	4,645,322	4,768,469	4,827,070

Table T45. Outcomes when $d_2 = 1.5$**.**

	VS	PVI	VI
r_1	303	231	204
r_2	303	287	207
$E\{w\}$	223	219	216
K_1	9,620	10,322	10,083
K_2	8,588	8,265	8,951
$K_1 + K_2$	18,208	18,587	19,034
$E\{q_1\}$	6,674	7,530	7,338
$E\{q_2\}$	6,435	6,182	7,011
$E\{q_1\} + E\{q_2\}$	13,109	13,712	14,349
$E\left\{\pi_1^G\right\}$	1,370,565	1,496,556	1,444,978
$E\left\{\pi_1^R\right\}$	45,979	12,526	-13,932
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,416,543	1,509,082	1,431,046
$E\left\{\pi_2^G\right\}$	1,160,074	1,091,137	1,208,038
$E\left\{\pi_2^R\right\}$	45,979	33,474	-10,045
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,206,052	1,124,612	1,197,994
$E\{CS\}$	2,331,535	2,429,906	2,507,691
$E\{CS + \pi\}$	4,954,130	5,063,600	5,136,730

Table T46. Outcomes when $k_1 = 17.5$**.**

	VS	PVI	VI
r_1	304	236	207
r_2	304	288	204
$E\{w\}$	226	219	216
K_1	8,846	9,294	8,951
K_2	8,846	9,287	10,083
$K_1 + K_2$	17,691	18,580	19,034
$E\{q_1\}$	6,497	7,244	7,011
$E\{q_2\}$	6,497	6,435	7,338
$E\{q_1\} + E\{q_2\}$	12,994	13,680	14,349
$E\left\{\pi_1^G\right\}$	1,196,606	1,264,083	1,208,038
$E\left\{\pi_1^R\right\}$	44,705	18,118	-10,045
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,241,312	1,282,201	1,197,994
$E\left\{\pi_2^G\right\}$	1,351,404	1,299,085	1,444,978
$E\left\{\pi_2^R\right\}$	44,705	34,594	-13,932
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,396,110	1,333,680	1,431,046
$E\{CS\}$	2,282,683	2,425,724	2,507,691
$E\{CS + \pi\}$	4,920,104	5,041,605	5,136,730

Table T47. Outcomes when $k_2 = 17.5$**.**

	VS	PVI	VI
r_1	306	234	207
r_2	306	290	211
$E\{w\}$	228	223	221
K_1	8,745	9,478	9,187
K_2	8,375	8,063	8,796
$K_1 + K_2$	17,120	17,542	17,983
$E\{q_1\}$	6,640	7,475	7,259
$E\{q_2\}$	6,284	6,037	6,869
$E\{q_1\} + E\{q_2\}$	12,924	13,512	14,128
$E\left\{\pi_1^G\right\}$	1,306,980	1,428,897	1,375,784
$E\left\{\pi_1^R\right\}$	43,973	10,706	-16,089
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,350,952	1,439,603	1,359,694
$E\left\{\pi_2^G\right\}$	1,178,923	1,112,538	1,238,322
$E\left\{\pi_2^R\right\}$	43,973	31,660	-11,631
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,222,896	1,144,198	1,226,691
$E\{CS\}$	2,239,633	2,332,489	2,401,917
$E\{CS + \pi\}$	4,813,480	4,916,289	4,988,302

Table T48. Outcomes when $c_1 = 10$**.**

	VS	PVI	VI
r_1	306	238	211
r_2	306	291	207
$E\{w\}$	228	223	221
K_1	8,375	9,092	8,796
K_2	8,745	8,436	9,187
$K_1 + K_2$	17,120	17,527	17,983
$E\{q_1\}$	6,284	7,096	6,869
$E\{q_2\}$	6,640	6,395	7,259
$E\{q_1\} + E\{q_2\}$	12,924	13,490	14,128
$E\left\{\pi_1^G\right\}$	1,178,923	1,289,722	1,238,322
$E\left\{\pi_1^R\right\}$	43,973	14,527	-11,631
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,222,896	1,304,249	1,226,691
$E\left\{\pi_2^G\right\}$	1,306,980	1,236,665	1,375,784
$E\left\{\pi_2^R\right\}$	43,973	32,396	-16,089
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,350,952	1,269,060	1,359,694
$E\{CS\}$	2,239,632	2,329,723	2,401,917
$E\{CS + \pi\}$	4,813,480	4,903,032	4,988,302

Table T49. Outcomes when $c_2 = 10$.

	VS	PVI	VI
r_1	307	238	212
r_2	307	293	212
$E\{w\}$	231	227	224
K_1	8,489	9,209	8,921
K_2	8,489	8,187	8,918
$K_1 + K_2$	16,979	17,396	17,839
$E\{q_1\}$	6,396	7,211	6,993
$E\{q_2\}$	6,396	6,154	6,991
$E\{q_1\} + E\{q_2\}$	12,792	13,365	13,984
$E\left\{\pi_1^G\right\}$	1,219,330	1,332,947	1,282,210
$E\left\{\pi_1^R\right\}$	42,768	12,089	-13,798
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,262,098	1,345,036	1,268,412
$E\left\{\pi_2^G\right\}$	1,219,330	1,152,375	1,281,818
$E\left\{\pi_2^R\right\}$	42,432	30,839	-14,069
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,261,762	1,183,214	1,267,750
$E\{CS\}$	2,199,701	2,289,878	2,358,503
$E\{CS + \pi\}$	4,723,562	4,818,127	4,894,665

Table T50. Outcomes when $c_1^r = 0.5$.

	VS	PVI	VI
r_1	307	239	212
r_2	307	292	212
$E\{w\}$	231	227	224
K_1	8,489	9,208	8,918
K_2	8,489	8,187	8,921
$K_1 + K_2$	16,979	17,395	17,839
$E\{q_1\}$	6,396	7,209	6,991
$E\{q_2\}$	6,396	6,155	6,993
$E\{q_1\} + E\{q_2\}$	12,792	13,364	13,984
$E\left\{\pi_1^G\right\}$	1,219,330	1,332,826	1,281,818
$E\left\{\pi_1^R\right\}$	42,432	11,739	-14,069
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,261,762	1,344,565	1,267,750
$E\left\{\pi_2^G\right\}$	1,219,330	1,152,608	1,282,210
$E\left\{\pi_2^R\right\}$	42,768	31,118	-13,798
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,262,098	1,183,726	1,268,412
$E\{CS\}$	2,199,701	2,289,481	2,358,503
$E\{CS + \pi\}$	4,723,562	4,817,772	4,894,665

Table T51. Outcomes when $c_2^r = 0.5$.

IIE. 20% Increases in Selected Pairs of Parameters.

	VS	PVI	VI
r_1	266	196	175
r_2	266	254	175
$E\{w\}$	226	224	223
K_1	8,388	9,097	8,874
K_2	8,388	8,120	8,874
$K_1 + K_2$	16,776	17,217	17,748
$E\{q_1\}$	6,262	7,059	6,913
$E\{q_2\}$	6,262	6,082	6,913
$E\{q_1\} + E\{q_2\}$	12,524	13,141	13,826
$E\left\{\pi_1^G\right\}$	1,163,840	1,287,740	1,261,980
$E\left\{\pi_1^R\right\}$	13,438	-27,968	-47,339
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,177,278	1,259,772	1,214,641
$E\left\{\pi_2^G\right\}$	1,163,840	1,122,699	1,261,980
$E\left\{\pi_2^R\right\}$	13,438	7,286	-47,339
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,177,278	1,129,985	1,214,641
$E\{CS\}$	2,233,374	2,291,125	2,329,877
$E\{CS + \pi\}$	4,587,929	4,680,882	4,759,158

Table T52. Outcomes when $b_1 = b_2 = 9.6$.

	VS	PVI	VI
r_1	309	241	214
r_2	309	294	214
$E\{w\}$	234	229	227
K_1	8,435	9,146	8,861
K_2	8,435	8,137	8,861
$K_1 + K_2$	16,870	17,283	17,722
$E\{q_1\}$	6,343	7,148	6,933
$E\{q_2\}$	6,343	6,105	6,933
$E\{q_1\} + E\{q_2\}$	12,686	13,253	13,866
$E\left\{\pi_1^G\right\}$	1,200,830	1,312,304	1,262,702
$E\left\{\pi_1^R\right\}$	41,346	11,150	-14,176
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,242,176	1,323,454	1,248,525
$E\left\{\pi_2^G\right\}$	1,200,830	1,135,521	1,262,702
$E\left\{\pi_2^R\right\}$	41,346	29,988	-14,176
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,242,176	1,165,508	1,248,525
$E\{CS\}$	2,168,845	2,256,725	2,323,793
$E\{CS + \pi\}$	4,653,197	4,745,687	4,820,843

Table T53. Outcomes when $c_1 = c_2 = 24$.

	VS	PVI	VI
r_1	310	242	217
r_2	310	295	217
$E\{w\}$	235	231	229
K_1	8,233	8,932	8,657
K_2	8,233	7,944	8,657
$K_1 + K_2$	16,466	16,876	17,314
$E\{q_1\}$	6,313	7,104	6,888
$E\{q_2\}$	6,313	6,077	6,888
$E\{q_1\} + E\{q_2\}$	12,626	13,181	13,777
$E\left\{\pi_1^G\right\}$	1,177,110	1,286,500	1,238,084
$E\left\{\pi_1^R\right\}$	40,729	11,152	-13,041
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,217,839	1,297,652	1,225,043
$E\left\{\pi_2^G\right\}$	1,177,110	1,113,910	1,238,084
$E\left\{\pi_2^R\right\}$	40,729	29,506	-13,041
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,217,839	1,143,416	1,225,043
$E\{CS\}$	2,134,768	2,219,205	2,283,061
$E\{CS + \pi\}$	4,570,446	4,660,273	4,733,147

Table T54. Outcomes when $k_1 = k_2 = 42$**.**

	VS	PVI	VI
r_1	307	244	216
r_2	307	294	216
$E\{w\}$	231	227	225
K_1	8,471	9,148	8,874
K_2	8,471	8,194	8,874
$K_1 + K_2$	16,943	17,342	17,747
$E\{q_1\}$	6,391	7,164	6,965
$E\{q_2\}$	6,391	6,159	6,965
$E\{q_1\} + E\{q_2\}$	12,782	13,323	13,929
$E\left\{\pi_1^G\right\}$	1,220,731	1,326,770	1,281,315
$E\left\{\pi_1^R\right\}$	42,273	17,046	-10,133
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,263,004	1,343,816	1,271,182
$E\left\{\pi_2^G\right\}$	1,220,731	1,155,072	1,281,315
$E\left\{\pi_2^R\right\}$	42,273	32,076	-10,133
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,263,004	1,187,148	1,271,182
$E\{CS\}$	2,195,028	2,281,275	2,345,441
$E\{CS + \pi\}$	4,721,035	4,812,238	4,887,805

Table T55. Outcomes when $d = 3.6$**.**

IIF. 20% Reductions in Selected Pairs of Parameters.

	VS	PVI	VI
r_1	374	308	274
r_2	374	356	274
$E\{w\}$	237	230	226
K_1	8,609	9,337	9,009
K_2	8,609	8,277	8,921
$K_1 + K_2$	17,219	17,614	17,931
$E\{q_1\}$	6,557	7,400	7,107
$E\{q_2\}$	6,557	6,245	7,077
$E\{q_1\} + E\{q_2\}$	13,113	13,645	14,185
$E\left\{\pi_1^G\right\}$	1,288,024	1,390,402	1,314,072
$E\left\{\pi_1^R\right\}$	112,701	92,218	55,353
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,400,725	1,482,619	1,369,425
$E\left\{\pi_2^G\right\}$	1,288,024	1,190,516	1,307,395
$E\left\{\pi_2^R\right\}$	112,701	93,257	55,492
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,400,725	1,283,773	1,362,886
$E\{CS\}$	2,194,204	2,316,916	2,413,040
$E\{CS + \pi\}$	4,995,654	5,083,309	5,145,351

Table T56. Outcomes when $b_1 = b_2 = 6.4$.

	VS	PVI	VI
r_1	306	237	209
r_2	306	291	209
$E\{w\}$	228	224	222
K_1	8,543	9,270	8,977
K_2	8,543	8,237	8,977
$K_1 + K_2$	17,085	17,507	17,954
$E\{q_1\}$	6,448	7,270	7,050
$E\{q_2\}$	6,448	6,204	7,050
$E\{q_1\} + E\{q_2\}$	12,896	13,474	14,099
$E\left\{\pi_1^G\right\}$	1,237,648	1,353,202	1,301,204
$E\left\{\pi_1^R\right\}$	43,655	12,475	-13,864
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,281,303	1,365,676	1,287,340
$E\left\{\pi_2^G\right\}$	1,237,648	1,169,426	1,301,204
$E\left\{\pi_2^R\right\}$	43,655	31,797	-13,864
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,281,303	1,201,223	1,287,340
$E\{CS\}$	2,230,899	2,322,768	2,393,176
$E\{CS + \pi\}$	4,793,504	4,889,668	4,967,856

Table T57. Outcomes when $c_1 = c_2 = 16$.

	VS	PVI	VI
r_1	305	235	207
r_2	305	290	207
$E\{w\}$	227	222	220
K_1	8,770	9,510	9,206
K_2	8,770	8,455	9,206
$K_1 + K_2$	17,540	17,965	18,411
$E\{q_1\}$	6,477	7,313	7,095
$E\{q_2\}$	6,477	6,231	7,095
$E\{q_1\} + E\{q_2\}$	12,954	13,545	14,190
$E\left\{\pi_1^G\right\}$	1,262,815	1,380,437	1,327,285
$E\left\{\pi_1^R\right\}$	44,263	12,402	-15,268
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,307,078	1,392,839	1,312,017
$E\left\{\pi_2^G\right\}$	1,262,815	1,192,307	1,327,285
$E\left\{\pi_2^R\right\}$	44,263	32,275	-15,268
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,307,078	1,224,581	1,312,017
$E\{CS\}$	2,265,954	2,361,915	2,436,024
$E\{CS + \pi\}$	4,880,110	4,979,335	5,060,058

Table T58. Outcomes when $k_1 = k_2 = 28$.

	VS	PVI	VI
r_1	308	233	208
r_2	308	292	208
$E\{w\}$	231	226	224
K_1	8,504	9,263	8,958
K_2	8,504	8,181	8,958
$K_1 + K_2$	17,008	17,444	17,916
$E\{q_1\}$	6,399	7,254	7,017
$E\{q_2\}$	6,399	6,151	7,017
$E\{q_1\} + E\{q_2\}$	12,799	13,404	14,033
$E\left\{\pi_1^G\right\}$	1,217,689	1,339,188	1,283,425
$E\left\{\pi_1^R\right\}$	42,676	6,289	-17,606
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,260,366	1,345,477	1,265,820
$E\left\{\pi_2^G\right\}$	1,217,689	1,150,554	1,283,425
$E\left\{\pi_2^R\right\}$	42,676	30,138	-17,606
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,260,366	1,180,692	1,265,820
$E\{CS\}$	2,203,972	2,297,321	2,368,889
$E\{CS + \pi\}$	4,724,704	4,823,491	4,900,529

Table T59. Outcomes when $d = 2.4$.

IIG. Additional Variation in $d_1 = d_2 \equiv d$.

	VS	PVI	VI
r_1	253	169	166
r_2	253	250	166
$E\{w\}$	224	224	224
K_1	8,284	8,938	8,758
K_2	8,284	8,068	8,758
$K_1 + K_2$	16,567	17,006	17,516
$E\{q_1\}$	6,166	6,921	6,813
$E\{q_2\}$	6,166	6,050	6,813
$E\{q_1\} + E\{q_2\}$	12,332	12,971	13,627
$E\left\{\pi_1^G\right\}$	1,132,480	1,264,031	1,248,370
$E\left\{\pi_1^R\right\}$	5,756	-48,250	-50,399
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,138,236	1,215,781	1,197,971
$E\left\{\pi_2^G\right\}$	1,132,480	1,117,128	1,248,370
$E\left\{\pi_2^R\right\}$	5,756	4,654	-50,399
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,138,236	1,121,782	1,197,971
$E\{CS\}$	2,254,767	2,287,170	2,321,230
$E\{CS + \pi\}$	4,531,239	4,624,733	4,717,172

Table T60. Outcomes when $d = 0.5$**.**

	VS	PVI	VI
r_1	262	182	174
r_2	262	257	174
$E\{w\}$	225	224	224
K_1	8,319	8,989	8,796
K_2	8,319	8,087	8,796
$K_1 + K_2$	16,639	17,076	17,592
$E\{q_1\}$	6,205	6,969	6,845
$E\{q_2\}$	6,205	6,067	6,845
$E\{q_1\} + E\{q_2\}$	12,410	13,035	13,690
$E\left\{\pi_1^G\right\}$	1,146,790	1,274,504	1,252,466
$E\left\{\pi_1^R\right\}$	9,630	-39,039	-45,128
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,156,421	1,235,465	1,207,338
$E\left\{\pi_2^G\right\}$	1,146,790	1,121,950	1,252,466
$E\left\{\pi_2^R\right\}$	9,630	7,205	-45,128
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,156,421	1,129,155	1,207,338
$E\{CS\}$	2,243,403	2,287,872	2,330,024
$E\{CS + \pi\}$	4,556,244	4,652,492	4,744,700

Table T61. Outcomes when $d = 1$**.**

	VS	PVI	VI
r_1	283	209	191
r_2	283	273	191
$E\{w\}$	228	225	224
K_1	8,398	9,097	8,865
K_2	8,398	8,127	8,865
$K_1 + K_2$	16,797	17,224	17,730
$E\{q_1\}$	6,293	7,078	6,914
$E\{q_2\}$	6,293	6,104	6,914
$E\{q_1\} + E\{q_2\}$	12,586	13,182	13,828
$E\left\{\pi_1^G\right\}$	1,179,566	1,300,201	1,264,174
$E\left\{\pi_1^R\right\}$	21,881	-17,080	-32,078
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,201,447	1,283,121	1,232,095
$E\left\{\pi_2^G\right\}$	1,179,566	1,134,561	1,264,174
$E\left\{\pi_2^R\right\}$	21,881	15,632	-32,078
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,201,447	1,150,192	1,232,095
$E\{CS\}$	2,220,831	2,288,342	2,345,882
$E\{CS + \pi\}$	4,623,724	4,721,655	4,810,072

Table T62. Outcomes when $d = 2$**.**

	VS	PVI	VI
r_1	336	273	237
r_2	336	318	237
$E\{w\}$	235	229	226
K_1	8,593	9,324	8,994
K_2	8,593	8,267	8,994
$K_1 + K_2$	17,187	17,590	17,989
$E\{q_1\}$	6,517	7,367	7,091
$E\{q_2\}$	6,517	6,219	7,091
$E\{q_1\} + E\{q_2\}$	13,035	13,586	14,181
$E\left\{\pi_1^G\right\}$	1,267,655	1,374,388	1,309,526
$E\left\{\pi_1^R\right\}$	75,620	51,473	12,351
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,343,275	1,425,861	1,321,877
$E\left\{\pi_2^G\right\}$	1,267,655	1,176,987	1,309,526
$E\left\{\pi_2^R\right\}$	75,620	57,272	12,351
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,343,275	1,234,259	1,321,877
$E\{CS\}$	2,182,697	2,293,708	2,368,522
$E\{CS + \pi\}$	4,869,247	4,953,828	5,012,276

Table T63. Outcomes when $d = 4$**.**

	VS	PVI	VI
r_1	370	313	270
r_2	370	349	270
$E\{w\}$	240	231	228
K_1	8,716	9,445	9,103
K_2	8,716	8,374	9,103
$K_1 + K_2$	17,431	17,818	18,207
$E\{q_1\}$	6,663	7,562	7,222
$E\{q_2\}$	6,663	6,303	7,222
$E\{q_1\} + E\{q_2\}$	13,327	13,865	14,444
$E\left\{\pi_1^G\right\}$	1,328,022	1,428,563	1,352,508
$E\left\{\pi_1^R\right\}$	127,836	108,390	52,929
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,455,858	1,536,953	1,405,438
$E\left\{\pi_2^G\right\}$	1,328,022	1,210,856	1,352,508
$E\left\{\pi_2^R\right\}$	127,836	101,833	52,929
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,455,858	1,312,689	1,405,438
$E\{CS\}$	2,174,128	2,304,363	2,375,151
$E\{CS + \pi\}$	5,085,844	5,154,005	5,186,027

Table T64. Outcomes when $d = 5$**.**

	VS	PVI	VI
r_1	463	421	376
r_2	463	442	376
$E\{w\}$	254	241	240
K_1	9,035	9,690	9,330
K_2	9,035	8,713	9,330
$K_1 + K_2$	18,071	18,402	18,661
$E\{q_1\}$	7,062	8,101	7,599
$E\{q_2\}$	7,062	6,566	7,599
$E\{q_1\} + E\{q_2\}$	14,124	14,667	15,198
$E\left\{\pi_1^G\right\}$	1,504,587	1,595,537	1,530,395
$E\left\{\pi_1^R\right\}$	340,088	326,840	230,115
$E\left\{\pi_1^G + \pi_1^R\right\}$	1,844,676	1,922,377	1,760,509
$E\left\{\pi_2^G\right\}$	1,504,587	1,326,976	1,530,395
$E\left\{\pi_2^R\right\}$	340,088	302,386	230,115
$E\left\{\pi_2^G + \pi_2^R\right\}$	1,844,676	1,629,363	1,760,509
$E\{CS\}$	2,220,985	2,377,367	2,415,338
$E\{CS + \pi\}$	5,910,337	5,929,107	5,936,357

Table T65. Outcomes when $d = 7$**.**

III. Detailed Proofs of Conclusions 7 and 9.

The Conclusions refer to Lemmas 1 – 3, which are reproduced from the text.

Lemma 1. *Suppose neither G1 nor G2 is capacity constrained. Then in equilibrium, for* $i, j \in \{1, 2\}$ $(j \neq i)$*:*

$$q_i^{0c}(\varepsilon) \;=\; \frac{b^w \left[\, a^I + (1 + 2\,\alpha_i)\, Q_i^r + (1 - \alpha_j)\, Q_j^r \,\right] + \varepsilon + c_j - 2\, c_i}{3\, b^w} \;; \;\; \text{and}$$

$$w^{0c}(\varepsilon) \;=\; \frac{1}{3} \left[\, b^w \left(\, a^I + [\, 1 - \alpha_1\,]\, Q_1^r + [\, 1 - \alpha_2\,]\, Q_2^r \right) + \varepsilon + c_1 + c_2 \,\right] \,.$$

Lemma 2. *Suppose only Gj is capacity constrained for* $\varepsilon \in (\, \varepsilon_0, \varepsilon_{12}\,)$*. Then in equilibrium, for* $i, j \in \{1, 2\}$ $(j \neq i)$*:*

$$q_j^{1c}(\varepsilon) \;=\; K_j \,; \;\; q_i^{1c}(\varepsilon) \;=\; \frac{1}{2} \left[\, a^I + Q_i^r\, (1 + \alpha_i) + Q_j^r \,\right] + \frac{1}{2\, b^w} \left[\, \varepsilon - c_i \,\right] - \frac{1}{2}\, K_j \,; \;\; \text{and}$$

$$w^{1c}(\varepsilon) \;=\; \frac{1}{2}\, b^w \left[\, a^I + Q_i^r\, (1 - \alpha_i) + Q_j^r \,\right] + \frac{1}{2} \left[\, \varepsilon + c_i - b^w\, K_j \,\right] .$$

Lemma 3. *Suppose G1 and G2 are both capacity constrained. Then in equilibrium:*

$$q_1^{2c}(\varepsilon) \;=\; K_1 \,; \;\; q_2^{2c}(\varepsilon) \;=\; K_2 \,; \;\text{and}\;\; w^{2c}(\varepsilon) \;=\; b^w \left[\, a^I + Q_1^r + Q_2^r - K_1 - K_2 \,\right] + \varepsilon \,.$$

Conclusion 7. *Suppose* $b^w\, a^I + \varepsilon \;>\; c$ *for all* $\varepsilon \in [\, \underline{\varepsilon}, \overline{\varepsilon}\,]$*. Then* $\left.\frac{\partial r_i}{\partial \alpha_i}\right|_{dK_i \,=\, dK_j \,=\, 0} < \; 0$ *and* $\left.\frac{\partial (r_i \,+\, r_j)}{\partial \alpha_i}\right|_{dK_i \,=\, dK_j \,=\, 0} < \; 0$ *in the symmetric setting.*

<u>Proof</u>. Given α_i, Ri seeks to maximize:

$$L^i \;\equiv\; \int_{\underline{\varepsilon}}^{\varepsilon_0} \left\{\, \alpha_i \left[\, w^{0c}(\varepsilon) - c_i \,\right] q_i^{0c}(\varepsilon) + \left[\, r_i - c_i^r - w^{0c}(\varepsilon) \,\right] Q_i^r(r_i, r_j) \,\right\} dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \left\{\, \alpha_i \left[\, w^{1c}(\varepsilon) - c_i \,\right] q_i^{1c}(\varepsilon) + \left[\, r_i - c_i^r - w^{1c}(\varepsilon) \,\right] Q_i^r(r_i, r_j) \,\right\} dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left\{\, \alpha_i \left[\, w^{2c}(\varepsilon) - c_i \,\right] q_i^{2c}(\varepsilon) + \left[\, r_i - c_i^r - w^{2c}(\varepsilon) \,\right] Q_i^r(r_i, r_j) \,\right\} dH(\varepsilon) - \alpha_i\, k_i\, K_i \,. \quad (237)$$

(237) implies that equilibrium choices of r_i and r_j are determined by:

$$\frac{\partial L^i}{\partial r_i} \;\equiv\; L^i_{r_i} \;=\; 0 \quad \text{and} \quad \frac{\partial L^j}{\partial r_j} \;\equiv\; L^j_{r_j} \;=\; 0\,. \tag{238}$$

(237) and the proof of Lemma 4 in the text imply:

$$\begin{aligned}
L^i_{r_i} \;\equiv\; & \int_{\underline{\varepsilon}}^{\varepsilon_0} \Bigg\{ \alpha_i \left[\, w^{0c}(\varepsilon) - c_i \,\right] \frac{dq_i^{0c}(\varepsilon)}{dr_i} + \alpha_i \, q_i^{0c}(\varepsilon) \, \frac{dw^{0c}(\varepsilon)}{dr_i} \\
& \quad + \left[\, r_i - c_i^r - w^{0c}(\varepsilon) \,\right] \frac{dQ_i^r(r_i, r_j)}{dr_i} + Q_i^r(r_i, r_j) \left[1 - \frac{dw^{0c}(\varepsilon)}{dr_i} \right] \Bigg\} \, dH(\varepsilon) \\
& + \int_{\varepsilon_0}^{\varepsilon_{12}} \Bigg\{ \alpha_i \left[\, w^{1c}(\varepsilon) - c_i \,\right] \frac{dq_i^{1c}(\varepsilon)}{dr_i} + \alpha_i \, q_i^{1c}(\varepsilon) \, \frac{dw^{1c}(\varepsilon)}{dr_i} \\
& \quad + \left[\, r_i - c_i^r - w^{1c}(\varepsilon) \,\right] \frac{dQ_i^r(r_i, r_j)}{dr_i} + Q_i^r(r_i, r_j) \left[1 - \frac{dw^{1c}(\varepsilon)}{dr_i} \right] \Bigg\} \, dH(\varepsilon) \\
& + \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \Bigg\{ \alpha_i \left[\, w^{2c}(\varepsilon) - c_i \,\right] \frac{dq_i^{2c}(\varepsilon)}{dK_i} + \alpha_i \, q_i^{2c}(\varepsilon) \, \frac{dw^{2c}(\varepsilon)}{dK_i} \\
& \quad + \left[\, r_i - c_i^r - w^{2c}(\varepsilon) \,\right] \frac{dQ_i^r(r_i, r_j)}{dr_i} + Q_i^r(r_i, r_j) \left[1 - \frac{dw^{2c}(\varepsilon)}{dr_i} \right] \Bigg\} \, dH(\varepsilon)\,.
\end{aligned} \tag{239}$$

Define $L^i_{xy} \equiv \frac{\partial^2 L^i}{\partial x \partial y}$. Then (239) implies:

$$\begin{aligned}
& L^i_{r_i r_i} \, dr_i + L^i_{r_i r_j} \, dr_j + L^i_{r_i \alpha_i} \, d\alpha_i \;=\; 0 \;\; \text{and} \\
& L^j_{r_j r_i} \, dr_i + L^j_{r_j r_j} \, dr_j + L^j_{r_j \alpha_i} \, d\alpha_i \;=\; 0 \\
\Leftrightarrow \quad & \begin{bmatrix} L^i_{r_i r_i} & L^i_{r_i r_j} \\ L^j_{r_j r_i} & L^j_{r_j r_j} \end{bmatrix} \begin{bmatrix} dr_i \\ dr_j \end{bmatrix} \;=\; - \begin{bmatrix} L^i_{r_i \alpha_i} \\ L^j_{r_j \alpha_i} \end{bmatrix} d\alpha_i\,.
\end{aligned} \tag{240}$$

We assume the standard sufficient conditions for r_i and r_j to be strict local maxima are satisfied. Formally:

$$L^i_{r_i r_i} \;<\; 0, \quad L^j_{r_j r_j} \;<\; 0, \quad \text{and} \quad L^i_{r_i r_i} \, L^j_{r_j r_j} - L^i_{r_i r_j} \, L^j_{r_j r_i} \;>\; 0\,. \tag{241}$$

(239) implies that $L^j_{r_j \alpha_i} = 0$. Therefore, (240), (241), and Cramer's Rule imply:

$$\frac{dr_i}{d\alpha_i} = \frac{\begin{vmatrix} -L^i_{r_i\alpha_i} & L^i_{r_ir_j} \\ 0 & L^j_{r_jr_j} \end{vmatrix}}{\begin{vmatrix} L^i_{r_ir_i} & L^i_{r_ir_j} \\ L^j_{r_jr_i} & L^j_{r_jr_j} \end{vmatrix}} = -\frac{L^i_{r_i\alpha_i}\, L^j_{r_jr_j}}{L^i_{r_ir_i}\, L^j_{r_jr_j} - L^i_{r_ir_j}\, L^j_{r_jr_i}}$$

$$\stackrel{s}{=} -\, L^i_{r_i\alpha_i}\, L^j_{r_jr_j} \stackrel{s}{=} L^i_{r_i\alpha_i}\,. \tag{242}$$

(239) implies:

$$L^i_{r_i\alpha_i} = \int_{\underline{\varepsilon}}^{\varepsilon_0} \Bigg\{ \alpha_i \left[w^{0c}(\varepsilon) - c_i \right] \frac{d}{d\alpha_i}\left(\frac{dq_i^{0c}(\varepsilon)}{dr_i} \right) + \alpha_i \frac{dq_i^{0c}(\varepsilon)}{dr_i} \frac{dw^{0c}(\varepsilon)}{d\alpha_i} + \alpha_i\, q_i^{0c}(\varepsilon) \frac{d}{d\alpha_i}\left(\frac{dw^{0c}(\varepsilon)}{dr_i} \right)$$
$$+ \alpha_i \frac{dw^{0c}(\varepsilon)}{dr_i} \frac{dq_i^{0c}(\varepsilon)}{d\alpha_i} + \left[w^{0c}(\varepsilon) - c_i \right] \frac{dq_i^{0c}(\varepsilon)}{dr_i} + q_i^{0c}(\varepsilon) \frac{dw^{0c}(\varepsilon)}{dr_i}$$
$$+ \left[r_i - c_i^r - w^{0c}(\varepsilon) \right] \frac{d}{d\alpha_i}\left(\frac{dQ_i^r}{dr_i} \right) - \frac{dQ_i^r}{dr_i} \frac{dw^{0c}(\varepsilon)}{d\alpha_i}$$
$$- Q_i^r(r_i, r_j) \frac{d}{d\alpha_i}\left(\frac{dw^{0c}(\varepsilon)}{dr_i} \right) + \left[1 - \frac{dw^{0c}(\varepsilon)}{dr_i} \right] \frac{dQ_i^r}{d\alpha_i} \Bigg\} dH(\varepsilon)$$
$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \Bigg\{ \alpha_i \left[w^{1c}(\varepsilon) - c_i \right] \frac{d}{d\alpha_i}\left(\frac{dq_i^{1c}(\varepsilon)}{dr_i} \right) + \alpha_i \frac{dq_i^{1c}(\varepsilon)}{dr_i} \frac{dw^{1c}(\varepsilon)}{d\alpha_i} + \alpha_i\, q_i^{1c}(\varepsilon) \frac{d}{d\alpha_i}\left(\frac{dw^{1c}(\varepsilon)}{dr_i} \right)$$
$$+ \alpha_i \frac{dw^{1c}(\varepsilon)}{dr_i} \frac{dq_i^{1c}(\varepsilon)}{d\alpha_i} + \left[w^{1c}(\varepsilon) - c_i \right] \frac{dq_i^{1c}(\varepsilon)}{dr_i} + q_i^{1c}(\varepsilon) \frac{dw^{1c}(\varepsilon)}{dr_i}$$
$$+ \left[r_i - c_i^r - w^{1c}(\varepsilon) \right] \frac{d}{d\alpha_i}\left(\frac{dQ_i^r}{dr_i} \right) - \frac{dQ_i^r}{dr_i} \frac{dw^{1c}(\varepsilon)}{d\alpha_i}$$
$$- Q_i^r(r_i, r_j) \frac{d}{d\alpha_i}\left(\frac{dw^{1c}(\varepsilon)}{dr_i} \right) + \left[1 - \frac{dw^{1c}(\varepsilon)}{dr_i} \right] \frac{dQ_i^r}{d\alpha_i} \Bigg\} dH(\varepsilon)$$
$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \Bigg\{ \alpha_i \left[w^{2c}(\varepsilon) - c_i \right] \frac{d}{d\alpha_i}\left(\frac{dq_i^{2c}(\varepsilon)}{dr_i} \right) + \alpha_i \frac{dq_i^{2c}(\varepsilon)}{dr_i} \frac{dw^{2c}(\varepsilon)}{d\alpha_i} + \alpha_i\, q_i^{2c}(\varepsilon) \frac{d}{d\alpha_i}\left(\frac{dw^{2c}(\varepsilon)}{dr_i} \right)$$
$$+ \alpha_i \frac{dw^{2c}(\varepsilon)}{dr_i} \frac{dq_i^{2c}(\varepsilon)}{d\alpha_i} + \left[w^{2c}(\varepsilon) - c_i \right] \frac{dq_i^{2c}(\varepsilon)}{dr_i} + q_i^{2c}(\varepsilon) \frac{dw^{2c}(\varepsilon)}{dr_i}$$
$$+ \left[r_i - c_i^r - w^{2c}(\varepsilon) \right] \frac{d}{d\alpha_i}\left(\frac{dQ_i^r}{dr_i} \right) - \frac{dQ_i^r}{dr_i} \frac{dw^{2c}(\varepsilon)}{d\alpha_i}$$

$$- \; Q_i^r(r_i, r_j) \, \frac{d}{d\alpha_i} \left(\frac{dw^{2c}(\varepsilon)}{dr_i} \right) + \left[1 - \frac{dw^{2c}(\varepsilon)}{dr_i} \right] \frac{dQ_i^r}{d\alpha_i} \right\} dH(\varepsilon) \, . \tag{243}$$

Lemmas 1, 2, and 3 imply that when K_i and K_j are held constant and when Gj is capacity constrained for $\varepsilon \in (\, \varepsilon_0, \varepsilon_{12} \,)$:

$$\frac{\partial q_i^{0c}(\varepsilon)}{\partial \alpha_i} \;=\; \frac{2}{3} \, Q_i^r \, ; \quad \frac{\partial q_j^{0c}(\varepsilon)}{\partial \alpha_i} \;=\; -\,\frac{1}{3} \, Q_i^r \, ; \quad \frac{\partial q_i^{0c}(\varepsilon)}{\partial r_i} \;=\; -\,\frac{1}{3} \, [\, (1 + 2\,\alpha_i) \, b_i - d_j \, (1 - \alpha_j) \,] \, ;$$

$$\frac{\partial w^{0c}(\varepsilon)}{\partial \alpha_i} \;=\; -\,\frac{b^w}{3} \, Q_i^r \, ; \quad \frac{\partial w^{0c}(\varepsilon)}{\partial r_i} \;=\; -\,\frac{b^w}{3} \, [\, (1 - \alpha_i) \, b_i - (1 - \alpha_j) \, d_j \,] \, ;$$

$$\frac{d}{d\alpha_i} \left(\frac{dq_i^{0c}(\varepsilon)}{dr_i} \right) \;=\; -\,\frac{2}{3} \, b_i \, ; \quad \frac{d}{d\alpha_i} \left(\frac{dw^{0c}(\varepsilon)}{dr_i} \right) \;=\; \frac{1}{3} \, b^w \, b_i \, ;$$

$$\frac{\partial q_i^{1c}(\varepsilon)}{\partial \alpha_i} \;=\; \frac{1}{2} \, Q_i^r \, ; \quad \frac{\partial q_j^{1c}(\varepsilon)}{\partial \alpha_i} \;=\; 0 \, ; \quad \frac{\partial q_i^{1c}(\varepsilon)}{\partial r_i} \;\; -\,\frac{1}{2} \, [\, (1 + \alpha_i) \, b_i - d_j \,] \, ;$$

$$\frac{\partial w^{1c}(\varepsilon)}{\partial \alpha_i} \;=\; -\,\frac{b^w}{2} \, Q_i^r \, ; \quad \frac{\partial w^{1c}(\varepsilon)}{\partial r_i} \;=\; -\,\frac{b^w}{2} \, [\, (1 - \alpha_i) \, b_i - d_j \,] \, ;$$

$$\frac{d}{d\alpha_i} \left(\frac{dq_i^{1c}(\varepsilon)}{dr_i} \right) \;=\; -\,\frac{1}{2} \, b_i \, ; \quad \frac{d}{d\alpha_i} \left(\frac{dw^{1c}(\varepsilon)}{dr_i} \right) \;=\; \frac{1}{2} \, b^w \, b_i \, ;$$

$$\frac{\partial q_i^{2c}(\varepsilon)}{\partial \alpha_i} \;=\; 0 \, ; \quad \frac{\partial q_j^{2c}(\varepsilon)}{\partial \alpha_i} \;=\; 0 \, ; \quad \frac{\partial q_i^{2c}(\varepsilon)}{\partial r_i} \;=\; 0 \, ; \quad \frac{\partial q_j^{2c}(\varepsilon)}{\partial r_i} \;=\; 0 \, ;$$

$$\frac{\partial w^{2c}(\varepsilon)}{\partial \alpha_i} \;=\; 0 \, ; \quad \frac{\partial w^{2c}(\varepsilon)}{\partial r_i} \;=\; -\, b^w \, [\, b_i - d_j \,] \, ;$$

$$\frac{d}{d\alpha_i} \left(\frac{dq_i^{2c}(\varepsilon)}{dr_i} \right) \;=\; 0 \, ; \quad \frac{d}{d\alpha_i} \left(\frac{dw^{2c}(\varepsilon)}{dr_i} \right) \;=\; 0 \, . \tag{244}$$

Furthermore, in this comparative static exercise:

$$\frac{dQ_i^r}{d\alpha_i} \;=\; 0 \quad \text{and} \quad \frac{dQ_i^r}{dr_i} \;=\; -\, b_i \quad \Rightarrow \quad \frac{d}{d\alpha_i} \left(\frac{dQ_i^r}{dr_i} \right) \;=\; 0 \, . \tag{245}$$

(243) – (245) imply that when Gj is capacity constrained for $\varepsilon \in (\, \varepsilon_0, \varepsilon_{12} \,)$:

$$L^i_{r_i \alpha_i} \;=\; \int_{\underline{\varepsilon}}^{\varepsilon_0} \left\{ \alpha_i \, [\, w^{0c}(\varepsilon) - c_i \,] \left[-\,\frac{2}{3} \, b_i \right] + \alpha_i \, \frac{dq_i^{0c}(\varepsilon)}{dr_i} \, \frac{dw^{0c}(\varepsilon)}{d\alpha_i} + \alpha_i \, q_i^{0c}(\varepsilon) \left[\frac{1}{3} \, b^w \, b_i \right] \right.$$

$$+ \;\; \alpha_i \, \frac{dw^{0c}(\varepsilon)}{dr_i} \, \frac{dq_i^{0c}(\varepsilon)}{d\alpha_i} + [\, w^{0c}(\varepsilon) - c_i \,] \, \frac{dq_i^{0c}(\varepsilon)}{dr_i} + q_i^{0c}(\varepsilon) \, \frac{dw^{0c}(\varepsilon)}{dr_i}$$

$$+ \left[r_i - c_i^r - w^{0c}(\varepsilon) \right] [\,0\,] - [\,-b_i\,] \frac{dw^{0c}(\varepsilon)}{d\alpha_i}$$

$$- Q_i^r \left[\frac{1}{3} b^w b_i \right] + \left[1 - \frac{dw^{0c}(\varepsilon)}{dr_i} \right] [\,0\,] \Bigg\} dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \Bigg\{ \alpha_i \left[w^{1c}(\varepsilon) - c_i \right] \left[-\frac{1}{2} b_i \right] + \alpha_i \frac{dq_i^{1c}(\varepsilon)}{dr_i} \frac{dw^{1c}(\varepsilon)}{d\alpha_i} + \alpha_i q_i^{1c}(\varepsilon) \left[\frac{1}{2} b^w b_i \right]$$

$$+ \alpha_i \frac{dw^{1c}(\varepsilon)}{dr_i} \frac{dq_i^{1c}(\varepsilon)}{d\alpha_i} + \left[w^{1c}(\varepsilon) - c_i \right] \frac{dq_i^{1c}(\varepsilon)}{dr_i} + q_i^{1c}(\varepsilon) \frac{dw^{1c}(\varepsilon)}{dr_i}$$

$$+ \left[r_i - c_i^r - w^{1c}(\varepsilon) \right] [\,0\,] - [\,-b_i\,] \frac{dw^{1c}(\varepsilon)}{d\alpha_i}$$

$$- Q_i^r \left[\frac{1}{2} b^w b_i \right] + \left[1 - \frac{dw^{1c}(\varepsilon)}{dr_i} \right] [\,0\,] \Bigg\} dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\bar{\varepsilon}} \Bigg\{ \alpha_i \left[w^{2c}(\varepsilon) - c_i \right] [\,0\,] + \alpha_i \frac{dq_i^{2c}(\varepsilon)}{dr_i} \frac{dw^{2c}(\varepsilon)}{d\alpha_i} + \alpha_i q_i^{2c}(\varepsilon) [\,0\,]$$

$$+ \alpha_i \frac{dw^{2c}(\varepsilon)}{dr_i} \frac{dq_i^{2c}(\varepsilon)}{d\alpha_i} + \left[w^{2c}(\varepsilon) - c_i \right] \frac{dq_i^{2c}(\varepsilon)}{dr_i} + q_i^{2c}(\varepsilon) \frac{dw^{2c}(\varepsilon)}{dr_i}$$

$$+ \left[r_i - c_i^r - w^{2c}(\varepsilon) \right] [\,0\,] - [\,-b_i\,] \frac{dw^{2c}(\varepsilon)}{d\alpha_i}$$

$$- Q_i^r [\,0\,] + \left[1 - \frac{dw^{2c}(\varepsilon)}{dr_i} \right] [\,0\,] \Bigg\} dH(\varepsilon)$$

$$= \int_{\underline{\varepsilon}}^{\varepsilon_0} \Bigg\{ -\frac{2}{3} \alpha_i b_i \left[w^{0c}(\varepsilon) - c_i \right] + \alpha_i \frac{dq_i^{0c}(\varepsilon)}{dr_i} \frac{dw^{0c}(\varepsilon)}{d\alpha_i} + \frac{1}{3} b^w b_i \alpha_i q_i^{0c}(\varepsilon)$$

$$+ \alpha_i \frac{dw^{0c}(\varepsilon)}{dr_i} \frac{dq_i^{0c}(\varepsilon)}{d\alpha_i} + \left[w^{0c}(\varepsilon) - c_i \right] \frac{dq_i^{0c}(\varepsilon)}{dr_i} + q_i^{0c}(\varepsilon) \frac{dw^{0c}(\varepsilon)}{dr_i}$$

$$+ b_i \frac{dw^{0c}(\varepsilon)}{d\alpha_i} - \frac{1}{3} b^w b_i Q_i^r \Bigg\} dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \Bigg\{ -\frac{1}{2} b_i \alpha_i \left[w^{1c}(\varepsilon) - c_i \right] + \alpha_i \frac{dq_i^{1c}(\varepsilon)}{dr_i} \frac{dw^{1c}(\varepsilon)}{d\alpha_i} + \frac{1}{2} b^w b_i \alpha_i q_i^{1c}(\varepsilon)$$

$$+ \alpha_i \frac{dw^{1c}(\varepsilon)}{dr_i} \frac{dq_i^{1c}(\varepsilon)}{d\alpha_i} + \left[w^{1c}(\varepsilon) - c_i \right] \frac{dq_i^{1c}(\varepsilon)}{dr_i} + q_i^{1c}(\varepsilon) \frac{dw^{1c}(\varepsilon)}{dr_i}$$

$$+\, b_i \, \frac{dw^{1c}(\varepsilon)}{d\alpha_i} - \frac{1}{2}\, b^w \, b_i \, Q_i^r \Bigg\} \, dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \Bigg\{ \alpha_i \, \frac{dq_i^{2c}(\varepsilon)}{dr_i} \, \frac{dw^{2c}(\varepsilon)}{d\alpha_i} + \alpha_i \, \frac{dw^{2c}(\varepsilon)}{dr_i} \, \frac{dq_i^{2c}(\varepsilon)}{d\alpha_i}$$

$$+ \left[\, w^{2c}(\varepsilon) - c_i \,\right] \frac{dq_i^{2c}(\varepsilon)}{dr_i} + q_i^{2c}(\varepsilon) \, \frac{dw^{2c}(\varepsilon)}{dr_i} + b_i \, \frac{dw^{2c}(\varepsilon)}{d\alpha_i} \Bigg\} \, dH(\varepsilon) \tag{246}$$

$$= \int_{\underline{\varepsilon}}^{\varepsilon_0} \Bigg\{ -\frac{2}{3}\, \alpha_i \, b_i \left[\, w^{0c}(\varepsilon) - c_i \,\right] + \frac{1}{9}\, b^w \, \alpha_i \left[\, (1 + 2\,\alpha_i)\, b_i - d_j \,(1 - \alpha_j) \,\right] Q_i^r$$

$$+ \frac{1}{3}\, b^w \, b_i \, \alpha_i \, q_i^{0c}(\varepsilon) - \frac{2}{9}\, b^w \, \alpha_i \left[\, (1 - \alpha_i)\, b_i - (1 - \alpha_j)\, d_j \,\right] Q_i^r$$

$$- \frac{1}{3} \left[\, (1 + 2\,\alpha_i)\, b_i - d_j \,(1 - \alpha_j) \,\right] \left[\, w^{0c}(\varepsilon) - c_i \,\right]$$

$$- \frac{1}{3}\, b^w \left[\, (1 - \alpha_i)\, b_i - (1 - \alpha_j)\, d_j \,\right] q_i^{0c}(\varepsilon) - \frac{1}{3}\, b^w \, b_i \, Q_i^r - \frac{1}{3}\, b^w \, b_i \, Q_i^r \Bigg\} \, dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \Bigg\{ -\frac{1}{2}\, b_i \, \alpha_i \left[\, w^{1c}(\varepsilon) - c_i \,\right] + \frac{1}{4}\, b^w \, \alpha_i \left[\, (1 + \alpha_i)\, b_i - d_j \,\right] Q_i^r + \frac{1}{2}\, b^w \, b_i \, \alpha_i \, q_i^{1c}(\varepsilon)$$

$$- \frac{1}{4}\, b^w \, \alpha_i \left[\, (1 - \alpha_i)\, b_i - d_j \,\right] Q_i^r - \frac{1}{2} \left[\, (1 + \alpha_i)\, b_i - d_j \,\right] \left[\, w^{1c}(\varepsilon) - c_i \,\right]$$

$$- \frac{1}{2}\, b^w \left[\, (1 - \alpha_i)\, b_i - d_j \,\right] q_i^{1c}(\varepsilon) - \frac{1}{2}\, b^w \, b_i \, Q_i^r - \frac{1}{2}\, b^w \, b_i \, Q_i^r \Bigg\} \, dH(\varepsilon)$$

$$- \int_{\varepsilon_{12}}^{\overline{\varepsilon}} b^w \left[\, b_i - d_j \,\right] q_i^{2c}(\varepsilon) \, dH(\varepsilon) \tag{247}$$

$$= \int_{\underline{\varepsilon}}^{\varepsilon_0} \Bigg\{ -\frac{1}{3} \left[\, (\, 1 + 4\,\alpha_i \,)\, b_i - d_j \,(\, 1 - \alpha_j \,) \,\right] \left[\, w^{0c}(\varepsilon) - c_i \,\right]$$

$$- \frac{1}{3}\, b^w \left[\, (\, 1 - 2\,\alpha_i \,)\, b_i - (\, 1 - \alpha_j \,)\, d_j \,\right] q_i^{0c}(\varepsilon)$$

$$- \frac{1}{9}\, b^w \, Q_i^r \left[\, 6\, b_i + \alpha_i \,(\, 2\,[\, 1 - \alpha_i \,]\, b_i - 2\,[\, 1 - \alpha_j \,]\, d_j \right.$$

$$\left. - \,[\, 1 + 2\,\alpha_i \,]\, b_i + [\, 1 - \alpha_j \,]\, d_j] \,) \right\} dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \Big\{ - \frac{1}{2} \, [\, (\, 1 + 2 \, \alpha_i \,) \, b_i - d_j \,] \, \big[\, w^{1c}(\varepsilon) - c_i \, \big] \, - \frac{1}{2} \, b^w \, [\, (\, 1 - 2 \, \alpha_i \,) \, b_i - d_j \,] \, q_i^{1c}(\varepsilon)$$

$$- \frac{1}{4} \, b^w \, Q_i^r \, [\, 4 \, b_i + \alpha_i \, (\, [\, 1 - \alpha_i \,] \, b_i - d_j - [\, 1 + \alpha_i \,] \, b_i + d_j \,) \Big\} \, dH(\varepsilon)$$

$$- \int_{\varepsilon_{12}}^{\overline{\varepsilon}} b^w \, [\, b_i - d_j \,] \, q_i^{2c}(\varepsilon) \, dH(\varepsilon) \tag{248}$$

$$= \int_{\underline{\varepsilon}}^{\varepsilon_0} \Big\{ - \frac{1}{3} \, [\, (1 + 4 \, \alpha_i) \, b_i - (\, 1 - \alpha_j \,) \, d_j \,] \, \big[\, w^{0c}(\varepsilon) - c_i \, \big]$$

$$- \frac{1}{3} \, b^w \, [\, (\, 1 - 2 \, \alpha_i \,) \, b_i - (\, 1 - \alpha_j \,) \, d_j \,] \, q_i^{0c}(\varepsilon)$$

$$- \frac{1}{9} \, b^w \, Q_i^r \, [\, 6 \, b_i + \alpha_i \, (\, [\, 1 - 4 \, \alpha_i \,] \, b_i - [\, 1 - \alpha_j \,] \, d_j \,) \,] \; \Big\} \, dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \Big\{ - \frac{1}{2} \, [\, (\, 1 + 2 \, \alpha_i \,) \, b_i - d_j \,] \, \big[\, w^{1c}(\varepsilon) - c_i \, \big] \, - \frac{1}{2} \, b^w \, [\, (\, 1 - 2 \, \alpha_i \,) \, b_i - d_j \,] \, q_i^{1c}(\varepsilon)$$

$$- \frac{1}{4} \, b^w \, Q_i^r \, \big[\, 4 \, b_i - 2 \, (\alpha_i)^2 \, b_i \, \big] \; \Big\} \; dH(\varepsilon)$$

$$- \int_{\varepsilon_{12}}^{\overline{\varepsilon}} b^w \, [\, b_i - d_j \,] \, q_i^{2c}(\varepsilon) \, dH(\varepsilon)$$

$$= \int_{\underline{\varepsilon}}^{\varepsilon_0} \Big\{ - \frac{1}{3} \, [\, (1 + 4 \, \alpha_i) \, b_i - (\, 1 - \alpha_j \,) \, d_j \,] \, \big[\, w^{0c}(\varepsilon) - c_i \, \big]$$

$$- \frac{1}{3} \, b^w \, [\, (\, 1 - 2 \, \alpha_i \,) \, b_i - (\, 1 - \alpha_j \,) \, d_j \,] \, q_i^{0c}(\varepsilon)$$

$$- \frac{1}{9} \, b^w \, Q_i^r \, [\, 6 \, b_i + \alpha_i \, (\, [\, 1 - 4 \, \alpha_i \,] \, b_i - [\, 1 - \alpha_j \,] \, d_j \,) \,] \; \Big\} \, dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \Big\{ - \frac{1}{2} \, [\, (\, 1 + 2 \, \alpha_i \,) \, b_i - d_j \,] \, \big[\, w^{1c}(\varepsilon) - c_i \, \big] \, - \frac{1}{2} \, b^w \, [\, (\, 1 - 2 \, \alpha_i \,) \, b_i - d_j \,] \, q_i^{1c}(\varepsilon)$$

$$- \frac{1}{2} \, b^w \, b_i \, Q_i^r \, \big[\, 2 - (\alpha_i)^2 \, \big] \; \Big\} \; dH(\varepsilon)$$

$$- \int_{\varepsilon_{12}}^{\overline{\varepsilon}} b^w \, [\, b_i - d_j \,] \, q_i^{2c}(\varepsilon) \, dH(\varepsilon) \, . \tag{249}$$

Lemma 1 implies that the first integral in (249) is:

$$\begin{aligned}
& \int_{\underline{\varepsilon}}^{\varepsilon_0} \Big\{ - \frac{1}{9} \, [\, (1 + 4\,\alpha_i) \, b_i - (\, 1 - \alpha_j \,) \, d_j \,] \\
& \qquad \cdot \Big[b^w \left(a^I + [\, 1 - \alpha_i \,] \, Q_i^r + [\, 1 - \alpha_j \,] \, Q_j^r \right) + \varepsilon + c_j - 2\, c_i \Big] \\
& \quad - \frac{1}{9} \, [\, (\, 1 - 2\,\alpha_i \,) \, b_i - (\, 1 - \alpha_j \,) \, d_j \,] \\
& \qquad \cdot \Big[b^w \left(a^I + [\, 1 + 2\,\alpha_i \,] \, Q_i^r + [\, 1 - \alpha_j \,] \, Q_j^r \right) + \varepsilon + c_j - 2\, c_i \Big] \\
& \quad - \frac{1}{9} \, b^w \, Q_i^r \, [\, 6\, b_i + \alpha_i \, (\, [\, 1 - 4\,\alpha_i \,] \, b_i - [\, 1 - \alpha_j \,] \, d_j \,) \,] \Big\} \, dH(\varepsilon) \\
= \; & - \frac{1}{9} \int_{\underline{\varepsilon}}^{\varepsilon_0} \Big\{ [\, (\, 1 + 4\,\alpha_i \,) \, b_i - (\, 1 - \alpha_j \,) \, d_j + (\, 1 - 2\,\alpha_i \,) \, b_i - (\, 1 - \alpha_j \,) \, d_j \,] \\
& \qquad \cdot \Big[b^w \, a^I + \varepsilon + c_j - 2\, c_i \Big] \\
& \qquad + b^w \, Q_i^r \Big[\, (\, [\, 1 + 4\,\alpha_i \,] \, b_i - [\, 1 - \alpha_j \,] \, d_j) \, (\, 1 - \alpha_i \,) \\
& \qquad\qquad + \, (\, [\, 1 - 2\,\alpha_i \,] \, b_i - [\, 1 - \alpha_j \,] \, d_j \,) \, (\, 1 + 2\,\alpha_i \,) \\
& \qquad\qquad + \, 6\, b_i + \alpha_i \, (\, 1 - 4\,\alpha_i \,) \, b_i - \alpha_i \, (\, 1 - \alpha_j \,) \, d_j \Big] \\
& \qquad + b^w \, Q_j^r \Big[\, (\, [\, 1 + 4\,\alpha_i \,] \, b_i - [\, 1 - \alpha_j \,] \, d_j) \, (\, 1 - \alpha_j \,) \\
& \qquad\qquad + \, (\, [\, 1 - 2\,\alpha_i \,] \, b_i - [\, 1 - \alpha_j \,] \, d_j \,) \, (\, 1 - \alpha_j \,) \Big] \Big\} \, dH(\varepsilon) \, .
\end{aligned} \tag{250}$$

The term multiplying $b^w \, a^I + \varepsilon + c_j - 2\, c_i$ in (250) is:

$$\begin{aligned}
& b_i \, [\, 1 + 4\,\alpha_i + 1 - 2\,\alpha_i \,] - 2 \, [\, 1 - \alpha_j \,] \, d_j \\
& \quad = \; b_i \, [\, 2 + 2\,\alpha_i \,] - 2 \, [\, 1 - \alpha_j \,] \, d_j \; = \; 2 \, [\, (\, 1 + \alpha_i \,) \, b_i - (\, 1 - \alpha_j \,) \, d_j \,] \, .
\end{aligned} \tag{251}$$

The term multiplying $b^w \, Q_i^r$ in (250) is:

$$b_i \; \{\, [\, 1 + 4\,\alpha_i \,] \, [\, 1 - \alpha_i \,] + [\, 1 - 2\,\alpha_i \,] \, [\, 1 + 2\,\alpha_i \,] + 6 + \alpha_i \, [\, 1 - 4\,\alpha_i \,] \,\}$$

$$- \; d_j \,[\, 1 - \alpha_j \,]\,[\, 1 - \alpha_i + 1 + 2\,\alpha_i + \alpha_i \,]$$

$$= \; b_i \left[\, 1 + 3\,\alpha_i - 4\,(\alpha_i)^2 + 1 - 4\,(\alpha_i)^2 + 6 + \alpha_i - 4\,(\alpha_i)^2 \,\right] - \; d_j \,[\, 1 - \alpha_j \,]\,[\, 2 + 2\,\alpha_i \,]$$

$$= \; b_i \left[\, 8 + 4\,\alpha_i - 12\,(\alpha_i)^2 \,\right] - \; 2\,d_j \,[\, 1 - \alpha_j \,]\,[\, 1 + \alpha_i \,]$$

$$= \; 4\,b_i \left[\, 2 + \alpha_i - 3\,(\alpha_i)^2 \,\right] - \; 2\,d_j \,[\, 1 - \alpha_j \,]\,[\, 1 + \alpha_i \,]$$

$$= \; 4\,b_i \,[\, 2 + 3\,\alpha_i \,]\,[\, 1 - \; \alpha_i \,] - \; 2\,d_j \,[\, 1 - \alpha_j \,]\,[\, 1 + \alpha_i \,]$$

$$= \; 2 \,\{\, 2\,[\, 2 + 3\,\alpha_i \,]\,[\, 1 - \; \alpha_i \,]\, b_i - [\, 1 - \alpha_j \,]\,[\, 1 + \alpha_i \,]\, d_j \,\} \,. \tag{252}$$

The term multiplying $b^w \, Q_j^r$ in (250) is:

$$b_i \,[\, 1 - \alpha_j \,]\,[\, 1 + 4\,\alpha_i + 1 - 2\,\alpha_i \,] - 2\,d_j \,[\, 1 - \alpha_j \,]^2$$

$$= \; 2\,b_i \,[\, 1 - \alpha_j \,]\,[\, 1 + \alpha_i \,] - 2\,d_j \,[\, 1 - \alpha_j \,]^2$$

$$= \; 2\,[\, 1 - \alpha_j \,]\,[\, (\, 1 + \alpha_i \,)\, b_i - (\, 1 - \alpha_j \,)\, d_j \,] \,. \tag{253}$$

(250) – (253) imply that the first integral in (249) is:

$$- \frac{1}{9} \int_{\underline{\varepsilon}}^{\varepsilon_0} \Big\{ 2\,[\, (\, 1 + \alpha_i \,)\, b_i - (\, 1 - \alpha_j \,)\, d_j \,] \left[\, b^w \, a^I + \varepsilon + c_j - 2\,c_i \,\right]$$

$$+ \; 2\,b^w \, Q_i^r \; [\, 2\,(\, 2 + 3\,\alpha_i \,)\,(\, 1 - \; \alpha_i \,)\, b_i - (\, 1 - \alpha_j \,)\,(\, 1 + \alpha_i \,) \; d_j \,]$$

$$+ \; 2\,b^w \, Q_j^r \,[\, 1 - \alpha_j \,]\,[\, (\, 1 + \alpha_i \,)\, b_i - (\, 1 - \alpha_j \,)\, d_j \,] \; \Big\} \, dH(\varepsilon) \,. \tag{254}$$

Lemma 2 implies that the second integral in (249) is:

$$- \; \frac{1}{2} \int_{\varepsilon_0}^{\varepsilon_{12}} \Big\{ \; [\, (\, 1 + 2\,\alpha_i \,)\, b_i - d_j \,] \, \frac{1}{2} \left[\, b^w \left(\, a^I + Q_i^r \,[\, 1 - \alpha_i \,] + Q_j^r - K_j \,\right) + \varepsilon - c_i \,\right]$$

$$+ \; b^w \,[\, (\, 1 - 2\,\alpha_i \,)\, b_i - d_j \,] \, \frac{1}{2\,b^w} \left[\, b^w \left(\, a^I + Q_i^r \,[\, 1 + \alpha_i \,] + Q_j^r - K_j \,\right) + \varepsilon - c_i \,\right]$$

$$+ \, b^w \, b_i \, Q_i^r \left[\, 2 - (\alpha_i)^2 \,\right] \; \Big\} \, dH(\varepsilon)$$

$$= \; - \frac{1}{2} \int_{\varepsilon_0}^{\varepsilon_{12}} \Big\{ \frac{1}{2} \,[\, (\, 1 + 2\,\alpha_i \,)\, b_i - d_j + (\, 1 - 2\,\alpha_i \,)\, b_i - d_j \,] \left[\, b^w \left(\, a^I - K_j \,\right) + \varepsilon - c_i \,\right]$$

$$+ \, b^w \, Q_i^r \Big[\frac{1}{2} \,(\, [\, 1 + 2\,\alpha_i \,]\, b_i - d_j \,)\,(\, 1 - \alpha_i \,)$$

$$+ \frac{1}{2} \left([\, 1 - 2\,\alpha_i \,]\, b_i - d_j \,\right) (\, 1 + \alpha_i \,) + b_i \left(2 - (\alpha_i)^2 \right) \Bigg]$$

$$+ \; b^w \, Q_j^r \left[\frac{1}{2} \left([\, 1 + 2\,\alpha_i \,]\, b_i - d_j \,\right) + \frac{1}{2} \left([\, 1 - 2\,\alpha_i \,]\, b_i - d_j \,\right) \right] \Bigg\} \, dH(\varepsilon) \, . \tag{255}$$

The coefficient on $\frac{1}{2} \left[b^w \left(a^I - K_j \right) + \varepsilon - c_i \right]$ in (255) is:

$$b_i \,[\, 1 + 2\,\alpha_i + 1 - 2\,\alpha_i \,] - 2\, d_j \;=\; 2\,[\, b_i - d_j \,] \, . \tag{256}$$

The coefficient on $\frac{1}{2}\, b^w \, Q_i^r$ in (255) is:

$$[\, (\, 1 + 2\,\alpha_i \,)\, b_i - d_j \,]\,[\, 1 - \alpha_i \,] + [\, (\, 1 - 2\,\alpha_i \,)\, b_i - d_j \,]\,[\, 1 + \alpha_i \,] + 2\, b_i \left[2 - (\alpha_i)^2 \right]$$

$$= \; b_i \left\{ [\, 1 + 2\,\alpha_i \,]\,[\, 1 - \alpha_i \,] + [\, 1 - 2\,\alpha_i \,]\,[\, 1 + \alpha_i \,] + 4 - 2\,(\alpha_i)^2 \right\}$$

$$- \; d_j \,[\, 1 - \alpha_i + 1 + \alpha_i \,]$$

$$= \; b_i \left[1 + \alpha_i - 2\,(\alpha_i)^2 + 1 - \alpha_i - 2\,(\alpha_i)^2 + 4 - 2\,(\alpha_i)^2 \right] - 2\, d_j$$

$$= \; b_i \left[6 - 6\,(\alpha_i)^2 \right] - 2\, d_j \;=\; 6\, b_i \left[1 - (\alpha_i)^2 \right] - 2\, d_j \;=\; 2 \left[3 \left(1 - (\alpha_i)^2 \right) b_i - d_j \right] . \tag{257}$$

The coefficient on $\frac{1}{2}\, b^w \, Q_j^r$ in (255) is:

$$b_i \,[\, 1 + 2\,\alpha_i + 1 - 2\,\alpha_i \,] - 2\, d_j \;=\; 2\,[\, b_i - d_j \,] \, . \tag{258}$$

(255) – (258) imply that the second integral in (249) is:

$$- \frac{1}{2} \int_{\varepsilon_0}^{\varepsilon_{12}} \Bigg\{ \; [\, b_i - d_j \,] \left[b^w \left(a^I - K_j \right) + \varepsilon - c_i \right]$$

$$+ \; b^w \, Q_i^r \left[3 \left(1 - (\alpha_i)^2 \right) b_i - d_j \right] + b^w \, Q_j^r \,[\, b_i - d_j \,] \Bigg\} \, dH(\varepsilon) \, . \tag{259}$$

(249), (254), and (259) imply that when Gj is capacity constrained for $\varepsilon \in (\, \varepsilon_0, \varepsilon_{12} \,)$:

$$L^i_{r_i \alpha_i} \;=\; - \; \frac{1}{9} \int_{\varepsilon}^{\varepsilon_0} \Bigg\{ 2 \,[\, (\, 1 + \alpha_i \,)\, b_i - (\, 1 - \alpha_j \,)\, d_j \,] \left[b^w \, a^I + \varepsilon + c_j - 2\, c_i \right]$$

$$+ \; 2\, b^w \, Q_i^r \; [\, (\, 2 + 3\,\alpha_i \,)\, (\, 1 - \; \alpha_i \,)\, b_i - (\, 1 - \alpha_j \,)\, (\, 1 + \alpha_i \,) \; d_j \,]$$

$$+ \; 2\, b^w \, Q_j^r \,[\, 1 - \alpha_j \,]\,[\, (\, 1 + \alpha_i \,)\, b_i - (\, 1 - \alpha_j \,)\, d_j \,] \Bigg\} \, dH(\varepsilon)$$

$$
- \frac{1}{2} \int_{\varepsilon_0}^{\varepsilon_{12}} \Big\{ \, [\, b_i - d_j \,] \left[b^w \left(a^I - K_j \right) + \varepsilon - c_i \right] + \; b^w \, Q_i^r \left[3 \left(1 - (\alpha_i)^2 \right) b_i - d_j \,\right] + b^w \, Q_j^r \, [\, b_i - d_j \,] \Big\} \, dH(\varepsilon)
$$

$$
- \int_{\varepsilon_{12}}^{\overline{\varepsilon}} b^w \, [\, b_i - d_j \,] \, q_i^{2c}(\varepsilon) \, dH(\varepsilon) \, . \tag{260}
$$

$H(\varepsilon_{12}) - H(\varepsilon_0) = 0$ in the symmetric setting. Therefore, (260) implies that in this setting:

$$
\begin{aligned}
L^i_{r_i \alpha_i} \;=\; & - \frac{1}{9} \int_{\underline{\varepsilon}}^{\varepsilon_0} \Big\{ 2 \, [\, (\, 1 + \alpha \,) \, b - (\, 1 - \alpha \,) \, d \,] \left[b^w \, a^I + \varepsilon - c \right] \\
& \quad + \; 2 \, b^w \, Q_i^r \, [\, 1 - \alpha \,] \, [\, (\, 2 + 3 \, \alpha \,) \, b - (\, 1 + \alpha \,) \; d \,] \\
& \quad + \; 2 \, b^w \, Q_j^r \, [\, 1 - \alpha \,] \, [\, (\, 1 + \alpha \,) \, b - (\, 1 - \alpha \,) \, d \,] \Big\} \, dH(\varepsilon) \\
& - \int_{\varepsilon_{12}}^{\overline{\varepsilon}} b^w \, [\, b - d \,] \, q_i^{2c}(\varepsilon) \, dH(\varepsilon) \;<\; 0 \, .
\end{aligned} \tag{261}
$$

The inequality in (261) holds because the first of the three terms in the $\int_{\underline{\varepsilon}}^{\varepsilon_0} (\cdot)$ integral in (261) is strictly positive and the other two terms are nonnegative (and $q_i^{2c}(\varepsilon) \geq 0$). Therefore, (242) implies that $\frac{\partial r_i}{\partial \alpha_i} \Big|_{K_i, K_j \text{ constant}} < 0$.

To prove that $\frac{\partial (r_i + r_j)}{\partial \alpha_i} \Big|_{K_i, K_j \text{ constant}} < 0$, observe from (238) that:

$$
\begin{aligned}
& L^i_{r_i r_i} \, dr_i + L^i_{r_i r_j} \, dr_j + L^i_{r_i \alpha_i} \, d\alpha_i \;=\; 0 \;\; \text{and} \\
& L^j_{r_j r_i} \, dr_i + L^j_{r_j r_j} \, dr_j + L^j_{r_j \alpha_i} \, d\alpha_i \;=\; 0 \\
\Leftrightarrow \quad & \begin{bmatrix} L^i_{r_i r_i} & L^i_{r_i r_j} \\ L^j_{r_j r_i} & L^j_{r_j r_j} \end{bmatrix} \begin{bmatrix} dr_i \\ dr_j \end{bmatrix} \;=\; - \begin{bmatrix} L^i_{r_i \alpha_i} \\ L^j_{r_j \alpha_i} \end{bmatrix} d\alpha_i \, .
\end{aligned} \tag{262}
$$

(237) implies that $L^j_{r_j \alpha_i} = 0$. Therefore, (262) and Cramer's Rule imply that if the

conditions in (241) hold:

$$\frac{dr_j}{d\alpha_i} \;=\; \frac{\begin{vmatrix} L^i_{r_i r_i} & -L^i_{r_i\alpha_i} \\ L^j_{r_j r_i} & 0 \end{vmatrix}}{\begin{vmatrix} L^i_{r_i r_i} & L^i_{r_i r_j} \\ L^j_{r_j r_i} & L^j_{r_j r_j} \end{vmatrix}} \;=\; \frac{L^i_{r_i\alpha_i}\, L^j_{r_j r_i}}{L^i_{r_i r_i}\, L^j_{r_j r_j} - L^i_{r_i r_j}\, L^j_{r_j r_i}} \;\stackrel{s}{=}\; L^i_{r_i\alpha_i}\, L^j_{r_j r_i}\,. \tag{263}$$

We demonstrate in (309) below that in the symmetric setting:

$$\begin{aligned}
\mathcal{L}^j_{r_j\, r_i} \;=\; & \left\{ \frac{\alpha_j\, b^w}{9} \left[(1 + 2\,\alpha_j)\, b - (1 - \alpha_i)\, d \right] \left[(1 - \alpha_i)\, b - (1 - \alpha_j)\, d \right] \right. \\
& + \frac{\alpha_j\, b^w}{9} \left[(1 - \alpha_i)\, b - (1 + 2\,\alpha_j)\, d \right] \left[(1 - \alpha_j)\, b - (1 - \alpha_i)\, d \right] \\
& - \frac{b\, b^w}{3} \left[(1 - \alpha_i)\, b - (1 - \alpha_j)\, d \right] \\
& \left. + \; d \left[1 + \frac{b^w}{3} \left([1 - \alpha_j]\, b - [1 - \alpha_i]\, d \right) \right] \right\} H(\varepsilon_0) \\
& + \left\{ -\, b\, b^w [\, b - d\,] + d \left[1 + b^w (\, b - d\,) \right] \right\} [\, 1 - H(\varepsilon_0)\,]\,.
\end{aligned} \tag{264}$$

Because $L^i_{r_i\alpha_i} < 0$ from (261), (242) and (263) imply:

$$\begin{aligned}
\frac{dr_i}{d\alpha_i} + \frac{dr_j}{d\alpha_i} \;&=\; \frac{1}{L^i_{r_i r_i}\, L^j_{r_j r_j} - L^i_{r_i r_j}\, L^j_{r_j r_i}} \left[-\, L^i_{r_i\alpha_i}\, L^j_{r_j r_j} + L^i_{r_i\alpha_i}\, L^j_{r_j r_i} \right] \\
&\stackrel{s}{=}\; -\, L^i_{r_i\alpha_i}\, L^j_{r_j r_j} + L^i_{r_i\alpha_i}\, L^j_{r_j r_i} \;=\; -\, L^i_{r_i\alpha_i} \left[L^j_{r_j r_j} - L^j_{r_j r_i} \right] \;\stackrel{s}{=}\; L^j_{r_j r_j} - L^j_{r_j r_i}.
\end{aligned} \tag{265}$$

We demonstrate in (295) below that in the symmetric setting:

$$\begin{aligned}
\mathcal{L}^j_{r_j\, r_j} \;=\; & 2 \left\{ \alpha_j\, \frac{b^w}{9} \left[(1 - \alpha_j)\, b - (1 - \alpha_i)\, d \right] \left[(1 - \alpha_j)\, b - (1 + 2\,\alpha_i)\, d \right] \right. \\
& \left. -\, b \left[1 + \frac{b^w}{3} \left([1 - \alpha_j]\, b - [1 - \alpha_i]\, d \right) \right] \right\} H(\varepsilon_0) \\
& -\, 2\, b \left[1 + b^w (\, b - d\,) \right] [\, 1 - H(\varepsilon_0)\,]\,.
\end{aligned} \tag{266}$$

(264) and (266) imply that in the symmetric setting:

$$L^j_{r_j r_j} - L^j_{r_j r_i} \;=\; H(\varepsilon_0) \left\{ 2\,\alpha_j\, \frac{b^w}{9} \left[(1 - \alpha_j)\, b - (1 - \alpha_i)\, d \right] \left[(1 - \alpha_j)\, b - (1 + 2\,\alpha_i)\, d \,\right] \right.$$

$$
\begin{aligned}
& - 2\,b \left[1 + \frac{b^w}{3}\,(\,[\,1-\alpha_j\,]\,b - [\,1-\alpha_i\,]\,d\,) \right] \\
& - \frac{\alpha_j\,b^w}{9}\,[\,(\,1+2\,\alpha_j\,)\,b - (\,1-\alpha_i\,)\,d\,]\,[\,(\,1-\alpha_i\,)\,b - (\,1-\alpha_j\,)\,d\,] \\
& - \frac{\alpha_j\,b^w}{9}\,[\,(\,1-\alpha_i\,)\,b - (\,1+2\,\alpha_j\,)\,d\,]\,[\,(\,1-\alpha_j\,)\,b - (\,1-\alpha_i\,)\,d\,] \\
& + \frac{b\,b^w}{3}\,[\,(\,1-\alpha_i\,)\,b - (\,1-\alpha_j\,)\,d\,] \\
& - d \left[1 + \frac{b^w}{3}\,(\,[\,1-\alpha_j\,]\,b - [\,1-\alpha_i\,]\,d\,) \right] \Bigg\} \\
+ \; & [\,1 - H(\varepsilon_0)\,]\,\{\,-2\,b\,[\,1 + b^w\,(\,b-d\,)\,] + b\,b^w\,[\,b-d\,] - d\,[\,1 + b^w\,(\,b-d\,)\,]\,\}\;. \qquad (267)
\end{aligned}
$$

(267) implies that under VI in the symmetric setting:

$$
\begin{aligned}
L^j_{r_j r_j} - L^j_{r_j r_i} \;=\; & H(\varepsilon_0)\,[\,-2\,b - d\,] \\
& + [\,1 - H(\varepsilon_0)\,]\,[\,-2\,b\; - 2\,b\,b^w(\,b-d\,) + \,b\,b^w\,(\,b-d\,) - d - d\,b^w\,(\,b-d\,)\,] \\
=\; & -H(\varepsilon_0)\,[\,2\,b + d\,] - [\,1 - H(\varepsilon_0)\,]\,[\,2\,b\, + b\,b^w(\,b-d\,) + d + d\,b^w\,(\,b-d\,)\,] \;<\; 0\,. \qquad (268)
\end{aligned}
$$

(267) implies that under VS in the symmetric setting:

$$
\begin{aligned}
L^j_{r_j r_j} - L^j_{r_j r_i} \;=\; & H(\varepsilon_0) \left\{ -2\,b \left[1 + \frac{b^w}{3}\,(\,b-d\,) \right] + \frac{b\,b^w}{3}\,[\,b-d\,] - \,d \left[1 + \frac{b^w}{3}\,(\,b-d\,) \right] \right\} \\
& + [\,1 - H(\varepsilon_0)\,]\,\{\,-2\,b\,[\,1 + b^w\,(\,b-d\,)\,] + \,b\,b^w\,[\,b-d\,] - d\,[\,1 + b^w\,(\,b-d\,)\,]\,\} \\
=\; & H(\varepsilon_0) \left\{ -2\,b - \frac{2\,b\,b^w}{3}\,[\,b-d\,] + \frac{b\,b^w}{3}\,[\,b-d\,] - d - d\,\frac{b^w}{3}\,[\,b-d\,] \right\} \\
& + [\,1 - H(\varepsilon_0)\,]\,\{\,-2\,b\; - 2\,b\,b^w\,[\,b-d\,] + \,b\,b^w\,[\,b-d\,] - d - d\,b^w\,[\,b-d\,]\,\} \\
=\; & H(\varepsilon_0) \left[-2\,b - \frac{b\,b^w}{3}(\,b-d\,) - d - d\,\frac{b^w}{3}\,(\,b-d\,) \right] \\
& + [\,1 - H(\varepsilon_0)\,]\,[\,-2\,b\; - b\,b^w(\,b-d\,) - d - d\,b^w\,(\,b-d\,)\,] \;<\; 0\,. \qquad (269)
\end{aligned}
$$

(265), (268), and (269) imply that $\frac{\partial(r_i + r_j)}{\partial\alpha_i}\Big|_{K_i, K_j \text{ constant}} < 0$. ■

Conclusion 9. $\frac{dr_i}{dK_i} < 0$ *and* $\frac{dr_j}{dK_i} < 0$ *for* $i,\, j \in \{1, 2\}$ $(j \neq i)$ *in the symmetric setting under both VS and VI.*

Proof. Let $\widetilde{\pi}^{Ri}$ denote Ri's objective when it fully values its own retail profit and derives benefit α_i from each dollar of Gi's upstream profit. Formally:

$$\widetilde{\pi}^{Ri} \;\equiv\; E\,\{\,\alpha_i\,[\,(\,w(\cdot) - c_i\,)\,q_i - k_i\,K_i\,] + [\,r_i - c_i^r - w(\cdot)\,]\,Q_i^r(r_i, r_j)\,\}\,. \tag{270}$$

Given α_i and α_j, the equilibrium values of r_i and r_j $(i, j \in \{1, 2\}$ $(j \neq i))$ are determined by:

$$\widetilde{\pi}^{Ri}_{r_i} \;\equiv\; \frac{\partial \widetilde{\pi}^{Ri}}{\partial r_i} \;=\; 0 \quad \text{and} \quad \widetilde{\pi}^{Ri}_{r_j} \;\equiv\; \frac{\partial \widetilde{\pi}^{Rj}}{\partial r_j} \;=\; 0\,. \tag{271}$$

Letting $\widetilde{\pi}^{Ri}_{xy}$ denote $\frac{\partial^2 \widetilde{\pi}^{Ri}}{\partial x \partial y}$, expression (271) implies:

$$\begin{bmatrix} \widetilde{\pi}^{Ri}_{r_i\,r_i} & \widetilde{\pi}^{Ri}_{r_i\,r_j} \\ \widetilde{\pi}^{Rj}_{r_j\,r_i} & \widetilde{\pi}^{Rj}_{r_j\,r_j} \end{bmatrix} \begin{bmatrix} dr_i \\ dr_j \end{bmatrix} \;=\; -\begin{bmatrix} \widetilde{\pi}^{Ri}_{r_i\,K_i} \\ \widetilde{\pi}^{Ri}_{r_j\,K_i} \end{bmatrix} dK_i\,. \tag{272}$$

(272) and Cramer's Rule imply:

$$\frac{dr_i}{dK_i} \;=\; \frac{1}{Z^R}\left[\widetilde{\pi}^{Ri}_{r_i\,r_j}\,\widetilde{\pi}^{Ri}_{r_j\,K_i} - \widetilde{\pi}^{Rj}_{r_j\,r_j}\,\widetilde{\pi}^{Ri}_{r_i\,K_i}\right] \text{ and } \frac{dr_j}{dK_i} \;=\; \frac{1}{Z^R}\left[\widetilde{\pi}^{Rj}_{r_j\,r_i}\,\widetilde{\pi}^{Ri}_{r_i\,K_i} - \widetilde{\pi}^{Ri}_{r_i\,r_i}\,\widetilde{\pi}^{Rj}_{r_j\,K_i}\right]$$

$$\text{where } \; Z^R \;\equiv\; \widetilde{\pi}^{Ri}_{r_i\,r_i}\,\widetilde{\pi}^{Rj}_{r_j\,r_j} - \widetilde{\pi}^{Ri}_{r_i\,r_j}\,\widetilde{\pi}^{Rj}_{r_j\,r_i}\,. \tag{273}$$

(273) implies:

$$\frac{dr_i}{dK_i} \;\stackrel{s}{=}\; \widetilde{\pi}^{Ri}_{r_i\,r_j}\,\widetilde{\pi}^{Ri}_{r_j\,K_i} - \widetilde{\pi}^{Rj}_{r_j\,r_j}\,\widetilde{\pi}^{Ri}_{r_i\,K_i} \;\text{ and }\; \frac{dr_j}{dK_i} \;\stackrel{s}{=}\; \widetilde{\pi}^{Rj}_{r_j\,r_i}\,\widetilde{\pi}^{Ri}_{r_i\,K_i} - \widetilde{\pi}^{Ri}_{r_i\,r_i}\,\widetilde{\pi}^{Rj}_{r_j\,K_i}\,. \tag{274}$$

Observe that:

$$\widetilde{\pi}^{Ri}_{r_i} \;=\; L^i_{r_i} \;\text{ and }\; \widetilde{\pi}^{Rj}_{r_j} \;=\; L^j_{r_j} \tag{275}$$

where $L^i_{r_i}$ is explicitly defined in (239) and $L^j_{r_j}$ is defined in (239) by substituting j for i.

Differentiating (275) provides:

$$\begin{aligned}
\widetilde{\pi}^{Ri}_{r_i\,K_i} \;=\; \int_{\underline{\varepsilon}}^{\varepsilon_0} \Bigg\{ & \alpha_i\,\big[\,w^{0c}(\varepsilon) - c_i\,\big]\,\frac{\partial}{\partial K_i}\left(\frac{\partial q_i^{0c}(\varepsilon)}{\partial r_i}\right) + \alpha_i\,\frac{\partial q_i^{0c}(\varepsilon)}{\partial r_i}\,\frac{\partial w^{0c}(\varepsilon)}{\partial K_i} \\
& + \alpha_i\,q_i^{0c}(\varepsilon)\,\frac{\partial}{\partial K_i}\left(\frac{\partial w^{0c}(\varepsilon)}{\partial r_i}\right) + \alpha_i\,\frac{\partial q_i^{0c}(\varepsilon)}{\partial K_i}\,\frac{\partial w^{0c}(\varepsilon)}{\partial r_i} \\
& + \big[\,r_i - c_i^r - w^{0c}(\varepsilon)\,\big]\,\frac{\partial}{\partial K_i}\left(\frac{\partial Q_i^r(r_i, r_j)}{\partial r_i}\right) - \frac{\partial Q_i^r(r_i, r_j)}{\partial r_i}\,\frac{\partial w^{0c}(\varepsilon)}{\partial K_i}
\end{aligned}$$

$$- Q_i^r(r_i,r_j)\,\frac{\partial}{\partial K_i}\left(\frac{\partial w^{0c}(\varepsilon)}{\partial r_i}\right) + \frac{\partial Q_i^r(r_i,r_j)}{\partial K_i}\left[1-\frac{\partial w^{0c}(\varepsilon)}{\partial r_i}\right]\Bigg\}\,dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \Bigg\{ \alpha_i\left[\,w^{1c}(\varepsilon)-c_i\,\right]\frac{\partial}{\partial K_i}\left(\frac{\partial q_i^{1c}(\varepsilon)}{\partial r_i}\right) + \alpha_i\,\frac{\partial q_i^{1c}(\varepsilon)}{\partial r_i}\,\frac{\partial w^{1c}(\varepsilon)}{\partial K_i}$$

$$+ \alpha_i\,q_i^{1c}(\varepsilon)\,\frac{\partial}{\partial K_i}\left(\frac{\partial w^{1c}(\varepsilon)}{\partial r_i}\right) + \alpha_i\,\frac{\partial q_i^{1c}(\varepsilon)}{\partial K_i}\,\frac{\partial w^{1c}(\varepsilon)}{\partial r_i}$$

$$+ \left[\,r_i - c_i^r - w^{1c}(\varepsilon)\,\right]\frac{\partial}{\partial K_i}\left(\frac{\partial Q_i^r(r_i,r_j)}{\partial r_i}\right) - \frac{\partial Q_i^r(r_i,r_j)}{\partial r_i}\,\frac{\partial w^{1c}(\varepsilon)}{\partial K_i}$$

$$- Q_i^r(r_i,r_j)\,\frac{\partial}{\partial K_i}\left(\frac{\partial w^{1c}(\varepsilon)}{\partial r_i}\right) + \frac{\partial Q_i^r(r_i,r_j)}{\partial K_i}\left[1-\frac{\partial w^{1c}(\varepsilon)}{\partial r_i}\right]\Bigg\}\,dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \Bigg\{ \alpha_i\left[\,w^{2c}(\varepsilon)-c_i\,\right]\frac{\partial}{\partial K_i}\left(\frac{\partial q_i^{2c}(\varepsilon)}{\partial r_i}\right) + \alpha_i\,\frac{\partial q_i^{2c}(\varepsilon)}{\partial r_i}\,\frac{\partial w^{2c}(\varepsilon)}{\partial K_i}$$

$$+ \alpha_i\,q_i^{2c}(\varepsilon)\,\frac{\partial}{\partial K_i}\left(\frac{\partial w^{2c}(\varepsilon)}{\partial r_i}\right) + \alpha_i\,\frac{\partial q_i^{2c}(\varepsilon)}{\partial K_i}\,\frac{\partial w^{2c}(\varepsilon)}{\partial r_i}$$

$$+ \left[\,r_i - c_i^r - w^{2c}(\varepsilon)\,\right]\frac{\partial}{\partial K_i}\left(\frac{\partial Q_i^r(r_i,r_j)}{\partial r_i}\right) - \frac{\partial Q_i^r(r_i,r_j)}{\partial r_i}\,\frac{\partial w^{2c}(\varepsilon)}{\partial K_i}$$

$$- Q_i^r(r_i,r_j)\,\frac{\partial}{\partial K_i}\left(\frac{\partial w^{2c}(\varepsilon)}{\partial r_i}\right) + \frac{\partial Q_i^r(r_i,r_j)}{\partial K_i}\left[1-\frac{\partial w^{2c}(\varepsilon)}{\partial r_i}\right]\Bigg\}\,dH(\varepsilon)\,; \tag{276}$$

$$\widetilde{\pi}^{Ri}_{r_i\,r_j} = \int_{\underline{\varepsilon}}^{\varepsilon_0} \Bigg\{ \alpha_i\left[\,w^{0c}(\varepsilon)-c_i\,\right]\frac{\partial}{\partial r_j}\left(\frac{\partial q_i^{0c}(\varepsilon)}{\partial r_i}\right) + \alpha_i\,\frac{\partial q_i^{0c}(\varepsilon)}{\partial r_i}\,\frac{\partial w^{0c}(\varepsilon)}{\partial r_j}$$

$$+ \alpha_i\,q_i^{0c}(\varepsilon)\,\frac{\partial}{\partial r_j}\left(\frac{\partial w^{0c}(\varepsilon)}{\partial r_i}\right) + \alpha_i\,\frac{\partial q_i^{0c}(\varepsilon)}{\partial r_j}\,\frac{\partial w^{0c}(\varepsilon)}{\partial r_i}$$

$$+ \left[\,r_i - c_i^r - w^{0c}(\varepsilon)\,\right]\frac{\partial}{\partial r_j}\left(\frac{\partial Q_i^r(r_i,r_j)}{\partial r_i}\right) - \frac{\partial Q_i^r(r_i,r_j)}{\partial r_i}\,\frac{\partial w^{0c}(\varepsilon)}{\partial r_j}$$

$$- Q_i^r(r_i,r_j)\,\frac{\partial}{\partial r_j}\left(\frac{\partial w^{0c}(\varepsilon)}{\partial r_i}\right) + \frac{\partial Q_i^r(r_i,r_j)}{\partial r_j}\left[1-\frac{\partial w^{0c}(\varepsilon)}{\partial r_i}\right]\Bigg\}\,dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \Bigg\{ \alpha_i\left[\,w^{1c}(\varepsilon)-c_i\,\right]\frac{\partial}{\partial r_j}\left(\frac{\partial q_i^{1c}(\varepsilon)}{\partial r_i}\right) + \alpha_i\,\frac{\partial q_i^{1c}(\varepsilon)}{\partial r_i}\,\frac{\partial w^{1c}(\varepsilon)}{\partial r_j}$$

$$+ \alpha_i \, q_i^{1c}(\varepsilon) \, \frac{\partial}{\partial r_j} \left(\frac{\partial w^{1c}(\varepsilon)}{\partial r_i} \right) + \alpha_i \, \frac{\partial q_i^{1c}(\varepsilon)}{\partial r_j} \, \frac{\partial w^{1c}(\varepsilon)}{\partial r_i}$$

$$+ \left[r_i - c_i^r - w^{1c}(\varepsilon) \right] \frac{\partial}{\partial r_j} \left(\frac{\partial Q_i^r(r_i, r_j)}{\partial r_i} \right) - \frac{\partial Q_i^r(r_i, r_j)}{\partial r_i} \, \frac{\partial w^{1c}(\varepsilon)}{\partial r_j}$$

$$- Q_i^r(r_i, r_j) \, \frac{\partial}{\partial r_j} \left(\frac{\partial w^{1c}(\varepsilon)}{\partial r_i} \right) + \frac{\partial Q_i^r(r_i, r_j)}{\partial r_j} \left[1 - \frac{\partial w^{1c}(\varepsilon)}{\partial r_i} \right] \Bigg\} \, dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\bar{\varepsilon}} \Bigg\{ \alpha_i \left[w^{2c}(\varepsilon) - c_i \right] \frac{\partial}{\partial r_j} \left(\frac{\partial q_i^{2c}(\varepsilon)}{\partial r_i} \right) + \alpha_i \, \frac{\partial q_i^{2c}(\varepsilon)}{\partial r_i} \, \frac{\partial w^{2c}(\varepsilon)}{\partial r_j}$$

$$+ \alpha_i \, q_i^{2c}(\varepsilon) \, \frac{\partial}{\partial r_j} \left(\frac{\partial w^{2c}(\varepsilon)}{\partial r_i} \right) + \alpha_i \, \frac{\partial q_i^{2c}(\varepsilon)}{\partial r_j} \, \frac{\partial w^{2c}(\varepsilon)}{\partial r_i}$$

$$+ \left[r_i - c_i^r - w^{2c}(\varepsilon) \right] \frac{\partial}{\partial r_j} \left(\frac{\partial Q_i^r(r_i, r_j)}{\partial r_i} \right) - \frac{\partial Q_i^r(r_i, r_j)}{\partial r_i} \, \frac{\partial w^{2c}(\varepsilon)}{\partial r_j}$$

$$- Q_i^r(r_i, r_j) \, \frac{\partial}{\partial r_j} \left(\frac{\partial w^{2c}(\varepsilon)}{\partial r_i} \right) + \frac{\partial Q_i^r(r_i, r_j)}{\partial r_j} \left[1 - \frac{\partial w^{2c}(\varepsilon)}{\partial r_i} \right] \Bigg\} \, dH(\varepsilon) \, ; \qquad (277)$$

$$\widetilde{\pi}_{r_j K_i}^{Rj} = \int_{\underline{\varepsilon}}^{\varepsilon_0} \Bigg\{ \alpha_j \left[w^{0c}(\varepsilon) - c_j \right] \frac{\partial}{\partial K_i} \left(\frac{\partial q_j^{0c}(\varepsilon)}{\partial r_j} \right) + \alpha_j \, \frac{\partial q_j^{0c}(\varepsilon)}{\partial r_j} \, \frac{\partial w^{0c}(\varepsilon)}{\partial K_i}$$

$$+ \alpha_j \, q_j^{0c}(\varepsilon) \, \frac{\partial}{\partial K_i} \left(\frac{\partial w^{0c}(\varepsilon)}{\partial r_j} \right) + \alpha_j \, \frac{\partial q_j^{0c}(\varepsilon)}{\partial K_i} \, \frac{\partial w^{0c}(\varepsilon)}{\partial r_j}$$

$$+ \left[r_j - c_j^r - w^{0c}(\varepsilon) \right] \frac{\partial}{\partial K_i} \left(\frac{\partial Q_j^r(r_j, r_i)}{\partial r_j} \right) - \frac{\partial Q_j^r(r_j, r_i)}{\partial r_j} \, \frac{\partial w^{0c}(\varepsilon)}{\partial K_i}$$

$$- Q_j^r(r_j, r_i) \, \frac{\partial}{\partial K_i} \left(\frac{\partial w^{0c}(\varepsilon)}{\partial r_j} \right) + \frac{\partial Q_j^r(r_j, r_i)}{\partial K_i} \left[1 - \frac{\partial w^{0c}(\varepsilon)}{\partial r_j} \right] \Bigg\} \, dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \Bigg\{ \alpha_j \left[w^{1c}(\varepsilon) - c_j \right] \frac{\partial}{\partial K_i} \left(\frac{\partial q_j^{1c}(\varepsilon)}{\partial r_j} \right) + \alpha_j \, \frac{\partial q_j^{1c}(\varepsilon)}{\partial r_j} \, \frac{\partial w^{1c}(\varepsilon)}{\partial K_i}$$

$$+ \alpha_j \, q_j^{1c}(\varepsilon) \, \frac{\partial}{\partial K_i} \left(\frac{\partial w^{1c}(\varepsilon)}{\partial r_j} \right) + \alpha_j \, \frac{\partial q_j^{1c}(\varepsilon)}{\partial K_i} \, \frac{\partial w^{1c}(\varepsilon)}{\partial r_j}$$

$$+ \left[r_j - c_j^r - w^{1c}(\varepsilon) \right] \frac{\partial}{\partial K_i} \left(\frac{\partial Q_j^r(r_j, r_i)}{\partial r_j} \right) - \frac{\partial Q_j^r(r_j, r_i)}{\partial r_j} \, \frac{\partial w^{1c}(\varepsilon)}{\partial K_i}$$

$$- Q_j^r(r_j,r_i)\,\frac{\partial}{\partial K_i}\left(\frac{\partial w^{1c}(\varepsilon)}{\partial r_j}\right) + \frac{\partial Q_j^r(r_j,r_i)}{\partial K_i}\left[1 - \frac{\partial w^{1c}(\varepsilon)}{\partial r_j}\right]\Bigg\}\,dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \Bigg\{ \alpha_j\left[\,w^{2c}(\varepsilon) - c_j\,\right]\frac{\partial}{\partial K_i}\left(\frac{\partial q_j^{2c}(\varepsilon)}{\partial r_j}\right) + \alpha_j\,\frac{\partial q_j^{2c}(\varepsilon)}{\partial r_j}\,\frac{\partial w^{2c}(\varepsilon)}{\partial K_i}$$

$$+ \alpha_j\,q_j^{2c}(\varepsilon)\,\frac{\partial}{\partial K_i}\left(\frac{\partial w^{2c}(\varepsilon)}{\partial r_j}\right) + \alpha_j\,\frac{\partial q_j^{2c}(\varepsilon)}{\partial K_i}\,\frac{\partial w^{2c}(\varepsilon)}{\partial r_j}$$

$$+ \left[\,r_j - c_j^r - w^{2c}(\varepsilon)\,\right]\frac{\partial}{\partial K_i}\left(\frac{\partial Q_j^r(r_j,r_i)}{\partial r_j}\right) - \frac{\partial Q_j^r(r_j,r_i)}{\partial r_j}\,\frac{\partial w^{2c}(\varepsilon)}{\partial K_i}$$

$$- Q_j^r(r_j,r_i)\,\frac{\partial}{\partial K_i}\left(\frac{\partial w^{2c}(\varepsilon)}{\partial r_j}\right) + \frac{\partial Q_j^r(r_j,r_i)}{\partial K_i}\left[1 - \frac{\partial w^{2c}(\varepsilon)}{\partial r_j}\right]\Bigg\}\,dH(\varepsilon)\,; \quad (278)$$

$$\widetilde{\pi}^{Rj}_{r_j\,r_j} \;=\; \int_{\underline{\varepsilon}}^{\varepsilon_0} \Bigg\{ \alpha_j\left[\,w^{0c}(\varepsilon) - c_j\,\right]\frac{\partial}{\partial r_j}\left(\frac{\partial q_j^{0c}(\varepsilon)}{\partial r_j}\right) + \alpha_j\,\frac{\partial q_j^{0c}(\varepsilon)}{\partial r_j}\,\frac{\partial w^{0c}(\varepsilon)}{\partial r_j}$$

$$+ \alpha_j\,q_j^{0c}(\varepsilon)\,\frac{\partial}{\partial r_j}\left(\frac{\partial w^{0c}(\varepsilon)}{\partial r_j}\right) + \alpha_j\,\frac{\partial q_j^{0c}(\varepsilon)}{\partial r_j}\,\frac{\partial w^{0c}(\varepsilon)}{\partial r_j}$$

$$+ \left[\,r_j - c_j^r - w^{0c}(\varepsilon)\,\right]\frac{\partial}{\partial r_j}\left(\frac{\partial Q_j^r(r_j,r_i)}{\partial r_j}\right) + \frac{\partial Q_j^r(r_j,r_i)}{\partial r_j}\left[1 - \frac{\partial w^{0c}(\varepsilon)}{\partial r_j}\right]$$

$$- Q_j^r(r_j,r_i)\,\frac{\partial}{\partial r_j}\left(\frac{\partial w^{0c}(\varepsilon)}{\partial r_j}\right) + \frac{\partial Q_j^r(r_j,r_i)}{\partial r_j}\left[1 - \frac{\partial w^{0c}(\varepsilon)}{\partial r_j}\right]\Bigg\}\,dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \Bigg\{ \alpha_j\left[\,w^{1c}(\varepsilon) - c_j\,\right]\frac{\partial}{\partial r_j}\left(\frac{\partial q_j^{1c}(\varepsilon)}{\partial r_j}\right) + \alpha_j\,\frac{\partial q_j^{1c}(\varepsilon)}{\partial r_j}\,\frac{\partial w^{1c}(\varepsilon)}{\partial r_j}$$

$$+ \alpha_j\,q_j^{1c}(\varepsilon)\,\frac{\partial}{\partial r_j}\left(\frac{\partial w^{1c}(\varepsilon)}{\partial r_j}\right) + \alpha_j\,\frac{\partial q_j^{1c}(\varepsilon)}{\partial r_j}\,\frac{\partial w^{1c}(\varepsilon)}{\partial r_j}$$

$$+ \left[\,r_j - c_j^r - w^{1c}(\varepsilon)\,\right]\frac{\partial}{\partial r_j}\left(\frac{\partial Q_j^r(r_j,r_i)}{\partial r_j}\right) + \frac{\partial Q_j^r(r_j,r_i)}{\partial r_j}\left[1 - \frac{\partial w^{1c}(\varepsilon)}{\partial r_j}\right]$$

$$- Q_j^r(r_j,r_i)\,\frac{\partial}{\partial r_j}\left(\frac{\partial w^{1c}(\varepsilon)}{\partial r_j}\right) + \frac{\partial Q_j^r(r_j,r_i)}{\partial r_j}\left[1 - \frac{\partial w^{1c}(\varepsilon)}{\partial r_j}\right]\Bigg\}\,dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \Bigg\{ \alpha_j\left[\,w^{2c}(\varepsilon) - c_j\,\right]\frac{\partial}{\partial r_j}\left(\frac{\partial q_j^{2c}(\varepsilon)}{\partial r_j}\right) + \alpha_j\,\frac{\partial q_j^{2c}(\varepsilon)}{\partial r_j}\,\frac{\partial w^{2c}(\varepsilon)}{\partial r_j}$$

$$+\ \alpha_j\, q_j^{2c}(\varepsilon)\ \frac{\partial}{\partial r_j}\left(\frac{\partial w^{2c}(\varepsilon)}{\partial r_j}\right) + \alpha_j\ \frac{\partial q_j^{2c}(\varepsilon)}{\partial r_j}\ \frac{\partial w^{2c}(\varepsilon)}{\partial r_j}$$

$$+\ \left[\, r_j - c_j^r - w^{2c}(\varepsilon)\,\right] \frac{\partial}{\partial r_j}\left(\frac{\partial Q_j^r(r_j, r_i)}{\partial r_j}\right) + \frac{\partial Q_j^r(r_j, r_i)}{\partial r_j}\left[1 - \frac{\partial w^{2c}(\varepsilon)}{\partial r_j}\right]$$

$$-\ Q_j^r(r_j, r_i)\ \frac{\partial}{\partial r_j}\left(\frac{\partial w^{2c}(\varepsilon)}{\partial r_j}\right) + \frac{\partial Q_j^r(r_j, r_i)}{\partial r_j}\left[1 - \frac{\partial w^{2c}(\varepsilon)}{\partial r_j}\right]\right\} dH(\varepsilon)\,. \tag{279}$$

Lemmas 1 – 3 imply that for $x \in \{r_j, K_i\}$ and $n \in \{\,0, 1, 2\,\}$:

$$\frac{\partial}{\partial x}\left(\frac{\partial w^{nc}(\varepsilon)}{\partial r_i}\right) \ = \ \frac{\partial}{\partial x}\left(\frac{\partial q_i^{nc}(\varepsilon)}{\partial r_i}\right) \ = \ \frac{\partial}{\partial x}\left(\frac{\partial w^{nc}(\varepsilon)}{\partial r_j}\right) \ = \ \frac{\partial}{\partial x}\left(\frac{\partial q_i^{nc}(\varepsilon)}{\partial r_j}\right) \ = \ 0\,. \tag{280}$$

Furthermore, (1) implies that for $x \in \{r_j, K_i\}$:

$$\frac{\partial Q_i^r(r_i, r_j)}{\partial r_i} \ = \ -\,b_i, \quad \frac{\partial Q_j^r(r_j, r_i)}{\partial r_j} \ = \ -\,b_j, \ \text{ and } \ \frac{\partial Q_i^r(r_i, r_j)}{\partial r_j} \ = \ d_i$$

$$\Rightarrow\ \frac{\partial}{\partial x}\left(\frac{\partial Q_i^r(r_i, r_j)}{\partial r_i}\right) \ = \ \frac{\partial}{\partial x}\left(\frac{\partial Q_j^r(r_j, r_i)}{\partial r_j}\right) \ = \ 0\,. \tag{281}$$

In addition, in this comparative static exercise:

$$\frac{\partial Q_i^r(r_i, r_j)}{\partial K_i} \ = \ \frac{\partial Q_j^r(r_j, r_i)}{\partial K_i} \ = \ 0\,. \tag{282}$$

(276), (280), (281), and (282) imply:

$$\widetilde{\pi}^{Ri}_{r_i\,K_i} \ = \ \int_{\underline{\varepsilon}}^{\varepsilon_0} \left\{ \alpha_i\, \frac{\partial q_i^{0c}(\varepsilon)}{\partial r_i}\ \frac{\partial w^{0c}(\varepsilon)}{\partial K_i} + \alpha_i\, \frac{\partial q_i^{0c}(\varepsilon)}{\partial K_i}\ \frac{\partial w^{0c}(\varepsilon)}{\partial r_i} + b_i\ \frac{\partial w^{0c}(\varepsilon)}{\partial K_i}\right\} dH(\varepsilon)$$

$$+\ \int_{\varepsilon_0}^{\varepsilon_{12}} \left\{ \alpha_i\, \frac{\partial q_i^{1c}(\varepsilon)}{\partial r_i}\ \frac{\partial w^{1c}(\varepsilon)}{\partial K_i} + \alpha_i\, \frac{\partial q_i^{1c}(\varepsilon)}{\partial K_i}\ \frac{\partial w^{1c}(\varepsilon)}{\partial r_i} + b_i\ \frac{\partial w^{1c}(\varepsilon)}{\partial K_i}\right\} dH(\varepsilon)$$

$$+\ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left\{ \alpha_i\, \frac{\partial q_i^{2c}(\varepsilon)}{\partial r_i}\ \frac{\partial w^{2c}(\varepsilon)}{\partial K_i} + \alpha_i\, \frac{\partial q_i^{2c}(\varepsilon)}{\partial K_i}\ \frac{\partial w^{2c}(\varepsilon)}{\partial r_i} + b_i\ \frac{\partial w^{2c}(\varepsilon)}{\partial K_i}\right\} dH(\varepsilon)\,. \tag{283}$$

(277), (280), and (281) imply:

$$\widetilde{\pi}^{Ri}_{r_i\,rj} \ = \ \int_{\underline{\varepsilon}}^{\varepsilon_0} \left\{ \alpha_i\, \frac{\partial q_i^{0c}(\varepsilon)}{\partial r_i}\ \frac{\partial w^{0c}(\varepsilon)}{\partial r_j} + \alpha_i\, \frac{\partial q_i^{0c}(\varepsilon)}{\partial r_j}\ \frac{\partial w^{0c}(\varepsilon)}{\partial r_i}\right.$$

$$+ \, b_i \, \frac{\partial w^{0c}(\varepsilon)}{\partial r_j} + d_i \left[1 - \frac{\partial w^{0c}(\varepsilon)}{\partial r_i} \right] \Bigg\} \, dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \Bigg\{ \alpha_i \, \frac{\partial q_i^{1c}(\varepsilon)}{\partial r_i} \, \frac{\partial w^{1c}(\varepsilon)}{\partial r_j} + \alpha_i \, \frac{\partial q_i^{1c}(\varepsilon)}{\partial r_j} \, \frac{\partial w^{1c}(\varepsilon)}{\partial r_i}$$

$$+ \, b_i \, \frac{\partial w^{1c}(\varepsilon)}{\partial r_j} + d_i \left[1 - \frac{\partial w^{1c}(\varepsilon)}{\partial r_i} \right] \Bigg\} \, dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \Bigg\{ \alpha_i \, \frac{\partial q_i^{2c}(\varepsilon)}{\partial r_i} \, \frac{\partial w^{2c}(\varepsilon)}{\partial r_j} + \alpha_i \, \frac{\partial q_i^{2c}(\varepsilon)}{\partial r_j} \, \frac{\partial w^{2c}(\varepsilon)}{\partial r_i}$$

$$+ \, b_i \, \frac{\partial w^{2c}(\varepsilon)}{\partial r_j} + d_i \left[1 - \frac{\partial w^{2c}(\varepsilon)}{\partial r_i} \right] \Bigg\} \, dH(\varepsilon) \, . \tag{284}$$

(278), (280), (281), and (282) imply:

$$\widetilde{\pi}^{Rj}_{r_j \, K_i} = \int_{\underline{\varepsilon}}^{\varepsilon_0} \Bigg\{ \alpha_j \, \frac{\partial q_j^{0c}(\varepsilon)}{\partial r_j} \, \frac{\partial w^{0c}(\varepsilon)}{\partial K_i} + \alpha_j \, \frac{\partial q_j^{0c}(\varepsilon)}{\partial K_i} \, \frac{\partial w^{0c}(\varepsilon)}{\partial r_j} + b_j \, \frac{\partial w^{0c}(\varepsilon)}{\partial K_i} \Bigg\} \, dH(\varepsilon)$$

$$+ \int_{\varepsilon_0}^{\varepsilon_{12}} \Bigg\{ \alpha_j \, \frac{\partial q_j^{1c}(\varepsilon)}{\partial r_j} \, \frac{\partial w^{1c}(\varepsilon)}{\partial K_i} + \alpha_j \, \frac{\partial q_j^{1c}(\varepsilon)}{\partial K_i} \, \frac{\partial w^{1c}(\varepsilon)}{\partial r_j} + b_j \, \frac{\partial w^{1c}(\varepsilon)}{\partial K_i} \Bigg\} \, dH(\varepsilon)$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \Bigg\{ \alpha_j \, \frac{\partial q_j^{2c}(\varepsilon)}{\partial r_j} \, \frac{\partial w^{2c}(\varepsilon)}{\partial K_i} + \alpha_j \, \frac{\partial q_j^{2c}(\varepsilon)}{\partial K_i} \, \frac{\partial w^{2c}(\varepsilon)}{\partial r_j} + b_j \, \frac{\partial w^{2c}(\varepsilon)}{\partial K_i} \Bigg\} \, dH(\varepsilon) \, . \tag{285}$$

(279), (280), and (281) imply:

$$\widetilde{\pi}^{Rj}_{r_j \, r_j} = 2 \int_{\underline{\varepsilon}}^{\varepsilon_0} \Bigg\{ \alpha_j \, \frac{\partial q_j^{0c}(\varepsilon)}{\partial r_j} \, \frac{\partial w^{0c}(\varepsilon)}{\partial r_j} - b_j \left[1 - \frac{\partial w^{0c}(\varepsilon)}{\partial r_j} \right] \Bigg\} \, dH(\varepsilon)$$

$$+ \; 2 \int_{\varepsilon_0}^{\varepsilon_{12}} \Bigg\{ \alpha_j \, \frac{\partial q_j^{1c}(\varepsilon)}{\partial r_j} \, \frac{\partial w^{1c}(\varepsilon)}{\partial r_j} - b_j \left[1 - \frac{\partial w^{1c}(\varepsilon)}{\partial r_j} \right] \Bigg\} \, dH(\varepsilon)$$

$$+ \; 2 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \Bigg\{ \alpha_j \, \frac{\partial q_j^{2c}(\varepsilon)}{\partial r_j} \, \frac{\partial w^{2c}(\varepsilon)}{\partial r_j} - b_j \left[1 - \frac{\partial w^{2c}(\varepsilon)}{\partial r_j} \right] \Bigg\} \, dH(\varepsilon) \, . \tag{286}$$

Lemma 1 implies:

$$
\begin{aligned}
\frac{\partial w^{0c}(\varepsilon)}{\partial r_i} &= -\frac{b^w}{3}\,[\,(\,1-\alpha_i\,)\,b_i - (\,1-\alpha_j\,)\,d_j\,]\,;\\
\frac{\partial w^{0c}(\varepsilon)}{\partial r_j} &= -\frac{b^w}{3}\,[\,(\,1-\alpha_j\,)\,b_j - (\,1-\alpha_i\,)\,d_i\,]\,;\\
\frac{\partial q_i^{0c}(\varepsilon)}{\partial r_i} &= -\frac{1}{3}\,[\,(\,1+2\,\alpha_i\,)\,b_i - (\,1-\alpha_j\,)\,d_j\,]\,;\\
\frac{\partial q_i^{0c}(\varepsilon)}{\partial r_j} &= -\frac{1}{3}\,[\,(\,1-\alpha_j\,)\,b_j - (\,1+2\,\alpha_i\,)\,d_i\,]\,;\\
\frac{\partial q_j^{0c}(\varepsilon)}{\partial r_i} &= -\frac{1}{3}\,[\,(\,1-\alpha_i\,)\,b_i - (\,1+2\,\alpha_j\,)\,d_j\,]\,;\\
\frac{\partial q_j^{0c}(\varepsilon)}{\partial r_j} &= -\frac{1}{3}\,[\,(\,1+2\,\alpha_j\,)\,b_j - (\,1-\alpha_i\,)\,d_i\,]\,;\\
\frac{\partial w^{0c}(\varepsilon)}{\partial K_i} &= \frac{\partial q_i^{0c}(\varepsilon)}{\partial K_i} = \frac{\partial q_j^{0c}(\varepsilon)}{\partial K_i} = 0\,. \qquad (287)
\end{aligned}
$$

Lemma 2 implies that when Gj is capacity constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$
\begin{aligned}
&\frac{\partial w^{1c}(\varepsilon)}{\partial r_i} = -\frac{b^w}{2}\,[\,(\,1-\alpha_i\,)\,b_i - \,d_j\,]\,; \quad \frac{\partial w^{1c}(\varepsilon)}{\partial r_j} = -\frac{b^w}{2}\,[\,b_j - (\,1-\alpha_i\,)\,d_i\,]\,;\\
&\frac{\partial q_i^{1c}(\varepsilon)}{\partial r_i} = -\frac{1}{2}\,[\,(\,1+\alpha_i\,)\,b_i - \,d_j\,]\,; \quad \frac{\partial q_i^{1c}(\varepsilon)}{\partial r_j} = -\frac{1}{2}\,[\,b_j - (\,1+\alpha_i\,)\,d_i\,]\,;\\
&\frac{\partial q_j^{1c}(\varepsilon)}{\partial r_i} = \frac{\partial q_j^{1c}(\varepsilon)}{\partial r_j} = \frac{\partial w^{1c}(\varepsilon)}{\partial K_i} = \frac{\partial q_i^{1c}(\varepsilon)}{\partial K_i} = \frac{\partial q_j^{1c}(\varepsilon)}{\partial K_i} = 0\,. \qquad (288)
\end{aligned}
$$

Lemma 3 implies that when both generators are capacity constrained:

$$
\begin{aligned}
&\frac{\partial w^{2c}(\varepsilon)}{\partial r_i} = -\,b^w\,[\,b_i - \,d_j\,]\,; \quad \frac{\partial w^{2c}(\varepsilon)}{\partial r_j} = -\,b^w\,[\,b_j - \,d_i\,]\,;\\
&\frac{\partial q_i^{2c}(\varepsilon)}{\partial r_i} = \frac{\partial q_i^{2c}(\varepsilon)}{\partial r_j} = \frac{\partial q_j^{2c}(\varepsilon)}{\partial r_i} = \frac{\partial q_j^{2c}(\varepsilon)}{\partial r_j} = 0\,;\\
&\frac{\partial w^{2c}(\varepsilon)}{\partial K_i} = -\,b^w\,; \quad \frac{\partial q_i^{2c}(\varepsilon)}{\partial K_i} = 1\,; \quad \frac{\partial q_j^{2c}(\varepsilon)}{\partial K_i} = 0\,. \qquad (289)
\end{aligned}
$$

(287), (288), and (289) imply that when Gj is capacity constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$\frac{\partial w^{0c}(\varepsilon)}{\partial K_i} \;=\; \frac{\partial q_i^{0c}(\varepsilon)}{\partial K_i} \;=\; \frac{\partial w^{1c}(\varepsilon)}{\partial K_i} \;=\; \frac{\partial q_i^{1c}(\varepsilon)}{\partial K_i} \;=\; \frac{\partial q_i^{2c}(\varepsilon)}{\partial r_i} \;=\; 0\,;$$

$$\frac{\partial q_i^{2c}(\varepsilon)}{\partial K_i} \;=\; 1\,; \quad \frac{\partial w^{2c}(\varepsilon)}{\partial K_i} \;=\; -\,b^w\,. \tag{290}$$

(283) and (290) imply that when Gj is capacity constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$\widetilde{\pi}^{Ri}_{r_i\,K_i} \;=\; \int_{\varepsilon_{12}}^{\overline{\varepsilon}} [\,\alpha_i\,(\,-\,b^w\,) + b_i\,(\,-\,b^w\,)\,]\,dH(\varepsilon) \;=\; -\,b^w\,[\,\alpha_i + b_i\,]\,[\,1 - H(\varepsilon_{12})\,]\,. \tag{291}$$

(284), (287), (288), and (289) imply that when Gj is capacity constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$\begin{aligned}
\widetilde{\pi}^{Ri}_{r_i\,r_j} \;=\; & \left\{ \frac{\alpha_i\,b^w}{9}\,[\,(\,1-\alpha_j\,)\,b_j - (\,1-\alpha_i\,)\,d_i\,]\,[\,(\,1+2\,\alpha_i\,)\,b_i - (\,1-\alpha_j\,)\,d_j\,] \right.\\
& \quad +\; \frac{\alpha_i\,b^w}{9}\,[\,(\,1-\alpha_i\,)\,b_i - (\,1-\alpha_j\,)\,d_j\,]\,[\,(\,1-\alpha_j\,)\,b_j - (\,1+2\,\alpha_i\,)\,d_i\,]\\
& \quad -\; \frac{b_i\,b^w}{3}\,[\,(\,1-\alpha_j\,)\,b_j - (\,1-\alpha_i\,)\,d_i\,]\\
& \quad \left. +\; d_i \left[1 + \frac{b^w}{3}\,(\,[\,1-\alpha_i\,]\,b_i - (\,1-\alpha_j\,)\,d_j\,) \right] \right\} H(\varepsilon_0)\\
& +\; \left\{ \frac{\alpha_i\,b^w}{4}\,[\,b_j - (\,1-\alpha_i\,)\,d_i\,]\,[\,(\,1+\alpha_i\,)\,b_i - d_j\,] \right.\\
& \quad +\; \frac{\alpha_i\,b^w}{4}\,[\,b_j - (\,1+\alpha_i\,)\,d_i\,]\,[\,(\,1-\alpha_i\,)\,b_i - d_j\,] - \frac{b_i\,b^w}{2}\,[\,b_j - (\,1-\alpha_i\,)\,d_i\,]\\
& \quad \left. +\; d_i \left[1 + \frac{b^w}{2}\,(\,[\,1-\alpha_i\,]\,b_i - d_j\,) \right] \right\} [\,H(\varepsilon_{12}) - H(\varepsilon_0)\,]\\
& +\; \{\,-\,b_i\,b^w\,[\,b_j - d_i\,] + d_i\,[\,1 + b^w\,(\,b_i - d_j\,)\,]\,\}\,[\,1 - H(\varepsilon_{12})\,]\,.
\end{aligned} \tag{292}$$

(287), (288), and (289) imply that when Gj is capacity constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$\frac{\partial w^{0c}(\varepsilon)}{\partial K_i} \;=\; \frac{\partial q_i^{0c}(\varepsilon)}{\partial K_i} \;=\; \frac{\partial w^{1c}(\varepsilon)}{\partial K_i} \;=\; \frac{\partial q_j^{1c}(\varepsilon)}{\partial K_i} \;=\; \frac{\partial q_j^{2c}(\varepsilon)}{\partial r_j} \;=\; \frac{\partial q_j^{2c}(\varepsilon)}{\partial K_i} \;=\; 0\,;$$

$$\frac{\partial w^{2c}(\varepsilon)}{\partial K_i} \;=\; -\,b^w\,. \tag{293}$$

(285) and (293) imply:

$$\widetilde{\pi}^{Rj}_{r_j K_i} = \int_{\varepsilon_{12}}^{\overline{\varepsilon}} b_j [- b^w] \, dH(\varepsilon) = - b_j \, b^w [1 - H(\varepsilon_{12})] . \tag{294}$$

(286), (287), (288), and (289) imply that when Gj is capacity constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$\begin{aligned} \widetilde{\pi}^{Rj}_{r_j r j} = \; & 2 \left\{ \alpha_j \frac{b^w}{9} [(1 - \alpha_j) b_j - (1 - \alpha_i) d_i] [(1 - \alpha_j) b_j - (1 + 2 \alpha_i) d_i] \right. \\ & \left. - b_j \left[1 + \frac{b^w}{3} ([1 - \alpha_j] b_j - [1 - \alpha_i] d_i) \right] \right\} H(\varepsilon_0) \\ & + 2 \left\{ \alpha_j \frac{b^w}{4} [b_j - (1 - \alpha_i) d_i] [b_j - (1 + \alpha_i) d_i] \right. \\ & \left. - b_j \left[1 + \frac{b^w}{2} (b_j - [1 - \alpha_i] d_i) \right] \right\} [H(\varepsilon_{12}) - H(\varepsilon_0)] \\ & - 2 b_j [1 + b^w (b_j - d_i)] [1 - H(\varepsilon_{12})] . \end{aligned} \tag{295}$$

(295) implies that when $\alpha_i = \alpha_j = 0$ and Gj is capacity constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$\begin{aligned} \widetilde{\pi}^{Rj}_{r_j r_j} = \; & - 2 b_j - \frac{2}{3} b^w b_j [b_j - d_i] H(\varepsilon_0) - b^w b_j [b_j - d_i] [H(\varepsilon_{12}) - H(\varepsilon_0)] \\ & - 2 b^w b_j [b_j - d_i] [1 - H(\varepsilon_{12})] < 0 . \end{aligned} \tag{296}$$

(295) implies that when $\alpha_i = \alpha_j = 1$ and Gj is capacity constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$\begin{aligned} \widetilde{\pi}^{Rj}_{r_j r_j} = \; & - 2 b_j + \frac{1}{2} b^w \left\{ b_j [b_j - 2 d_i] - 2 (b_j)^2 \right\} [H(\varepsilon_{12}) - H(\varepsilon_0)] \\ & - 2 b^w b_j [b_j - d_i] [1 - H(\varepsilon_{12})] \\ = \; & - 2 b_j + \frac{1}{2} b^w b_j [b_j - 2 d_i - 2 b_j] [H(\varepsilon_{12}) - H(\varepsilon_0)] \\ & - 2 b^w b_j [b_j - d_i] [1 - H(\varepsilon_{12})] \\ = \; & - 2 b_j - \frac{1}{2} b^w b_j [b_j + 2 d_i] \, [H(\varepsilon_{12}) - H(\varepsilon_0)] \\ & - 2 b^w b_j [b_j - d_i] [1 - H(\varepsilon_{12})] < 0 . \end{aligned} \tag{297}$$

(292) implies that when $\alpha_i = \alpha_j = 0$ and Gj is capacity constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$\widetilde{\pi}^{Ri}_{r_i r_j} = d_i + \left\{ - \frac{1}{3} b_i b^w [b_j - d_i] + d_i \frac{1}{3} b^w [b_i - d_j] \right\} H(\varepsilon_0)$$

$$+ \left\{ -\frac{1}{2} b_i b^w [b_j - d_i] + d_i \frac{1}{2} b^w [b_i - d_j] \right\} [H(\varepsilon_{12}) - H(\varepsilon_0)]$$

$$+ \{ -b_i b^w [b_j - d_i] + d_i b^w [b_i - d_j] \} [1 - H(\varepsilon_{12})]$$

$$= d_i + \frac{1}{3} b^w \{ d_i [b_i - d_j] - b_i [b_j - d_i] \} H(\varepsilon_0)$$

$$+ \frac{1}{2} b^w \{ d_i [b_i - d_j] - b_i [b_j - d_i] \} [H(\varepsilon_{12}) - H(\varepsilon_0)]$$

$$+ b^w \{ d_i [b_i - d_j] - b_i [b_j - d_i] \} [1 - H(\varepsilon_{12})]$$

$$= d_i - b^w [b_i b_j - 2 b_i d_i + d_i d_j] \left[\frac{1}{3} H(\varepsilon_0) + \frac{1}{3} (H(\varepsilon_{12}) - H(\varepsilon_0)) + 1 - H(\varepsilon_{12}) \right]. \qquad (298)$$

(292) implies that when $\alpha_i = \alpha_j = 1$ and Gj is capacity constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$\widetilde{\pi}^{Ri}_{r_i r_j} = d_i + \left\{ \frac{1}{4} b_j b^w [2 b_i - d_j] + \frac{1}{4} b^w [b_j - 2 d_i] [-d_j] - \frac{1}{2} b^w b_i b_j + \frac{1}{2} b^w d_i [-d_j] \right\} [H(\varepsilon_{12}) - H(\varepsilon_0)]$$

$$+ \{ -b_i b^w [b_j - d_i] + b^w d_i [b_i - d_j] \} [1 - H(\varepsilon_{12})]$$

$$= d_i + \frac{1}{4} b^w \{ b_j [2 b_i - d_j] - d_j [b_j - 2 d_i] - 2 b_i b_j - 2 d_i d_j \} [H(\varepsilon_{12}) - H(\varepsilon_0)]$$

$$+ b^w \{ d_i [b_i - d_j] - b_i [b_j - d_i] \} [1 - H(\varepsilon_{12})]$$

$$= d_i + \frac{1}{4} b^w \{ 2 b_i b_j - b_j d_j - b_j d_j + 2 d_i d_j - 2 b_i b_j - 2 d_i d_j \} [H(\varepsilon_{12}) - H(\varepsilon_0)]$$

$$+ b^w \{ d_i [b_i - d_j] - b_i [b_j - d_i] \} [1 - H(\varepsilon_{12})]$$

$$= d_i + \frac{1}{4} b^w [-2 b_j d_j] [H(\varepsilon_{12}) - H(\varepsilon_0)] - b^w [b_i b_j - 2 b_i d_i + d_i d_j] [1 - H(\varepsilon_{12})]$$

$$= d_i - \frac{1}{2} b^w b_j d_j [H(\varepsilon_{12}) - H(\varepsilon_0)] - b^w [b_i b_j - 2 b_i d_i + d_i d_j] [1 - H(\varepsilon_{12})]. \qquad (299)$$

Observation 1. $\frac{dr_i}{dK_i} < 0$ *under VS in the symmetric setting.*

Proof. (291) implies that at a symmetric equilibrium:

$$\widetilde{\pi}^{Ri}_{r_i K_i} = -b^w [\alpha_i + b_i] [1 - H(\varepsilon_{12})] < 0 \quad \text{and} \quad \widetilde{\pi}^{Rj}_{r_j K_i} = -b_j b^w [1 - H(\varepsilon_{12})] < 0$$

$$\Rightarrow \quad \left| \widetilde{\pi}^{Ri}_{r_i K_i} \right| \ \geq \ \left| \widetilde{\pi}^{Rj}_{r_j K_i} \right| . \tag{300}$$

(274) and (300) imply:

$$\frac{dr_i}{dK_i} \ \stackrel{s}{=} \ \widetilde{\pi}^{Rj}_{r_j r_j} \left| \widetilde{\pi}^{Ri}_{r_i K_i} \right| - \widetilde{\pi}^{Ri}_{r_i r_j} \left| \widetilde{\pi}^{Rj}_{r_j K_i} \right| \ \stackrel{s}{=} \ \frac{\left| \widetilde{\pi}^{Ri}_{r_i K_i} \right|}{\left| \widetilde{\pi}^{Rj}_{r_j K_i} \right|} \widetilde{\pi}^{Rj}_{r_j r_j} - \widetilde{\pi}^{Ri}_{r_i r_j} . \tag{301}$$

(296) and (299) imply that under VS:

$$\widetilde{\pi}^{Rj}_{r_j r_j} \ = \ -2b - 2b^w b[b-d] \left[\frac{1}{3} H(\varepsilon_0) + 1 - H(\varepsilon_{12}) \right] \quad \text{and}$$

$$\widetilde{\pi}^{Ri}_{r_i r_j} \ = \ d - b^w [b-d]^2 \left[\frac{1}{3} H(\varepsilon_0) + 1 - H(\varepsilon_{12}) \right] . \tag{302}$$

If $\widetilde{\pi}^{Ri}_{r_i r_j} \geq 0$, then (301) and (302) imply $\frac{dr_i}{dK_i} < 0$. If $\widetilde{\pi}^{Ri}_{r_i r_j} < 0$, then (301) and (302) imply:

$$\frac{dr_i}{dK_i} \ < \ 0 \quad \text{if} \quad \left| \widetilde{\pi}^{Rj}_{r_j r_j} \right| \ > \ \left| \widetilde{\pi}^{Ri}_{r_i r_j} \right|$$

$$\Leftrightarrow \quad 2b + 2b^w b[b-d] \left[\frac{1}{3} H(\varepsilon_0) + 1 - H(\varepsilon_{12}) \right]$$

$$> \ b^w [b-d]^2 \left[\frac{1}{3} H(\varepsilon_0) + 1 - H(\varepsilon_{12}) \right] - d$$

$$\Leftrightarrow \quad b^w [b-d] \left[\frac{1}{3} H(\varepsilon_0) + 1 - H(\varepsilon_{12}) \right] [2b - (b-d)] + 2b + d \ > \ 0$$

$$\Leftrightarrow \quad b^w [b-d][b+d] \left[\frac{1}{3} H(\varepsilon_0) + 1 - H(\varepsilon_{12}) \right] + 2b + d \ > \ 0 .$$

This inequality holds, so $\frac{dr_i}{dK_i} < 0$. □

Observation 2. $\frac{dr_i}{dK_i} < 0$ *under VI in the symmetric setting.*

Proof. (297) and (299) imply that under VI:

$$\widetilde{\pi}^{Rj}_{r_j r_j} \ = \ -2b - 2b^w b[b-d][1 - H(\varepsilon_{12})] \ < \ 0 \quad \text{and}$$

$$\widetilde{\pi}^{Ri}_{r_i r_j} \ = \ d - b^w [b-d]^2 [1 - H(\varepsilon_{12})] . \tag{303}$$

If $\widetilde{\pi}^{Ri}_{r_i r_j} \geq 0$, then (301) and (303) imply $\frac{dr_i}{dK_i} < 0$. If $\widetilde{\pi}^{Ri}_{r_i r_j} < 0$, then (301) and (303) imply $\frac{dr_i}{dK_i} < 0$ if:

$$\left|\widetilde{\pi}^{Rj}_{r_j r_j}\right| > \left|\widetilde{\pi}^{Ri}_{r_i r_j}\right| \;\Leftrightarrow\; 2\,b + 2\,b^w\,b\,[\,b-d\,]\,[\,1-H(\varepsilon_{12})\,] \;>\; b^w\,[\,b-d\,]^2\,[\,1-H(\varepsilon_{12})\,]-d$$

$$\Leftrightarrow\; b^w\,[\,b-d\,]\,[\,1-H(\varepsilon_{12})\,]\,[\,2\,b-(\,b-d\,)\,]+2\,b+d \;>\; 0$$

$$\Leftrightarrow\; b^w\,[\,b-d\,]\,[\,b+d\,]\,[\,1-H(\varepsilon_{12})\,]+2\,b+d \;>\; 0\,.$$

This inequality holds, so $\frac{dr_i}{dK_i} < 0$. □

To determine the sign of $\frac{dr_j}{dK_i}$, first note it can be verified that:

$$\begin{aligned}
\widetilde{\pi}^{Ri}_{r_i r_i} \;=\; & \int_{\underline{\varepsilon}}^{\varepsilon_0} \Bigg\{ \alpha_i\,[\,w^{0c}(\varepsilon)-c_i\,]\,\frac{\partial}{\partial r_i}\left(\frac{\partial q_i^{0c}(\varepsilon)}{\partial r_i}\right)+\alpha_i\,\frac{\partial q_i^{0c}(\varepsilon)}{\partial r_i}\,\frac{\partial w^{0c}(\varepsilon)}{\partial r_i} \\
& +\alpha_i\,q_i^{0c}(\varepsilon)\,\frac{\partial}{\partial r_i}\left(\frac{\partial w^{0c}(\varepsilon)}{\partial r_i}\right)+\alpha_i\,\frac{\partial q_i^{0c}(\varepsilon)}{\partial r_i}\,\frac{\partial w^{0c}(\varepsilon)}{\partial r_i} \\
& +[\,r_i-c_i^r-w^{0c}(\varepsilon)\,]\,\frac{\partial}{\partial r_i}\left(\frac{\partial Q_i^r(r_i,r_j)}{\partial r_i}\right)+\frac{\partial Q_i^r(r_i,r_j)}{\partial r_i}\left[1-\frac{\partial w^{0c}(\varepsilon)}{\partial r_i}\right] \\
& -Q_i^r(r_i,r_j)\,\frac{\partial}{\partial r_i}\left(\frac{\partial w^{0c}(\varepsilon)}{\partial r_i}\right)+\frac{\partial Q_i^r(r_i,r_j)}{\partial r_i}\left[1-\frac{\partial w^{0c}(\varepsilon)}{\partial r_i}\right]\Bigg\}\,dH(\varepsilon) \\
+ & \int_{\varepsilon_0}^{\varepsilon_{12}} \Bigg\{ \alpha_i\,[\,w^{1c}(\varepsilon)-c_i\,]\,\frac{\partial}{\partial r_i}\left(\frac{\partial q_i^{1c}(\varepsilon)}{\partial r_i}\right)+\alpha_i\,\frac{\partial q_i^{1c}(\varepsilon)}{\partial r_i}\,\frac{\partial w^{1c}(\varepsilon)}{\partial r_i} \\
& +\alpha_i\,q_i^{1c}(\varepsilon)\,\frac{\partial}{\partial r_i}\left(\frac{\partial w^{1c}(\varepsilon)}{\partial r_i}\right)+\alpha_i\,\frac{\partial q_i^{1c}(\varepsilon)}{\partial r_i}\,\frac{\partial w^{1c}(\varepsilon)}{\partial r_i} \\
& +[\,r_i-c_i^r-w^{1c}(\varepsilon)\,]\,\frac{\partial}{\partial r_i}\left(\frac{\partial Q_i^r(r_i,r_j)}{\partial r_i}\right)+\frac{\partial Q_i^r(r_i,r_j)}{\partial r_i}\left[1-\frac{\partial w^{1c}(\varepsilon)}{\partial r_i}\right] \\
& -Q_i^r(r_i,r_j)\,\frac{\partial}{\partial r_i}\left(\frac{\partial w^{1c}(\varepsilon)}{\partial r_i}\right)+\frac{\partial Q_i^r(r_i,r_j)}{\partial r_i}\left[1-\frac{\partial w^{1c}(\varepsilon)}{\partial r_i}\right]\Bigg\}\,dH(\varepsilon) \\
+ & \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \Bigg\{ \alpha_i\,[\,w^{2c}(\varepsilon)-c_i\,]\,\frac{\partial}{\partial r_i}\left(\frac{\partial q_i^{2c}(\varepsilon)}{\partial r_i}\right)+\alpha_i\,\frac{\partial q_i^{2c}(\varepsilon)}{\partial r_i}\,\frac{\partial w^{2c}(\varepsilon)}{\partial r_i} \\
& +\alpha_i\,q_i^{2c}(\varepsilon)\,\frac{\partial}{\partial r_i}\left(\frac{\partial w^{2c}(\varepsilon)}{\partial r_i}\right)+\alpha_i\,\frac{\partial q_i^{2c}(\varepsilon)}{\partial r_i}\,\frac{\partial w^{2c}(\varepsilon)}{\partial r_i}
\end{aligned}$$

$$+ \left[r_i - c_i^r - w^{2c}(\varepsilon) \right] \frac{\partial}{\partial r_i} \left(\frac{\partial Q_i^r(r_i, r_j)}{\partial r_i} \right) + \frac{\partial Q_i^r(r_i, r_j)}{\partial r_i} \left[1 - \frac{\partial w^{2c}(\varepsilon)}{\partial r_i} \right]$$

$$- Q_i^r(r_i, r_j) \frac{\partial}{\partial r_i} \left(\frac{\partial w^{2c}(\varepsilon)}{\partial r_i} \right) + \frac{\partial Q_i^r(r_i, r_j)}{\partial r_i} \left[1 - \frac{\partial w^{2c}(\varepsilon)}{\partial r_i} \right] \Bigg\} dH(\varepsilon) \,. \qquad (304)$$

(280) – (282) and (304) imply:

$$\widetilde{\pi}_{r_i r_i}^{Ri} = 2 \int_{\underline{\varepsilon}}^{\varepsilon_0} \left\{ \alpha_i \frac{\partial q_i^{0c}(\varepsilon)}{\partial r_i} \frac{\partial w^{0c}(\varepsilon)}{\partial r_i} - b_i \left[1 - \frac{\partial w^{0c}(\varepsilon)}{\partial r_i} \right] \right\} dH(\varepsilon)$$

$$+ 2 \int_{\varepsilon_0}^{\varepsilon_{12}} \left\{ \alpha_i \frac{\partial q_i^{1c}(\varepsilon)}{\partial r_i} \frac{\partial w^{1c}(\varepsilon)}{\partial r_i} - b_i \left[1 - \frac{\partial w^{1c}(\varepsilon)}{\partial r_i} \right] \right\} dH(\varepsilon)$$

$$+ 2 \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left\{ \alpha_i \frac{\partial q_i^{2c}(\varepsilon)}{\partial r_i} \frac{\partial w^{2c}(\varepsilon)}{\partial r_i} - b_i \left[1 - \frac{\partial w^{2c}(\varepsilon)}{\partial r_i} \right] \right\} dH(\varepsilon) \,. \qquad (305)$$

(287), (288), (289), and (305) imply that when Gj is capacity constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$\widetilde{\pi}_{r_i r_i}^{Ri} = 2 \Bigg\{ \alpha_i \frac{b^w}{9} [(1 - \alpha_i) b_i - (1 - \alpha_j) d_j] [(1 + 2 \alpha_i) b_i - (1 - \alpha_j) d_j]$$

$$- b_i \left[1 + \frac{b^w}{3} ([1 - \alpha_i] b_i - [1 - \alpha_j] d_j) \right] \Bigg\} H(\varepsilon_0)$$

$$+ 2 \Bigg\{ \alpha_i \frac{b^w}{4} [(1 + \alpha_i) b_i - d_j] [(1 - \alpha_i) b_i - d_j]$$

$$- b_i \left[1 + \frac{b^w}{2} ([1 - \alpha_i] b_i - d_j) \right] \Bigg\} [H(\varepsilon_{12}) - H(\varepsilon_0)]$$

$$- 2 b_i [1 + b^w (b_i - d_j)] [1 - H(\varepsilon_{12})] \,. \qquad (306)$$

Differentiating (275) provides:

$$\widetilde{\pi}_{r_j r_i}^{Rj} = \int_{\underline{\varepsilon}}^{\varepsilon_0} \Bigg\{ \alpha_j [w^{0c}(\varepsilon) - c_j] \frac{\partial}{\partial r_i} \left(\frac{\partial q_j^{0c}(\varepsilon)}{\partial r_j} \right) + \alpha_j \frac{\partial q_j^{0c}(\varepsilon)}{\partial r_j} \frac{\partial w^{0c}(\varepsilon)}{\partial r_i}$$

$$+ \alpha_j q_j^{0c}(\varepsilon) \frac{\partial}{\partial r_i} \left(\frac{\partial w^{0c}(\varepsilon)}{\partial r_j} \right) + \alpha_j \frac{\partial q_j^{0c}(\varepsilon)}{\partial r_i} \frac{\partial w^{0c}(\varepsilon)}{\partial r_j}$$

$$+ \left[r_j - c_j^r - w^{0c}(\varepsilon) \right] \frac{\partial}{\partial r_i} \left(\frac{\partial Q_j^r(r_j, r_i)}{\partial r_j} \right) - \frac{\partial Q_j^r(r_j, r_i)}{\partial r_j} \frac{\partial w^{0c}(\varepsilon)}{\partial r_i}$$

$$
- \, Q^r_j(r_j, r_i) \, \frac{\partial}{\partial r_i} \left(\frac{\partial w^{0c}(\varepsilon)}{\partial r_j} \right) + \frac{\partial Q^r_j(r_j, r_i)}{\partial r_i} \left[1 - \frac{\partial w^{0c}(\varepsilon)}{\partial r_j} \right] \Bigg\} \, dH(\varepsilon)
$$

$$
+ \int_{\varepsilon_0}^{\varepsilon_{12}} \Bigg\{ \alpha_j \left[w^{1c}(\varepsilon) - c_j \right] \frac{\partial}{\partial r_i} \left(\frac{\partial q^{1c}_j(\varepsilon)}{\partial r_j} \right) + \alpha_j \, \frac{\partial q^{1c}_j(\varepsilon)}{\partial r_j} \, \frac{\partial w^{1c}(\varepsilon)}{\partial r_i}
$$

$$
+ \, \alpha_j \, q^{1c}_j(\varepsilon) \, \frac{\partial}{\partial r_i} \left(\frac{\partial w^{1c}(\varepsilon)}{\partial r_j} \right) + \alpha_j \, \frac{\partial q^{1c}_j(\varepsilon)}{\partial r_i} \, \frac{\partial w^{1c}(\varepsilon)}{\partial r_j}
$$

$$
+ \, \left[r_j - c^r_j - w^{1c}(\varepsilon) \right] \frac{\partial}{\partial r_i} \left(\frac{\partial Q^r_j(r_j, r_i)}{\partial r_j} \right) - \frac{\partial Q^r_j(r_j, r_i)}{\partial r_j} \, \frac{\partial w^{1c}(\varepsilon)}{\partial r_i}
$$

$$
- \, Q^r_j(r_j, r_i) \, \frac{\partial}{\partial r_i} \left(\frac{\partial w^{1c}(\varepsilon)}{\partial r_j} \right) + \frac{\partial Q^r_j(r_j, r_i)}{\partial r_i} \left[1 - \frac{\partial w^{1c}(\varepsilon)}{\partial r_j} \right] \Bigg\} \, dH(\varepsilon)
$$

$$
+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \Bigg\{ \alpha_j \left[w^{2c}(\varepsilon) - c_j \right] \frac{\partial}{\partial r_i} \left(\frac{\partial q^{2c}_j(\varepsilon)}{\partial r_j} \right) + \alpha_j \, \frac{\partial q^{2c}_j(\varepsilon)}{\partial r_j} \, \frac{\partial w^{2c}(\varepsilon)}{\partial r_i}
$$

$$
+ \, \alpha_j \, q^{2c}_j(\varepsilon) \, \frac{\partial}{\partial r_i} \left(\frac{\partial w^{2c}(\varepsilon)}{\partial r_j} \right) + \alpha_j \, \frac{\partial q^{2c}_j(\varepsilon)}{\partial r_i} \, \frac{\partial w^{2c}(\varepsilon)}{\partial r_j}
$$

$$
+ \, \left[r_j - c^r_j - w^{2c}(\varepsilon) \right] \frac{\partial}{\partial r_i} \left(\frac{\partial Q^r_j(r_j, r_i)}{\partial r_j} \right) - \frac{\partial Q^r_j(r_j, r_i)}{\partial r_j} \, \frac{\partial w^{2c}(\varepsilon)}{\partial r_i}
$$

$$
- \, Q^r_j(r_j, r_i) \, \frac{\partial}{\partial r_i} \left(\frac{\partial w^{2c}(\varepsilon)}{\partial r_j} \right) + \frac{\partial Q^r_j(r_j, r_i)}{\partial r_i} \left[1 - \frac{\partial w^{2c}(\varepsilon)}{\partial r_j} \right] \Bigg\} \, dH(\varepsilon) \, . \qquad (307)
$$

(280) – (282) and (307) imply:

$$
\widetilde{\pi}^{Rj}_{r_j \, r_i} \; = \; \int_{\underline{\varepsilon}}^{\varepsilon_0} \Bigg\{ \alpha_j \, \frac{\partial q^{0c}_j(\varepsilon)}{\partial r_j} \, \frac{\partial w^{0c}(\varepsilon)}{\partial r_i} + \alpha_j \, \frac{\partial q^{0c}_j(\varepsilon)}{\partial r_i} \, \frac{\partial w^{0c}(\varepsilon)}{\partial r_j}
$$

$$
+ \, b_j \, \frac{\partial w^{0c}(\varepsilon)}{\partial r_i} + d_j \left[1 - \frac{\partial w^{0c}(\varepsilon)}{\partial r_j} \right] \Bigg\} \, dH(\varepsilon)
$$

$$
+ \int_{\varepsilon_0}^{\varepsilon_{12}} \Bigg\{ \alpha_j \, \frac{\partial q^{1c}_j(\varepsilon)}{\partial r_j} \, \frac{\partial w^{1c}(\varepsilon)}{\partial r_i} + \alpha_j \, \frac{\partial q^{1c}_j(\varepsilon)}{\partial r_i} \, \frac{\partial w^{1c}(\varepsilon)}{\partial r_j}
$$

$$
+ \, b_j \, \frac{\partial w^{1c}(\varepsilon)}{\partial r_i} + d_j \left[1 - \frac{\partial w^{1c}(\varepsilon)}{\partial r_j} \right] \Bigg\} \, dH(\varepsilon)
$$

$$+ \int_{\varepsilon_{12}}^{\overline{\varepsilon}} \left\{ \alpha_j \frac{\partial q_j^{2c}(\varepsilon)}{\partial r_j} \frac{\partial w^{2c}(\varepsilon)}{\partial r_i} + \alpha_j \frac{\partial q_j^{2c}(\varepsilon)}{\partial r_i} \frac{\partial w^{2c}(\varepsilon)}{\partial r_j} \right.$$

$$\left. + b_j \frac{\partial w^{2c}(\varepsilon)}{\partial r_i} + d_j \left[1 - \frac{\partial w^{2c}(\varepsilon)}{\partial r_j} \right] \right\} dH(\varepsilon) . \qquad (308)$$

(287), (288), (289), and (308) imply that when Gj is capacity constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$\begin{aligned} \widetilde{\pi}^{Rj}_{r_j r_i} \;=\; & \left\{ \frac{\alpha_j b^w}{9} [(1 + 2 \alpha_j) b_j - (1 - \alpha_i) d_i] [(1 - \alpha_i) b_i - (1 - \alpha_j) d_j] \right. \\ & + \frac{\alpha_j b^w}{9} [(1 - \alpha_i) b_i - (1 + 2 \alpha_j) d_j] [(1 - \alpha_j) b_j - (1 - \alpha_i) d_i] \\ & - \frac{b_j b^w}{3} [(1 - \alpha_i) b_i - (1 - \alpha_j) d_j] \\ & \left. + d_j \left[1 + \frac{b^w}{3} ([1 - \alpha_j] b_j - (1 - \alpha_i) d_i) \right] \right\} H(\varepsilon_0) \\ & + \left\{ - \frac{b_j b^w}{2} [(1 - \alpha_i) b_i - d_j] \right. \\ & \left. + d_j \left[1 + \frac{b^w}{2} (b_j - [1 - \alpha_i] d_i) \right] \right\} [H(\varepsilon_{12}) - H(\varepsilon_0)] \\ & + \{ - b_j b^w [b_i - d_j] + d_j [1 + b^w (b_j - d_i)] \} [1 - H(\varepsilon_{12})] . \end{aligned} \qquad (309)$$

(306) implies that when $\alpha_i = \alpha_j = 0$ and Gj is capacity constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$\begin{aligned} \widetilde{\pi}^{Ri}_{r_i r_i} \;=\; & - 2 b_i - \frac{2}{3} b^w b_i [b_i - d_j] H(\varepsilon_0) - b^w b_i [b_i - d_j] [H(\varepsilon_{12}) - H(\varepsilon_0)] \\ & - 2 b^w b_i [b_i - d_j] [1 - H(\varepsilon_{12})] \;<\; 0 . \end{aligned} \qquad (310)$$

(306) implies that when $\alpha_i = \alpha_j = 1$ and Gj is capacity constrained for $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$:

$$\begin{aligned} \widetilde{\pi}^{Ri}_{r_i r_i} \;=\; & - 2 b_i + 2 \left\{ \frac{b^w}{4} [2 b_i - d_j] [- d_j] - \frac{b^w}{2} b_i [- d_j] \right\} [H(\varepsilon_{12}) - H(\varepsilon_0)] \\ & - 2 b^w b_i [b_i - d_j] [1 - H(\varepsilon_{12})] \\ =\; & - 2 b_i - \frac{1}{2} b^w d_j [2 b_i - d_j - 2 b_i] [H(\varepsilon_{12}) - H(\varepsilon_0)] \\ & - 2 b^w b_i [b_i - d_j] [1 - H(\varepsilon_{12})] \\ =\; & - 2 b_i + \frac{1}{2} b^w d_j^2 [H(\varepsilon_{12}) - H(\varepsilon_0)] - 2 b^w b_i [b_i - d_j] [1 - H(\varepsilon_{12})] . \end{aligned} \qquad (311)$$

Observation 3. $\frac{dr_j}{dK_i} \; < \; 0$ *in the symmetric setting.*

Proof. (291) and (294) imply that at a symmetric equilibrium:

$$\widetilde{\pi}^{Ri}_{r_i K_i} \; = \; - b^w [\, \alpha_i + b \,] [\, 1 - H(\varepsilon_{12}) \,] \; < \; 0 \quad \text{and} \quad \widetilde{\pi}^{Rj}_{r_j K_i} \; = \; - b \, b^w [\, 1 - H(\varepsilon_{12}) \,] \; < \; 0$$

$$\Rightarrow \quad \left| \widetilde{\pi}^{Ri}_{r_i K_i} \right| \; \geq \; \left| \widetilde{\pi}^{Rj}_{r_j K_i} \right| . \tag{312}$$

(274) and (312) imply:

$$\frac{dr_j}{dK_i} \; \overset{s}{=} \; \widetilde{\pi}^{Ri}_{r_i r_i} \left| \widetilde{\pi}^{Rj}_{r_j K_i} \right| - \widetilde{\pi}^{Rj}_{r_j r_i} \left| \widetilde{\pi}^{Ri}_{r_i K_i} \right| \; \overset{s}{=} \; \frac{\left| \widetilde{\pi}^{Rj}_{r_j K_i} \right|}{\left| \widetilde{\pi}^{Ri}_{r_i K_i} \right|} \, \widetilde{\pi}^{Ri}_{r_i r_i} - \widetilde{\pi}^{Rj}_{r_j r_i} . \tag{313}$$

(310) implies that at a symmetric equilibrium under VS:

$$\widetilde{\pi}^{Ri}_{r_i r_i} \; = \; - 2 \, b - 2 \, b^w \, b \, [\, b - d \,] \left[\frac{1}{3} \, H(\varepsilon_0) + 1 - H(\varepsilon_{12}) \right] \; < \; 0 \, . \tag{314}$$

(309) implies that at a symmetric equilibrium under VS:

$$\begin{aligned} \widetilde{\pi}^{Rj}_{r_j r_i} \; &= \; d + \left\{ - \frac{b \, b^w}{3} \, [\, b - d \,] + d \left[\frac{b^w}{3} \, (\, b - d \,) \right] \right\} H(\varepsilon_0) \\ &\quad - \, b^w \, [\, b \, (\, b - d \,) - d \, (\, b - d \,) \,] \, [\, 1 - H(\varepsilon_{12}) \,] \\ &= \; d - b^w \, [\, b - d \,]^2 \left[\frac{1}{3} \, H(\varepsilon_0) + 1 - H(\varepsilon_{12}) \right] . \end{aligned} \tag{315}$$

First suppose $\widetilde{\pi}^{Rj}_{r_j r_i} \geq 0$. Then (313) and (314) imply $\frac{dr_j}{dK_i} \; < \; 0$.

Now suppose $\widetilde{\pi}^{Rj}_{r_j r_i} < 0$. (312) implies that $\left| \widetilde{\pi}^{Rj}_{r_j K_i} \right| = \left| \widetilde{\pi}^{Ri}_{r_i K_i} \right|$ under VS. Therefore, (313) implies:

$$\begin{aligned} \frac{dr_j}{dK_i} \; < \; 0 \quad \text{if} \quad & \left| \widetilde{\pi}^{Ri}_{r_i r_i} \right| \; > \; \left| \widetilde{\pi}^{Rj}_{r_j r_i} \right| \\ \Leftrightarrow \quad & 2 \, b + 2 \, b^w \, b \, [\, b - d \,] \left[\frac{1}{3} \, H(\varepsilon_0) + 1 - H(\varepsilon_{12}) \right] \\ & \quad > \; b^w \, [\, b - d \,]^2 \left[\frac{1}{3} \, H(\varepsilon_0) + 1 - H(\varepsilon_{12}) \right] - d \\ \Leftrightarrow \quad & b^w \, [\, b - d \,] \left[\frac{1}{3} \, H(\varepsilon_0) + 1 - H(\varepsilon_{12}) \right] [\, 2 \, b - (\, b - d \,) \,] + 2 \, b + d \; > \; 0 \\ \Leftrightarrow \quad & b^w \, [\, b - d \,] \, [\, b + d \,] \left[\frac{1}{3} \, H(\varepsilon_0) + 1 - H(\varepsilon_{12}) \right] + 2 \, b + d \; > \; 0 \, . \end{aligned}$$

This inequality holds, so $\frac{dr_j}{dK_i} < 0$. □

Observation 4. $\frac{dr_j}{dK_i} < 0$ *under VI in the symmetric setting.*

Proof. (311) implies that at a symmetric equilibrium under VI:

$$\widetilde{\pi}^{Ri}_{r_i\, r_i} = -2\,b - 2\,b^w\,b\,[\,b-d\,]\,[\,1 - H(\varepsilon_{12})\,] < 0. \tag{316}$$

(309) implies that at a symmetric equilibrium under VI:

$$\widetilde{\pi}^{Rj}_{r_j\, r_i} = d + \{-b\,b^w\,[\,b-d\,] + d\,b^w\,[\,b-d\,]\}\,[\,1 - H(\varepsilon_{12})\,]$$

$$= d - b^w\,[\,b-d\,]^2\,[\,1 - H(\varepsilon_{12})\,]\,. \tag{317}$$

First suppose $\widetilde{\pi}^{Rj}_{r_j\, r_i} \geq 0$. Then (313) and (316) imply $\frac{dr_j}{dK_i} < 0$.

Now suppose $\widetilde{\pi}^{Rj}_{r_j\, r_i} < 0$. (312) implies that under VI:

$$\frac{\left|\,\widetilde{\pi}^{Rj}_{r_j\, K_i}\,\right|}{\left|\,\widetilde{\pi}^{Ri}_{r_i\, K_i}\,\right|} = \frac{b\,b^w\,[\,1 - H(\varepsilon_{12})\,]}{b^w\,[\,1+b\,]\,[\,1 - H(\varepsilon_{12})\,]} = \frac{b}{1+b}\,. \tag{318}$$

(313) and (318) imply:

$$\frac{dr_j}{dK_i} < 0 \text{ if } b\left|\,\widetilde{\pi}^{Ri}_{r_i\, r_i}\,\right| > [\,1+b\,]\left|\,\widetilde{\pi}^{Rj}_{r_j\, r_i}\,\right|$$

$$\Leftrightarrow \; b\,\{\,2\,b + 2\,b^w\,b\,[\,b-d\,]\,[\,1 - H(\varepsilon_{12})\,]\,\} > [\,1+b\,]\left\{\,b^w\,[\,b-d\,]^2\,[\,1 - H(\varepsilon_{12})\,] - d\,\right\}$$

$$\Leftrightarrow \; 2\,b^2 + 2\,b^2\,b^w\,[\,b-d\,]\,[\,1 - H(\varepsilon_{12})\,] > b^w\,[\,b-d\,]^2\,[\,1 - H(\varepsilon_{12})\,] - d$$
$$+ \; b\,b^w\,[\,b-d\,]^2\,[\,1 - H(\varepsilon_{12})\,] - b\,d$$

$$\Leftrightarrow \; 2\,b^2 + 2\,b\,d + d + b^w\,b\,[\,b-d\,]\,[\,1 - H(\varepsilon_{12})\,]\,[\,2\,b - (\,b-d\,)\,] > 0\,.$$

This inequality holds, so $\frac{dr_j}{dK_i} < 0$. □ ■

IV. Benchmark Setting where Vertical Integration Does Not Eliminate a Double Marginalization Problem.

The ensuing analysis demonstrates that vertical integration can continue to enhance incentives to compete aggressively (and so can increase capacity investment and wholesale output) even when it does not eliminate a double marginalization problem. The analysis considers the setting where $H(\varepsilon)$ is the uniform distribution and where $r_i = E\{\, w(\varepsilon)\,\} + c_i^r$ for $i \in \{1, 2\}$. Thus, the expected retail profit margin is always zero under VS, PVI, and VI.

Case 1. Vertical Separation ($\alpha_1^R = \alpha_2^R = \alpha_1^G = \alpha_2^G = 0$) and $G2$ is capacity-constrained when $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$.

Lemmas 1 – 3 imply that in this case:

$$E\{w(\varepsilon)\} = \frac{1}{3}\left[\, b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \,\right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{6} \left[\frac{(\varepsilon_0)^2 - (\underline{\varepsilon})^2}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$
$$+ \frac{1}{2}\left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_2 \right) + c_1 \,\right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{4} \left[\frac{(\varepsilon_{12})^2 - (\varepsilon_0)^2}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$
$$+ b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{2} \left[\frac{(\overline{\varepsilon})^2 - (\varepsilon_{12})^2}{\overline{\varepsilon} - \underline{\varepsilon}} \right]; \tag{319}$$

$$E\{q_1^*\} = \frac{1}{3\, b^w} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r \right) + c_2 - 2\, c_1 \,\right] + \frac{1}{6\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right]$$
$$+ \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_2 - \frac{1}{b^w}\, c_1 \right]$$
$$+ \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right] + K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]; \tag{320}$$

$$E\{q_2^*\} = \frac{1}{3\, b^w} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 - 2\, c_2 \,\right]$$
$$+ \frac{1}{6\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right] + K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]; \tag{321}$$

$$E\{\, w(Q^*)\, q_1^* \,\} = \frac{1}{9\, b^w} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \,\right]$$
$$\cdot \left[\, b^w \left(a^L + Q_1^r + Q_2^r \right) + c_2 - 2\, c_1 \,\right]$$

$$+ \frac{1}{18\,b^w}\left[\,2\,b^w\left(a^L + Q_1^r + Q_2^r\right) + 2\,c_2 - c_1\,\right]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2 - (\underline{\varepsilon})^2\,\right]$$

$$+ \frac{1}{27\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^3 - (\underline{\varepsilon})^3\,\right]$$

$$+ \frac{1}{4}\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,b^w\left(a^L + Q_1^r + Q_2^r - K_2\right) + c_1\,\right]\left[a^L + Q_1^r + Q_2^r - K_2 - \frac{1}{b^w}\,c_1\right]$$

$$+ \frac{1}{4}\left[\,a^L + Q_1^r + Q_2^r - K_2\,\right]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2 - (\varepsilon_0)^2\,\right]$$

$$+ \frac{1}{12\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^3 - (\varepsilon_0)^3\,\right]$$

$$+ b^w\left[\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right]K_1\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]$$

$$+ \frac{1}{2}\,K_1\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2 - (\varepsilon_{12})^2\,\right]; \qquad (322)$$

$$E\{\,w(Q^*)\,q_2^*\,\} \;=\; \frac{1}{9\,b^w}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,b^w\left(a^L + Q_1^r + Q_2^r\right) + c_1 + c_2\,\right]\left[\,b^w\left(a^L + Q_1^r + Q_2^r\right) + c_1 - 2\,c_2\,\right]$$

$$+ \frac{1}{18\,b^w}\left[\,2\,b^w\left(a^L + Q_1^r + Q_2^r\right) + 2\,c_1 - c_2\,\right]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2 - (\underline{\varepsilon})^2\,\right]$$

$$+ \frac{1}{27\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^3 - (\underline{\varepsilon})^3\,\right]$$

$$+ \frac{1}{2}\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,b^w\left(a^L + Q_1^r + Q_2^r - K_2\right) + c_1\,\right]K_2$$

$$+ \frac{1}{4}\,K_2\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2 - (\varepsilon_0)^2\,\right]$$

$$+ \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]b^w\left[\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right]K_2$$

$$+ \frac{1}{2}\,K_2\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2 - (\varepsilon_{12})^2\,\right]. \qquad (323)$$

$$E\{\pi_i^R\} \;=\; \left[\,r_i - c_i^r - E\{\,w(Q^*)\,\}\,\right]Q_i^r(r_1, r_2) \quad \text{and}$$

$$E\{\pi_i^w\} \;=\; E\{\, w(Q^*)\, q_i^* \,\} - \; c_i\, E\{\, q_i^* \,\} - z_i\, k_i \, , \tag{324}$$

where $Q_i^r(r_1, r_2)$ is defined in (1).

(18) and (20) imply that in this case:

$$\varepsilon_0 \;=\; 3\, b^w\, K_2 + 2\, c_2 - c_1 - b^w \left[\, a^L + \; Q_2^r + Q_1^r \,\right] ;$$

$$\varepsilon_{12} \;=\; 2\, b^w\, K_1 + b^w\, K_2 + c_1 - b^w \left[\, a^L + Q_1^r + Q_2^r \,\right] . \tag{325}$$

(324) implies that firm i's optimal choice on k_i in the current setting satisfies:

$$\frac{dE\{\pi_i^w\}}{dk_i} \;=\; \frac{dE\{\, w(Q^*)\, q_i^* \,\}}{dk_i} - \; c_i\, \frac{dE\{\, q_i^* \,\}}{dk_i} - z_i \;=\; 0\, . \tag{326}$$

Because $r_i \,=\, E\{\, w(\varepsilon)\} + c_i^r$, (1) implies:

$$\frac{\partial Q_i^r}{\partial k_i} \;=\; -\, b_i\, \frac{\partial r_i}{\partial k_i} + d_i\, \frac{\partial r_j}{\partial k_i} \;=\; -\, [\, b_i - d_i \,]\, \frac{\partial E\{w\}}{\partial k_i} \, ;$$

$$\frac{\partial Q_j^r}{\partial k_i} \;=\; -\, b_j\, \frac{\partial r_j}{\partial k_i} + d_j\, \frac{\partial r_i}{\partial k_i} \;=\; -\, [\, b_j - d_j \,]\, \frac{\partial E\{w\}}{\partial k_i} \, . \tag{327}$$

(325) and (327) imply:

$$\frac{\partial \varepsilon_0}{\partial K_1} \;=\; -\, b^w \left[\frac{\partial Q_1^r}{\partial K_1} + \frac{\partial Q_2^r}{\partial K_1} \right] \;=\; b^w\, [\, b_1 - d_1 + b_2 - d_2 \,]\, \frac{\partial E\{w\}}{\partial K_1} \, ;$$

$$\frac{\partial \varepsilon_0}{\partial K_2} \;=\; 3\, b^w - b^w \left[\frac{\partial Q_1^r}{\partial K_2} + \frac{\partial Q_2^r}{\partial K_2} \right] \;=\; 3\, b^w + b^w\, [\, b_1 - d_1 + b_2 - d_2 \,]\, \frac{\partial E\{w\}}{\partial K_2} \, ;$$

$$\frac{\partial \varepsilon_{12}}{\partial K_1} \;=\; 2\, b^w - b^w \left[\frac{\partial Q_1^r}{\partial K_1} + \frac{\partial Q_2^r}{\partial K_1} \right] \;=\; 2\, b^w + b^w\, [\, b_1 - d_1 + b_2 - d_2 \,]\, \frac{\partial E\{w\}}{\partial K_1} \, ;$$

$$\frac{\partial \varepsilon_{12}}{\partial K_2} \;=\; b^w - b^w \left[\frac{\partial Q_1^r}{\partial K_2} + \frac{\partial Q_2^r}{\partial K_2} \right] \;=\; b^w + b^w\, [\, b_1 - d_1 + b_2 - d_2 \,]\, \frac{\partial E\{w\}}{\partial K_2} \, . \tag{328}$$

Because $r_i = E\{\, w(\varepsilon)\} + c_i^r$, (1), (319), (327), and (328) imply:

$$\frac{\partial E\,\{w\}}{\partial K_1} \;=\; \frac{1}{3} \left[\, b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{\partial \varepsilon_0}{\partial K_1} + \frac{1}{3} \left[b^w \left(\frac{\partial Q_1^r}{\partial K_1} + \frac{\partial Q_2^r}{\partial K_1} \right) \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \; \frac{1}{6} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 2\, \varepsilon_0\, \frac{\partial \varepsilon_0}{\partial K_1} + \frac{1}{2} \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_2 \right) + c_1 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{\partial \varepsilon_{12}}{\partial K_1} - \frac{\partial \varepsilon_0}{\partial K_1} \right]$$

$$+ \; \frac{1}{2} \left[b^w \left(\frac{\partial Q_1^r}{\partial K_1} + \frac{\partial Q_2^r}{\partial K_1} \right) \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[2\, \varepsilon_{12}\, \frac{\partial \varepsilon_{12}}{\partial K_1} - 2\, \varepsilon_0\, \frac{\partial \varepsilon_0}{\partial K_1} \right]$$

$$- \; b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{\partial \varepsilon_{12}}{\partial K_1}$$

$$+ \; b^w \left[\frac{\partial Q_1^r}{\partial K_1} + \frac{\partial Q_2^r}{\partial K_1} - 1 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 2 \, \varepsilon_{12} \frac{\partial \varepsilon_{12}}{\partial K_1}$$

$$\Leftrightarrow \quad \frac{\partial E\{w\}}{\partial K_1} \; = \; \frac{1}{3} \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\, b_1 - d_1 + b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_1}$$

$$- \; \frac{b^w}{3} \left(\, b_1 - d_1 + b_2 - d_2 \right) \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{\partial E\{w\}}{\partial K_1}$$

$$+ \; \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \, b^w \left[\, b_1 - d_1 + b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_1}$$

$$+ \; \frac{1}{2} \left[b^w \left(a^L + Q_1^r + Q_2^r - K_2 \right) + c_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$\cdot \left[2 \, b^w + b^w \left(\, b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_1} - b^w \left(\, b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_1} \right]$$

$$- \; \frac{1}{2} \left[b^w \left(\, b_1 - d_1 + b_2 - d_2 \right) \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{\partial E\{w\}}{\partial K_1}$$

$$+ \; \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \left(2 \, b^w + b^w \left[\, b_1 - d_1 + b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_1} \right) \right.$$

$$\left. - \; \varepsilon_0 \left(b^w \left[\, b_1 - d_1 + b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_1} \right) \right]$$

$$- \; b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$\cdot \left[2 \, b^w + b^w \left(\, b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_1} \right]$$

$$+ \; b^w \left[- \left(\, b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_1} - 1 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \; \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \left[2 \, b^w + b^w \left(\, b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_1} \right]$$

$$\Leftrightarrow \quad \frac{\partial E\{w\}}{\partial K_1} \left\{ 1 - \frac{b^w}{3} \left[\, b_1 - d_1 + b_2 - d_2 \right] \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \right.$$

$$+ \; \frac{b^w}{3} \left[b_1 - d_1 + b_2 - d_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - \; \frac{b^w}{3} \left[\, b_1 - d_1 + b_2 - d_2 \right] \left[\frac{\varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+\ \frac{b^w}{2}\left[b_1 - d_1 + b_2 - d_2 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$-\ \frac{b^w}{2}\left[b_1 - d_1 + b_2 - d_2 \right] \left[\varepsilon_{12} - \varepsilon_0 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+\ (b^w)^2 \left[b_1 - d_1 + b_2 - d_2 \right] \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+\ b^w \left[b_1 - d_1 + b_2 - d_2 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b^w \left[b_1 - d_1 + b_2 - d_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \Bigg\}$$

$$=\ b^w \left[b^w \left(a^L + Q_1^r + Q_2^r - K_2 \right) + c_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b^w \varepsilon_{12} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$-\ 2\,(b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] -\ b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - 2\, b^w \varepsilon_{12} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$\Leftrightarrow\quad \frac{\partial E\{w\}}{\partial K_1} \Bigg\{ \overline{\varepsilon} - \underline{\varepsilon} - \frac{b^w}{3}\left[b_1 - d_1 + b_2 - d_2 \right] \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right]$$

$$+\ \frac{b^w}{3}\left[b_1 - d_1 + b_2 - d_2 \right] \left[\varepsilon_0 - \underline{\varepsilon} \right] -\ \frac{b^w}{3}\left[b_1 - d_1 + b_2 - d_2 \right] \left[\varepsilon_0 \right]$$

$$+\ \frac{b^w}{2}\left[b_1 - d_1 + b_2 - d_2 \right] \left[\varepsilon_{12} - \varepsilon_0 \right]$$

$$-\ \frac{b^w}{2}\left[b_1 - d_1 + b_2 - d_2 \right] \left[\varepsilon_{12} - \varepsilon_0 \right]$$

$$+\ (b^w)^2 \left[b_1 - d_1 + b_2 - d_2 \right] \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right]$$

$$+\ b^w \left[b_1 - d_1 + b_2 - d_2 \right] \left[\overline{\varepsilon} - \varepsilon_{12} \right] +\ b^w \left[b_1 - d_1 + b_2 - d_2 \right] \varepsilon_{12} \Bigg\}$$

$$=\ b^w \left[b^w \left(a^L + Q_1^r + Q_2^r - K_2 \right) + c_1 \right] + b^w \varepsilon_{12}$$

$$-\ 2\,(b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - b^w \left[\overline{\varepsilon} - \varepsilon_{12} \right] - 2\, b^w \varepsilon_{12}$$

$$\Leftrightarrow\quad \frac{\partial E\{w\}}{\partial K_1} \Bigg\{ \overline{\varepsilon} - \underline{\varepsilon} - \frac{b^w}{3}\left[b_1 - d_1 + b_2 - d_2 \right] \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right]$$

$$-\ \frac{b^w}{3}\left[b_1 - d_1 + b_2 - d_2 \right] \underline{\varepsilon}$$

$$+\ (b^w)^2 \left[b_1 - d_1 + b_2 - d_2 \right] \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] +\ b^w \left[b_1 - d_1 + b_2 - d_2 \right] \overline{\varepsilon} \Bigg\}$$

$$= \; b^w \left[b^w \left(a^L + Q_1^r + Q_2^r - K_2 \right) + c_1 \right]$$

$$- \; 2\,(b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - b^w\,\overline{\varepsilon}$$

$$\Leftrightarrow \quad \frac{\partial E\{w\}}{\partial K_1} \Bigg\{ \overline{\varepsilon} - \underline{\varepsilon} + \frac{b^w}{3} \left[b_1 - d_1 + b_2 - d_2 \right] \left[3\,\overline{\varepsilon} - \underline{\varepsilon} \right]$$

$$+ \; (b^w)^2 \left[\, b_1 - d_1 + b_2 - d_2 \right] \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right]$$

$$- \; \frac{b^w}{3} \left[\, b_1 - d_1 + b_2 - d_2 \right] \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right] \Bigg\}$$

$$= \; b^w \left[b^w \left(a^L + Q_1^r + Q_2^r - K_2 \right) + c_1 \right]$$

$$- \; 2\,(b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - b^w\,\overline{\varepsilon}$$

$$\Leftrightarrow \quad \frac{\partial E\{w\}}{\partial K_1} \Bigg\{ \overline{\varepsilon} - \underline{\varepsilon} + \frac{b^w}{3} \left[b_1 - d_1 + b_2 - d_2 \right] \Bigg[3\,\overline{\varepsilon} - \underline{\varepsilon}$$

$$+ 3\,b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) - b^w \left(a^L + Q_1^r + Q_2^r \right) - c_1 - c_2 \Bigg] \Bigg\}$$

$$= \; (b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_2 \right] + b^w\,c_1$$

$$- \; 2\,(b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - b^w\,\overline{\varepsilon}$$

$$\Leftrightarrow \quad \frac{\partial E\{w\}}{\partial K_1} \Bigg\{ \overline{\varepsilon} - \underline{\varepsilon} + \frac{b^w}{3} \left[b_1 - d_1 + b_2 - d_2 \right] \Bigg[3\,\overline{\varepsilon} - \underline{\varepsilon}$$

$$+ \; 2\,b^w \left(a^L + Q_1^r + Q_2^r \right) - \; 3\,b^w \left(\, K_1 + K_2 \, \right) - c_1 - c_2 \Bigg] \Bigg\}$$

$$= \; (b^w)^2 \; \left[a^L + Q_1^r + Q_2^r - K_2 - 2\,a^L - 2\,Q_1^r - 2\,Q_2^r + 2\,K_1 + 2\,K_2 \right] + b^w\,c_1 - b^w\,\overline{\varepsilon}$$

$$\Leftrightarrow \quad \frac{\partial E\{w\}}{\partial K_1} \Bigg\{ \overline{\varepsilon} - \underline{\varepsilon} + \frac{b^w}{3} \left[b_1 - d_1 + b_2 - d_2 \right] \Bigg[3\,\overline{\varepsilon} - \underline{\varepsilon}$$

$$+ \; 2\,b^w \left(a^L + Q_1^r + Q_2^r \right) - \; 3\,b^w \left(\, K_1 + K_2 \, \right) - c_1 - c_2 \Bigg] \Bigg\}$$

$$= \; -\,(b^w)^2 \left[a^L + Q_1^r + Q_2^r - 2\,K_1 - K_2 \right] - b^w \left[\overline{\varepsilon} - c_1 \right]$$

$$\Leftrightarrow \quad \frac{\partial E\{w\}}{\partial K_1} \; = \; \frac{Y_1}{Y_2} \tag{329}$$

where:

$$Y_1 \;\equiv\; -\,(b^w)^2\left[\,a^L + Q_1^r + Q_2^r - 2\,K_1 - K_2\,\right] - b^w\left[\,\overline{\varepsilon} - c_1\,\right]; \tag{330}$$

$$\begin{aligned} Y_2 \;\equiv\; & \overline{\varepsilon} - \underline{\varepsilon} + \frac{b^w}{3}\left[\,b_1 - d_1 + b_2 - d_2\,\right] \\ & \cdot\left[\,3\,\overline{\varepsilon} - \underline{\varepsilon} + \;2\,b^w\left(a^L + Q_1^r + Q_2^r\right) - 3\,b^w\,(\,K_1 + K_2\,) - c_1 - c_2\,\right]. \end{aligned} \tag{331}$$

Because $r_i = E\{\,w(\varepsilon)\} + c_i^r,$ (1), (319), (327), and (328) imply:

$$\begin{aligned} \frac{\partial E\,\{w\}}{\partial K_2} \;=\; & \frac{1}{3}\left[\,b^w\left(a^L + Q_1^r + Q_2^r\right) + c_1 + c_2\,\right]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{\partial \varepsilon_0}{\partial K_2} + \frac{1}{3}\left[\,b^w\left(\frac{\partial Q_1^r}{\partial K_2} + \frac{\partial Q_2^r}{\partial K_2}\right)\right]\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] \\ & + \frac{1}{6}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right] 2\,\varepsilon_0\,\frac{\partial \varepsilon_0}{\partial K_2} + \frac{1}{2}\left[\,b^w\left(a^L + Q_1^r + Q_2^r - K_2\right) + c_1\,\right]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\frac{\partial \varepsilon_{12}}{\partial K_2} - \frac{\partial \varepsilon_0}{\partial K_2}\right] \\ & + \frac{1}{2}\left[\,b^w\left(\frac{\partial Q_1^r}{\partial K_2} + \frac{\partial Q_2^r}{\partial K_2} - 1\right)\right]\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right] + \frac{1}{4}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[2\,\varepsilon_{12}\,\frac{\partial \varepsilon_{12}}{\partial K_2} - 2\,\varepsilon_0\,\frac{\partial \varepsilon_0}{\partial K_2}\right] \\ & - b^w\left[\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\frac{\partial \varepsilon_{12}}{\partial K_2} \\ & + b^w\left[\frac{\partial Q_1^r}{\partial K_2} + \frac{\partial Q_2^r}{\partial K_2} - 1\right]\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] - \frac{1}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right] 2\,\varepsilon_{12}\,\frac{\partial \varepsilon_{12}}{\partial K_2} \end{aligned}$$

$$\begin{aligned} \Leftrightarrow\quad \frac{\partial E\,\{w\}}{\partial K_2} \;=\; & \frac{1}{3}\left[\,b^w\left(a^L + Q_1^r + Q_2^r\right) + c_1 + c_2\,\right]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[3\,b^w + b^w\,(\;b_1 - d_1 + b_2 - d_2\,)\,\frac{\partial E\{w\}}{\partial K_2}\right] \\ & - \frac{b^w}{3}\left[\,b_1 - d_1 + b_2 - d_2\,\right]\frac{\partial E\{w\}}{\partial K_2}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] \\ & + \frac{1}{3}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\varepsilon_0\left[3\,b^w + b^w\,(\;b_1 - d_1 + b_2 - d_2\,)\,\frac{\partial E\{w\}}{\partial K_2}\right] \\ & + \frac{1}{2}\left[\,b^w\left(a^L + Q_1^r + Q_2^r - K_2\right) + c_1\,\right]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right] \\ & \quad\cdot\left[b^w + b^w\,(\;b_1 - d_1 + b_2 - d_2\,)\,\frac{\partial E\{w\}}{\partial K_2} - 3\,b^w - b^w\,(\;b_1 - d_1 + b_2 - d_2\,)\,\frac{\partial E\{w\}}{\partial K_2}\right] \\ & + \frac{b^w}{2}\left[-\,(\,b_1 - d_1\,)\;\frac{\partial E\{w\}}{\partial K_2} - (\,b_2 - d_2\,)\;\frac{\partial E\{w\}}{\partial K_2} - 1\right]\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right] \\ & + \frac{1}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[\varepsilon_{12}\,\left(b^w + b^w\left[\;b_1 - d_1 + b_2 - d_2\,\right]\;\frac{\partial E\{w\}}{\partial K_2}\right)\right. \\ & \qquad\qquad \left. -\;\varepsilon_0\,\left(3\,b^w + b^w\left[\;b_1 - d_1 + b_2 - d_2\,\right]\;\frac{\partial E\{w\}}{\partial K_2}\right)\;\right] \end{aligned}$$

$$- b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w + b^w \left[b_1 - d_1 + b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_2} \right]$$

$$+ b^w \left[- (b_1 - d_1) \frac{\partial E\{w\}}{\partial K_2} - (b_2 - d_2) \frac{\partial E\{w\}}{\partial K_2} - 1 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \left[b^w + b^w \left[b_1 - d_1 + b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_2} \right]$$

$$\Leftrightarrow \quad \frac{\partial E\{w\}}{\partial K_2} \Bigg\{ 1 - \frac{1}{3} \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[b_1 - d_1 + b_2 - d_2 \right]$$

$$+ \frac{b^w}{3} \left[b_1 - d_1 + b_2 - d_2 \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \, b^w \left[b_1 - d_1 + b_2 - d_2 \right]$$

$$+ \frac{b^w}{2} \left[b_1 - d_1 + b_2 - d_2 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \frac{b^w}{2} \left[b_1 - d_1 + b_2 - d_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} - \varepsilon_0 \right]$$

$$+ (b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b_1 - d_1 + b_2 - d_2 \right]$$

$$+ b^w \left[b_1 - d_1 + b_2 - d_2 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ b^w \left[b_1 - d_1 + b_2 - d_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \Bigg\}$$

$$= b^w \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0$$

$$- b^w \left[b^w \left(a^L + Q_1^r + Q_2^r - K_2 \right) + c_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - \frac{b^w}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \frac{b^w}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} - 3\,\varepsilon_0 \right] - (b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12}$$

$$\Leftrightarrow \quad \frac{\partial E\{w\}}{\partial K_2} \Big\{ \overline{\varepsilon} - \underline{\varepsilon} - \frac{1}{3} \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right] b^w \left[b_1 - d_1 + b_2 - d_2 \right]$$

$$+ \frac{b^w}{3} \left[b_1 - d_1 + b_2 - d_2 \right] \left[\varepsilon_0 - \underline{\varepsilon} \right]$$

$$- \frac{b^w}{3} \varepsilon_0 \left[b_1 - d_1 + b_2 - d_2 \right]$$

$$+ (b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[b_1 - d_1 + b_2 - d_2 \right]$$

$$+ b^w \left[b_1 - d_1 + b_2 - d_2 \right] \left[\overline{\varepsilon} - \varepsilon_{12} \right] + b^w \left[b_1 - d_1 + b_2 - d_2 \right] \varepsilon_{12} \Big\}$$

$$= b^w \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right] + b^w \varepsilon_0$$

$$- b^w \left[b^w \left(a^L + Q_1^r + Q_2^r - K_2 \right) + c_1 \right] - \frac{b^w}{2} \left[\varepsilon_{12} - \varepsilon_0 \right]$$

$$+ \frac{b^w}{2} \left[\varepsilon_{12} - 3\,\varepsilon_0 \right] - (b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right]$$

$$- b^w \left[\overline{\varepsilon} - \varepsilon_{12} \right] - b^w \varepsilon_{12}$$

$$\Leftrightarrow \quad \frac{\partial E\{w\}}{\partial K_2} \Big\{ \overline{\varepsilon} - \underline{\varepsilon} + \frac{b^w}{3} \left[b_1 - d_1 + b_2 - d_2 \right] \Big[- \left(b^w \left[a^L + Q_1^r + Q_2^r \right] + c_1 + c_2 \right)$$

$$+ \varepsilon_0 - \underline{\varepsilon} - \varepsilon_0 + 3\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right)$$

$$+ 3 \left(\overline{\varepsilon} - \varepsilon_{12} \right) + 3\, \varepsilon_{12} \Big] \Big\}$$

$$= b^w \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 - b^w \left(a^L + Q_1^r + Q_2^r - K_2 \right) - c_1 \right]$$

$$+ b^w \left[\varepsilon_0 - \frac{1}{2} \left(\varepsilon_{12} - \varepsilon_0 \right) + \frac{1}{2} \left(\varepsilon_{12} - 3\, \varepsilon_0 \right) - \left(\overline{\varepsilon} - \varepsilon_{12} \right) - \varepsilon_{12} \right]$$

$$- (b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right]$$

$$\Leftrightarrow \quad \frac{\partial E\{w\}}{\partial K_2} \Big\{ \overline{\varepsilon} - \underline{\varepsilon} + \frac{b^w}{3} \left[b_1 - d_1 + b_2 - d_2 \right] \Big[3\, \overline{\varepsilon} - \underline{\varepsilon}$$

$$+ b^w \left[3\, a^L + 3\, Q_1^r + 3\, Q_2^r - 3\, K_1 - 3\, K_2 - a^L - Q_1^r - Q_2^r \right] - c_1 - c_2 \Big\}$$

$$= b^w \left[c_2 + b^w K_2 \right] - (b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - b^w \overline{\varepsilon}$$

$$\Leftrightarrow \quad \frac{\partial E\{w\}}{\partial K_2} \Big\{ \overline{\varepsilon} - \underline{\varepsilon} + \frac{b^w}{3} [\, b_1 - d_1 + b_2 - d_2] \Big[3\,\overline{\varepsilon} - \underline{\varepsilon}$$

$$+ \, b^w \left(2\,a^L + 2\,Q_1^r + 2\,Q_2^r - 3\,K_1 - 3\,K_2 \right) - c_1 - c_2 \Big] \Big\}$$

$$= \; b^w \left[c_2 + b^w K_2 - b^w \left(a^L + Q_1^r + Q_2^r - K_1 - K_2 \right) \right] - b^w \, \overline{\varepsilon}$$

$$\Leftrightarrow \quad \frac{\partial E\{w\}}{\partial K_2} \; = \; \frac{Y_3}{Y_4} \tag{332}$$

where

$$Y_3 \; \equiv \; - \, b^w \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 - 2\,K_2 \right) - c_2 \right] - b^w \, \overline{\varepsilon} \, ; \tag{333}$$

$$Y_4 \; \equiv \; \overline{\varepsilon} - \underline{\varepsilon} + \frac{b^w}{3} [\, b_1 - d_1 + b_2 - d_2 \,] \Big[3\,\overline{\varepsilon} - \underline{\varepsilon}$$

$$+ \; b^w \left(2\,a^L + 2\,Q_1^r + 2\,Q_2^r - 3\,K_1 - 3\,K_2 \right) - c_1 - c_2 \Big] . \tag{334}$$

Because $r_i \, = \, E\{\, w(\varepsilon)\} + c_i^r$, (1), (320), (321), (325) – (329), and (332) imply:

$$\frac{\partial E\{q_1^*\}}{\partial K_1} \; = \; \frac{1}{3\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_2 - 2\,c_1 \right] \frac{\partial \varepsilon_0}{\partial K_1}$$

$$+ \; \frac{1}{3\,b^w} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\frac{\partial Q_1^r}{\partial K_1} + \frac{\partial Q_2^r}{\partial K_1} \right] + \frac{1}{6\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 2\,\varepsilon_0 \, \frac{\partial \varepsilon_0}{\partial K_1}$$

$$+ \; \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_2 - \frac{1}{b^w} \, c_1 \right] \left[\frac{\partial \varepsilon_{12}}{\partial K_1} - \frac{\partial \varepsilon_0}{\partial K_1} \right]$$

$$+ \; \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{\partial Q_1^r}{\partial K_1} + \frac{\partial Q_2^r}{\partial K_1} \right] + \; \frac{1}{4\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[2\,\varepsilon_{12} \, \frac{\partial \varepsilon_{12}}{\partial K_1} - 2\,\varepsilon_0 \, \frac{\partial \varepsilon_0}{\partial K_1} \right]$$

$$+ \; \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{\partial \varepsilon_{12}}{\partial K_1}$$

$$= \; \frac{1}{3\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_2 - 2\,c_1 \right] b^w [\, b_1 - d_1 + b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_1}$$

$$- \; \frac{1}{3\,b^w} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w [\, b_1 - d_1 + b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_1}$$

$$+ \; \frac{1}{6\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 2\,\varepsilon_0 \left[b^w \, (\, b_1 - d_1 + b_2 - d_2 \,) \, \frac{\partial E\{w\}}{\partial K_1} \right]$$

$$+\ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+Q_1^r+Q_2^r-K_2-\frac{1}{b^w}\,c_1\right]$$

$$\cdot\left[2\,b^w+b^w\,(\,b_1-d_1+b_2-d_2)\,\frac{\partial E\{w\}}{\partial K_1}-b^w\,(\,b_1-d_1+b_2-d_2)\,\frac{\partial E\{w\}}{\partial K_1}\right]$$

$$-\ \frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b_1-d_1+b_2-d_2\,]\,\frac{\partial E\{w\}}{\partial K_1}$$

$$+\ \frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\Bigg\{\varepsilon_{12}\left[2\,b^w+b^w\,(b_1-d_1+b_2-d_2)\,\frac{\partial E\{w\}}{\partial K_1}\right]$$

$$-\,\varepsilon_0\left[b^w\,(b_1-d_1+b_2-d_2)\,\frac{\partial E\{w\}}{\partial K_1}\right]\Bigg\}$$

$$+\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]-K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[2\,b^w+b^w\,[\,b_1-d_1+b_2-d_2\,]\,\frac{\partial E\{w\}}{\partial K_1}\right]$$

$$=\ \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r\right)+c_2-2\,c_1\right][\,b_1-d_1+b_2-d_2\,]\,\frac{Y_1}{Y_2}$$

$$-\ \frac{1}{3}\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b_1-d_1+b_2-d_2\,]\,\frac{Y_1}{Y_2}$$

$$+\ \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\,[\,b_1-d_1+b_2-d_2\,]\,\frac{Y_1}{Y_2}$$

$$+\ b^w\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+Q_1^r+Q_2^r-K_2-\frac{1}{b^w}\,c_1\right]$$

$$-\ \frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b_1-d_1+b_2-d_2\,]\,\frac{Y_1}{Y_2}$$

$$+\ \frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\Bigg\{\varepsilon_{12}\left[2\,b^w+b^w\,(\,b_1-d_1+b_2-d_2\,)\,\frac{Y_1}{Y_2}\right]$$

$$-\,\varepsilon_0\left[b^w\,(\,b_1-d_1+b_2-d_2\,)\,\frac{Y_1}{Y_2}\right]\Bigg\}$$

$$+\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]-K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[2\,b^w+b^w\,(\,b_1-d_1+b_2-d_2\,)\,\frac{Y_1}{Y_2}\right]; \tag{335}$$

$$\frac{\partial E\{q_2^*\}}{\partial K_2}\ =\ \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+\ Q_1^r+Q_2^r\right)+c_1-2\,c_2\,\right]\,\frac{\partial\varepsilon_0}{\partial K_2}$$

$$+\ \frac{1}{3\,b^w}\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[b^w\left(\frac{\partial Q_1^r}{\partial K_2}+\frac{\partial Q_2^r}{\partial K_2}\right)\right]+\frac{1}{6\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\,2\,\varepsilon_0\,\frac{\partial\varepsilon_0}{\partial K_2}$$

$$+ \left[\frac{\overline{\varepsilon} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{\partial \varepsilon_0}{\partial K_2}$$

$$= \frac{1}{3\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 - 2\,c_2 \right] \left[3\,b^w + b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_2} \right]$$

$$- \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b_1 - d_1 + b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_2}$$

$$+ \frac{1}{3\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \left[3\,b^w + b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_2} \right]$$

$$+ \left[\frac{\overline{\varepsilon} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[3\,b^w + b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_2} \right]$$

$$= \frac{1}{3\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 - 2\,c_2 \right] \left[3\,b^w + b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{Y_3}{Y_4} \right]$$

$$- \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b_1 - d_1 + b_2 - d_2 \right] \frac{Y_3}{Y_4}$$

$$+ \frac{1}{3\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \left[3\,b^w + b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{Y_3}{Y_4} \right]$$

$$+ \left[\frac{\overline{\varepsilon} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[3\,b^w + b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{Y_3}{Y_4} \right]. \qquad (336)$$

Because $r_i = E\{ w(\varepsilon) \} + c_i^r$, (1), (322), (323), (325) – (329), and (332) imply:

$$\frac{\partial E\{ w(Q^*) \, q_1^* \}}{\partial K_1} = \frac{1}{9\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right]$$

$$\cdot \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_2 - 2\,c_1 \right] \frac{\partial \varepsilon_0}{\partial K_1}$$

$$+ \frac{1}{9\,b^w} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(\frac{\partial Q_1^r}{\partial K_1} + \frac{\partial Q_2^r}{\partial K_1} \right) \right] \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_2 - 2\,c_1 \right]$$

$$+ \frac{1}{9\,b^w} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right] \left[b^w \left(\frac{\partial Q_1^r}{\partial K_1} + \frac{\partial Q_2^r}{\partial K_1} \right) \right]$$

$$+ \frac{1}{18\,b^w} \left[2\,b^w \left(a^L + Q_1^r + Q_2^r \right) + 2\,c_2 - c_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 2\,\varepsilon_0 \frac{\partial \varepsilon_0}{\partial K_1}$$

$$+ \frac{1}{18\,b^w} \left[2\,b^w \left(\frac{\partial Q_1^r}{\partial K_1} + \frac{\partial Q_2^r}{\partial K_1} \right) \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]$$

$$+\ \frac{1}{27\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]3\,(\varepsilon_0)^2\,\frac{\partial\varepsilon_0}{\partial K_1}$$

$$+\ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_2\right)+c_1\,\right]$$

$$\cdot\left[a^L+Q_1^r+Q_2^r-\ K_2\ -\frac{1}{b^w}\,c_1\right]\left[\frac{\partial\varepsilon_{12}}{\partial K_1}-\frac{\partial\varepsilon_0}{\partial K_1}\right]$$

$$+\ \frac{1}{4}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[b^w\left(\frac{\partial Q_1^r}{\partial K_1}+\frac{\partial Q_2^r}{\partial K_1}\right)\right]\left[a^L+Q_1^r+Q_2^r-\ K_2\ -\frac{1}{b^w}\,c_1\right]$$

$$+\ \frac{1}{4}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_2\right)+c_1\,\right]\left[\frac{\partial Q_1^r}{\partial K_1}+\frac{\partial Q_2^r}{\partial K_1}\right]$$

$$+\ \frac{1}{4}\left[\,a^L+Q_1^r+Q_2^r-\ K_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[2\,\varepsilon_{12}\,\frac{\partial\varepsilon_{12}}{\partial K_1}-2\,\varepsilon_0\,\frac{\partial\varepsilon_0}{\partial K_1}\right]$$

$$+\ \frac{1}{4}\left[\frac{\partial Q_1^r}{\partial K_1}+\frac{\partial Q_2^r}{\partial K_1}\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right]$$

$$+\ \frac{1}{12\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[3\,(\varepsilon_{12})^2\,\frac{\partial\varepsilon_{12}}{\partial K_1}-3\,(\varepsilon_0)^2\,\frac{\partial\varepsilon_0}{\partial K_1}\right]$$

$$+\ b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$-\ b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{\partial\varepsilon_{12}}{\partial K_1}$$

$$+\ b^w\left[\frac{\partial Q_1^r}{\partial K_1}+\frac{\partial Q_2^r}{\partial K_1}-1\right]K_1\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+\ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2-(\varepsilon_{12})^2\,\right]-\ \frac{1}{2}\,K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\,2\,\varepsilon_{12}\,\frac{\partial\varepsilon_{12}}{\partial K_1}$$

$$=\ \frac{1}{9\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r\right)+c_1+c_2\,\right]$$

$$\cdot\ \left[\,b^w\left(a^L+Q_1^r+Q_2^r\right)+c_2-2\,c_1\,\right]\,b^w\left[\,b_1-d_1+b_2-d_2\,\right]\,\frac{\partial E\{w\}}{\partial K_1}$$

$$-\ \frac{1}{9\,b^w}\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\,b^w\,(\,b_1-d_1+b_2-d_2\,)\,\frac{\partial E\{w\}}{\partial K_1}\left[\,b^w\left(a^L+Q_1^r+Q_2^r\right)+c_2-2\,c_1\,\right]$$

$$-\ \frac{1}{9\,b^w}\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r\right)+c_1+c_2\,\right]\,b^w\left[\,b_1-d_1+b_2-d_2\,\right]\frac{\partial E\{w\}}{\partial K_1}$$

$$+\ \frac{1}{9}\left[\,2\,b^w\left(a^L+Q_1^r+Q_2^r\right)+2\,c_2-c_1\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\,\varepsilon_0\,[\,b_1-d_1+b_2-d_2\,]\,\frac{\partial E\{w\}}{\partial K_1}$$

$$-\ \frac{1}{9}\left[\,(\,b_1-d_1+b_2-d_1\,)\,\frac{\partial E\{w\}}{\partial K_1}\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,]$$

$$+\ \frac{1}{27\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]3\,(\varepsilon_0)^2\,b^w\,[\,b_1-d_1+b_2-d_2\,]\,\frac{\partial E\{w\}}{\partial K_1}$$

$$+\ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b^w\left(a^L+Q_1^r+Q_2^r-K_2\right)+c_1\,]$$

$$\cdot\left[a^L+Q_1^r+Q_2^r-\ K_2\ -\frac{1}{b^w}\,c_1\right]\left[2\,b^w+b^w\,(\,b_1-d_1+b_2-d_2\,)\,\frac{\partial E\{w\}}{\partial K_1}\right.$$

$$\left.-\ b^w\,(\,b_1-d_1+b_2-d_2\,)\,\frac{\partial E\{w\}}{\partial K_1}\right]$$

$$-\ \frac{1}{4}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]b^w\,[\,b_1-d_1+b_2-d_2\,]\,\frac{\partial E\{w\}}{\partial K_1}\left[a^L+Q_1^r+Q_2^r-\ K_2\ -\frac{1}{b^w}\,c_1\right]$$

$$-\ \frac{1}{4}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b^w\left(a^L+Q_1^r+Q_2^r-K_2\right)+c_1\,]\,[\,b_1-d_1+b_2-d_2\,]\,\frac{\partial E\{w\}}{\partial K_1}$$

$$+\ \frac{1}{2}\left[\,a^L+Q_1^r+Q_2^r-\ K_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\left(2\,b^w+b^w\,[\,b_1-d_1+b_2-d_2\,]\,\frac{\partial E\{w\}}{\partial K_1}\right)\right.$$

$$\left.-\ \varepsilon_0\left(b^w\,[\,b_1-d_1+b_2-d_2\,]\,\frac{\partial E\{w\}}{\partial K_1}\right)\right]$$

$$-\ \frac{1}{4}\,[\,b_1-d_1+b_2-d_2\,]\,\frac{\partial E\{w\}}{\partial K_1}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,]$$

$$+\ \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[(\varepsilon_{12})^2\left(2\,b^w+b^w\,[\,b_1-d_1+b_2-d_2\,]\,\frac{\partial E\{w\}}{\partial K_1}\right)\right.$$

$$\left.-\ (\varepsilon_0)^2\left(b^w\,[\,b_1-d_1+b_2-d_2\,]\,\frac{\partial E\{w\}}{\partial K_1}\right)\right]$$

$$+\ b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$-\ b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[2\,b^w+b^w\,(\,b_1-d_1+b_2-d_2\,)\,\frac{\partial E\{w\}}{\partial K_1}\right]$$

$$-\ b^w\left[\,(\,b_1-d_1+b_2-d_2\,)\,\frac{\partial E\{w\}}{\partial K_1}+1\right]K_1\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+\ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2-(\varepsilon_{12})^2\,\right]-\ K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_{12}\left[2\,b^w+b^w\,(\,b_1-d_1+b_2-d_2\,)\ \frac{\partial E\{w\}}{\partial K_1}\right]$$

$$=\ \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r\right)+c_1+c_2\,\right]$$

$$\cdot\ \left[\,b^w\left(a^L+Q_1^r+Q_2^r\right)+c_2-2\,c_1\,\right]\left[\,b_1-d_1+b_2-d_2\,\right]\ \frac{Y_1}{Y_2}$$

$$-\ \frac{1}{9}\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b_1-d_1+b_2-d_2\,\right]\frac{Y_1}{Y_2}\ \left[\,b^w\left(a^L+Q_1^r+Q_2^r\right)+c_2-2\,c_1\,\right]$$

$$-\ \frac{1}{9}\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r\right)+c_1+c_2\,\right]\left[\,b_1-d_1+b_2-d_2\,\right]\ \frac{Y_1}{Y_2}$$

$$+\ \frac{1}{9}\left[\,2\,b^w\left(a^L+Q_1^r+Q_2^r\right)+2\,c_2-c_1\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\ \varepsilon_0\left[\ b_1-d_1+b_2-d_2\,\right]\ \frac{Y_1}{Y_2}$$

$$-\ \frac{1}{9}\left[\,b_1-d_1+b_2-d_1\,\right]\ \frac{Y_1}{Y_2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right]$$

$$+\ \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right](\varepsilon_0)^2\ \left[\ b_1-d_1+b_2-d_2\,\right]\ \frac{Y_1}{Y_2}$$

$$+\ \ \frac{b^w}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_2\right)+c_1\,\right]\left[a^L+Q_1^r+Q_2^r-\ K_2\ -\frac{1}{b^w}\,c_1\right]$$

$$-\ \frac{1}{4}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]b^w\left[\,b_1-d_1+b_2-d_2\,\right]\ \frac{Y_1}{Y_2}\ \left[a^L+Q_1^r+Q_2^r-\ K_2\ -\frac{1}{b^w}\,c_1\right]$$

$$-\ \frac{1}{4}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_2\right)+c_1\,\right]\left[\,b_1-d_1+b_2-d_2\,\right]\ \frac{Y_1}{Y_2}$$

$$+\ \frac{1}{2}\left[\,a^L+Q_1^r+Q_2^r-\ K_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\left(2\,b^w+b^w\left[\,b_1-d_1+b_2-d_2\,\right]\frac{Y_1}{Y_2}\right)\right.$$

$$\left.-\ \varepsilon_0\left(b^w\left[\,b_1-d_1+b_2-d_2\,\right]\frac{Y_1}{Y_2}\right)\right]$$

$$-\ \frac{1}{4}\left[\,b_1-d_1+b_2-d_2\,\right]\ \frac{Y_1}{Y_2}\ \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right]$$

$$+\ \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[(\varepsilon_{12})^2\left(2\,b^w+b^w\left[\,b_1-d_1+b_2-d_2\,\right]\ \frac{Y_1}{Y_2}\right)\right.$$

$$\left.-\ (\varepsilon_0)^2\ b^w\left[\,b_1-d_1+b_2-d_2\,\right]\ \frac{Y_1}{Y_2}\right]$$

$$+\; b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$-\; b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[2\, b^w + b^w \left(\, b_1 - d_1 + b_2 - d_2 \,\right) \frac{Y_1}{Y_2} \right]$$

$$-\; b^w \left[\left(\, b_1 - d_1 + b_2 - d_2 \,\right) \frac{Y_1}{Y_2} + 1 \right] K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \,\right]$$

$$-\; K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \left[2\, b^w + b^w \left(\, b_1 - d_1 + b_2 - d_2 \,\right) \frac{Y_1}{Y_2} \right]; \tag{337}$$

$$\begin{aligned}
\frac{\partial E\{\, w(Q^*)\, q_2^* \,\}}{\partial K_2} \;=\; & \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \,\right] \\
& \cdot \left[\, b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 - 2\, c_2 \,\right] \frac{\partial \varepsilon_0}{\partial K_2} \\
& + \frac{1}{9\, b^w} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(\frac{\partial Q_1^r}{\partial K_2} + \frac{\partial Q_2^r}{\partial K_2} \right) \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 - 2\, c_2 \,\right] \\
& + \frac{1}{9\, b^w} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \,\right] b^w \left[\frac{\partial Q_1^r}{\partial K_2} + \frac{\partial Q_2^r}{\partial K_2} \right] \\
& + \frac{1}{18\, b^w} \left[\, 2\, b^w \left(a^L + Q_1^r + Q_2^r \right) + 2\, c_1 - c_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 2\, \varepsilon_0 \, \frac{\partial \varepsilon_0}{\partial K_2} \\
& + \frac{1}{18\, b^w} \left[2\, b^w \left(\frac{\partial Q_1^r}{\partial K_2} + \frac{\partial Q_2^r}{\partial K_2} \right) \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right] \\
& + \frac{1}{27\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 3\, (\varepsilon_0)^2 \, \frac{\partial \varepsilon_0}{\partial K_2} \\
& + \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_2 \right) + c_1 \,\right] \\
& + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_2 \right) + c_1 \,\right] K_2 \left[\frac{\partial \varepsilon_{12}}{\partial K_2} - \frac{\partial \varepsilon_0}{\partial K_2} \right] \\
& + \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w\, K_2 \left[\frac{\partial Q_1^r}{\partial K_2} + \frac{\partial Q_2^r}{\partial K_2} - 1 \right] \qquad (338) \\
& + \frac{1}{4}\, K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[2\, \varepsilon_{12} \, \frac{\partial \varepsilon_{12}}{\partial K_2} - 2\, \varepsilon_0 \, \frac{\partial \varepsilon_0}{\partial K_2} \right] + \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right] \\
& - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] K_2 \, \frac{\partial \varepsilon_{12}}{\partial K_2}
\end{aligned}$$

$$
+ \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] + \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\frac{\partial Q_1^r}{\partial K_2} + \frac{\partial Q_2^r}{\partial K_2} - 1 \right] K_2
$$

$$
+ \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right] - \frac{1}{2} K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 2\, \varepsilon_{12} \, \frac{\partial \varepsilon_{12}}{\partial K_2}
$$

$$
= \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right]
$$

$$
\cdot \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 - 2\, c_2 \right] \left[3\, b^w + b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_2} \right]
$$

$$
- \frac{1}{9} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b_1 - d_1 + b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_2} \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 - 2\, c_2 \right]
$$

$$
- \frac{1}{9} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right] \left[b_1 - d_1 + b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_2}
$$

$$
+ \frac{\varepsilon_0}{9\, b^w} \left[2\, b^w \left(a^L + Q_1^r + Q_2^r \right) + 2\, c_1 - c_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right]
$$

$$
\cdot \left[3\, b^w + b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_2} \right]
$$

$$
- \frac{1}{9} \left[\left(b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_2} \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]
$$

$$
+ \frac{1}{9\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2 \left[3\, b^w + b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_2} \right]
$$

$$
+ \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_2 \right) + c_1 \right]
$$

$$
+ \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_2 \right) + c_1 \right] K_2
$$

$$
\cdot \left[b^w + b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_2} - 3\, b^w - b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_2} \right]
$$

$$
- \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w K_2 \left[\left(b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_2} + 1 \right] \tag{339}
$$

$$
+ \frac{1}{2} K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \left(b^w + b^w \left[b_1 - d_1 + b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_2} \right) \right.
$$

$$
\left. - \varepsilon_0 \left(3\, b^w + b^w \left[b_1 - d_1 + b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_2} \right) \right]
$$

$$
\begin{aligned}
& + \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right] \\
& - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] K_2 \left[b^w + b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_2} \right] \\
& + \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \\
& - \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\left(b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_2} + 1 \right] K_2 + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right] \\
& - K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \left[b^w + b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_2} \right] \\
= & \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right] \\
& \quad \cdot \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 - 2\, c_2 \right] \left[3 + \left(b_1 - d_1 + b_2 - d_2 \right) \frac{Y_3}{Y_4} \right] \\
& - \frac{1}{9} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b_1 - d_1 + b_2 - d_2 \right] \frac{Y_3}{Y_4} \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 - 2\, c_2 \right] \\
& - \frac{1}{9} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r \right) + c_1 + c_2 \right] \left[b_1 - d_1 + b_2 - d_2 \right] \frac{Y_3}{Y_4} \\
& + \frac{\varepsilon_0}{9} \left[2\, b^w \left(a^L + Q_1^r + Q_2^r \right) + 2\, c_1 - c_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[3 + \left(b_1 - d_1 + b_2 - d_2 \right) \frac{Y_3}{Y_4} \right] \\
& - \frac{1}{9} \left[b_1 - d_1 + b_2 - d_2 \right] \frac{Y_3}{Y_4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right] \\
& + \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2 \left[3 + \left(b_1 - d_1 + b_2 - d_2 \right) \frac{Y_3}{Y_4} \right] \\
& + \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_2 \right) + c_1 \right] \\
& - b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_2 \right) + c_1 \right] K_2 \\
& - \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w K_2 \left[\left(b_1 - d_1 + b_2 - d_2 \right) \frac{Y_3}{Y_4} + 1 \right] \qquad (340) \\
& + \frac{1}{2} K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \left(b^w + b^w \left[b_1 - d_1 + b_2 - d_2 \right] \frac{Y_3}{Y_4} \right) \right.
\end{aligned}
$$

$$- \varepsilon_0 \left(3\, b^w + b^w \left[b_1 - d_1 + b_2 - d_2 \right] \frac{Y_3}{Y_4} \right) \Big]$$

$$+ \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$- \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] K_2 \left[b^w + b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{Y_3}{Y_4} \right]$$

$$+ \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right]$$

$$- \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\left(b_1 - d_1 + b_2 - d_2 \right) \frac{Y_3}{Y_4} + 1 \right] K_2$$

$$+ \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right] - K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \left[b^w + b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{Y_3}{Y_4} \right]. \quad (341)$$

Case 2. Vertical Integration ($\alpha_1^R = \alpha_2^R = \alpha_1^G = \alpha_2^G = 1$) and $G2$ is capacity-constrained when $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$.

Lemmas 1 – 3 imply that in this case:

$$E\{ w(\varepsilon) \} = \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w a^L + c_1 + c_2 \right] + \frac{1}{6} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]$$

$$+ \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{1}{2} b^w \left(a^L + a_2 - b_2 r_2 + d_2 r_1 - K_2 \right) + \frac{1}{2} c_1 \right]$$

$$+ \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$+ \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[a^L + a_1 - b_1 r_1 + d_1 r_2 + a_2 - b_2 r_2 + d_2 r_1 - K_1 - K_2 \right]$$

$$+ \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right]; \quad (342)$$

$$E\{q_1^*\} = \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ \frac{1}{3} \left[a^L + 3 \left(a_1 - b_1 r_1 + d_1 r_2 \right) \right] + \frac{1}{3\, b^w} \left[c_2 - 2\, c_1 \right] \right\}$$

$$+ \frac{1}{6\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]$$

$$+ \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ \frac{1}{2} \left[a^L + 2 \left(a_1 - b_1 r_1 + d_1 r_2 \right) + \left(a_2 - b_2 r_2 + d_2 r_1 \right) - K_2 \right] - \frac{1}{2\, b^w} c_1 \right\}$$

$$+ \frac{1}{4\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right] + K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]; \tag{343}$$

$$E\{q_2^*\} = \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left\{ \frac{1}{3} \left[\, a^L + 3\,(\, a_2 - b_2\, r_2 + d_2\, r_1 \,) \right] + \frac{1}{3\,b^w} \left[\, c_1 - 2\, c_2 \,\right] \right\}$$

$$+ \frac{1}{6\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right] + K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]; \tag{344}$$

$$E\{\, w(Q^*)\, q_1^* \,\} = \frac{1}{9} \left[\, b^w\, a^L + c_1 + c_2 \,\right] \left[a^L + 3\, Q_1^r + \frac{1}{b^w}\,(\, c_2 - 2\, c_1 \,) \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \frac{1}{18\,b^w} \left[\, b^w\, a^L + c_1 + c_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right]$$

$$+ \frac{1}{18} \left[a^L + 3\, Q_1^r + \frac{1}{b^w}\,(\, c_2 - 2\, c_1 \,) \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right]$$

$$+ \frac{1}{27\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^3 - (\underline{\varepsilon})^3 \,\right]$$

$$+ \frac{1}{4} \left[\, b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \,\right] \left[a^L + 2\, Q_1^r + Q_2^r - K_2 - \frac{1}{b^w}\, c_1 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \frac{1}{8\,b^w} \left[\, b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right]$$

$$+ \frac{1}{8} \left[a^L + 2\, Q_1^r + Q_2^r - K_2 - \frac{1}{b^w}\, c_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right]$$

$$+ \frac{1}{12\,b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^3 - (\varepsilon_0)^3 \,\right]$$

$$+ b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \frac{1}{2}\, K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \,\right]; \tag{345}$$

$$E\{\, w(Q^*)\, q_2^* \,\} = \frac{1}{9} \left[\, b^w\, a^L + c_1 + c_2 \,\right] \left[a^L + 3\, Q_2^r + \frac{1}{b^w}\,(\, c_1 - 2\, c_2 \,) \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \frac{1}{18\,b^w} \left[\, b^w\, a^L + c_1 + c_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right]$$

$$+\ \frac{1}{18}\left[a^L + 3\,Q_2^r + \frac{1}{b^w}\,(\,c_1 - 2\,c_2\,)\right]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\,(\varepsilon_0)^2 - (\underline{\varepsilon})^2\,]$$

$$+\ \frac{1}{27\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\,(\varepsilon_0)^3 - (\underline{\varepsilon})^3\,]$$

$$+\ \frac{1}{2}\left[\,b^w\left(a^L + Q_2^r - K_2\right) + c_1\,\right] K_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]$$

$$+\ \frac{1}{4}\,K_2\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\,(\varepsilon_{12})^2 - (\varepsilon_0)^2\,]$$

$$+\ b^w\left[\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right] K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]$$

$$+\ \frac{1}{2}\,K_2\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\,(\overline{\varepsilon})^2 - (\varepsilon_{12})^2\,]; \tag{346}$$

$$E\{\pi_i^R\} \;=\; [\,r_i - c_i^r - E\{\,w(Q^*)\,\}\,]\,Q_i^r(r_1, r_2) \quad \text{and}$$

$$E\{\pi_i^w\} \;=\; E\{\,w(Q^*)\,q_i^*\,\} - \;c_i\,E\{\,q_i^*\,\}\; - z_i\,k_i\,. \tag{347}$$

where $Q_i^r(r_1, r_2)$ is defined by (1).

(51) implies that in this case:

$$\varepsilon_0 \;=\; 3\,b^w\,K_2 + 2\,c_2 - c_1 - b^w\left[\,a^L + 3\,Q_2^r\,\right];$$

$$\varepsilon_{12} \;=\; 2\,b^w\,K_1 + b^w\,K_2 + c_1 - b^w\left[\,a^L + 2\,Q_1^r + Q_2^r\,\right]. \tag{348}$$

Because $r_i = E\{w(\varepsilon)\} + c_i^r$, (347) implies that firm i's optimal choice of k_i in the current setting satisfies:

$$\frac{d\left(E\{\pi_i^w\} + E\{\pi_i^R\}\right)}{dk_i} \;=\; \frac{dE\{\,w(Q^*)\,q_i^*\,\}}{dk_i} - \;c_i\,\frac{dE\{\,q_i^*\,\}}{dk_i} - z_i$$

$$+\ [\,r_i - c_i^r - E\{\,w(Q^*)\,\}\,]\,\frac{dQ_i^r(r_1, r_2)}{dk_i} + Q_i^r(r_1, r_2)\left[\frac{dr_i}{dk_i} - \frac{E\{\,w(Q^*)\,\}}{dk_i}\right] \;=\; 0$$

$$\Rightarrow\quad \frac{dE\{\,w(Q^*)\,q_i^*\,\}}{dk_i} - \;c_i\,\frac{dE\{\,q_i^*\,\}}{dk_i} - z_i \;=\; 0\,. \tag{349}$$

(369) reflects the fact that $r_i \;=\; E\{\,w(\varepsilon)\} + c_i^r$ (and so $\frac{dr_i}{dk_i} = \frac{E\{\,w(Q^*)\,\}}{dk_i}$).

(327) and (348) imply:

$$\frac{\partial \varepsilon_0}{\partial K_1} = -\, b^w \left[3 \, \frac{\partial Q_2^r}{\partial K_1} \right] = 3 \, b^w \, [\, b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_1} \, ;$$

$$\frac{\partial \varepsilon_{12}}{\partial K_1} = 2 \, b^w - b^w \left[2 \, \frac{\partial Q_1^r}{\partial K_1} + \frac{\partial Q_2^r}{\partial K_1} \right] = 2 \, b^w + 2 \, b^w \left[b_1 - d_1 + \frac{1}{2} \, (b_2 - d_2) \right] \frac{\partial E\{w\}}{\partial K_1} \, ;$$

$$\frac{\partial \varepsilon_0}{\partial K_2} = 3 \, b^w - b^w \left[3 \, \frac{\partial Q_2^r}{\partial K_2} \right] = 3 \, b^w + 3 \, b^w \, [\, b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_2} \, ;$$

$$\frac{\partial \varepsilon_{12}}{\partial K_2} = b^w - b^w \left[2 \, \frac{\partial Q_1^r}{\partial K_2} + \frac{\partial Q_2^r}{\partial K_2} \right] = b^w + 2 \, b^w \left[b_1 - d_1 + \frac{1}{2} \, (b_2 - d_2) \right] \frac{\partial E\{w\}}{\partial K_2} \, . \qquad (350)$$

Because $r_i = E\{w(\varepsilon)\} + c_i^r$, (1), (342), and (350) imply:

$$\begin{aligned}
\frac{\partial E\{w\}}{\partial K_1} = \; & \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \, a^L + c_1 + c_2 \,\right] \, \frac{\partial \varepsilon_0}{\partial K_1} + \frac{1}{6} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 2 \, \varepsilon_0 \, \frac{\partial \varepsilon_0}{\partial K_1} \\
& + \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{1}{2} \, b^w \left(a^L + Q_2^r - K_2 \right) + \frac{1}{2} \, c_1 \right] \left[\frac{\partial \varepsilon_{12}}{\partial K_1} - \frac{\partial \varepsilon_0}{\partial K_1} \right] \\
& + \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{1}{2} \, b^w \left(-\, b_2 \, \frac{\partial r_2}{\partial K_1} + d_2 \, \frac{\partial r_1}{\partial K_1} \right) \right] \\
& + \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[2 \, \varepsilon_{12} \, \frac{\partial \varepsilon_{12}}{\partial K_1} - 2 \, \varepsilon_0 \, \frac{\partial \varepsilon_0}{\partial K_1} \right] \\
& - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \frac{\partial \varepsilon_{12}}{\partial K_1} \\
& + \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[-\, b_1 \, \frac{\partial r_1}{\partial K_1} + d_1 \, \frac{\partial r_2}{\partial K_1} - b_2 \, \frac{\partial r_2}{\partial K_1} + d_2 \, \frac{\partial r_1}{\partial K_1} - 1 \right] \\
& - \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 2 \, \varepsilon_{12} \, \frac{\partial \varepsilon_{12}}{\partial K_1}
\end{aligned}$$

$$\begin{aligned}
\Leftrightarrow \quad \frac{\partial E\{w\}}{\partial K_1} = \; & \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \, a^L + c_1 + c_2 \,\right] \, b^w \, [\, b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_1} \\
& + \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \, b^w \, [\, b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_1} \\
& + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \,\right]
\end{aligned}$$

$$\cdot \left[2\, b^w + 2\, b^w \left(b_1 - d_1 + \frac{1}{2}\,[\, b_2 - d_2\,] \right) \frac{\partial E\{w\}}{\partial K_1} - \, 3\, b^w\, (b_2 - d_2)\, \frac{\partial E\{w\}}{\partial K_1} \right]$$

$$- \, \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w\, [\, b_2 - d_2\,]\, \frac{\partial E\{w\}}{\partial K_1}$$

$$+ \, \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \Bigg[\varepsilon_{12} \left(2\, b^w + 2\, b^w \left[b_1 - d_1 + \frac{1}{2}\,(b_2 - d_2) \right] \frac{\partial E\{w\}}{\partial K_1} \right)$$

$$- \, \varepsilon_0 \left(3\, b^w\, [\, b_2 - d_2\,]\, \frac{\partial E\{w\}}{\partial K_1} \right) \Bigg]$$

$$- \, 2\, (b^w)^2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2\, \right] \left[1 + \left(\, b_1 - d_1 + \frac{1}{2}\,[\, b_2 - d_2\,] \right) \frac{\partial E\{w\}}{\partial K_1} \right]$$

$$- \, \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[(b_1 - \, d_1 \, + b_2 - d_2)\, \frac{\partial E\{w\}}{\partial K_1} + 1 \right]$$

$$- \, \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \left[2\, b^w + 2\, b^w \left(\, b_1 - d_1 + \frac{1}{2}\,[\, b_2 - d_2\,] \right) \frac{\partial E\{w\}}{\partial K_1} \right]$$

$$\Leftrightarrow \quad \frac{\partial E\{w\}}{\partial K_1} \Bigg\{ 1 - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w\, a^L + c_1 + c_2\, \right]\, b^w\, [\, b_2 - d_2\,] - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0\, b^w\, [\, b_2 - d_2\,]$$

$$- \, \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(\, a^L + Q_2^r - K_2 \right) + c_1 \, \right] [\, 2\, b^w\, (b_1 - d_1) - 2\, b^w\, (b_2 - d_2)\,]$$

$$+ \, \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \, b^w\, [\, b_2 - d_2\,]$$

$$- \, \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, 2\, b^w\, \varepsilon_{12}\, (b_1 - d_1) + \varepsilon_{12}\, b^w\, (b_2 - d_2) - \varepsilon_0\, 3\, b^w\, (\, b_2 - d_2)\,]$$

$$+ \, 2\, (b^w)^2 \, \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2\, \right] \left[\, b_1 - d_1 + \frac{1}{2}\,(b_2 - d_2) \right]$$

$$+ \, \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \, [\, b_1 - \, d_1 \, + b_2 - d_2\,]$$

$$+ \, 2\, b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \, \varepsilon_{12} \, \left[\, b_1 - d_1 + \frac{1}{2}\,(b_2 - d_2) \right] \Bigg\}$$

$$= \, b^w \, \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(\, a^L + Q_2^r - K_2 \right) + c_1 \, \right] + b^w \, \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \, \varepsilon_{12}$$

$$- \, 2(b^w)^2 \, \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2\, \right] - b^w \, \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- 2\, b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12}$$

$$\Leftrightarrow \quad \frac{\partial E\{w\}}{\partial K_1} \Big\{ \overline{\varepsilon} - \underline{\varepsilon} - \left[b^w a^L + c_1 + c_2 \right] b^w [b_2 - d_2] - \varepsilon_0 b^w [b_2 - d_2]$$

$$- b^w \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right] [b_1 - d_1 - b_2 + d_2]$$

$$+ \frac{1}{2} [\varepsilon_{12} - \varepsilon_0] b^w [b_2 - d_2] - \frac{1}{2} \left[2\, b^w \varepsilon_{12} (b_1 - d_1) + b^w (b_2 - d_2) (\varepsilon_{12} - 3\, \varepsilon_0) \right]$$

$$+ 2\, (b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[b_1 - d_1 + \frac{1}{2} (b_2 - d_2) \right]$$

$$+ [\overline{\varepsilon} - \varepsilon_{12}] b^w [b_1 - d_1 + b_2 - d_2] + 2\, b^w \varepsilon_{12} \left[b_1 - d_1 + \frac{1}{2} (b_2 - d_2) \right] \Big\}$$

$$= b^w \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right] + b^w \varepsilon_{12}$$

$$- 2\, (b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - b^w [\overline{\varepsilon} - \varepsilon_{12}] - 2\, b^w \varepsilon_{12}$$

$$\Leftrightarrow \quad \frac{\partial E\{w\}}{\partial K_1} = \frac{Y_5}{Y_6} \tag{351}$$

where:

$$Y_5 \equiv b^w \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right] - 2\, (b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - b^w \, \overline{\varepsilon}\, ; \tag{352}$$

$$Y_6 \equiv \overline{\varepsilon} - \underline{\varepsilon} - \left[b^w a^L + c_1 + c_2 \right] b^w [b_2 - d_2] - \varepsilon_0 b^w [b_2 - d_2]$$

$$- b^w \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right] [b_1 - d_1 - b_2 + d_2]$$

$$+ \frac{1}{2} [\varepsilon_{12} - \varepsilon_0] b^w [b_2 - d_2] - \frac{1}{2} \left[2\, b^w \varepsilon_{12} (b_1 - d_1) + b^w (b_2 - d_2) (\varepsilon_{12} - 3\, \varepsilon_0) \right]$$

$$+ 2\, (b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[b_1 - d_1 + \frac{1}{2} (b_2 - d_2) \right]$$

$$+ [\overline{\varepsilon} - \varepsilon_{12}] b^w [b_1 - d_1 + b_2 - d_2] + 2\, b^w \varepsilon_{12} \left[b_1 - d_1 + \frac{1}{2} (b_2 - d_2) \right]. \tag{353}$$

Because $r_i = E\{w(\varepsilon)\} + c_i^r$, (1), (342), and (350) imply:

$$\frac{\partial E\{w(\varepsilon)\}}{\partial K_2} = \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w a^L + c_1 + c_2 \right] \frac{\partial \varepsilon_0}{\partial K_2} + \frac{1}{6} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 2\, \varepsilon_0 \frac{\partial \varepsilon_0}{\partial K_2}$$

$$+ \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right] \left[\frac{\partial \varepsilon_{12}}{\partial K_2} - \frac{\partial \varepsilon_0}{\partial K_2} \right]$$

$$+\frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right] b^w \left[\frac{\partial Q_2^r}{\partial K_2}-1\right]$$

$$+\frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[2\,\varepsilon_{12}\,\frac{\partial \varepsilon_{12}}{\partial K_2}-2\,\varepsilon_0\,\frac{\partial \varepsilon_0}{\partial K_2}\right]$$

$$-\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] b^w \left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]\frac{\partial \varepsilon_{12}}{\partial K_2}$$

$$+\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] b^w \left[\frac{\partial Q_1^r}{\partial K_2}+\frac{\partial Q_2^r}{\partial K_2}-1\right]-\frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]2\,\varepsilon_{12}\frac{\partial \varepsilon_{12}}{\partial K_2}$$

$$\Leftrightarrow \quad \frac{\partial E\{w(\varepsilon)\}}{\partial K_2} = \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w a^L+c_1+c_2\,\right]\left[3\,b^w+3\,b^w\,(b_2-d_2)\,\frac{\partial E\{w\}}{\partial K_2}\right]$$

$$+\frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\left[3\,b^w+3\,b^w\,(b_2-d_2)\,\frac{\partial E\{w\}}{\partial K_2}\right]$$

$$+\frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(\,a^L+Q_2^r-K_2\,\right)+\,c_1\,\right]$$

$$\cdot\left[\left(b^w+2\,b^w\left[b_1-d_1+\frac{1}{2}\,(b_2-d_2)\right]\frac{\partial E\{w\}}{\partial K_2}\right)\right.$$

$$\left.-\left(3\,b^w+3\,b^w\,[\,b_2-d_2\,]\,\frac{\partial E\{w\}}{\partial K_2}\right)\right]$$

$$-\frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right] b^w\left[(\,b_2-d_2\,)\frac{\partial E\{w\}}{\partial K_2}+1\right]$$

$$+\frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\left(b^w+2\,b^w\left[b_1-d_1+\frac{1}{2}\,(b_2-d_2)\right]\frac{\partial E\{w\}}{\partial K_2}\right)\right.$$

$$\left.-\,\varepsilon_0\left(3\,b^w+3\,b^w\,[\,b_2-d_2\,]\,\frac{\partial E\{w\}}{\partial K_2}\right)\right]$$

$$-\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right] b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]$$

$$\cdot\left[b^w+2\,b^w\left(b_1-d_1+\frac{1}{2}\,[\,b_2-d_2\,]\right)\frac{\partial E\{w\}}{\partial K_2}\right]$$

$$-\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] b^w\left[(\,b_1-d_1\,)\frac{\partial E\{w\}}{\partial K_2}+(\,b_2-d_2\,)\frac{\partial E\{w\}}{\partial K_2}+1\right]$$

$$- \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \left[b^w + 2\, b^w \left(b_1 - d_1 + \frac{1}{2} \left[b_2 - d_2 \right] \right) \frac{\partial E\{w\}}{\partial K_2} \right]$$

$$\Leftrightarrow \quad \frac{\partial E\{w(\varepsilon)\}}{\partial K_2} \Bigg\{ 1 - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w\, a^L + c_1 + c_2 \right] b^w \left[b_2 - d_2 \right]$$

$$- \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0\, b^w \left[b_2 - d_2 \right]$$

$$- \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right]$$

$$\cdot \left[2\, b^w \left(b_1 - d_1 + \frac{1}{2} \left[b_2 - d_2 \right] \right) - 3\, b^w \left(b_2 - d_2 \right) \right]$$

$$+ \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[b_2 - d_2 \right]$$

$$- \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12}\, 2\, b^w \left(b_1 - d_1 + \frac{1}{2} \left[b_2 - d_2 \right] \right) - \varepsilon_0\, 3\, b^w \left(b_2 - d_2 \right) \right]$$

$$+ 2 \left(b^w \right)^2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[b_1 - d_1 + \frac{1}{2} \left(b_2 - d_2 \right) \right]$$

$$+ \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[b_1 - d_1 + b_2 - d_2 \right]$$

$$+ 2\, b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \left[b_1 - d_1 + \frac{1}{2} \left(b_2 - d_2 \right) \right] \Bigg\}$$

$$= b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w\, a^L + c_1 + c_2 \right] + b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0$$

$$- b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right]$$

$$- \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w + \frac{1}{2}\, b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} - 3\, \varepsilon_0 \right]$$

$$- \left(b^w \right)^2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right]$$

$$- \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w - \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w\, \varepsilon_{12}$$

$$\Leftrightarrow \quad \frac{\partial E\{w(\varepsilon)\}}{\partial K_2} \Bigg\{ \overline{\varepsilon} - \underline{\varepsilon} - \left[b^w\, a^L + c_1 + c_2 \right] b^w \left[b_2 - d_2 \right] - \varepsilon_0\, b^w \left[b_2 - d_2 \right]$$

$$- \frac{1}{2} \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right]$$

$$\cdot \left[2\, b^w \left(b_1 - d_1 + \frac{1}{2} [\, b_2 - d_2 \,] \right) - 3\, b^w \; (b_2 - d_2) \right]$$

$$+ \frac{1}{2} [\, \varepsilon_{12} - \varepsilon_0 \,] \; b^w [\, b_2 - d_2 \,]$$

$$- \frac{1}{2} b^w \left[2\, \varepsilon_{12} (\, b_1 - d_1 \,) + (b_2 - d_2) \, (\, \varepsilon_{12} - 3\, \varepsilon_0 \,) \right]$$

$$+ 2\, (b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[b_1 - d_1 + \frac{1}{2} (b_2 - d_2) \right]$$

$$+ [\, \overline{\varepsilon} - \varepsilon_{12} \,] \, b^w \, [\, b_1 - d_1 + b_2 - d_2 \,]$$

$$+ \; 2\, b^w \, \varepsilon_{12} \left[b_1 - d_1 + \frac{1}{2} (b_2 - d_2) \right] \Bigg\}$$

$$= \; b^w \left[b^w \, a^L + c_1 + c_2 \right] + b^w \, \varepsilon_0 - b^w \; \left[b^w \left(a^L + Q_2^r - K_2 \right) + \; c_1 \right]$$

$$- \; (b^w)^2 \; \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right]$$

$$- \; \frac{1}{2} b^w \; [\, \varepsilon_{12} - \varepsilon_0 \,] + \frac{1}{2} b^w \, [\, \varepsilon_{12} \; - 3\, \varepsilon_0 \,] - [\, \overline{\varepsilon} - \varepsilon_{12} \,] \, b^w - \; b^w \, \varepsilon_{12}$$

$$\Leftrightarrow \quad \frac{\partial E\{w(\varepsilon)\}}{\partial K_2} \; = \; \frac{Y_7}{Y_8} \tag{354}$$

where:

$$Y_7 \; \equiv \; b^w \left[b^w \, a^L + c_1 + c_2 - b^w \left(a^L + Q_2^r - K_2 \right) - \; c_1 \right]$$

$$- \; (b^w)^2 \; \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right]$$

$$+ \; b^w \left[\varepsilon_0 - \; \varepsilon_{12} - \overline{\varepsilon} + \varepsilon_{12} - \frac{1}{2} \varepsilon_{12} + \frac{1}{2} \varepsilon_0 + \frac{1}{2} \varepsilon_{12} - \frac{3}{2} \varepsilon_0 \right]$$

$$= \; b^w \; [\, c_2 - b^w \, (\, Q_2^r - K_2 \,) \,] - (b^w)^2 \; \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - b^w \, \overline{\varepsilon} \, ; \tag{355}$$

$$Y_8 \; \equiv \; \overline{\varepsilon} - \underline{\varepsilon} - \left[b^w \, a^L + c_1 + c_2 \right] b^w \, [\; b_2 - d_2] - \varepsilon_0 \, b^w \, [\; b_2 - d_2]$$

$$- \; \frac{1}{2} \; \left[b^w \left(a^L + Q_2^r - K_2 \right) + \; c_1 \right]$$

$$\cdot \left[2\, b^w \left(b_1 - d_1 + \frac{1}{2} [\, b_2 - d_2 \,] \right) - 3\, b^w \, (b_2 - d_2) \right]$$

$$+ \frac{1}{2} [\, \varepsilon_{12} - \varepsilon_0 \,] \; b^w [\, b_2 - d_2 \,]$$

$$- \frac{1}{2} \, b^w \left[2\, \varepsilon_{12} (b_1 - d_1) + (b_2 - d_2) \, (\varepsilon_{12} - 3\, \varepsilon_0) \right]$$

$$+ 2\, (b^w)^2 \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \left[b_1 - d_1 + \frac{1}{2} (b_2 - d_2) \right]$$

$$+ [\, \overline{\varepsilon} - \varepsilon_{12} \,] \, b^w \, [\, b_1 - d_1 + b_2 - d_2 \,]$$

$$+ 2\, b^w \, \varepsilon_{12} \left[b_1 - d_1 + \frac{1}{2} (b_2 - d_2) \right] . \tag{356}$$

Because $r_i = E\{w\} + c_i^r$, (1), (343), (344), and (350) – (356) imply:

$$\frac{\partial E\{q_1^*\}}{\partial K_1} = \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{1}{3} \left(a^L + 3\, Q_1^r \right) + \frac{1}{3\, b^w} (\, c_2 - 2\, c_1 \,) \right] \frac{\partial \varepsilon_0}{\partial K_1}$$

$$+ \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{\partial Q_1^r}{\partial K_1} + \frac{1}{6\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 2\, \varepsilon_0 \, \frac{\partial \varepsilon_0}{\partial K_1}$$

$$+ \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{1}{2} \left(a^L + 2\, Q_1^r + Q_2^r - K_2 \right) - \frac{1}{2\, b^w} \, c_1 \right] \left[\frac{\partial \varepsilon_{12}}{\partial K_1} - \frac{\partial \varepsilon_0}{\partial K_1} \right]$$

$$+ \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[2 \, \frac{\partial Q_1^r}{\partial K_1} + \frac{\partial Q_2^r}{\partial K_1} \right]$$

$$+ \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[2\, \varepsilon_{12} \, \frac{\partial \varepsilon_{12}}{\partial K_1} - 2\, \varepsilon_0 \, \frac{\partial \varepsilon_0}{\partial K_1} \right]$$

$$+ \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{\partial \varepsilon_{12}}{\partial K_1}$$

$$= \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\, Q_1^r + \frac{1}{b^w} (\, c_2 - 2\, c_1 \,) \right] \left[3\, b^w \, (b_2 - d_2) \, \frac{\partial E\{w\}}{\partial K_1} \right]$$

$$- \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b_1 - d_1 \,] \, \frac{\partial E\{\, w \,\}}{\partial K_1}$$

$$+ \frac{1}{3\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \left[3\, b^w \, (b_2 - d_2) \, \frac{\partial E\{w\}}{\partial K_1} \right]$$

$$
\begin{aligned}
&+\ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+2\,Q_1^r+Q_2^r-K_2-\frac{1}{b^w}\,c_1\right]\\
&\quad\cdot\left[2\,b^w+2\,b^w\left(b_1-d_1+\frac{1}{2}\,[\,b_2-d_2\,]\right)\frac{\partial E\{w\}}{\partial K_1}-3\,b^w\,(b_2-d_2)\,\frac{\partial E\{w\}}{\partial K_1}\right]\\
&-\ \frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,2\,(b_1-d_1)+b_2-d_2\,]\,\frac{\partial E\{w\}}{\partial K_1}\\
&+\frac{1}{2\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\left(2\,b^w+2\,b^w\left[\,b_1-d_1+\frac{1}{2}\,(b_2-d_2)\right]\frac{\partial E\{w\}}{\partial K_1}\right)\right.\\
&\qquad\qquad\left.-\,\varepsilon_0\,\left(3\,b^w\,[\,b_2-d_2]\,\frac{\partial E\{w\}}{\partial K_1}\right)\,\right]\\
&+\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]-K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[2\,b^w+2\,b^w\left(b_1-d_1+\frac{1}{2}\,[\,b_2-d_2\,]\right)\frac{\partial E\{w\}}{\partial K_1}\right]\\
&=\ \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+3\,Q_1^r+\frac{1}{b^w}\,(c_2-2\,c_1)\right]\,b^w\,[\,b_2-d_2\,]\,\frac{Y_5}{Y_6}\\
&\quad-\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b_1-d_1\,]\,\frac{Y_5}{Y_6}+\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\,\varepsilon_0\,[\,b_2-d_2\,]\,\frac{Y_5}{Y_6}\\
&\quad+\ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+2\,Q_1^r+Q_2^r-K_2-\frac{1}{b^w}\,c_1\right]\\
&\qquad\cdot\left[2\,b^w+2\,b^w\left(b_1-d_1+\frac{1}{2}\,[\,b_2-d_2\,]\right)\,\frac{Y_5}{Y_6}-3\,b^w\,(\,b_2-d_2\,)\,\frac{Y_5}{Y_6}\right]\\
&\quad-\ \frac{1}{2}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,2\,(b_1-d_1)+b_2-d_2\,]\,\frac{Y_5}{Y_6}\\
&\quad+\frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\varepsilon_{12}\left(2+2\left[\,b_1-d_1+\frac{1}{2}\,(b_2-d_2)\right]\frac{Y_5}{Y_6}\right)\right.\\
&\qquad\qquad\left.-\,3\,\varepsilon_0\,(b_2-d_2)\,\frac{Y_5}{Y_6}\,\right]\\
&\quad+\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]-2\,b^w\,K_1\,\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[1+\left(b_1-d_1+\frac{1}{2}\,[\,b_2-d_2\,]\right)\frac{Y_5}{Y_6}\,\right]; \qquad (357)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E\{q_2^*\}}{\partial K_2}\ &=\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{1}{3}\,[\,a^L+3\,Q_2^r\,]+\frac{1}{3\,b^w}\,(\,c_1-2\,c_2\,)\right]\,\frac{\partial\varepsilon_0}{\partial K_2}+\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\,\frac{\partial Q_2^r}{\partial K_2}\\
&\quad+\frac{1}{6\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\,2\,\varepsilon_0\,\frac{\partial\varepsilon_0}{\partial K_2}+\left[\frac{\overline{\varepsilon}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]-K_2\,\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{\partial\varepsilon_0}{\partial K_2}
\end{aligned}
$$

$$= \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L + 3\,Q_2^r + \frac{1}{b^w}\,(\,c_1 - 2\,c_2\,)\right]\left[3\,b^w + 3\,b^w\,(b_2 - d_2)\,\frac{\partial E\{w\}}{\partial K_2}\right]$$

$$- \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b_2 - d_2\,]\,\frac{\partial E\{w\}}{\partial K_2}$$

$$+ \frac{1}{3\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\left[3\,b^w + 3\,b^w\,(b_2 - d_2)\,\frac{\partial E\{w\}}{\partial K_2}\right]$$

$$+ \left[\frac{\overline{\varepsilon}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right] - K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[3\,b^w + 3\,b^w\,(b_2 - d_2)\,\frac{\partial E\{w\}}{\partial K_2}\right]$$

$$= b^w\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L + 3\,Q_2^r + \frac{1}{b^w}\,(c_1 - 2\,c_2)\right]\left[1 + (b_2 - d_2)\,\frac{Y_7}{Y_8}\right]$$

$$- \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b_2 - d_2\,]\,\frac{Y_7}{Y_8} + \left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\varepsilon_0\left[1 + (b_2 - d_2)\,\frac{Y_7}{Y_8}\right]$$

$$+ \left[\frac{\overline{\varepsilon}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right] - 3\,b^w\,K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[1 + (b_2 - d_2)\,\frac{Y_7}{Y_8}\right]. \tag{358}$$

Because $r_i = E\{w\} + c_i^r$, (1), (345), (346), and (350) – (356) imply:

$$\frac{\partial E\{\,w(Q^*)\,q_1^*\,\}}{\partial K_1} = \frac{1}{9}\,[\,b^w\,a^L + c_1 + c_2\,]\left[a^L + 3\,Q_1^r + \frac{1}{b^w}\,(\,c_2 - 2\,c_1\,)\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{\partial \varepsilon_0}{\partial K_1}$$

$$+ \frac{1}{9}\,[\,b^w\,a^L + c_1 + c_2\,]\left[3\,\frac{\partial Q_1^r}{\partial K_1}\right]\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+ \frac{1}{18\,b^w}\,[\,b^w\,a^L + c_1 + c_2\,]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]2\,\varepsilon_0\,\frac{\partial \varepsilon_0}{\partial K_1}$$

$$+ \frac{1}{18}\left[a^L + 3\,Q_1^r + \frac{1}{b^w}\,(\,c_2 - 2\,c_1\,)\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]2\,\varepsilon_0\,\frac{\partial \varepsilon_0}{\partial K_1}$$

$$+ \frac{1}{18}\left[3\,\frac{\partial Q_1^r}{\partial K_1}\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,(\varepsilon_0)^2 - (\underline{\varepsilon})^2\,]$$

$$+ \frac{1}{27\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]3\,(\varepsilon_0)^2\,\frac{\partial \varepsilon_0}{\partial K_1}$$

$$+ \frac{1}{4}\,[\,b^w\left(a^L + Q_2^r - K_2\right) + c_1\,]\left[a^L + 2\,Q_1^r + Q_2^r - K_2 - \frac{1}{b^w}\,c_1\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\frac{\partial \varepsilon_{12}}{\partial K_1} - \frac{\partial \varepsilon_0}{\partial K_1}\right]$$

$$+ \frac{1}{4}\left[b^w\,\frac{\partial Q_2^r}{\partial K_1}\right]\left[a^L + 2\,Q_1^r + Q_2^r - K_2 - \frac{1}{b^w}\,c_1\right]\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+\ \frac{1}{4}\left[\, b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \,\right] \left[2\,\frac{\partial Q_1^r}{\partial K_1} + \frac{\partial Q_2^r}{\partial K_1} \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+\ \frac{1}{8\,b^w}\left[\, b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[2\,\varepsilon_{12}\,\frac{\partial \varepsilon_{12}}{\partial K_1} - 2\,\varepsilon_0\,\frac{\partial \varepsilon_0}{\partial K_1} \right]$$

$$+\ \frac{1}{8\,b^w}\left[b^w\,\frac{\partial Q_2^r}{\partial K_1} \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right]$$

$$+\ \frac{1}{8}\left[a^L + 2\,Q_1^r + Q_2^r - K_2 - \frac{1}{b^w}\,c_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[2\,\varepsilon_{12}\,\frac{\partial \varepsilon_{12}}{\partial K_1} - 2\,\varepsilon_0\,\frac{\partial \varepsilon_0}{\partial K_1} \right]$$

$$+\ \frac{1}{8}\left[2\,\frac{\partial Q_1^r}{\partial K_1} + \frac{\partial Q_2^r}{\partial K_1} \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right]$$

$$+\ \frac{1}{12\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[3\,(\varepsilon_{12})^2\,\frac{\partial \varepsilon_{12}}{\partial K_1} - 3\,(\varepsilon_0)^2\,\frac{\partial \varepsilon_0}{\partial K_1} \right]$$

$$-\ b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{\partial \varepsilon_{12}}{\partial K_1}$$

$$+\ b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+\ b^w \left[\frac{\partial Q_1^r}{\partial K_1} + \frac{\partial Q_2^r}{\partial K_1} - 1 \right] K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+\ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \,\right] - \frac{1}{2}\,K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 2\,\varepsilon_{12}\,\frac{\partial \varepsilon_{12}}{\partial K_1}$$

$$=\ \frac{1}{3}\left[\, b^w\,a^L + c_1 + c_2 \,\right] \left[a^L + 3\,Q_1^r + \frac{1}{b^w}\,(\,c_2 - 2\,c_1\,) \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\, b_2 - d_2 \,\right] \frac{\partial E\{w\}}{\partial K_1}$$

$$-\ \frac{1}{3}\left[\, b^w\,a^L + c_1 + c_2 \,\right] \left[\, b_1 - d_1 \,\right] \frac{\partial E\{w\}}{\partial K_1} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+\ \frac{1}{3}\left[\, b^w\,a^L + c_1 + c_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \left[\, b_2 - d_2 \,\right] \frac{\partial E\{w\}}{\partial K_1}$$

$$+\ \frac{1}{3}\left[a^L + 3\,Q_1^r + \frac{1}{b^w}\,(\,c_2 - 2\,c_1\,) \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0\,b^w \left[\, b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_1}$$

$$-\ \frac{1}{6}\left[\, b_1 - d_1 \,\right] \frac{\partial E\{w\}}{\partial K_1} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right]$$

$$+\ \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right](\varepsilon_0)^2\ [\,b_2-d_2\,]\,\frac{\partial E\{w\}}{\partial K_1}$$

$$+\ \frac{1}{4}\left[\,b^w\left(\,a^L+Q_2^r-K_2\,\right)+c_1\,\right]\left[a^L+2\,Q_1^r+Q_2^r-K_2-\frac{1}{b^w}\,c_1\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$\cdot\left[2\,b^w+2\,b^w\left(b_1-d_1+\frac{1}{2}\,[\,b_2-d_2\,]\right)\frac{\partial E\{w\}}{\partial K_1}-3\,b^w\,(b_2-d_2)\,\frac{\partial E\{w\}}{\partial K_1}\right]$$

$$-\ \frac{1}{4}\,b^w\ [\,b_2-d_2\,]\,\frac{\partial E\{w\}}{\partial K_1}\left[a^L+2\,Q_1^r+Q_2^r-K_2-\frac{1}{b^w}\,c_1\right]\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$-\ \frac{1}{4}\left[\,b^w\left(\,a^L+Q_2^r-K_2\,\right)+c_1\,\right][\,2\,(b_1-d_1)+b_2-d_2\,]\,\frac{\partial E\{w\}}{\partial K_1}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$+\ \frac{1}{4\,b^w}\left[\,b^w\left(\,a^L+Q_2^r-K_2\,\right)+c_1\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$\cdot\left[\varepsilon_{12}\left(2\,b^w+2\,b^w\left[b_1-d_1+\frac{1}{2}\,(b_2-d_2)\right]\frac{\partial E\{w\}}{\partial K_1}\right)\right.$$

$$\left.-\ \varepsilon_0\,3\,b^w\ (b_2-d_2)\ \frac{\partial E\{w\}}{\partial K_1}\right]$$

$$-\ \frac{1}{8}\,[\,b_2-d_2\,]\,\frac{\partial E\{w\}}{\partial K_1}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,]$$

$$+\ \frac{1}{4}\left[a^L+2\,Q_1^r+Q_2^r-K_2-\frac{1}{b^w}\,c_1\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$\cdot\left[\varepsilon_{12}\left(2\,b^w+2\,b^w\left[\,b_1-d_1+\frac{1}{2}\,(b_2-d_2)\right]\frac{\partial E\{w\}}{\partial K_1}\right)\right.$$

$$\left.-\ \varepsilon_0\,3\,b^w\,(b_2-d_2)\,\frac{\partial E\{w\}}{\partial K_1}\right]$$

$$-\ \frac{1}{8}\,[\,2\,(b_1-d_1)+b_2-d_2\,]\,\frac{\partial E\{w\}}{\partial K_1}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,]$$

$$+\ \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[(\varepsilon_{12})^2\ \left(2\,b^w+2\,b^w\left[b_1-d_1+\frac{1}{2}\,(b_2-d_2)\right]\frac{\partial E\{w\}}{\partial K_1}\right)\right.$$

$$\left.-\ (\varepsilon_0)^2\,3\,b^w\,(b_2-d_2)\ \frac{\partial E\{w\}}{\partial K_1}\right]$$

$$-\,2\,(b^w)^2\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$\cdot \left[1 + \left(b_1 - d_1 + \frac{1}{2} [b_2 - d_2] \right) \frac{\partial E\{w\}}{\partial K_1} \right]$$

$$+ \; b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \; b^w \left[(b_1 - d_1 + b_2 - d_2) \frac{\partial E\{w\}}{\partial K_1} + 1 \right] K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \; \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [(\overline{\varepsilon})^2 - (\varepsilon_{12})^2]$$

$$- \; K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \left[2 \, b^w + 2 \, b^w \left(b_1 - d_1 + \frac{1}{2} [b_2 - d_2] \right) \frac{\partial E\{w\}}{\partial K_1} \right]$$

$$= \; \frac{1}{3} \left[b^w a^L + c_1 + c_2 \right] \left[a^L + 3 \, Q_1^r + \frac{1}{b^w} (c_2 - 2 \, c_1) \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w [\, b_2 - d_2] \, \frac{Y_5}{Y_6}$$

$$- \; \frac{1}{3} \left[b^w a^L + c_1 + c_2 \right] [b_1 - d_1] \, \frac{Y_5}{Y_6} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \; \frac{1}{3} \left[b^w a^L + c_1 + c_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \; [\, b_2 - d_2] \, \frac{Y_5}{Y_6}$$

$$+ \; \frac{1}{3} \left[a^L + 3 \, Q_1^r + \frac{1}{b^w} (c_2 - 2 \, c_1) \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \, b^w [b_2 - d_2] \, \frac{Y_5}{Y_6}$$

$$- \; \frac{1}{6} [b_1 - d_1] \, \frac{Y_5}{Y_6} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [(\varepsilon_0)^2 - (\underline{\varepsilon})^2]$$

$$+ \; \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2 \; [b_2 - d_2] \, \frac{Y_5}{Y_6}$$

$$+ \; \frac{1}{4} \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right] \left[a^L + 2 \, Q_1^r + Q_2^r - K_2 - \frac{1}{b^w} \, c_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$\cdot \left[2 \, b^w + 2 \, b^w \left(b_1 - d_1 + \frac{1}{2} [b_2 - d_2] \right) \frac{Y_5}{Y_6} - 3 \, b^w \; (b_2 - d_2) \, \frac{Y_5}{Y_6} \right]$$

$$- \; \frac{1}{4} \, b^w \; [b_2 - d_2] \, \frac{Y_5}{Y_6} \left[a^L + 2 \, Q_1^r + Q_2^r - K_2 - \frac{1}{b^w} \, c_1 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \; \frac{1}{4} \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right] [2 \, (b_1 - d_1) + b_2 - d_2] \, \frac{Y_5}{Y_6} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \; \frac{1}{4 \, b^w} \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$\cdot \Bigg[\varepsilon_{12} \Bigg(2\, b^w + 2\, b^w \left[b_1 - d_1 + \frac{1}{2}\,(b_2 - d_2) \right] \frac{Y_5}{Y_6} \Bigg)$$

$$- \varepsilon_0\, 3\, b^w\, (b_2 - d_2)\, \frac{Y_5}{Y_6} \Bigg]$$

$$- \frac{1}{8}\,[\, b_2 - d_2\,]\, \frac{Y_5}{Y_6} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, (\varepsilon_{12})^2 - (\varepsilon_0)^2\,]$$

$$+ \frac{1}{4} \left[a^L + 2\, Q_1^r + Q_2^r - K_2 - \frac{1}{b^w}\, c_1 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$\cdot \Bigg[\varepsilon_{12} \Bigg(2\, b^w + 2\, b^w \left[b_1 - d_1 + \frac{1}{2}\,(b_2 - d_2) \right] \frac{Y_5}{Y_6} \Bigg)$$

$$- \varepsilon_0\, 3\, b^w\, (b_2 - d_2)\, \frac{Y_5}{Y_6} \Bigg]$$

$$- \frac{1}{8}\,[\, 2\,(b_1 - d_1) + b_2 - d_2\,]\, \frac{Y_5}{Y_6} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, (\varepsilon_{12})^2 - (\varepsilon_0)^2\,]$$

$$+ \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \Bigg[(\varepsilon_{12})^2 \Bigg(2\, b^w + 2\, b^w \left[b_1 - d_1 + \frac{1}{2}\,(b_2 - d_2) \right] \frac{Y_5}{Y_6} \Bigg)$$

$$- (\varepsilon_0)^2\, 3\, b^w\, (b_2 - d_2)\, \frac{Y_5}{Y_6} \Bigg]$$

$$- 2\,(b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$\cdot \left[1 + \left(b_1 - d_1 + \frac{1}{2}\,[\, b_2 - d_2\,] \right) \frac{Y_5}{Y_6} \right]$$

$$+ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b^w \left[(\, b_1 - d_1 + b_2 - d_2\,)\, \frac{Y_5}{Y_6} + 1 \right] K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2\,]$$

$$- K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \left[2\, b^w + 2\, b^w \left(b_1 - d_1 + \frac{1}{2}\,[\, b_2 - d_2\,] \right) \frac{Y_5}{Y_6} \right]; \qquad (359)$$

$$\frac{\partial E\{\, w(Q^*)\, q_2^*\,\}}{\partial K_2} = \frac{1}{9}\,[\, b^w\, a^L + c_1 + c_2\,] \left[a^L + 3\, Q_2^r + \frac{1}{b^w}\,(\, c_1 - 2\, c_2\,) \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{\partial \varepsilon_0}{\partial K_2}$$

$$+ \frac{1}{9}\left[\, b^w\, a^L + c_1 + c_2 \,\right] \left[3\, \frac{\partial Q_2^r}{\partial K_2} \right] \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \frac{1}{18\, b^w}\left[\, b^w\, a^L + c_1 + c_2 \,\right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 2\, \varepsilon_0\, \frac{\partial \varepsilon_0}{\partial K_2}$$

$$+ \frac{1}{18}\left[a^L + 3\, Q_2^r + \frac{1}{b^w}\,(\, c_1 - 2\, c_2 \,) \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 2\, \varepsilon_0\, \frac{\partial \varepsilon_0}{\partial K_2}$$

$$+ \frac{1}{18}\left[3\, \frac{\partial Q_2^r}{\partial K_2} \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,]$$

$$+ \frac{1}{27\, b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 3\, (\varepsilon_0)^2\, \frac{\partial \varepsilon_0}{\partial K_2}$$

$$+ \frac{1}{2}\left[\, b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \,\right] K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\frac{\partial \varepsilon_{12}}{\partial K_2} - \frac{\partial \varepsilon_0}{\partial K_2} \right]$$

$$+ \frac{1}{2}\left[\, b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \,\right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \frac{1}{2}\, b^w \left[\frac{\partial Q_2^r}{\partial K_2} - 1 \right] K_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \frac{1}{4}\, K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[2\, \varepsilon_{12}\, \frac{\partial \varepsilon_{12}}{\partial K_2} - 2\, \varepsilon_0\, \frac{\partial \varepsilon_0}{\partial K_2} \right]$$

$$+ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,]$$

$$- b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \frac{\partial \varepsilon_{12}}{\partial K_2}$$

$$+ b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ b^w \left[\frac{\partial Q_1^r}{\partial K_2} + \frac{\partial Q_2^r}{\partial K_2} - 1 \right] K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \frac{1}{2}\, K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 2\, \varepsilon_{12}\, \frac{\partial \varepsilon_{12}}{\partial K_2} + \frac{1}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \,]$$

$$= \frac{1}{3}\, b^w \left[\, b^w\, a^L + c_1 + c_2 \,\right] \left[a^L + 3\, Q_2^r + \frac{1}{b^w}\,(\, c_1 - 2\, c_2 \,) \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[1 + (b_2 - d_2)\, \frac{\partial E\{w\}}{\partial K_2} \right]$$

$$- \frac{1}{3} \left[b^w a^L + c_1 + c_2 \right] \left[b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_2} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \frac{1}{3} \left[b^w a^L + c_1 + c_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \left[1 + (b_2 - d_2) \frac{\partial E\{w\}}{\partial K_2} \right]$$

$$+ \frac{1}{3} b^w \left[a^L + 3\, Q_2^r + \frac{1}{b^w} (c_1 - 2\, c_2) \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \left[1 + (b_2 - d_2) \frac{\partial E\{w\}}{\partial K_2} \right]$$

$$- \frac{1}{6} \left[b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]$$

$$+ \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2 \left[1 + (b_2 - d_2) \frac{\partial E\{w\}}{\partial K_2} \right]$$

$$+ \frac{1}{2} \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right] K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$\cdot \left[b^w + 2\, b^w \left(b_1 - d_1 + \frac{1}{2} \left[b_2 - d_2 \right] \right) \frac{\partial E\{w\}}{\partial K_2} - 3\, b^w - 3\, b^w (b_2 - d_2) \frac{\partial E\{w\}}{\partial K_2} \right]$$

$$+ \frac{1}{2} \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \frac{1}{2} b^w \left[(b_2 - d_2) \frac{\partial E\{w\}}{\partial K_2} + 1 \right] K_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \frac{1}{2} K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \left(b^w + 2\, b^w \left[b_1 - d_1 + \frac{1}{2} (b_2 - d_2) \right] \frac{\partial E\{w\}}{\partial K_2} \right) \right.$$

$$\left. - \varepsilon_0 \left(3\, b^w + 3\, b^w \left[b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_2} \right) \right]$$

$$+ \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$- (b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$\cdot \left[1 + 2 \left(b_1 - d_1 + \frac{1}{2} \left[b_2 - d_2 \right] \right) \frac{\partial E\{w\}}{\partial K_2} \right]$$

$$+ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- b^w \left[(b_1 - d_1 + b_2 - d_2) \frac{\partial E\{w\}}{\partial K_2} + 1 \right] K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \left[b^w + 2\, b^w \left(b_1 - d_1 + \frac{1}{2} \left[b_2 - d_2 \right] \right) \frac{\partial E\{w\}}{\partial K_2} \right]$$

$$+ \, \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right]$$

$$= \frac{1}{3}\, b^w \left[b^w\, a^L + c_1 + c_2 \right] \left[a^L + 3\, Q_2^r + \frac{1}{b^w} \left(c_1 - 2\, c_2 \right) \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[1 + (b_2 - d_2) \, \frac{Y_7}{Y_8} \right]$$

$$- \, \frac{1}{3} \left[b^w\, a^L + c_1 + c_2 \right] \left[b_2 - d_2 \right] \frac{Y_7}{Y_8} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \, \frac{1}{3} \left[b^w\, a^L + c_1 + c_2 \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \left[1 + (b_2 - d_2) \, \frac{Y_7}{Y_8} \right]$$

$$+ \, \frac{1}{3}\, b^w \left[a^L + 3\, Q_2^r + \frac{1}{b^w} \left(c_1 - 2\, c_2 \right) \right] \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0 \left[1 + (b_2 - d_2) \, \frac{Y_7}{Y_8} \right]$$

$$- \, \frac{1}{6} \left[b_2 - d_2 \right] \frac{Y_7}{Y_8} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right]$$

$$+ \, \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2 \left[1 + (b_2 - d_2) \, \frac{Y_7}{Y_8} \right]$$

$$+ \, \frac{1}{2} \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right] K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$\cdot \left[b^w + 2\, b^w \left(b_1 - d_1 + \frac{1}{2} \left[b_2 - d_2 \right] \right) \frac{Y_7}{Y_8} - 3\, b^w - 3\, b^w \, (b_2 - d_2) \, \frac{Y_7}{Y_8} \right]$$

$$+ \, \frac{1}{2} \left[b^w \left(a^L + Q_2^r - K_2 \right) + c_1 \right] \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, \frac{1}{2}\, b^w \left[(b_2 - d_2) \, \frac{Y_7}{Y_8} + 1 \right] K_2 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$+ \, \frac{1}{2}\, K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \left(b^w + 2\, b^w \left[b_1 - d_1 + \frac{1}{2} \, (b_2 - d_2) \right] \frac{Y_7}{Y_8} \right) \right.$$

$$\left. - \, \varepsilon_0 \left(3\, b^w + 3\, b^w \left[b_2 - d_2 \right] \frac{Y_7}{Y_8} \right) \right]$$

$$+ \, \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$- \, (b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$\cdot \left[1 + 2 \left(b_1 - d_1 + \frac{1}{2} [\, b_2 - d_2 \,] \right) \frac{Y_7}{Y_8} \right]$$

$$+ \, b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, b^w \left[(b_1 - d_1 + b_2 - d_2) \frac{Y_7}{Y_8} + 1 \right] K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \left[b^w + 2\, b^w \left(b_1 - d_1 + \frac{1}{2} [\, b_2 - d_2 \,] \right) \frac{Y_7}{Y_8} \right]$$

$$+ \, \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \,]. \tag{360}$$

Case 3. Partial Vertical Integration ($\alpha_1^R = \alpha_1^G = 1; \alpha_2^R = \alpha_2^G = 0$) and $G1$ is capacity-constrained when $\varepsilon \in (\varepsilon_0, \varepsilon_{12})$.

Lemmas 1 – 3 imply that in this case:

$$E\{\, w(Q^*) \,\} \;=\; \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \right] + \frac{1}{6} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,]$$

$$+ \, \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \right]$$

$$+ \, \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,] + b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right]$$

$$+ \, \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \,] \,; \tag{361}$$

$$E\{q_1^*\} \;=\; \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\, Q_1^r + Q_2^r + \frac{1}{b^w} (\, c_2 - 2\, c_1 \,) \right]$$

$$+ \, \frac{1}{6\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,] + K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]; \tag{362}$$

$$E\{q_2^*\} \;=\; \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_2^r + \frac{1}{b^w} (\, c_1 - 2\, c_2 \,) \right] + \frac{1}{6\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,]$$

$$+ \, \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 - \frac{c_2}{b^w} \right]$$

$$+\ \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right]+K_2\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]; \tag{363}$$

$$\begin{aligned}
E\{\,w(Q^*)\,q_1^*\,\} \;=\; & \frac{1}{9}\left[\,b^w\left(a^L+Q_2^r\right)+c_1+c_2\,\right]\left[a^L+3\,Q_1^r+Q_2^r+\frac{1}{b^w}\,(\,c_2-2\,c_1\,)\right]\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \\
& +\ \frac{1}{18\,b^w}\left[\,b^w\left(a^L+Q_2^r\right)+c_1+c_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right] \\
& +\ \frac{1}{18}\left[a^L+3\,Q_1^r+Q_2^r+\frac{1}{b^w}\,(\,c_2-2\,c_1\,)\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right] \\
& +\ \frac{1}{27\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^3-(\underline{\varepsilon})^3\,\right] \\
& +\ \frac{1}{2}\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1\right)+c_2\,\right]K_1\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \\
& +\ \frac{1}{4}\,K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right] \\
& +\ b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]K_1\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \\
& +\ \frac{1}{2}\,K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\overline{\varepsilon})^2-(\varepsilon_{12})^2\,\right];
\end{aligned} \tag{364}$$

$$\begin{aligned}
E\{\,w(Q^*)\,q_2^*\,\} \;=\; & \frac{1}{9}\left[\,b^w\left(a^L+Q_2^r\right)+c_1+c_2\,\right]\left[a^L+Q_2^r+\frac{1}{b^w}\,(\,c_1-2\,c_2\,)\right]\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \\
& +\ \frac{1}{18\,b^w}\left[\,b^w\left(a^L+Q_2^r\right)+c_1+c_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right] \\
& +\ \frac{1}{18}\left[a^L+Q_2^r+\frac{1}{b^w}\,(\,c_1-2\,c_2\,)\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right] \\
& +\ \frac{1}{27\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^3-(\underline{\varepsilon})^3\,\right] \\
& +\ \frac{1}{4}\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1\right)+c_2\,\right]\left[a^L+Q_1^r+Q_2^r-K_1\,-\,\frac{1}{b^w}\,c_2\right]\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right] \\
& +\ \frac{1}{8\,b^w}\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1\right)+c_2\,\right]\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right]
\end{aligned}$$

$$+\ \frac{1}{8}\left[a^L + Q_1^r + Q_2^r - K_1\ - \frac{1}{b^w}\,c_2\right]\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\,(\varepsilon_{12})^2 - (\varepsilon_0)^2\,]$$

$$+\ \frac{1}{12\,b^w}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\,(\varepsilon_{12})^3 - (\varepsilon_0)^3\,]$$

$$+\ b^w\left[\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,\right]K_2\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]$$

$$+\ \frac{1}{2}\,K_2\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\,(\overline{\varepsilon})^2 - (\varepsilon_{12})^2\,]. \tag{365}$$

The expected retail profit and wholesale profit of firm i are, respectively:

$$E\{\pi_i^R\} \;=\; [\,r_i - c_i^r - E\{\,w(Q^*)\,\}\,]\,Q_i^r(r_1, r_2) \;\text{ and}$$

$$E\{\pi_i^w\} \;=\; E\{\,w(Q^*)\,q_i^*\,\} - \,c_i\,E\{\,q_i^*\,\} \,-\, z_i\,k_i\,, \tag{366}$$

where $Q_i^r(r_1, r_2)$ is defined by (1).

(23) and (25) imply that in this case:

$$\varepsilon_0 \;=\; 3\,b^w\,K_1 + 2\,c_1 - c_2 - b^w\left[\,a^L + 3\,Q_1^r + Q_2^r\,\right];$$

$$\varepsilon_{12} \;=\; 2\,b^w\,K_2 + b^w\,K_1 + c_2 - b^w\left[\,a^L + Q_1^r + Q_2^r\,\right]. \tag{367}$$

(366) implies that firm 2's optimal choice of K_2 is determined by:

$$\frac{dE\{\pi_2^w\}}{dK_2} \;=\; \frac{dE\{\,w(Q^*)\,q_2^*\,\}}{dK_2} -\, c_2\,\frac{dE\{\,q_2^*\,\}}{dK_2} - z_2 \;=\; 0\,. \tag{368}$$

(366) implies that firm 1's optimal choice of K_1 is determined by:

$$\frac{d\left(E\{\pi_1^w\} + E\{\pi_1^R\}\right)}{dK_1} \;=\; \frac{dE\{\,w(Q^*)\,q_1^*\,\}}{dK_1} -\, c_1\,\frac{dE\{\,q_1^*\,\}}{dK_1} - z_1$$

$$+\ [\,r_1 - c_1^r - E\{\,w(Q^*)\,\}\,]\,\frac{dQ_1^r(r_1, r_2)}{dK_1} + Q_i^r(r_1, r_2)\left[\,\frac{dr_1}{dK_1} - \frac{E\{\,w(Q^*)\,\}}{dK_1}\,\right] \;=\; 0$$

$$\Rightarrow\quad \frac{dE\{\,w(Q^*)\,q_1^*\,\}}{dK_1} -\, c_1\,\frac{dE\{\,q_1^*\,\}}{dK_1} - z_1 \;=\; 0\,. \tag{369}$$

(369) reflects the fact that $r_i \;=\; E\{\,w(\varepsilon)\} + c_i^r$ (and so $\frac{dr_1}{dK_1} \;=\; \frac{E\{\,w(Q^*)\,\}}{dK_1}$).

Because $r_i \;=\; E\{\,w(\varepsilon)\} + c_i^r$, (1) implies:

$$\frac{\partial Q_i^r}{\partial k_i} \;=\; -\,b_i\,\frac{\partial r_i}{\partial k_i} + d_i\,\frac{\partial r_j}{\partial k_i} \;=\; -\,[\,b_i - d_i\,]\;\frac{\partial E\{w\}}{\partial k_i}\,;$$

$$\frac{\partial Q_j^r}{\partial k_i} \;=\; -\,b_j\,\frac{\partial r_j}{\partial k_i} + d_j\,\frac{\partial r_i}{\partial k_i} \;=\; -\,[\,b_j - d_j\,]\,\frac{\partial E\{w\}}{\partial k_i}\,. \tag{370}$$

(367) and (370) imply:

$$\begin{aligned}
\frac{\partial \varepsilon_0}{\partial K_1} &= 3\,b^w - b^w\left[3\,\frac{\partial Q_1^r}{\partial K_1} + \frac{\partial Q_2^r}{\partial K_1}\right] = 3\,b^w + b^w\,[\,3\,(b_1 - d_1) + b_2 - d_2\,]\,\frac{\partial E\{w\}}{\partial K_1};\\
\frac{\partial \varepsilon_0}{\partial K_2} &= -\,b^w\left[3\,\frac{\partial Q_1^r}{\partial K_2} + \frac{\partial Q_2^r}{\partial K_2}\right] = b^w\,[\,3\,(b_1 - d_1) + b_2 - d_2\,]\,\frac{\partial E\{w\}}{\partial K_2}\,;\\
\frac{\partial \varepsilon_{12}}{\partial K_1} &= b^w - b^w\left[\frac{\partial Q_1^r}{\partial K_1} + \frac{\partial Q_2^r}{\partial K_1}\right] = b^w + b^w\,[\,b_1 - d_1 + b_2 - d_2\,]\,\frac{\partial E\{w\}}{\partial K_1}\,;\\
\frac{\partial \varepsilon_{12}}{\partial K_2} &= 2\,b^w - b^w\left[\frac{\partial Q_1^r}{\partial K_2} + \frac{\partial Q_2^r}{\partial K_2}\right] = 2\,b^w + b^w\,[\,b_1 - d_1 + b_2 - d_2\,]\,\frac{\partial E\{w\}}{\partial K_2}\,.
\end{aligned} \tag{371}$$

(361), (370), and (371) imply:

$$\begin{aligned}
\frac{dE\{w\}}{dK_1} &= \frac{1}{3}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\,b^w\left(a^L + Q_2^r\right) + c_1 + c_2\,]\,\frac{d\varepsilon_0}{dK_1}\\
&- \frac{1}{3}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\,b^w\,(\,b_2 - d_2\,)\,]\,\frac{\partial E\{w\}}{\partial K_1} + \frac{1}{6}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]2\,\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\\
&+ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\,b^w\left(a^L + Q_1^r + Q_2^r - K_1\right) + c_2\,]\left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1}\right] - \frac{1}{2}\,b^w\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\\
&- \frac{1}{2}\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}}\right]b^w\,[\,b_1 - d_1 + b_2 - d_2\,]\,\frac{\partial E\{w\}}{\partial K_1} + \frac{1}{4}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]\left[2\,\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1} - 2\,\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}\right]\\
&- b^w\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\,a^L + Q_1^r + Q_2^r - K_1 - K_2\,]\,\frac{d\varepsilon_{12}}{dK_1}\\
&- b^w\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\,b_1 - d_1 + b_2 - d_2\,]\,\frac{\partial E\{w\}}{\partial K_1} - b^w\left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}}\right] - \frac{1}{2}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]2\,\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_1}
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{3}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right][\,b^w\left(a^L + Q_2^r\right) + c_1 + c_2\,]\left[3\,b^w + b^w\,(\,3\,[\,b_1 - d_1\,] + b_2 - d_2\,)\,\frac{\partial E\{w\}}{\partial K_1}\right]\\
&\quad - \frac{1}{3}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}\right]b^w\,[\,b_2 - d_2\,]\,\frac{\partial E\{w\}}{\partial K_1}\\
&\quad + \frac{1}{6}\left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}}\right]2\,\varepsilon_0\left[3\,b^w + b^w\,(\,3\,[\,b_1 - d_1\,] + b_2 - d_2\,)\,\frac{\partial E\{w\}}{\partial K_1}\right]
\end{aligned}$$

$$
\begin{aligned}
&+ \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \right] \\
&\qquad \cdot \left[b^w + b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_1} \right. \\
&\qquad\qquad \left. - \left(3\, b^w + b^w \left[3 \left(b_1 - d_1 \right) + b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_1} \right) \right] \\
&- \frac{1}{2} b^w \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[b_1 - d_1 + b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_1} \\
&+ \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \left(b^w + b^w \left[b_1 - d_1 + b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_1} \right) \right. \\
&\qquad\qquad \left. - \varepsilon_0 \left(3\, b^w + b^w \left[3 \left(b_1 - d_1 \right) + b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_1} \right) \right] \\
&- b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[b^w + b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_1} \right] \\
&- b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b_1 - d_1 + b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_1} - b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \\
&- \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \left[b^w + b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_1} \right] . \qquad (372)
\end{aligned}
$$

(372) implies:

$$
\begin{aligned}
\frac{dE\{w\}}{dK_1} \Bigg\{ & 1 - \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \right] b^w \left[3 \left(b_1 - d_1 \right) + b_2 - d_2 \right] \\
&+ \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[b_2 - d_2 \right] - \frac{1}{3} \left[\frac{\varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[3 \left(b_1 - d_1 \right) + b_2 - d_2 \right] \\
&- \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \right] \\
&\quad \cdot \left[b^w \left(b_1 - d_1 + b_2 - d_2 \right) - b^w \left(3 \left[b_1 - d_1 \right] + b_2 - d_2 \right) \right] \\
&+ \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[b_1 - d_1 + b_2 - d_2 \right] \\
&- \frac{b^w}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \left(b_1 - d_1 + b_2 - d_2 \right) - \varepsilon_0 \left(3 \left[b_1 - d_1 \right] + b_2 - d_2 \right) \right] \\
&+ b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] b^w \left[b_1 - d_1 + b_2 - d_2 \right]
\end{aligned}
$$

$$+ \ b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b_1 - d_1 + b_2 - d_2 \,] + \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_{12} \, b^w \, [\, b_1 - d_1 + b_2 - d_2 \,] \Bigg\}$$

$$= \ b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \right] + b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \varepsilon_0$$

$$- \ b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \right]$$

$$- \ \frac{1}{2} \, b^w \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] + \frac{b^w}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, \varepsilon_{12} - 3 \ \varepsilon_0 \,]$$

$$- \ (b^w)^2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w \left[\frac{\varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] . \qquad (373)$$

(373) implies:

$$\frac{dE\{w\}}{dK_1} \Bigg\{ \overline{\varepsilon} - \underline{\varepsilon} - \frac{b^w}{3} \left[b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \right] [\, 3 \, (b_1 - d_1) + b_2 - d_2 \,]$$

$$+ \ \frac{1}{3} \, [\, \varepsilon_0 - \underline{\varepsilon} \,] \, b^w \, [\, b_2 - d_2 \,] - \frac{1}{3} \, [\, \varepsilon_0 \,] \, b^w \, [\, 3 \, (\, b_1 - d_1 \,) + b_2 - d_2 \,]$$

$$- \ \frac{1}{2} \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \right]$$

$$\cdot \, [\, b^w \, (\, b_1 - d_1 + b_2 - d_2 \,) - b^w \, (\, 3 \, [\, b_1 - d_1 \,] + b_2 - d_2 \,) \,]$$

$$+ \ \frac{1}{2} \, b^w \, [\, \varepsilon_{12} - \varepsilon_0 \,] \, [\, b_1 - d_1 + b_2 - d_2 \,]$$

$$- \ \frac{b^w}{2} \, [\, \varepsilon_{12} \, (\, b_1 - d_1 + b_2 - d_2 \,) - \varepsilon_0 \, (\, 3 \, [\, b_1 - d_1 \,] + b_2 - d_2 \,) \,]$$

$$+ \ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] b^w \, [\, b_1 - d_1 + b_2 - d_2 \,]$$

$$+ \ b^w \, [\, \overline{\varepsilon} - \varepsilon_{12} \,] \, [\, b_1 - d_1 + b_2 - d_2 \,] + \varepsilon_{12} \, b^w \, [\, b_1 - d_1 + b_2 - d_2 \,] \Bigg\}$$

$$= \ b^w \left[b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \right] + b^w \, \varepsilon_0$$

$$- \ b^w \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \right] - \frac{1}{2} \, b^w \, [\, \varepsilon_{12} - \varepsilon_0 \,]$$

$$+ \ \frac{b^w}{2} \, [\, \varepsilon_{12} - 3 \ \varepsilon_0 \,] - (b^w)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - b^w \, \overline{\varepsilon}$$

$$\Rightarrow \quad \frac{dE\{w\}}{dK_1} \left\{ \overline{\varepsilon} - \underline{\varepsilon} - \frac{b^w}{3} \left[b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \right] \left[3 \left(b_1 - d_1 \right) + b_2 - d_2 \right] \right.$$

$$- \frac{1}{3} \underline{\varepsilon} \, b^w \left[b_2 - d_2 \right] - b^w \varepsilon_0 \left[b_1 - d_1 \right]$$

$$+ b^w \left[b_1 - d_1 \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \right]$$

$$- \frac{1}{2} b^w \varepsilon_0 \left[b_1 - d_1 + b_2 - d_2 \right] + \frac{1}{2} b^w \varepsilon_0 \left[3 \left(b_1 - d_1 \right) + b_2 - d_2 \right]$$

$$+ \left(b^w \right)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[b_1 - d_1 + b_2 - d_2 \right]$$

$$\left. + b^w \overline{\varepsilon} \left[b_1 - d_1 + b_2 - d_2 \right] \right\}$$

$$= b^w c_1 - \left(b^w \right)^2 \left[Q_1^r - K_1 \right] + b^w \varepsilon_0 - \frac{1}{2} b^w \left[\varepsilon_{12} - \varepsilon_0 \right]$$

$$+ \frac{b^w}{2} \left[\varepsilon_{12} - 3 \, \varepsilon_0 \right] - \left(b^w \right)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - b^w \overline{\varepsilon} \tag{374}$$

$$\Rightarrow \quad \frac{dE\{w\}}{dK_1} \left\{ \overline{\varepsilon} - \underline{\varepsilon} - \frac{b^w}{3} \left[b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \right] \left[3 \left(b_1 - d_1 \right) + b_2 - d_2 \right] \right.$$

$$- \frac{1}{3} \underline{\varepsilon} \, b^w \left[b_2 - d_2 \right] - b^w \varepsilon_0 \left[b_1 - d_1 \right]$$

$$+ b^w \left[b_1 - d_1 \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \right] + b^w \varepsilon_0 \left[b_1 - d_1 \right]$$

$$+ \left(b^w \right)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[b_1 - d_1 + b_2 - d_2 \right]$$

$$\left. + b^w \overline{\varepsilon} \left[b_1 - d_1 + b_2 - d_2 \right] \right\}$$

$$= b^w c_1 - b^w \overline{\varepsilon} - \left(b^w \right)^2 \left[Q_1^r - K_1 \right] - \left(b^w \right)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right]. \tag{375}$$

(374) implies:

$$\frac{dE\{w\}}{dK_1} = \frac{Y_{3B1}}{Y_{3B2}} \tag{376}$$

where:

$$Y_{3B1} \equiv c_1 - \overline{\varepsilon} - b^w \left[Q_1^r - K_1 \right] - b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right];$$

$$Y_{3B2} \equiv \frac{\overline{\varepsilon} - \underline{\varepsilon}}{b^w} - \frac{1}{3} \left[b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \right] \left[3 \left(b_1 - d_1 \right) + b_2 - d_2 \right]$$

$$
\begin{aligned}
&- \ \frac{1}{3}\, \underline{\varepsilon}\, [\, b_2 - d_2 \,] + [\, b_1 - d_1 \,]\, \Big[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \,\Big] \\
&+ \ b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] [\, b_1 - d_1 + b_2 - d_2 \,] \\
&+ \overline{\varepsilon}\, [\, b_1 - d_1 + b_2 - d_2 \,]\,. \qquad (377)
\end{aligned}
$$

(361), (370), and (371) imply:

$$
\begin{aligned}
\frac{dE\{w\}}{dK_2} \ = \ & \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \,\right] \frac{d\varepsilon_0}{dK_2} \\
& - \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b^w \,(\, b_2 - d_2 \,)\,]\, \frac{\partial E\{w\}}{\partial K_2} + \frac{1}{6} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 2\, \varepsilon_0\, \frac{d\varepsilon_0}{dK_2} \\
& + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \,\right] \left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2} \right] \\
& - \ \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w\, [\, b_1 - d_1 + b_2 - d_2 \,]\ \frac{\partial E\{w\}}{\partial K_2} + \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[2\, \varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_2} - 2\, \varepsilon_0\, \frac{d\varepsilon_0}{dK_2} \right] \\
& - b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \frac{d\varepsilon_{12}}{dK_2} \\
& - \ b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b_1 - d_1 + b_2 - d_2 \,]\, \frac{\partial E\{w\}}{\partial K_2} - b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 2\ \varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_2}
\end{aligned}
$$

$$
\begin{aligned}
= \ & \frac{1}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \,\right]\ b^w\, [\, 3\, (b_1 - d_1) + b_2 - d_2 \,]\, \frac{\partial E\{w\}}{\partial K_2} \\
& - \ \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w\, [\, b_2 - d_2 \,]\, \frac{\partial E\{w\}}{\partial K_2} + \frac{1}{6} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 2\, \varepsilon_0\ b^w\, [\, 3\, (b_1 - d_1) + b_2 - d_2 \,]\, \frac{\partial E\{w\}}{\partial K_2} \\
& + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \,\right] \\
& \qquad \cdot \left[2\, b^w + b^w\, (\, b_1 - d_1 + b_2 - d_2 \,)\, \frac{\partial E\{w\}}{\partial K_2} - b^w\, (\, 3\, [\, b_1 - d_1 \,] + b_2 - d_2 \,)\, \frac{\partial E\{w\}}{\partial K_2} \right] \\
& - \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w\, [\, b_1 - d_1 + b_2 - d_2 \,]\ \frac{\partial E\{w\}}{\partial K_2} \\
& + \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \Bigg[2\, \varepsilon_{12} \left(2\, b^w + b^w\, [\, b_1 - d_1 + b_2 - d_2 \,]\, \frac{\partial E\{w\}}{\partial K_2} \right) \\
& \qquad\qquad - 2\, \varepsilon_0 \left(b^w\, [\, 3\, (b_1 - d_1) + b_2 - d_2 \,]\, \frac{\partial E\{w\}}{\partial K_2} \right) \Bigg]
\end{aligned}
$$

$$
- \, b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \, \right] \left[2 \, b^w + b^w \, (\, b_1 - d_1 + b_2 - d_2 \,) \, \frac{\partial E\{w\}}{\partial K_2} \right]
$$

$$
- \, b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b_1 - d_1 + b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_2} - b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]
$$

$$
- \left[\frac{\varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[2 \, b^w + b^w \, (\, b_1 - d_1 + b_2 - d_2 \,) \, \frac{\partial E\{w\}}{\partial K_2} \right] . \tag{378}
$$

(378) implies:

$$
\frac{dE\{w\}}{dK_2} \left\{ 1 - \frac{b^w}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(\, a^L + Q_2^r \right) + c_1 + c_2 \, \right] [\, 3 \, (b_1 - d_1) + b_2 - d_2 \,] \right.
$$

$$
+ \, \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \, [\, b_2 - d_2 \,] - \frac{\varepsilon_0 \, b^w}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, 3 \, (b_1 - d_1) + b_2 - d_2 \,]
$$

$$
- \, \frac{b^w}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(\, a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \, \right]
$$

$$
\cdot \, [\, b_1 - d_1 + b_2 - d_2 \, - (\, 3 \, [\, b_1 - d_1 \,] + b_2 - d_2 \,) \,]
$$

$$
+ \, \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \, [\, b_1 - d_1 + b_2 - d_2 \,]
$$

$$
- \, \frac{b^w}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, \varepsilon_{12} \, (\, b_1 - d_1 + b_2 - d_2 \,) - \, \varepsilon_0 \, (\, 3 \, [\, b_1 - d_1] + b_2 - d_2 \,) \,]
$$

$$
+ \, b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \, \right] b^w \, [\, b_1 - d_1 + b_2 - d_2 \,]
$$

$$
\left. + \, b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b_1 - d_1 + b_2 - d_2 \,] + \left[\frac{\varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \, [\, b_1 - d_1 + b_2 - d_2 \,] \right\}
$$

$$
= \, b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(\, a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \, \right] + b^w \left[\frac{\varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]
$$

$$
- \, 2 \, (b^w)^2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \, \right] - b^w \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - 2 \, b^w \left[\frac{\varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] . \tag{379}
$$

(379) implies:

$$
\frac{dE\{w\}}{dK_2} \left\{ \overline{\varepsilon} - \underline{\varepsilon} - \frac{b^w}{3} \left[\, b^w \left(\, a^L + Q_2^r \right) + c_1 + c_2 \, \right] [\, 3 \, (b_1 - d_1) + b_2 - d_2 \,] \right.
$$

$$
+ \, \frac{1}{3} \, [\, \varepsilon_0 - \underline{\varepsilon} \,] \, b^w \, [\, b_2 - d_2 \,] - \frac{\varepsilon_0 \, b^w}{3} \, [\, 3 \, (b_1 - d_1) + b_2 - d_2 \,]
$$

$$
\begin{aligned}
& - \frac{b^w}{2} \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \right] \\
& \qquad \cdot \left[b_1 - d_1 + b_2 - d_2 \ - (3 \left[b_1 - d_1 \right] + b_2 - d_2) \right] \\
& + \frac{1}{2} \left[\varepsilon_{12} - \varepsilon_0 \right] b^w \left[b_1 - d_1 + b_2 - d_2 \right] \\
& - \frac{b^w}{2} \left[\varepsilon_{12} \left(b_1 - d_1 + b_2 - d_2 \right) - \ \varepsilon_0 \left(3 \left[b_1 - d_1 \right] + b_2 - d_2 \right) \right] \\
& + b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] b^w \left[b_1 - d_1 + b_2 - d_2 \right] \\
& + b^w \left[\overline{\varepsilon} - \varepsilon_{12} \right] \left[b_1 - d_1 + b_2 - d_2 \right] + \varepsilon_{12} \, b^w \left[b_1 - d_1 + b_2 - d_2 \right] \Bigg\} \\
= \ & b^w \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \right] + b^w \varepsilon_{12} \\
& - 2 \left(b^w \right)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] - b^w \left[\overline{\varepsilon} + \varepsilon_{12} \right]
\end{aligned} \tag{380}
$$

$$
\begin{aligned}
\Leftrightarrow \quad & \frac{dE\{w\}}{dK_2} \Bigg\{ \overline{\varepsilon} - \underline{\varepsilon} - \frac{b^w}{3} \left[b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \right] \left[3 \left(b_1 - d_1 \right) + b_2 - d_2 \right] \\
& \quad + \frac{1}{3} \left[\varepsilon_0 - \underline{\varepsilon} \right] b^w \left[b_2 - d_2 \right] - \frac{\varepsilon_0 \, b^w}{3} \left[3 \left(b_1 - d_1 \right) + b_2 - d_2 \right] \\
& \quad + b^w \left[b_1 - d_1 \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \right] \\
& \quad - \ \frac{1}{2} \, \varepsilon_0 \, b^w \left[b_1 - d_1 + b_2 - d_2 \right] + \frac{b^w}{2} \varepsilon_0 \left[3 \left(b_1 - d_1 \right) + b_2 - d_2 \right] \\
& \quad + b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] b^w \left[b_1 - d_1 + b_2 - d_2 \right] \\
& \quad + b^w \, \overline{\varepsilon} \, \left[b_1 - d_1 + b_2 - d_2 \right] \Bigg\} \\
& = \ b^w c_2 - \left(b^w \right)^2 \left[a^L + Q_1^r + Q_2^r - K_1 - 2 \, K_2 \right] - b^w \, \overline{\varepsilon}
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow \quad & \frac{dE\{w\}}{dK_2} \Bigg\{ \overline{\varepsilon} - \underline{\varepsilon} - \frac{b^w}{3} \left[b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \right] \left[3 \left(b_1 - d_1 \right) + b_2 - d_2 \right] \\
& \quad + b^w \left[b_1 - d_1 \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \right] \\
& \quad + b^w \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] b^w \left[b_1 - d_1 + b_2 - d_2 \right] \\
& \quad - \ \frac{1}{2} \, \varepsilon_0 \, b^w \left[b_1 - d_1 + b_2 - d_2 \right] + \frac{b^w}{6} \varepsilon_0 \left[3 \left(b_1 - d_1 \right) + b_2 - d_2 \right]
\end{aligned}
$$

$$+\ \frac{1}{3}\left[\,\varepsilon_0-\underline{\varepsilon}\,\right]b^w\left[\,b_2-d_2\,\right]+\ b^w\,\overline{\varepsilon}\,\left[\,b_1-d_1+b_2-d_2\,\right]\ \Big\}$$

$$=\ b^w\left[\,c_2-\ \overline{\varepsilon}\,\right]-(b^w)^2\left[\,a^L+Q_1^r+Q_2^r-K_1-2\,K_2\,\right]$$

$$\Leftrightarrow\quad \frac{dE\{w\}}{dK_2}\Big\{\,\overline{\varepsilon}-\underline{\varepsilon}-\frac{b^w}{3}\left[\,b^w\left(a^L+Q_2^r\right)+c_1+c_2\,\right]\left[\,3\,(b_1-d_1)+b_2-d_2\,\right]$$

$$+\ b^w\left[\,b_1-d_1\,\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1\right)+c_2\,\right]$$

$$+\ \ (b^w)^2\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]\left[\,b_1-d_1+b_2-d_2\,\right]$$

$$-\ \frac{1}{3}\,\underline{\varepsilon}\,b^w\left[\,b_2-d_2\,\right]+b^w\,\overline{\varepsilon}\,\left[\,b_1-d_1+b_2-d_2\,\right]\ \Big\}$$

$$=\ b^w\left[\,c_2-\ \overline{\varepsilon}\,\right]-(b^w)^2\left[\,a^L+Q_1^r+Q_2^r-K_1-2\,K_2\,\right]\ . \tag{381}$$

(381) implies:

$$\frac{dE\{w\}}{dK_2}\ =\ \frac{Y_{3B3}}{Y_{3B4}} \tag{382}$$

where:

$$Y_{3B3}\ \equiv\ c_2-\ \overline{\varepsilon}\ -b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-2\,K_2\,\right]\ ;$$

$$Y_{3B4}\ \equiv\ \frac{\overline{\varepsilon}-\underline{\varepsilon}}{b^w}-\frac{1}{3}\left[\,b^w\left(a^L+Q_2^r\right)+c_1+c_2\,\right]\left[\,3\,(b_1-d_1)+b_2-d_2\,\right]$$

$$+\ \left[\,b_1-d_1\,\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1\right)+c_2\,\right]$$

$$+\ b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]\left[\,b_1-d_1+b_2-d_2\,\right]$$

$$-\ \frac{1}{3}\,\underline{\varepsilon}\left[\,b_2-d_2\,\right]+\ \overline{\varepsilon}\left[\,b_1-d_1+b_2-d_2\,\right]. \tag{383}$$

(362), (370), and (371) imply:

$$\frac{\partial E\{q_1^*\}}{\partial K_1}\ =\ \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,a^L+3\,Q_1^r+Q_2^r+\frac{1}{b^w}\,(\,c_2-2\,c_1\,)\right]\frac{d\varepsilon_0}{dK_1}$$

$$-\ \frac{1}{3}\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,3\,(b_1-d_1)+b_2-d_2\,\right]\frac{\partial E\{w\}}{\partial K_1}$$

$$+\ \frac{1}{6\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\,2\,\varepsilon_0\,\frac{d\varepsilon_0}{dK_1}+\frac{\overline{\varepsilon}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}-K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_0}{dK_1}$$

$$= \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L + 3\,Q_1^r + Q_2^r + \frac{1}{b^w}\,(\,c_2 - 2\,c_1\,)\right]$$

$$\cdot \left[3\,b^w + b^w\,(\,3\,[\,b_1 - d_1\,] + b_2 - d_2\,)\,\frac{\partial E\{w\}}{\partial K_1}\right]$$

$$- \frac{1}{3}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,3\,(b_1 - d_1) + b_2 - d_2\,]\,\frac{\partial E\{w\}}{\partial K_1}$$

$$+ \frac{1}{3\,b^w}\left[\frac{\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[3\,b^w + b^w\,(\,3\,[\,b_1 - d_1\,] + b_2 - d_2\,)\,\frac{\partial E\{w\}}{\partial K_1}\right] + \frac{\overline{\varepsilon}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}$$

$$- K_1\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[3\,b^w + b^w\,(\,3\,[\,b_1 - d_1\,] + b_2 - d_2\,)\,\frac{\partial E\{w\}}{\partial K_1}\right]$$

$$= \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L + 3\,Q_1^r + Q_2^r + \frac{1}{b^w}\,(\,c_2 - 2\,c_1\,)\right]$$

$$\cdot \left[3\,b^w + b^w\,(\,3\,[\,b_1 - d_1\,] + b_2 - d_2\,)\,\frac{Y_{3B1}}{Y_{3B2}}\right]$$

$$- \frac{1}{3}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,3\,(b_1 - d_1) + b_2 - d_2\,]\,\frac{Y_{3B1}}{Y_{3B2}} + \frac{\overline{\varepsilon}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}$$

$$+ \frac{1}{3\,b^w}\left[\frac{\varepsilon_0 - 3\,b^w\,K_1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[3\,b^w + b^w\,(\,3\,[\,b_1 - d_1\,] + b_2 - d_2\,)\,\frac{Y_{3B1}}{Y_{3B2}}\right]. \tag{384}$$

(363), (370), and (371) imply:

$$\frac{\partial E\{q_2^*\}}{\partial K_2} = \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L + Q_2^r + \frac{1}{b^w}\,(\,c_1 - 2\,c_2\,)\right]\frac{d\varepsilon_0}{dK_2}$$

$$- \frac{1}{3}\left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b_2 - d_2\,]\,\frac{\partial E\{w\}}{\partial K_2} + \frac{1}{6\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]2\,\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}$$

$$+ \frac{1}{2}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L + Q_1^r + Q_2^r - K_1 - \frac{c_2}{b^w}\right]\left[\frac{d\varepsilon_{12}}{dK_2} - \frac{d\varepsilon_0}{dK_2}\right]$$

$$- \frac{1}{2}\left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b_1 - d_1 + b_2 - d_2\,]\,\frac{\partial E\{w\}}{\partial K_2} + \frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}$$

$$+ \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[2\,\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2} - 2\,\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}\right] - K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\frac{d\varepsilon_{12}}{dK_2}$$

$$= \frac{1}{3}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L + Q_2^r + \frac{1}{b^w}\,(\,c_1 - 2\,c_2\,)\right]b^w\,[\,3\,(b_1 - d_1) + b_2 - d_2\,]\,\frac{\partial E\{w\}}{\partial K_2}$$

$$
\begin{aligned}
&- \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_2} \\
&+ \frac{1}{6\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 2\, \varepsilon_0 \, b^w \, [\, 3\, (b_1 - d_1) + b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_2} \\
&+ \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 - \frac{c_2}{b^w} \right] \\
&\qquad \cdot \left[2\, b^w + b^w \, (\, b_1 - d_1 + b_2 - d_2 \,) \, \frac{\partial E\{w\}}{\partial K_2} \right. \\
&\qquad\qquad \left. - \, b^w \, (\, 3\, [\, b_1 - d_1 \,] + b_2 - d_2 \,) \, \frac{\partial E\{w\}}{\partial K_2} \right] \\
&- \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b_1 - d_1 + b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_2} + \frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \\
&+ \frac{1}{2\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \left(2\, b^w + b^w \, [\, b_1 - d_1 + b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_2} \right) \right. \\
&\qquad\qquad \left. - \, \varepsilon_0 \, b^w \, (\, 3\, [\, b_1 - d_1 \,] + b_2 - d_2 \,) \, \frac{\partial E\{w\}}{\partial K_2} \right] \\
&- K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[2\, b^w + b^w \, (\, b_1 - d_1 + b_2 - d_2 \,) \, \frac{\partial E\{w\}}{\partial K_2} \right] . \qquad (385)
\end{aligned}
$$

(383) and (385) imply:

$$
\begin{aligned}
\frac{\partial E\{q_2^*\}}{\partial K_2} \; = \; & \frac{b^w}{3} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_2^r + \frac{1}{b^w} \, (\, c_1 - 2\, c_2 \,) \right] [\, 3\, (b_1 - d_1) + b_2 - d_2 \,] \, \frac{Y_{3B3}}{Y_{3B4}} \\
& - \frac{1}{3} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b_2 - d_2 \,] \, \frac{Y_{3B3}}{Y_{3B4}} + \frac{1}{3} \left[\frac{\varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, 3\, (b_1 - d_1) + b_2 - d_2 \,] \, \frac{Y_{3B3}}{Y_{3B4}} \\
& + b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 - \frac{c_2}{b^w} \right] \left[1 - (\, b_1 - d_1 \,) \, \frac{Y_{3B3}}{Y_{3B4}} \right] \\
& - \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b_1 - d_1 + b_2 - d_2 \,] \, \frac{Y_{3B3}}{Y_{3B4}} + \frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \\
& + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \left(2 + [\, b_1 - d_1 + b_2 - d_2 \,] \, \frac{Y_{3B3}}{Y_{3B4}} \right) \right. \\
& \qquad\qquad \left. - \, \varepsilon_0 \, (\, 3\, [\, b_1 - d_1 \,] + b_2 - d_2 \,) \, \frac{Y_{3B3}}{Y_{3B4}} \right]
\end{aligned}
$$

$$- \; K_2 \, b^w \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[2 + (\, b_1 - d_1 + b_2 - d_2 \,) \, \frac{Y_{3B3}}{Y_{3B4}} \right] . \tag{386}$$

(364) and (370) imply:

$$\frac{dE\{\, w(Q^*) \, q_1^* \,\}}{dK_1} \;=\; \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \,\right]$$

$$\cdot \left[a^L + 3\, Q_1^r + Q_2^r + \frac{1}{b^w} \, (\, c_2 - 2\, c_1 \,) \right] \frac{d\varepsilon_0}{dK_1}$$

$$- \; \frac{1}{9} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \,\right] \left[\, 3\, (b_1 - d_1) + b_2 - d_2 \,\right] \frac{\partial E\{w\}}{\partial K_1}$$

$$- \; \frac{1}{9} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\, Q_1^r + Q_2^r + \frac{1}{b^w} \, (\, c_2 - 2\, c_1 \,) \right] b^w \left[\, b_2 - d_2 \,\right] \frac{\partial E\{w\}}{\partial K_1}$$

$$+ \; \frac{1}{18\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \,\right] \; 2\, \varepsilon_0 \, \frac{d\varepsilon_0}{dK_1}$$

$$- \; \frac{1}{18\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\, b_2 - d_2 \,\right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right] \frac{\partial E\{w\}}{\partial K_1}$$

$$+ \; \frac{1}{18} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\, Q_1^r + Q_2^r + \frac{1}{b^w} \, (\, c_2 - 2\, c_1 \,) \right] \; 2\, \varepsilon_0 \, \frac{d\varepsilon_0}{dK_1}$$

$$- \; \frac{1}{18} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, 3\, (b_1 - d_1) + b_2 - d_2 \,\right] \left[\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,\right] \frac{\partial E\{w\}}{\partial K_1}$$

$$+ \; \frac{1}{27\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 3\, (\varepsilon_0)^2 \; \frac{d\varepsilon_0}{dK_1}$$

$$+ \; \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \,\right] K_1 \left[\frac{d\varepsilon_{12}}{dK_1} - \frac{d\varepsilon_0}{dK_1} \right]$$

$$+ \; \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \,\right] - \frac{1}{2} \, b^w \, K_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \; \frac{1}{2} \, K_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[\, b_1 - d_1 + b_2 - d_2 \,\right] \frac{\partial E\{w\}}{\partial K_1}$$

$$+ \; \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right] + \frac{1}{4} \, K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[2\, \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dK_1} - 2\, \varepsilon_0 \, \frac{d\varepsilon_0}{dK_1} \right]$$

$$+ \; b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] - b^w \, K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, b^w \, K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \, \right] \frac{d\varepsilon_{12}}{dK_1}$$

$$- \, b^w \, K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b_1 - d_1 + b_2 - d_2 \,] \frac{\partial E\{w\}}{\partial K_1}$$

$$+ \, \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \,] - \frac{1}{2} \, K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \, 2 \, \varepsilon_{12} \, \frac{d\varepsilon_{12}}{dK_1} \, . \tag{387}$$

(371) and (387) imply:

$$\frac{dE\{ \, w(Q^*) \, q_1^* \, \}}{dK_1} = \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(\, a^L + Q_2^r \right) + c_1 + c_2 \, \right] \left[a^L + 3 \, Q_1^r + Q_2^r + \frac{1}{b^w} \, (\, c_2 - 2 \, c_1 \,) \right]$$

$$\cdot \left[3 \, b^w + b^w \, (\, 3 \, [\, b_1 - d_1 \,] + b_2 - d_2 \,) \, \frac{\partial E\{w\}}{\partial K_1} \right]$$

$$- \, \frac{1}{9} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(\, a^L + Q_2^r \right) + c_1 + c_2 \, \right] [\, 3 \, (b_1 - d_1) + b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_1}$$

$$- \, \frac{1}{9} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3 \, Q_1^r + Q_2^r + \frac{1}{b^w} \, (\, c_2 - 2 \, c_1 \,) \right] b^w \, [\, b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_1}$$

$$+ \, \frac{1}{9 \, b^w} \left[\frac{\varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(\, a^L + Q_2^r \right) + c_1 + c_2 \, \right] \left[3 \, b^w + b^w \, (\, 3 \, [\, b_1 - d_1 \,] + b_2 - d_2 \,) \, \frac{\partial E\{w\}}{\partial K_1} \right]$$

$$- \, \frac{1}{18} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b_2 - d_2 \,] \, [\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,] \, \frac{\partial E\{w\}}{\partial K_1}$$

$$+ \, \frac{1}{9} \left[\frac{\varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3 \, Q_1^r + Q_2^r + \frac{1}{b^w} \, (\, c_2 - 2 \, c_1 \,) \right]$$

$$\cdot \left[3 \, b^w + b^w \, (\, 3 \, [\, b_1 - d_1 \,] + b_2 - d_2 \,) \, \frac{\partial E\{w\}}{\partial K_1} \right]$$

$$- \, \frac{1}{18} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, 3 \, (b_1 - d_1) + b_2 - d_2 \,] \, [\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,] \frac{\partial E\{w\}}{\partial K_1}$$

$$+ \, \frac{1}{9 \, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2 \left[3 \, b^w + b^w \, (\, 3 \, (b_1 - d_1) + b_2 - d_2 \,) \, \frac{\partial E\{w\}}{\partial K_1} \right]$$

$$+ \, \frac{1}{2} \, K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(\, a^L + Q_1^r + Q_2^r - K_1 \, \right) + c_2 \, \right]$$

$$\cdot \left[b^w + b^w \, (\, b_1 - d_1 + b_2 - d_2 \,) \, \frac{\partial E\{w\}}{\partial K_1} \right.$$

$$\left. - \left(3 \, b^w + b^w \, [\, 3 \, (b_1 - d_1) + b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_1} \right) \right]$$

$$+ \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - 2\,K_1 \right) + c_2 \right]$$

$$- \frac{1}{2} K_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[b_1 - d_1 + b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_1} + \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]$$

$$+ \frac{1}{2} K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \left(b^w + b^w \left[b_1 - d_1 + b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_1} \right) \right.$$

$$\left. - \, \varepsilon_0 \left(3\,b^w + b^w \left[3\,(b_1 - d_1) + b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_1} \right) \right]$$

$$+ \, b^w \left[a^L + Q_1^r + Q_2^r - 2\,K_1 - K_2 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, b^w K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[b^w + b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_1} \right]$$

$$- \, b^w K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b_1 - d_1 + b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_1}$$

$$+ \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right] - K_1 \left[\frac{\varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w + b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{\partial E\{w\}}{\partial K_1} \right]. \quad (388)$$

(376) and (388) imply:

$$\frac{dE\{\, w(Q^*)\, q_1^* \,\}}{dK_1} = \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \right] \left[a^L + 3\,Q_1^r + Q_2^r + \frac{1}{b^w} \left(c_2 - 2\,c_1 \right) \right]$$

$$\cdot \left[3\,b^w + b^w \left(3 \left[b_1 - d_1 \right] + b_2 - d_2 \right) \frac{Y_{3B1}}{Y_{3B2}} \right]$$

$$- \frac{1}{9} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \right] \left[3\,(b_1 - d_1) + b_2 - d_2 \right] \frac{Y_{3B1}}{Y_{3B2}}$$

$$- \frac{1}{9} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\,Q_1^r + Q_2^r + \frac{1}{b^w} \left(c_2 - 2\,c_1 \right) \right] b^w \left[b_2 - d_2 \right] \frac{Y_{3B1}}{Y_{3B2}}$$

$$+ \frac{1}{9} \left[\frac{\varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \right] \left[3 + \left(3 \left[b_1 - d_1 \right] + b_2 - d_2 \right) \frac{Y_{3B1}}{Y_{3B2}} \right]$$

$$- \frac{1}{18} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b_2 - d_2 \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right] \frac{Y_{3B1}}{Y_{3B2}}$$

$$+ \frac{1}{9} \left[\frac{\varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + 3\,Q_1^r + Q_2^r + \frac{1}{b^w} \left(c_2 - 2\,c_1 \right) \right]$$

$$
\cdot \left[3\, b^w + b^w \left(3 \left[b_1 - d_1 \right] + b_2 - d_2 \right) \frac{Y_{3B1}}{Y_{3B2}} \right]
$$

$$
- \frac{1}{18} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[3 \left(b_1 - d_1 \right) + b_2 - d_2 \right] \left[(\varepsilon_0)^2 - (\underline{\varepsilon})^2 \right] \frac{Y_{3B1}}{Y_{3B2}}
$$

$$
+ \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2 \left[3 + \left(3 \left[b_1 - d_1 \right] + b_2 - d_2 \right) \frac{Y_{3B1}}{Y_{3B2}} \right]
$$

$$
- \; b^w K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \right] \left[1 + (b_1 - d_1) \, \frac{Y_{3B1}}{Y_{3B2}} \right]
$$

$$
+ \frac{1}{2} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_1^r + Q_2^r - 2\, K_1 \right) + c_2 \right]
$$

$$
- \frac{1}{2} K_1 \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] b^w \left[b_1 - d_1 + b_2 - d_2 \right] \frac{Y_{3B1}}{Y_{3B2}} + \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right]
$$

$$
+ \frac{1}{2} K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\varepsilon_{12} \left(b^w + b^w \left[\, b_1 - d_1 + b_2 - d_2 \right] \frac{Y_{3B1}}{Y_{3B2}} \right) \right.
$$

$$
\left. - \; \varepsilon_0 \left(3\, b^w + b^w \left[3 \left(b_1 - d_1 \right) + b_2 - d_2 \right] \frac{Y_{3B1}}{Y_{3B2}} \right) \right]
$$

$$
+ \; b^w \left[a^L + Q_1^r + Q_2^r - 2\, K_1 - K_2 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]
$$

$$
- \; b^w K_1 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[b^w + b^w \left(b_1 - d_1 + b_2 - d_2 \right) \frac{Y_{3B1}}{Y_{3B2}} \right]
$$

$$
- \; b^w K_1 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b_1 - d_1 + b_2 - d_2 \right] \frac{Y_{3B1}}{Y_{3B2}}
$$

$$
+ \; \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right] - K_1 \left[\frac{\varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w + b^w \left(\, b_1 - d_1 + b_2 - d_2 \right) \frac{Y_{3B1}}{Y_{3B2}} \right] . \quad (389)
$$

(365) and (370) imply:

$$
\frac{dE\{ \, w(Q^*) \, q_2^* \, \}}{dK_2} \; = \; \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \right] \left[a^L + Q_2^r + \frac{1}{b^w} \left(c_1 - 2\, c_2 \right) \right] \frac{d\varepsilon_0}{dK_2}
$$

$$
- \frac{1}{9} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \right] \left[b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_2}
$$

$$
- \frac{1}{9} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_2^r + \frac{1}{b^w} \left(c_1 - 2\, c_2 \right) \right] b^w \left[b_2 - d_2 \right] \frac{\partial E\{w\}}{\partial K_2}
$$

$$+\ \frac{1}{18\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_2^r\right)+c_1+c_2\,\right]2\,\varepsilon_0\ \frac{d\varepsilon_0}{dK_2}$$

$$-\ \frac{1}{18\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right]b^w\left[\,b_2-d_2\,\right]\frac{\partial E\{w\}}{\partial K_2}$$

$$+\ \frac{1}{18}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+Q_2^r+\frac{1}{b^w}\left(\,c_1-2\,c_2\,\right)\right]2\,\varepsilon_0\ \frac{d\varepsilon_0}{dK_2}$$

$$-\ \frac{1}{18}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,\right]\left[\,b_2-d_2\,\right]\frac{\partial E\{w\}}{\partial K_2}+\frac{1}{27\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]3\,(\varepsilon_0)^2\ \frac{d\varepsilon_0}{dK_2}$$

$$+\ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1\right)+c_2\,\right]$$

$$\cdot\left[a^L+Q_1^r+Q_2^r-K_1\ -\frac{1}{b^w}\,c_2\right]\left[\frac{d\varepsilon_{12}}{dK_2}-\frac{d\varepsilon_0}{dK_2}\right]$$

$$-\ \frac{1}{4}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1\right)+c_2\,\right]\left[\,b_1-d_1+b_2-d_2\,\right]\frac{\partial E\{w\}}{\partial K_2}$$

$$-\ \frac{1}{4}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+Q_1^r+Q_2^r-K_1\ -\frac{1}{b^w}\,c_2\right]b^w\left[\,b_1-d_1+b_2-d_2\,\right]\frac{\partial E\{w\}}{\partial K_2}$$

$$+\ \frac{1}{8\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b^w\left(a^L+Q_1^r+Q_2^r-K_1\right)+c_2\,\right]\left[2\,\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2}-2\,\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}\right]$$

$$-\ \frac{1}{8\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right]b^w\left[\,b_1-d_1+b_2-d_2\,\right]\frac{\partial E\{w\}}{\partial K_2}$$

$$+\ \frac{1}{8}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+Q_1^r+Q_2^r-K_1\ -\frac{1}{b^w}\,c_2\right]\left[2\,\varepsilon_{12}\,\frac{d\varepsilon_{12}}{dK_2}-2\,\varepsilon_0\,\frac{d\varepsilon_0}{dK_2}\right]$$

$$-\ \frac{1}{8}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,\right]\left[\,b_1-d_1+b_2-d_2\,\right]\frac{\partial E\{w\}}{\partial K_2}$$

$$+\ \frac{1}{12\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[3\,(\varepsilon_{12})^2\ \frac{d\varepsilon_{12}}{dK_2}-3\,(\varepsilon_0)^2\ \frac{d\varepsilon_0}{dK_2}\right]$$

$$+\ b^w\left[\,a^L+Q_1^r+Q_2^r-K_1-2\,K_2\,\right]\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]$$

$$-\ b^w\,K_2\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,a^L+Q_1^r+Q_2^r-K_1-K_2\,\right]\frac{d\varepsilon_{12}}{dK_2}$$

$$-\ b^w\,K_2\left[\frac{\overline{\varepsilon}-\varepsilon_{12}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[\,b_1-d_1+b_2-d_2\,\right]\frac{\partial E\{w\}}{\partial K_2}$$

$$
- \ \frac{1}{2}\, K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] 2\, \varepsilon_{12}\, \frac{d\varepsilon_{12}}{dK_2} + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \,] \,. \tag{390}
$$

(371) and (390) imply:

$$
\begin{aligned}
\frac{dE\{\, w(Q^*)\, q_2^* \,\}}{dK_2} \;\; = \;\; & \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \,\right] \\
& \cdot \left[a^L + Q_2^r + \frac{1}{b^w}\, (\, c_1 - 2\, c_2\,) \right] b^w \, [\, 3\, (b_1 - d_1) + b_2 - d_2 \,]\, \frac{\partial E\{w\}}{\partial K_2} \\
& - \frac{1}{9} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \,\right] [\, b_2 - d_2 \,]\, \frac{\partial E\{w\}}{\partial K_2} \\
& - \frac{1}{9} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_2^r + \frac{1}{b^w}\, (\, c_1 - 2\, c_2\,) \right] b^w \, [\, b_2 - d_2 \,]\, \frac{\partial E\{w\}}{\partial K_2} \\
& + \frac{1}{9} \left[\frac{\varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \,\right] [\, 3\, (b_1 - d_1) + b_2 - d_2 \,]\, \frac{\partial E\{w\}}{\partial K_2} \\
& - \frac{1}{18\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,]\, b^w \, [\, b_2 - d_2 \,]\, \frac{\partial E\{w\}}{\partial K_2} \\
& + \frac{b^w}{9} \left[\frac{\varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_2^r + \frac{1}{b^w}\, (\, c_1 - 2\, c_2\,) \right] [\, 3\, (b_1 - d_1) + b_2 - d_2 \,]\, \frac{\partial E\{w\}}{\partial K_2} \\
& - \frac{1}{18} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, (\varepsilon_0)^2 - (\underline{\varepsilon})^2 \,]\, [\, b_2 - d_2 \,]\, \frac{\partial E\{w\}}{\partial K_2} \\
& + \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] (\varepsilon_0)^2 \; [\, 3\, (b_1 - d_1) + b_2 - d_2 \,]\, \frac{\partial E\{w\}}{\partial K_2} \\
& + \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \,\right] \left[a^L + Q_1^r + Q_2^r - K_1 \, - \frac{1}{b^w}\, c_2 \right] \\
& \cdot \left[2\, b^w + b^w\, (\, b_1 - d_1 + b_2 - d_2 \,)\, \frac{\partial E\{w\}}{\partial K_2} - b^w\, (\, 3\, [\, b_1 - d_1 \,] + b_2 - d_2 \,)\, \frac{\partial E\{w\}}{\partial K_2} \right] \\
& - \frac{1}{4} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \,\right] [\, b_1 - d_1 + b_2 - d_2 \,]\, \frac{\partial E\{w\}}{\partial K_2} \\
& - \frac{b^w}{4} \left[\frac{\varepsilon_{12} - \varepsilon_0}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 \, - \frac{1}{b^w}\, c_2 \right] [\, b_1 - d_1 + b_2 - d_2 \,]\, \frac{\partial E\{w\}}{\partial K_2} \\
& + \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_1^r + Q_2^r - K_1 \right) + c_2 \,\right]
\end{aligned}
$$

$$\cdot \left[\varepsilon_{12} \left(2\, b^w + b^w \, [\, b_1 - d_1 + b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_2} \right) \right.$$

$$\left. - \, \varepsilon_0 \, b^w \, [\, 3\, (b_1 - d_1) + b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_2} \right]$$

$$- \, \frac{1}{8\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right] b^w \, [\, b_1 - d_1 + b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_2}$$

$$+ \, \frac{1}{4} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 - \frac{1}{b^w} \, c_2 \right]$$

$$\cdot \left[\varepsilon_{12} \left(2\, b^w + b^w \, [\, b_1 - d_1 + b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_2} \right) \right.$$

$$\left. - \, \varepsilon_0 \, b^w \, [\, 3\, (b_1 - d_1) + b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_2} \right]$$

$$- \, \frac{1}{8} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\varepsilon_{12})^2 - (\varepsilon_0)^2 \,\right] [\, b_1 - d_1 + b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_2}$$

$$+ \, \frac{1}{4\, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 \left(2\, b^w + b^w \, [\, b_1 - d_1 + b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_2} \right) \right.$$

$$\left. - \, (\varepsilon_0)^2 \, b^w \, [\, 3\, (b_1 - d_1) + b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_2} \right]$$

$$+ \, b^w \left[\, a^L + Q_1^r + Q_2^r - K_1 - 2\, K_2 \,\right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]$$

$$- \, b^w \, K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, a^L + Q_1^r + Q_2^r - K_1 - K_2 \,\right] \left[2\, b^w + b^w \, (\, b_1 - d_1 + b_2 - d_2 \,) \, \frac{\partial E\{w\}}{\partial K_2} \right]$$

$$- \, b^w \, K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] [\, b_1 - d_1 + b_2 - d_2 \,] \, \frac{\partial E\{w\}}{\partial K_2} + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, (\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \,\right]$$

$$- \, K_2 \left[\frac{\varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[2\, b^w + b^w \, (\, b_1 - d_1 + b_2 - d_2 \,) \, \frac{\partial E\{w\}}{\partial K_2} \right] . \tag{391}$$

(382) and (391) imply:

$$\frac{dE\{\, w(Q^*) \, q_2^* \,\}}{dK_2} = \frac{1}{9} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \,\right]$$

$$\cdot \left[a^L + Q_2^r + \frac{1}{b^w} \, (\, c_1 - 2\, c_2 \,) \right] b^w \, [\, 3\, (b_1 - d_1) + b_2 - d_2 \,] \, \frac{Y_{3B3}}{Y_{3B4}}$$

$$- \, \frac{1}{9} \left[\frac{\varepsilon_0 - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[\, b^w \left(a^L + Q_2^r \right) + c_1 + c_2 \,\right] [\, b_2 - d_2 \,] \, \frac{Y_{3B3}}{Y_{3B4}}$$

$$- \frac{1}{9}\left[\frac{\varepsilon_0-\underline{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+Q_2^r+\frac{1}{b^w}\,(\,c_1-2\,c_2\,)\right]b^w\,[\,b_2-d_2\,]\,\frac{Y_{3B3}}{Y_{3B4}}$$

$$+ \frac{1}{9}\left[\frac{\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b^w\left(a^L+Q_2^r\right)+c_1+c_2\,]\,[\,3\,(b_1-d_1)+b_2-d_2\,]\,\frac{Y_{3B3}}{Y_{3B4}}$$

$$- \frac{1}{18\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,]\,b^w\,[\,b_2-d_2\,]\,\frac{Y_{3B3}}{Y_{3B4}}$$

$$+ \frac{b^w}{9}\left[\frac{\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+Q_2^r+\frac{1}{b^w}\,(\,c_1-2\,c_2\,)\right][\,3\,(b_1-d_1)+b_2-d_2\,]\,\frac{Y_{3B3}}{Y_{3B4}}$$

$$- \frac{1}{18}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,(\varepsilon_0)^2-(\underline{\varepsilon})^2\,]\,[\,b_2-d_2\,]\,\frac{Y_{3B3}}{Y_{3B4}}$$

$$+ \frac{1}{9}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right](\varepsilon_0)^2\;[\,3\,(b_1-d_1)+b_2-d_2\,]\,\frac{Y_{3B3}}{Y_{3B4}}$$

$$+ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b^w\left(a^L+Q_1^r+Q_2^r-K_1\right)+c_2\,]\left[a^L+Q_1^r+Q_2^r-K_1\;-\frac{1}{b^w}\,c_2\right]$$

$$\cdot\left[2\,b^w+b^w\,(\,b_1-d_1+b_2-d_2\,)\,\frac{Y_{3B3}}{Y_{3B4}}-b^w\,(\,3\,[\,b_1-d_1\,]+b_2-d_2\,)\,\frac{Y_{3B3}}{Y_{3B4}}\right]$$

$$- \frac{1}{4}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b^w\left(a^L+Q_1^r+Q_2^r-K_1\right)+c_2\,]\,[\,b_1-d_1+b_2-d_2\,]\,\frac{Y_{3B3}}{Y_{3B4}}$$

$$- \frac{b^w}{4}\left[\frac{\varepsilon_{12}-\varepsilon_0}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+Q_1^r+Q_2^r-K_1\;-\frac{1}{b^w}\,c_2\right][\,b_1-d_1+b_2-d_2\,]\,\frac{Y_{3B3}}{Y_{3B4}}$$

$$+ \frac{1}{4\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,b^w\left(a^L+Q_1^r+Q_2^r-K_1\right)+c_2\,]$$

$$\cdot\left[\varepsilon_{12}\left(2\,b^w+b^w\,[\,b_1-d_1+b_2-d_2\,]\,\frac{Y_{3B3}}{Y_{3B4}}\right)\right.$$

$$\left.-\;\varepsilon_0\;b^w\,[\,3\,(b_1-d_1)+b_2-d_2\,]\,\frac{Y_{3B3}}{Y_{3B4}}\right]$$

$$- \frac{1}{8\,b^w}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right][\,(\varepsilon_{12})^2-(\varepsilon_0)^2\,]\,b^w\,[\,b_1-d_1+b_2-d_2\,]\,\frac{Y_{3B3}}{Y_{3B4}}$$

$$+ \frac{1}{4}\left[\frac{1}{\overline{\varepsilon}-\underline{\varepsilon}}\right]\left[a^L+Q_1^r+Q_2^r-K_1\;-\frac{1}{b^w}\,c_2\right]$$

$$\cdot\left[\varepsilon_{12}\left(2\,b^w+b^w\,[\,b_1-d_1+b_2-d_2\,]\,\frac{Y_{3B3}}{Y_{3B4}}\right)\right.$$

$$
- \varepsilon_0 \, b^w \left[3 \, (b_1 - d_1) + b_2 - d_2 \right] \frac{Y_{3B3}}{Y_{3B4}} \Bigg]
$$

$$
- \frac{1}{8} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\varepsilon_{12})^2 - (\varepsilon_0)^2 \right] \left[b_1 - d_1 + b_2 - d_2 \right] \frac{Y_{3B3}}{Y_{3B4}}
$$

$$
+ \frac{1}{4 \, b^w} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \Bigg[(\varepsilon_{12})^2 \left(2 \, b^w + b^w \left[b_1 - d_1 + b_2 - d_2 \right] \frac{Y_{3B3}}{Y_{3B4}} \right)
$$

$$
- \; (\varepsilon_0)^2 \, b^w \left[3 \, (b_1 - d_1) + b_2 - d_2 \right] \frac{Y_{3B3}}{Y_{3B4}} \Bigg]
$$

$$
+ \, b^w \left[a^L + Q_1^r + Q_2^r - K_1 - 2 \, K_2 \right] \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right]
$$

$$
- \; b^w \, K_2 \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[a^L + Q_1^r + Q_2^r - K_1 - K_2 \right] \left[2 \, b^w + b^w \, (\, b_1 - d_1 + b_2 - d_2 \,) \frac{Y_{3B3}}{Y_{3B4}} \right]
$$

$$
- \, b^w \, K_2 \left[\frac{\overline{\varepsilon} - \varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[b_1 - d_1 + b_2 - d_2 \right] \frac{Y_{3B3}}{Y_{3B4}} + \frac{1}{2} \left[\frac{1}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[(\overline{\varepsilon})^2 - (\varepsilon_{12})^2 \right]
$$

$$
- \; K_2 \left[\frac{\varepsilon_{12}}{\overline{\varepsilon} - \underline{\varepsilon}} \right] \left[2 \, b^w + b^w \, (\, b_1 - d_1 + b_2 - d_2 \,) \frac{Y_{3B3}}{Y_{3B4}} \right] . \tag{392}
$$

The foregoing calculations allow us to generate numerical solutions that characterize equilibrium outcomes in the present setting. The outcomes that arise when model parameters are as specified in the baseline setting are reported in Table T66.

	VS	**PVI**	**VI**
r_1	237	230	225
r_2	237	230	225
$E\{w\}$	236	229	224
K_1	8,861	9,311	8,897
K_2	8,861	8,525	8,897
$E\{q_1\}$	6,669	7,267	6,940
$E\{q_1\}$	6,669	6,364	6,940
$E\{\pi_1^G\}$	1,295,042	1,357,631	1,267,430
$E\{\pi_1^R\}$	0	0	0
$E\{\pi_2^G\}$	1,295,042	1,202,450	1,267,430
$E\{\pi_2^R\}$	0	0	0

Table T66. Outcomes in the Baseline Setting when $r_i = E\{w\} + c_i^r$ for $i \in \{1, 2\}$.