Technical Appendix to Accompany

"Market Structure, Risk Preferences, and Forward Contracting Incentives"

by David P. Brown and David E. M. Sappington

Part A of this Technical Appendix provides more detailed proofs of the formal conclusions in the body of the paper. Part B provides more detailed proofs of the formal conclusions in Appendix B of the paper. Part C presents additional formal conclusions and their proofs. Part D extends the analysis in Section 3.5 of the paper.

Equations and Definitions from the Text

In settings with a single buyer:

$$Q(\cdot) \;=\; a^I - b^I\, w + \overline{Q} + \eta\,. \tag{1}$$

$$w(Q, \varepsilon) \;=\; a + \varepsilon - b\, Q \quad \text{where } a \;=\; \frac{a^I + \overline{Q}}{b^I},\;\; \varepsilon \;=\; \frac{\eta}{b^I},\; \text{and } b \;=\; \frac{1}{b^I}\,. \tag{2}$$

$$\pi^B(\varepsilon) \;=\; r\left[\,\overline{Q} + b^I\,\varepsilon\,\right] - w(\varepsilon)\left[\,\overline{Q} + b^I\,\varepsilon - F\,\right] - p^F\,F\,. \tag{3}$$

$$EU^B \;\equiv\; E\left\{\,\pi^B\,\right\} - A_B\,V_B\,, \;\text{ where } A_B \;\geq\; 0\,. \tag{4}$$

$$\frac{\partial E\{\,\pi^B(\varepsilon)\,\}}{\partial F} \;=\; -\,\gamma\,\overline{Q}\left[-\frac{1}{2\,b^I}\right] \;=\; \frac{\gamma}{2\,b^I}\,\overline{Q}\,. \tag{5}$$

In settings with a single generator:

$$\pi^G(\varepsilon) \;=\; w(\varepsilon)\,[\,q - F\,] + p^F\,F - c\,q\,. \tag{6}$$

$$EU^G \;\equiv\; E\left\{\pi^G(\varepsilon)\right\} - A_G\,V_G \;\text{ where } A_G \;\geq\; 0\,. \tag{7}$$

In settings with two generators:

$$EU^{Gi} \;=\; E\left\{\pi^{Gi}(\varepsilon)\right\} - A_{Gi}\,V_{Gi} \;\text{ where } A_{Gi} \;\geq\; 0\,. \tag{8}$$

$$\frac{\partial E\{\,\pi^B(\varepsilon)\,\}}{\partial F_i} \;=\; -\,\gamma\,\overline{Q}\left[-\frac{1}{3\,b^I}\right] \;=\; \frac{\gamma}{3\,b^I}\,\overline{Q}\,. \tag{9}$$

$$\delta_i \;\equiv\; \frac{\left[\,24\,b^I A_{Gi}\,(\overline{\varepsilon})^2 + 81\,\right]\left[\,a^I + \overline{Q} \,-\, b^I\,(\,2\,c_i \,-\, c_j\,)\,\right]}{24\,b^I\,A_{Gi}\,(\overline{\varepsilon})^2 + 324}. \tag{10}$$

$$\beta_i \;\equiv\; \frac{24\,b^I A_{Gi}\,(\overline{\varepsilon})^2 + 81}{24\,b^I A_{Gi}\,(\overline{\varepsilon})^2 + 324} \;\text{ for } i, j \in \{1, 2\} \;\; (j \neq i). \tag{11}$$

In settings with two buyers:

$$w(\cdot) \;=\; a - b\,Q + \varepsilon \quad \text{where} \;\; a \;=\; \frac{a^I + \overline{Q}_1 + \overline{Q}_2}{b^I}, \;\; \varepsilon \;=\; \frac{\eta_1 + \eta_2}{b^I}, \;\text{and}\; b \;=\; \frac{1}{b^I}\,. \tag{12}$$

A. Detailed Proofs of Formal Conclusions in the Text.

Lemma 1. *In equilibrium in the dual monopoly setting:*

$$w(\varepsilon) \;=\; \frac{1}{2\,b^I}\left[\,a^I + \overline{Q} - F + b^I\,(c+\varepsilon)\,\right], \quad p^f \;=\; \frac{1}{2\,b^I}\left[\,a^I + \overline{Q} - F + b^I c\,\right]$$

$$q(\varepsilon) \;=\; \frac{1}{2}\left[\,a^I + \overline{Q} + F - b^I\,(c-\varepsilon)\,\right], \;\; \textit{and}$$

$$E\left\{\pi^G(\varepsilon)\right\} \;=\; \frac{1}{4\,b^I}\left[\,a^I + \overline{Q} - F - b^I c\,\right]\left[\,a^I + \overline{Q} + F - b^I c\,\right] + \frac{b^I\,(\overline{\varepsilon})^2}{12}$$

$$\Rightarrow \quad \frac{\partial E\left\{\pi^G(\varepsilon)\right\}}{\partial F} \;\stackrel{s}{=}\; -\,2\,F\,.$$

<u>Proof</u>. (6) implies that when ε is realized, G's problem is:

$$\underset{q \geq 0}{Maximize} \;\; \pi^G(\varepsilon) \;=\; w(\varepsilon)\,[\,q - F\,] - c\,q + p^F\,F\,. \tag{13}$$

(2) and (13) imply that the necessary conditions for an interior optimum include:

$$\frac{\partial \pi^G(\varepsilon)}{\partial q} \;=\; w(\varepsilon) + [\,q - F\,]\,\frac{\partial w(\cdot)}{\partial Q} - c \;=\; 0$$

$$\Rightarrow \;\; a + \varepsilon - b\,q - b\,[\,q - F\,] - c \;=\; 0$$

$$\Rightarrow \;\; q(\varepsilon) \;=\; \frac{1}{2\,b}\,[\,a + \varepsilon - c + b\,F\,] \;=\; \frac{b^I}{2}\left[\,\frac{a^I + \overline{Q} + F}{b^I} - (c-\varepsilon)\right]$$

$$=\; \frac{1}{2}\left[\,a^I + \overline{Q} + F - b^I\,(c-\varepsilon)\,\right]. \tag{14}$$

(2) and (14) imply:

$$w(\varepsilon) \;=\; a + \varepsilon - b\,q(\varepsilon) \;=\; \frac{a^I + \overline{Q}}{b^I} + \varepsilon - \frac{b}{2}\left[\,a^I + \overline{Q} + F - b^I\,(c-\varepsilon)\,\right]$$

$$=\; \frac{2}{2\,b^I}\left[\,a^I + \overline{Q} + b^I\varepsilon\,\right] - \frac{1}{2\,b^I}\left[\,a^I + \overline{Q} + F - b^I\,(c-\varepsilon)\,\right]$$

$$=\; \frac{1}{2\,b^I}\left[\,a^I + \overline{Q} - F + b^I\,(c+\varepsilon)\,\right]$$

$$\Rightarrow \quad p^F \;=\; E\{w(\varepsilon)\} \;=\; \frac{1}{2\,b^I}\left[\,a^I + \overline{Q} - F + b^I c\,\right]. \tag{15}$$

(6), (14), and (15) imply:

$$\pi^G(\varepsilon) \;=\; [\,w(\varepsilon) - c\,]\,q(\varepsilon) + \left[\,p^F - w(\varepsilon)\,\right] F$$

$$= \; \frac{1}{4\,b^I}\left[\,a^I + \overline{Q} - F - b^I\,(c-\varepsilon)\,\right]\left[\,a^I + \overline{Q} + F - b^I\,(c-\varepsilon)\,\right] - \frac{\varepsilon}{2\,b^I}\;F\,. \tag{16}$$

Because ε has a uniform density on $[\,-\overline{\varepsilon}, \overline{\varepsilon}\,]$:

$$E\{\varepsilon\} \;=\; 0 \;\text{ and }\; E\left\{\varepsilon^2\right\} \;=\; \frac{1}{2\,\overline{\varepsilon}}\int_{-\overline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon^2\,d\varepsilon \;=\; \frac{1}{6\,\overline{\varepsilon}}\left[\,(\overline{\varepsilon})^3 - (-\overline{\varepsilon})^3\,\right] \;=\; \frac{(\overline{\varepsilon})^2}{3}\,. \tag{17}$$

(16) and (17) imply:

$$E\left\{\pi^G(\varepsilon)\right\} \;=\; E\left\{\frac{1}{4\,b^I}\left[\,a^I + \overline{Q} - F - b^I\,(c-\varepsilon)\,\right]\left[\,a^I + \overline{Q} + F - b^I\,(c-\varepsilon)\,\right]\right\}$$

$$= \; \frac{1}{4\,b^I}\left[\,a^I + \overline{Q} - F - b^I c\,\right]\left[\,a^I + \overline{Q} + F - b^I c\,\right] + \frac{b^I}{4}\,E\left\{\varepsilon^2\right\}$$

$$= \; \frac{1}{4\,b^I}\left[\,a^I + \overline{Q} - F - b^I c\,\right]\left[\,a^I + \overline{Q} + F - b^I c\,\right] + \frac{b^I\,(\overline{\varepsilon})^2}{12}\,. \tag{18}$$

Differentiating (18) provides:

$$\frac{\partial E\left\{\pi^G(\varepsilon)\right\}}{\partial F} \;\stackrel{s}{=}\; a^I + \overline{Q} - F - b^I c - \left[\,a^I + \overline{Q} + F - b^I c\,\right] \;=\; -\,2\,F\,. \quad \blacksquare$$

Lemma 2. *$V_G \;=\; \frac{(\overline{\varepsilon})^2}{36}\left[3\left(a^I + \overline{Q} - F - b^I c\right)^2 + \frac{\left(b^I\right)^2}{5}\,(\overline{\varepsilon})^2\right]$, so $\frac{\partial V_G}{\partial F} < 0$ for all $F < E\{\,q(\varepsilon)\,\}$,*[1] *in the dual monopoly setting.*

Proof. (2) implies:

$$a^I + \overline{Q} \pm F - b^I\,[\,c - \varepsilon\,] \;=\; \frac{1}{b}\,[\,a + \varepsilon - c \pm\, F\,] \;\;\text{and}\;\; \frac{b^I\,(\overline{\varepsilon})^2}{12} \;=\; \frac{(\overline{\varepsilon})^2}{12\,b}\,. \tag{19}$$

(16), (18), (19), and Lemma 1 imply:

$$\pi^G(\varepsilon) - E\left\{\,\pi^G(\varepsilon)\right\} \;=\; \frac{1}{4\,b}\,[\,a + \varepsilon - c - b\,F\,]\,[\,a + \varepsilon - c + b\,F\,] - \frac{\varepsilon}{2}\;F$$

[1]Lemma 1 implies that $F < E\{\,q(\varepsilon)\,\} \;\Leftrightarrow\; F < \frac{1}{2}\left[\,a^I + \overline{Q} + F - b^I c\,\right] \;\Leftrightarrow\; \frac{1}{2}\left[\,a^I + \overline{Q} - F - b^I c\,\right] > 0$.

$$- \frac{1}{4\,b}\,[\,a-c-b\,F\,]\,[\,a-c+b\,F\,] - \frac{(\,\overline{\varepsilon}\,)^2}{12\,b}$$

$$= \frac{1}{4\,b}\,[\,a-c-b\,F\,]\,[\,a-c+b\,F\,] + \frac{\varepsilon}{4\,b}\,[\,a-c+b\,F+a-c-b\,F\,]$$

$$+ \frac{\varepsilon^2}{4\,b} - \frac{\varepsilon}{2}\,F - \frac{1}{4\,b}\,[\,a-c-b\,F\,]\,[\,a-c+b\,F\,] - \frac{(\,\overline{\varepsilon}\,)^2}{12\,b}$$

$$= \frac{\varepsilon}{2\,b}\,[\,a-c\,] + \frac{\varepsilon^2}{4\,b} - \frac{\varepsilon}{2}\,F - \frac{(\,\overline{\varepsilon}\,)^2}{12\,b}$$

$$= \frac{3\,\varepsilon^2}{12\,b} + \frac{6\,\varepsilon}{12\,b}\,[\,a-c\,] - \frac{6\,b\,\varepsilon\,F}{12\,b} - \frac{(\,\overline{\varepsilon}\,)^2}{12\,b} \;=\; \frac{1}{12\,b}\left[\,3\,\varepsilon^2 + 6\,\varepsilon\,X - (\,\overline{\varepsilon}\,)^2\,\right] \tag{20}$$

where $X \;=\; a-c-b\,F \;=\; \frac{1}{b^I}\left[\,a^I + \overline{Q} - F - b^I c\,\right]$.

Observe that:

$$\left[\,3\,\varepsilon^2 + 6\,\varepsilon\,X - (\,\overline{\varepsilon}\,)^2\,\right]^2$$

$$= 9\,\varepsilon^4 + 36\,X\,\varepsilon^3 + 6\left[\,6\,X^2 - (\,\overline{\varepsilon}\,)^2\,\right]\varepsilon^2 - 12\,X\,(\,\overline{\varepsilon}\,)^2\,\varepsilon + (\,\overline{\varepsilon}\,)^4\,. \tag{21}$$

(17) and (21) imply:

$$E\left\{\left[\,3\,\varepsilon^2 + 6\,\varepsilon\,X - (\,\overline{\varepsilon}\,)^2\,\right]^2\right\}$$

$$= 9\,E\left\{\varepsilon^4\right\} + 36\,X\,E\left\{\varepsilon^3\right\} + 6\left[\,6\,X^2 - (\,\overline{\varepsilon}\,)^2\,\right]E\left\{\varepsilon^2\right\} + (\,\overline{\varepsilon}\,)^4$$

$$= \frac{9}{10\,\overline{\varepsilon}}\left[\,(\,\overline{\varepsilon}\,)^5 - (\,-\overline{\varepsilon}\,)^5\,\right] + \frac{36\,X}{8\,\overline{\varepsilon}}\left[\,(\,\overline{\varepsilon}\,)^4 - (\,-\overline{\varepsilon}\,)^4\,\right]$$

$$+ \frac{1}{6\,\overline{\varepsilon}}\,6\left[\,6\,X^2 - (\,\overline{\varepsilon}\,)^2\,\right]\left[\,(\,\overline{\varepsilon}\,)^3 - (\,-\overline{\varepsilon}\,)^3\,\right] + (\,\overline{\varepsilon}\,)^4$$

$$= \frac{9\,(\,\overline{\varepsilon}\,)^5}{5\,\overline{\varepsilon}} + \frac{\left[\,6\,X^2 - (\,\overline{\varepsilon}\,)^2\,\right]2\,(\,\overline{\varepsilon}\,)^3}{\overline{\varepsilon}} + (\,\overline{\varepsilon}\,)^4$$

$$= \frac{14}{5}\,(\,\overline{\varepsilon}\,)^4 + 2\left[\,6\,X^2 - (\,\overline{\varepsilon}\,)^2\,\right](\,\overline{\varepsilon}\,)^2 \;=\; \frac{4}{5}\,(\,\overline{\varepsilon}\,)^4 + 12\,(\,\overline{\varepsilon}\,)^2\,X^2$$

$$= 4\,(\,\overline{\varepsilon}\,)^2\left[\,3\,X^2 - \frac{4}{5}\,(\,\overline{\varepsilon}\,)^2\,\right]. \tag{22}$$

(20) and (22) imply:

$$V_G \;=\; \frac{\left(b^I\right)^2(\,\overline{\varepsilon}\,)^2}{36}\left[\,3\,X^2 + \frac{1}{5}\,(\,\overline{\varepsilon}\,)^2\,\right]. \tag{23}$$

(23) implies:

$$\frac{\partial V_G}{\partial F} \overset{s}{=} X \, \frac{\partial X}{\partial F} = -\frac{X}{b^I} \, . \quad \blacksquare$$

Proposition 1. *The value of F that maximizes G's expected utility in the dual monopoly setting is $F_{GDM} \equiv \frac{A_G\,(\overline{\varepsilon})^2\, b^I \left[\, a^I + \overline{Q} - b^I c \,\right]}{3 + b^I A_G\,(\overline{\varepsilon})^2}$. F_{GDM} is: (i) increasing in A_G, $\overline{\varepsilon}$, $\overline{Q}$, and a^I; and (ii) decreasing in c.*

Proof. (7), (18), (19), (23), and Lemma 1 imply that G's equilibrium expected utility is:

$$\begin{aligned}
EU^G &= \frac{1}{4\,b}\,[\,a - c - b\,F\,]\,[\,a - c + b\,F\,] + \frac{(\overline{\varepsilon})^2}{12\,b} \\
&\quad - \frac{A_G\,(\overline{\varepsilon})^2}{144\,b^2}\left[\,\frac{14}{5}\,(\overline{\varepsilon})^2 + 2\left(6\,X^2 - (\overline{\varepsilon})^2\right)\right] \\
\Rightarrow \quad \frac{\partial EU^G}{\partial F} &= \frac{1}{4\,b}\,[\,b\,(\,a - c - b\,F\,) - b\,(\,a - c + b\,F\,)\,] - \frac{A_G\,(\overline{\varepsilon})^2\; 24\,X}{144\,b^2}\,\frac{\partial X}{\partial F} \\
&= \frac{1}{4\,b}\left[\,-2\,b^2\,F\,\right] - \frac{A_G\,(\overline{\varepsilon})^2\; X}{6\,b^2}\,[\,-b\,] \\
&= -\frac{b}{2}\,F + \frac{A_G\,(\overline{\varepsilon})^2\,[\,a - c - b\,F\,]}{6\,b} \\
&= -\left[\,\frac{b}{2} + \frac{A_G\,(\overline{\varepsilon})^2}{6}\,\right] F + \frac{A_G\,(\overline{\varepsilon})^2\,[\,a - c\,]}{6\,b}\,. \qquad (24)
\end{aligned}$$

(2) and (24) imply that the level of F that maximizes G's expected utility is given by:

$$\begin{aligned}
& A_G\,(\overline{\varepsilon})^2\,[\,a - c\,] = b\left[\,3\,b + A_G\,(\overline{\varepsilon})^2\,\right] F \\
\Leftrightarrow \quad F &= \frac{A_G\,(\overline{\varepsilon})^2\,[\,a - c\,]}{b\left[\,3\,b + A_G\,(\overline{\varepsilon})^2\,\right]} = \frac{A_G\,(\overline{\varepsilon})^2\left[\,\frac{a^I + \overline{Q}}{b^I} - c\,\right]}{b\left[\,3\,b + A_G\,(\overline{\varepsilon})^2\,\right]} = \frac{A_G\,(\overline{\varepsilon})^2\left[\,a^I + \overline{Q} - b^I c\,\right]}{3\,b + A_G\,(\overline{\varepsilon})^2} \\
&= \frac{A_G\,(\overline{\varepsilon})^2\left[\,a^I + \overline{Q} - b^I c\,\right]}{\frac{3}{b^I} + A_G\,(\overline{\varepsilon})^2} = \frac{A_G\,(\overline{\varepsilon})^2\, b^I\left[\,a^I + \overline{Q} - b^I c\,\right]}{3 + b^I A_G\,(\overline{\varepsilon})^2} = F_{GDM}\,. \qquad (25)
\end{aligned}$$

It is apparent from (25) that F_{GDM} is increasing in a^I and $\overline{Q}$ and decreasing in c. Furthermore, because $a^I + \overline{Q} - b^I c > 0$ by assumption, (25) implies:

$$\frac{\partial F_{GDM}}{\partial A_G} \overset{s}{=} 3 + b^I A_G\,(\overline{\varepsilon})^2 - b^I A_G\,(\overline{\varepsilon})^2 = 3 > 0\,; \text{ and}$$

$$\frac{\partial F_{GDM}}{\partial \overline{\varepsilon}} \overset{s}{=} \left[3 + b^I A_G(\overline{\varepsilon})^2 \right] 2\,\overline{\varepsilon} - 2\,\overline{\varepsilon}\, b^I A_G(\overline{\varepsilon})^2 = 6\,\overline{\varepsilon} > 0 . \;\blacksquare$$

Proposition 2. *In the dual monopoly setting,* $\frac{\partial V_B}{\partial F} \lesseqgtr 0 \Leftrightarrow F \lesseqgtr \underline{F}_{DM}$*, where* $\underline{F}_{DM} \equiv \overline{Q} - \left[\frac{\gamma}{1+\gamma} \right] \left[b^I (2\,r_0 - c) - a^I \right]$*. When* $\gamma > 0$*,* $\underline{F}_{DM}$ *is: (i) strictly less than* $\overline{Q}$*; (ii) increasing in* a^I *and* c*; and (iii) decreasing in* r_0*,* b^I*, and* γ*.*

Proof. Because ε has a uniform distribution on $[-\overline{\varepsilon}, \overline{\varepsilon}]$:

$$E\{\varepsilon\} = 0 \quad \text{and} \quad E\{\varepsilon^2\} = \left[\frac{1}{2\,\overline{\varepsilon}} \right] \frac{1}{3} \left[(\overline{\varepsilon})^3 - (-\overline{\varepsilon})^3 \right] = \frac{(\overline{\varepsilon})^2}{3} . \tag{26}$$

(19) and Lemma 1 imply that in equilibrium in this setting:

$$q(\varepsilon) = \frac{1}{2\,b} [\, a + \varepsilon - c + b\,F \,] , \quad w(\varepsilon) = \frac{1}{2} [\, a + \varepsilon + c - b\,F \,] ,$$

$$\text{and} \quad p^F = E\{w(\varepsilon)\} = \frac{1}{2} [\, a + c - b\,F \,] . \tag{27}$$

(26) and (27) imply:

$$E\{\varepsilon\, w(\varepsilon)\} = E\left\{ \frac{1}{2} [\, a + c - b\,F \,]\, \varepsilon + \frac{1}{2} \varepsilon^2 \right\} = \frac{1}{2} E\{\varepsilon^2\} = \frac{(\overline{\varepsilon})^2}{6} . \tag{28}$$

(27) and (28) imply:

$$\frac{\partial E\{w(\varepsilon)\}}{\partial F} = -\frac{b}{2} \quad \text{and} \quad \frac{\partial E\{\varepsilon\, w(\varepsilon)\}}{\partial F} = 0 . \tag{29}$$

(3) implies:

$$\pi^B(\varepsilon) = [\, \gamma\, r_0 + (1 - \gamma)\, E\{ w(\varepsilon) \} \,] \left[\overline{Q} + b^I \varepsilon \right] - w(\varepsilon) \left[\overline{Q} + b^I \varepsilon - F \right] - p^F F . \tag{30}$$

(30) implies that because $p^F = E\{ w(\varepsilon) \}$:

$$\begin{aligned} E\{ \pi^B(\varepsilon) \} &= [\, \gamma\, r_0 + (1 - \gamma)\, E\{ w(\varepsilon) \} \,]\, \overline{Q} - E\left\{ w(\varepsilon) \left[\overline{Q} + b^I \varepsilon \right] \right\} + F \left[E\{ w(\varepsilon) \} - p^F \right] \\ &= [\, \gamma\, r_0 + (1 - \gamma)\, E\{ w(\varepsilon) \} \,]\, \overline{Q} - E\{ w(\varepsilon) \}\, \overline{Q} - b^I E\{ \varepsilon\, w(\varepsilon) \} . \end{aligned} \tag{31}$$

(30) and (31) imply:

$$\begin{aligned} \pi^B(\varepsilon) - E\{ \pi^B(\varepsilon) \} = {} & [\, \gamma\, r_0 + (1 - \gamma)\, E\{ w(\varepsilon) \} \,]\, b^I \varepsilon - w(\varepsilon) \left[\overline{Q} + b^I \varepsilon - F \right] \\ & - p^F F + E\{ w(\varepsilon) \}\, \overline{Q} + b^I E\{ \varepsilon\, w(\varepsilon) \} \end{aligned}$$

$$= \; [\,\gamma\, r_0 + (\,1-\gamma\,)\, E\{\, w(\varepsilon)\,\}\,]\, b^I\, \varepsilon \; - [\, w(\varepsilon) - E\,\{\, w(\varepsilon)\,\}\,]\, \overline{Q}$$

$$- \;\; b^I\, [\,\varepsilon\, w(\varepsilon) - E\,\{\,\varepsilon\, w(\varepsilon)\,\}\,] + [\, w(\varepsilon) - E\,\{\, w(\varepsilon)\,\}\,]\, F\,. \tag{32}$$

Because $b\, b^I = 1$, (27), (29), and (32) imply:

$$\frac{\partial\left(\,\pi^B(\varepsilon) - E\left\{\,\pi^B(\varepsilon)\,\right\}\right)}{\partial F} \;=\; w(\varepsilon) - E\,\{\, w(\varepsilon)\,\} + [\,1-\gamma\,]\, b^I\, \varepsilon\, \frac{\partial E\{\, w(\varepsilon)\,\}}{\partial F}$$

$$-\; \left[\, \frac{\partial w(\varepsilon)}{\partial F} - \frac{\partial E\{\, w(\varepsilon)\,\}}{\partial F} \,\right] [\,\overline{Q} - F\,] - b^I \left[\, \varepsilon\, \frac{\partial w(\varepsilon)}{\partial F} - \frac{\partial\, E\,\{\,\varepsilon\, w(\varepsilon)\,\}}{\partial F} \,\right]$$

$$=\; w(\varepsilon) - E\{\, w(\varepsilon)\,\} + [\,1-\gamma\,]\, b^I\, \varepsilon \left[-\frac{b}{2} \right] - \left[\left(-\frac{b}{2} \right) - \left(-\frac{b}{2} \right) \right] [\,\overline{Q} - F\,]$$

$$-\;\; b^I \left[\left(-\,\varepsilon\, \frac{b}{2} \right) - 0 \right] \;=\; w(\varepsilon) - E\{\, w(\varepsilon)\,\} - \frac{1}{2}\,[\,1-\gamma\,]\,\varepsilon + \frac{1}{2}\,\varepsilon$$

$$=\; \frac{\gamma}{2}\,\varepsilon + \; w(\varepsilon) - E\{\, w(\varepsilon)\,\} \;=\; \frac{\gamma}{2}\,\varepsilon + \frac{1}{2}\,\varepsilon \;=\; \left[\frac{1+\gamma}{2} \right] \varepsilon\,. \tag{33}$$

(27) and (28) imply that (32) can be written as:

$$\pi^B(\varepsilon) - E\{\pi^B(\varepsilon)\} \;=\; \left[\gamma\, r_0 + (\,1-\gamma\,)\, \frac{1}{2}\,(\,a + c - b\, F\,) \right] b^I\, \varepsilon$$

$$-\;\; \frac{1}{2}\,\varepsilon\, \overline{Q} - b^I \left[\frac{\varepsilon}{2}\,(\,a + \varepsilon + c - b\, F\,) - \frac{(\,\overline{\varepsilon}\,)^2}{6} \right] + \frac{1}{2}\,\varepsilon\, F\,. \tag{34}$$

Because $V_B = E\,\{\,[\,\pi^B(\varepsilon) - E\left\{\,\pi^B(\varepsilon)\right\}\,]^2\,\}$, (26), (33), and (34) imply:

$$\frac{\partial V_B}{\partial F} \;=\; \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} 2\,[\,\pi^B(\varepsilon) - E\{\pi^B(\varepsilon)\}\,]\; \frac{\partial\left(\,\pi^B(\varepsilon) - E\{\pi^B(\varepsilon)\}\,\right)}{\partial F}\, dH(\varepsilon) \tag{35}$$

$$= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} 2 \Bigg\{ \left[\gamma\, r_0 + (\,1-\gamma\,)\, \frac{1}{2}\,(\,a + c - b\, F\,) \right] b^I\, \varepsilon$$

$$-\;\; \frac{1}{2}\,\varepsilon\, \overline{Q} - b^I \left[\frac{\varepsilon}{2}\,(\,a + \varepsilon + c - b\, F\,) - \frac{(\,\overline{\varepsilon}\,)^2}{6} \right] + \frac{1}{2}\,\varepsilon\, F \Bigg\} \left[\frac{1+\gamma}{2} \right] \varepsilon\, dH(\varepsilon)$$

$$=\; 2 \left[\frac{1+\gamma}{2} \right] \left[\gamma\, r_0 + (\,1-\gamma\,)\, \frac{1}{2}\,(\,a + c - b\, F\,) \right] b^I \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon^2\, dH(\varepsilon)$$

$$-\, \overline{Q} \left[\frac{1+\gamma}{2} \right] \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon^2\, d\, H(\varepsilon) + \; F \left[\frac{1+\gamma}{2} \right] \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon^2\, dH(\varepsilon)$$

$$- 2 \left[\frac{1+\gamma}{2} \right] b^I \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \left[\frac{\varepsilon^2}{2} (a + \varepsilon + c - b F) - \frac{(\overline{\varepsilon})^2}{6} \varepsilon \right] dH(\varepsilon)$$

$$= [1+\gamma] \left[\gamma r_0 + (1-\gamma) \frac{1}{2} (a + c - b F) \right] b^I E\{ \varepsilon^2 \} - \overline{Q} \left[\frac{1+\gamma}{2} \right] E\{ \varepsilon^2 \}$$

$$+ F \left[\frac{1+\gamma}{2} \right] E\{ \varepsilon^2 \} - \left[\frac{1+\gamma}{2} \right] b^I [a + c - b F] E\{ \varepsilon^2 \}$$

$$- \left[\frac{1+\gamma}{2} \right] b^I E\{ \varepsilon^3 \} + [1+\gamma] b^I \frac{(\overline{\varepsilon})^2}{6} E\{ \varepsilon \}$$

$$= [1+\gamma] \left[\gamma r_0 + (1-\gamma) \frac{1}{2} (a + c - b F) \right] b^I E\{ \varepsilon^2 \}$$

$$- \overline{Q} \left[\frac{1+\gamma}{2} \right] E\{ \varepsilon^2 \} + F \left[\frac{1+\gamma}{2} \right] E\{ \varepsilon^2 \} - \left[\frac{1+\gamma}{2} \right] b^I [a + c - b F] E\{ \varepsilon^2 \}$$

$$= b^I \left[\frac{1+\gamma}{2} \right] E\{ \varepsilon^2 \} \left\{ 2 \gamma r_0 + [1-\gamma] [a + c - b F] + b \left[F - \overline{Q} \right] - [a + c - b F] \right\}$$

$$= b^I \left[\frac{1+\gamma}{2} \right] E\{ \varepsilon^2 \} \left[2 \gamma r_0 - \gamma (a + c - b F) + b \left(F - \overline{Q} \right) \right] . \tag{36}$$

Because $b \, b^I = 1$, (36) implies:

$$\frac{\partial^2 V_B}{\partial F^2} = \left[\frac{1+\gamma}{2} \right] E\{ \varepsilon^2 \} [\gamma + 1] > 0 . \tag{37}$$

(2), (36), and (37) imply that the value of F at which V_B is minimized is determined by:

$$2 \gamma r_0 - \gamma [a + c] - b \overline{Q} + b [1 + \gamma] F = 0$$

$$\Rightarrow F = \frac{1}{b [1 + \gamma]} \left[\gamma (a + c) + b \overline{Q} - 2 \gamma r_0 \right] = \frac{1}{1+\gamma} \left[b^I \gamma (a + c) + \overline{Q} - 2 \gamma b^I r_0 \right]$$

$$= \frac{1}{1+\gamma} \left[b^I \gamma \left(\frac{a^I + \overline{Q}}{b^I} + c \right) + \overline{Q} - 2 \gamma b^I r_0 \right]$$

$$= \frac{1}{1+\gamma} \left[\gamma \left(a^I - b^I [2 r_0 - c] \right) + (1+\gamma) \overline{Q} \right]$$

$$= \overline{Q} + \left[\frac{\gamma}{1+\gamma} \right] \left[a^I - b^I (2 r_0 - c) \right] = \underline{F}_{DM} . \tag{38}$$

(38) implies that $\underline{F}_{DM} < \overline{Q}$ when $\gamma > 0$ because Lemma 1 implies that $r_0 > E\{w(\varepsilon)\}$ for all $F \geq 0$ if and only if:

$$r_0 - \frac{1}{2\,b^I}\left[\,a^I + \overline{Q} + b^I c\,\right] \;>\; 0 \quad\Leftrightarrow\quad a^I + \overline{Q} + b^I c \;<\; 2\,b^I r_0$$

$$\Leftrightarrow\quad a^I - b^I\,[\,2\,r_0 - c\,] \;<\; -\,\overline{Q} \quad\Rightarrow\quad a^I - b^I\,[\,2\,r_0 - c\,] \;<\; 0\,. \tag{39}$$

It is apparent from (38) that $\frac{\partial F_{DM}}{\partial \overline{Q}} > 0$ and that $\frac{\partial F_{DM}}{\partial a^I} > 0$, $\frac{\partial F_{DM}}{\partial c} > 0$, and $\frac{\partial F_{DM}}{\partial r_0} < 0$ if $\gamma > 0$. (38) also implies that when $\gamma > 0$: (i) $\frac{\partial F_{DM}}{\partial b^I} \overset{s}{=} -\,(\,2\,r_0 - c\,) < 0$ (because $r_0 > c$); and, from (39), (ii) $\frac{\partial F_{DM}}{\partial \gamma} \overset{s}{=} -\,\frac{\partial}{\partial \gamma}\left(\frac{\gamma}{1+\gamma}\right) \;=\; -\,\frac{1}{[\,1+\gamma\,]^2} < 0$. ■

Proposition 3. *In the dual monopoly setting, EU^B does not vary with F if $\gamma = A_B = 0$. If $\gamma > 0$, then: (i) EU^B is strictly increasing in F if $A_B = 0$; whereas if $A_B > 0$, (ii) EU^B is maximized at $F_{BDM} \;\equiv\; \underline{F}_{DM} + \frac{\gamma}{[1+\gamma]^2}\,\frac{3\,\overline{Q}}{b^I A_B(\,\overline{\varepsilon}\,)^2}$, and (iii) F_{BDM} is increasing in $\overline{Q}$, a^I, and c, and decreasing in r_0, b^I, A_B, and $\overline{\varepsilon}$.*

Proof. (5) implies that EU^B does not vary with F if $\gamma = A_B = 0$. (5) also implies that EU^B is strictly increasing in F if $A_B = 0$ and $\gamma > 0$.

(2), (4), (5), (26), (36), and (38) imply that if $A_B > 0$, then EU^B is maximized where:

$$\frac{\partial EU^B}{\partial F} \;=\; \frac{\partial E\{\,\pi^B\,\}}{\partial F} - A_B\,\frac{\partial V_B}{\partial F} \;=\; 0$$

$$\Leftrightarrow\quad \gamma\,\frac{b}{2}\,\overline{Q} - A_B\,b^I\left[\frac{1+\gamma}{2}\right] E\{\,\varepsilon^2\,\}\left[\,2\,\gamma\,r_0 - \gamma\,(\,a + c - b\,F\,) + b\left(F - \overline{Q}\right)\right] \;=\; 0$$

$$\Leftrightarrow\quad \gamma\,\frac{b}{2}\,\overline{Q} - \frac{A_B}{2}\,[\,1+\gamma\,]\,E\{\,\varepsilon^2\,\}\left[\,2\,b^I\,\gamma\,r_0 - \gamma\,(\,a + c - b\,F\,)\,b^I - \overline{Q} \,+\, F\,\right] \;=\; 0$$

$$\Leftrightarrow\quad \gamma\,\frac{b}{2}\,\overline{Q} - \frac{A_B}{2}\,[\,1+\gamma\,]\,E\{\,\varepsilon^2\,\}\left[\,-\,\gamma\,(\,a + c - 2\,r_0\,)\,b^I - \overline{Q} \,+\, F\,(\,1+\gamma\,)\right] \;=\; 0$$

$$\Leftrightarrow\quad F\,\frac{A_B}{2}\,[\,1+\gamma\,]^2\,E\{\,\varepsilon^2\,\} \;=\; \gamma\,\frac{b}{2}\,\overline{Q} + \frac{A_B}{2}\,[\,1+\gamma\,]\,E\{\,\varepsilon^2\,\}\left[\,\gamma\,(\,a + c - 2\,r_0\,)\,b^I + \overline{Q}\,\right]$$

$$\Leftrightarrow\quad F\;A_B\,[\,1+\gamma\,]^2 \;=\; \frac{\gamma\,b\,\overline{Q}}{E\{\,\varepsilon^2\,\}} + A_B\,[\,1+\gamma\,]\left[\,\gamma\,(\,a + c - 2\,r_0\,)\,b^I + \overline{Q}\,\right]$$

$$\Leftrightarrow\quad F \;=\; \frac{3\,b\,\gamma\,\overline{Q}}{A_B\,[\,1+\gamma\,]^2\,(\,\overline{\varepsilon}\,)^2} + \frac{1}{1+\gamma}\left[\,\gamma\,(\,a + c - 2\,r_0\,)\,b^I + \overline{Q}\,\right]$$

$$=\; \frac{1}{1+\gamma}\left[\,\gamma\left(\frac{a^I + \overline{Q}}{b^I} + c - 2\,r_0\right) b^I + \overline{Q}\,\right] + \frac{3\,b\,\gamma\,\overline{Q}}{A_B\,[\,1+\gamma\,]^2\,(\,\overline{\varepsilon}\,)^2}$$

$$= \overline{Q} + \frac{\gamma}{1+\gamma} \left[a^I - b^I \left(2\, r_0 - c \right) \right] + \frac{\gamma}{[1+\gamma]^2} \frac{3\, \overline{Q}}{b^I A_B \left(\overline{\varepsilon} \right)^2}$$

$$= \underline{F}_{DM} + \frac{\gamma}{[1+\gamma]^2} \frac{3\, \overline{Q}}{b^I A_B \left(\overline{\varepsilon} \right)^2} = F_{BDM} \, . \tag{40}$$

Conclusion (iii) follows immediately from (38) and (40). ■

Corollary 1. *The set of parameter values for which F_{BDM} exceeds F_{GDM} expands as c increases or as r_0, A_B, or A_G declines.*

Proof. The conclusions follows directly from Propositions 1 and 3 for the reasons explained in the text. ■

Corollary 2. *Suppose $A_B = A_G \equiv A > 0$ and $\gamma > 0$. Then there exists an $\widehat{A} > 0$ such that $F_{BDM} \gtreqless F_{GDM} \Leftrightarrow A \lesseqgtr \widehat{A}$.*

Proof. (25), (38), and (40) imply that when $A_B = A_G = A$, $F_{BDM} \gtreqless F_{GDM}$

$$\Leftrightarrow \overline{Q} + \frac{\gamma}{1+\gamma} \left[a^I - b^I \left(2\, r_0 - c \right) \right] + \frac{\gamma}{[1+\gamma]^2} \left[\frac{3\, \overline{Q}}{b^I A \left(\overline{\varepsilon} \right)^2} \right]$$

$$\gtreqless \frac{A \left(\overline{\varepsilon} \right)^2 b^I \left[a^I + \overline{Q} - b^I c \right]}{3 + b^I A \left(\overline{\varepsilon} \right)^2}$$

$$\Leftrightarrow [1+\gamma]^2 \, \overline{Q} + \gamma [1+\gamma] \left[a^I - b^I \left(2\, r_0 - c \right) \right] + \gamma \left[\frac{3\, \overline{Q}}{b^I A \left(\overline{\varepsilon} \right)^2} \right]$$

$$\gtreqless \frac{[1+\gamma]^2 A \left(\overline{\varepsilon} \right)^2 b^I \left[a^I + \overline{Q} - b^I c \right]}{3 + b^I A \left(\overline{\varepsilon} \right)^2}$$

$$\Leftrightarrow [1+\gamma]^2 \, \overline{Q} \, b^I A \left(\overline{\varepsilon} \right)^2 \left[3 + b^I A \left(\overline{\varepsilon} \right)^2 \right]$$

$$+ \gamma [1+\gamma] \left[a^I - b^I \left(2\, r_0 - c \right) \right] b^I A \left(\overline{\varepsilon} \right)^2 \left[3 + b^I A \left(\overline{\varepsilon} \right)^2 \right]$$

$$+ \, 3\, \gamma \, \overline{Q} \left[3 + b^I A \left(\overline{\varepsilon} \right)^2 \right] \gtreqless [1+\gamma]^2 A \left(\overline{\varepsilon} \right)^2 b^I \left[a^I + \overline{Q} - b^I c \right] b^I A \left(\overline{\varepsilon} \right)^2$$

$$\Leftrightarrow 3 [1+\gamma]^2 \, \overline{Q} \, b^I A \left(\overline{\varepsilon} \right)^2 + [1+\gamma]^2 \, \overline{Q} \left(b^I \right)^2 A^2 \left(\overline{\varepsilon} \right)^4$$

$$+ \, 3\, \gamma [1+\gamma] \left[a^I - b^I \left(2\, r_0 - c \right) \right] b^I A \left(\overline{\varepsilon} \right)^2$$

$$+\ \gamma\,[\,1+\gamma\,]\,\big[\,a^I - b^I\,(\,2\,r_0 - c\,)\,\big]\,\big(b^I\big)^2\,A^2\,(\,\overline{\varepsilon}\,)^4 + 9\,\gamma\,\overline{Q}\, + 3\,\gamma\,\overline{Q}\,b^I\,A\,(\,\overline{\varepsilon}\,)^2$$

$$\gtreqless\ [\,1+\gamma\,]^2\,\big[\,a^I + \overline{Q} - b^I\,c\,\big]\,\big(b^I\big)^2\,A^2\,(\,\overline{\varepsilon}\,)^4$$

$$\Leftrightarrow\quad [\,1+\gamma\,]^2\ \overline{Q}\ \big(b^I\big)^2\,A^2\,(\,\overline{\varepsilon}\,)^4 + \gamma\,[\,1+\gamma\,]\,\big[\,a^I - b^I\,(\,2\,r_0 - c\,)\,\big]\,\big(b^I\big)^2\,A^2\,(\,\overline{\varepsilon}\,)^4$$

$$-\ [\,1+\gamma\,]^2\,\big[\,a^I + \overline{Q} - b^I\,c\,\big]\,\big(b^I\big)^2\,A^2\,(\,\overline{\varepsilon}\,)^4$$

$$+\ 3\,[\,1+\gamma\,]^2\ \overline{Q}\,b^I A\,(\,\overline{\varepsilon}\,)^2 + 3\,\gamma\,[\,1+\gamma\,]\,\big[\,a^I - b^I\,(\,2\,r_0 - c\,)\,\big]\,b^I A\,(\,\overline{\varepsilon}\,)^2$$

$$+\ 3\,\gamma\,\overline{Q}\,b^I\,A\,(\,\overline{\varepsilon}\,)^2 + 9\,\gamma\,\overline{Q}\ \ >\ 0$$

$$\Leftrightarrow\quad \big\{\,[\,1+\gamma\,]^2\ \overline{Q}\ + \gamma\,[\,1+\gamma\,]\,\big[\,a^I - b^I\,(\,2\,r_0 - c\,)\,\big] - [\,1+\gamma\,]^2\,\big[\,a^I + \overline{Q} - b^I\,c\,\big]\,\big\}$$

$$\cdot\ \big(b^I\big)^2\,A^2\,(\,\overline{\varepsilon}\,)^4$$

$$+\ \big\{\,3\,[\,1+\gamma\,]^2\ \overline{Q}\ + 3\,\gamma\,[\,1+\gamma\,]\,\big[\,a^I - b^I\,(\,2\,r_0 - c\,)\,\big] + 3\,\gamma\,\overline{Q}\ \big\}\,b^I A\,(\,\overline{\varepsilon}\,)^2$$

$$+\ 9\,\gamma\,\overline{Q}\ \ \gtreqless\ 0$$

$$\Leftrightarrow\quad [\,1+\gamma\,]\,\big\{\,\gamma\ \big[\,a^I - b^I\,(\,2\,r_0 - c\,)\,\big] - [\,1+\gamma\,]\,\big[\,a^I - b^I\,c\,\big]\,\big\}\,\big(b^I\big)^2\,A^2\,(\,\overline{\varepsilon}\,)^4$$

$$+\ 3\,\big\{\,\big[\,1 + 3\,\gamma + \gamma^2\,\big]\ \overline{Q}\ + \gamma\,[\,1+\gamma\,]\,\big[\,a^I - b^I\,(\,2\,r_0 - c\,)\,\big]\,\big\}\,b^I A\,(\,\overline{\varepsilon}\,)^2$$

$$+\ 9\,\gamma\,\overline{Q}\ \ \gtreqless\ 0\,. \tag{41}$$

Observe that:

$$\gamma\ \big[\,a^I - b^I\,(\,2\,r_0 - c\,)\,\big] - [\,1+\gamma\,]\,\big[\,a^I - b^I\,c\,\big]$$

$$=\ -\,a^I - \gamma\,b^I\,[\,2\,r_0 - c\,] + [\,1+\gamma\,]\,b^I\,c\ =\ -\,a^I + b^I\,c - \gamma\,b^I\,[\,2\,r_0 - 2\,c\,]$$

$$=\ -\ \big[\,a^I - b^I\,c + 2\,\gamma\,b^I\,(\,r_0 - c\,)\,\big]\,. \tag{42}$$

(41) and (42) imply that $F_{BDM} \gtreqless F_{GDM} \Leftrightarrow$

$$\Gamma(A)\ \equiv\ [\,1+\gamma\,]\,\big[\,a^I - b^I\,c + 2\,\gamma\,b^I\,(\,r_0 - c\,)\,\big]\,\big(b^I\big)^2\,A^2\,(\,\overline{\varepsilon}\,)^4$$

$$-\ 3\,\big\{\,\big[\,1 + 3\,\gamma + \gamma^2\,\big]\ \overline{Q}\ + \gamma\,[\,1+\gamma\,]\,\big[\,a^I - b^I\,(\,2\,r_0 - c\,)\,\big]\,\big\}\,b^I A\,(\,\overline{\varepsilon}\,)^2$$

$$-\ 9\,\gamma\,\overline{Q}\ \ \lesseqgtr\ 0\,. \tag{43}$$

(43) implies that if $a^I - b^I\,c + 2\,\gamma\,b^I\,[\,r_0 - c\,] > 0$, then:

$$\Gamma(0)\ <\ 0,\ \ \lim_{A\to\infty}\Gamma(A)\ =\ \infty,\ \text{and}\ \ \Gamma''(A)\ >\ 0\,. \tag{44}$$

(44) implies there exists a unique $\widehat{A} > 0$ such that:

$$\Gamma(A) \lesseqgtr 0 \;\Leftrightarrow\; A \lesseqgtr \widehat{A}\,. \tag{45}$$

Propositions 1 and 3 imply that $F_{BDM} > F_{GDM}$ when $A = 0$. Therefore, because $F_{BDM} \to \underline{F}_{DM}$ as $A \to \infty$, the Corollary follows from (43) and (45) because (25) and Proposition 2 imply:

$$\lim_{A \to \infty} F_{GDM} = \lim_{A \to \infty} \frac{(\overline{\varepsilon})^2 \, b^I \left[a^I + \overline{Q} - b^I c \right]}{3 \,/\, A + b^I \, (\overline{\varepsilon})^2}$$

$$= a^I + \overline{Q} - b^I c > \overline{Q} > \underline{F}_{DM}\,. \;\blacksquare$$

Proposition 4. *Suppose $A_{G1} = A_{G2} \equiv A_G$ and $c_1 = c_2 = c$. Then the aggregate level of forward contracting chosen by the generator(s) is higher in the duopoly generator setting than in the dual monopoly setting if and only if $A_G < \frac{54}{11\, b^I (\overline{\varepsilon})^2}$.*

Proof. The conclusion follows from (25) and (97) because:

$$\frac{2\left[24\,A\,(\overline{\varepsilon})^2 + 81\,b\right]\left[a - c\right]}{b\left[48\,A\,(\overline{\varepsilon})^2 + 405\,b\right]} > \frac{A_G\,(\overline{\varepsilon})^2\left[a - c\right]}{b\left[A\,(\overline{\varepsilon})^2 + 3\,b\right]}$$

$$\Leftrightarrow \frac{48\,A_G\,(\overline{\varepsilon})^2 + 162\,b}{48\,A_G\,(\overline{\varepsilon})^2 + 405\,b} > \frac{A_G\,(\overline{\varepsilon})^2}{A_G\,(\overline{\varepsilon})^2 + 3\,b}$$

$$\Leftrightarrow \left[48\,A_G\,(\overline{\varepsilon})^2 + 162\,b\right]\left[A_G\,(\overline{\varepsilon})^2 + 3\,b\right] > A_G\,(\overline{\varepsilon})^2\left[48\,A_G\,(\overline{\varepsilon})^2 + 405\,b\right]$$

$$\Leftrightarrow 48\,(A_G)^2\,(\overline{\varepsilon})^4 + 144\,A_G\,(\overline{\varepsilon})^2\,b + 162\,A_G\,(\overline{\varepsilon})^2\,b + 486\,b^2$$

$$> 48\,(A_G)^2\,(\overline{\varepsilon})^4 + 405\,A_G\,(\overline{\varepsilon})^2\,b$$

$$\Leftrightarrow 486\,b^2 > 99\,A_G\,(\overline{\varepsilon})^2\,b \;\Leftrightarrow\; A_G < \frac{54\,b}{11\,(\overline{\varepsilon})^2} = \frac{54}{11\,b^I\,(\overline{\varepsilon})^2}\,. \;\blacksquare$$

Lemma 3. *Suppose $c_1 = c_2 = c$ and $\gamma > 0$. Then $\underline{F}_{DG} < \underline{F}_{DM}$, so the aggregate level of forward contracting that minimizes V_B is smaller in the duopoly generator setting than in the dual monopoly setting.*

Proof. (38) and (111) imply that under the specified conditions, the aggregate level of forward contracting that minimizes V_B is lower in the duopoly generator setting than in the dual monopoly setting if and only if:

$$\overline{Q} + \left[\frac{\gamma}{1+\gamma} \right] \left[a^I - b^I \left(3\, r_0 - 2\, c \right) \right] \; < \; \overline{Q} + \left[\frac{\gamma}{1+\gamma} \right] \left[a^I - b^I \left(2\, r_0 - c \right) \right]$$

$$\Leftrightarrow \;\; a^I - b^I \left[2\, r_0 - c \right] \; > \; a^I - b^I \left[3\, r_0 - 2\, c \right]$$

$$\Leftrightarrow \;\; 3\, r_0 - 2\, c \; > \; 2\, r_0 - c \;\; \Leftrightarrow \;\; r_0 \; > \; c\,. \;\; \blacksquare$$

Proposition 5. *Suppose* $A_B > 0$*,* $\gamma > 0$*, and* $c_1 = c_2 = c$*. Then* $F_{BDG} < F_{BDM}$ *if and only if* $A_B > \frac{3\, \overline{Q}}{2 \left[1+\gamma \right] \left(b^I \right)^2 \left(\overline{\varepsilon} \right)^2 \left[r_0 - c \right]}$ *, so* B *prefers a smaller level of aggregate forward contracting in the duopoly generator setting than in the dual monopoly setting if and only if* B*'s aversion to profit variation is sufficiently pronounced.*

Proof. (38), (40), and (113) imply that under the specified conditions:

$$F_{BDM} \; > \; F_{BDG} \;\; \Leftrightarrow \;\; \overline{Q} + \frac{\gamma}{1+\gamma} \left[a^I - b^I \left(2\, r_0 - c \right) \right] + \frac{\gamma}{\left[1+\gamma \right]^2} \left[\frac{3\, \overline{Q}}{b^I A_B \left(\overline{\varepsilon} \right)^2} \right]$$

$$> \; \overline{Q} + \frac{\gamma}{1+\gamma} \left[a^I - b^I \left(3\, r_0 - 2\, c \right) \right] + \frac{\gamma}{\left[1+\gamma \right]^2} \left[\frac{9\, \overline{Q}}{2\, b^I A_B \left(\overline{\varepsilon} \right)^2} \right]$$

$$\Leftrightarrow \;\; \left[1+\gamma \right] \left[a^I - b^I \left(2\, r_0 - c \right) \right] + \frac{3\, \overline{Q}}{b^I A_B \left(\overline{\varepsilon} \right)^2}$$

$$> \; \left[1+\gamma \right] \left[a^I - b^I \left(3\, r_0 - 2\, c \right) \right] + \frac{9\, \overline{Q}}{2\, b^I A_B \left(\overline{\varepsilon} \right)^2}$$

$$\Leftrightarrow \;\; \left[1+\gamma \right] \left[b^I \left(3\, r_0 - 2\, c \right) - b^I \left(2\, r_0 - c \right) \right] \; > \; \frac{9\, \overline{Q}}{2\, b^I A_B \left(\overline{\varepsilon} \right)^2} - \frac{3\, \overline{Q}}{b^I A_B \left(\overline{\varepsilon} \right)^2}$$

$$\Leftrightarrow \;\; \left[1+\gamma \right] b^I \left[r_0 - c \right] \; > \; \frac{3\, \overline{Q}}{2\, b^I A_B \left(\overline{\varepsilon} \right)^2} \;\; \Leftrightarrow \;\; A_B \; > \; \frac{3\, \overline{Q}}{2 \left[1+\gamma \right] \left(b^I \right)^2 \left(\overline{\varepsilon} \right)^2 \left[r_0 - c \right]}\,. \;\; \blacksquare$$

Lemma 4. *Suppose* $c_1 = c_2 = c$*. Then the convexity of the variance of a buyer's profit is smaller in the duopoly buyer setting than in the dual monopoly setting.*

Proof. (121) implies that under the specified conditions in the duopoly buyer setting:

$$\frac{\partial^2 V_{Bi}}{\partial \left(F_i \right)^2} \; = \; \frac{1}{2} \left[\frac{1 + \frac{1}{2}\gamma}{2} \right] \; E\left\{ \varepsilon^2 \right\} \left[\gamma + 2 \right]$$

$$= \frac{1}{8}\,[\,2+\gamma\,]\;E\left\{\varepsilon^2\right\}[\,\gamma+2\,] \;=\; \frac{1}{8}\,[\,2+\gamma\,]^2\,E\{\,\varepsilon^2\,\}. \tag{46}$$

(37) implies that under the specified conditions in the dual monopoly setting:

$$\frac{\partial^2 V_B}{\partial F^2} \;=\; \frac{1}{2}\,[\,1+\gamma\,]^2\;E\{\,\varepsilon^2\,\}\,. \tag{47}$$

The conclusion follows from (46) and (47) because:

$$\frac{1}{8}\,[\,2+\gamma\,]^2 \;<\; \frac{1}{2}\,[\,1+\gamma\,]^2 \quad\Leftrightarrow\quad [\,2+\gamma\,]^2 \;<\; 8\,[\,1+\gamma\,]^2$$

$$\Leftrightarrow\quad 4+4\,\gamma+\gamma^2 \;<\; 8+16\,\gamma+8\,\gamma^2 \quad\Leftrightarrow\quad 4+12\,\gamma+7\,\gamma^2 \;>\; 0\,. \quad \blacksquare$$

Proposition 6. *When the buyers choose the levels of forward contracting in the duopoly buyer setting, Bi's equilibrium level of forward contracting is, for* $i,\,j \in \{1,2\}$ $(j \neq i)$, $F_{BiDB} = \frac{1+\alpha_j\,\gamma_j}{1+\alpha_i\,\gamma_i+\alpha_j\,\gamma_j}\,[\,\Psi_i\,(\,1+\alpha_i\,\gamma_i\,)-\alpha_i\,\gamma_i\,\Psi_j\,]$. *If* $\gamma > 0$ *and* $B1$ *and* $B2$ *are symmetric, then the aggregate equilibrium level of forward contracting is higher in the duopoly buyer setting than in the dual monopoly setting, i.e.,* $F_{B1DB}+F_{B2DB} > F_{BDM}$.

Proof. Define:

$$\Psi_i \;\equiv\; \overline{Q}_i - \frac{\alpha_i\,\gamma_i}{1+\alpha_i\,\gamma_i}\left[\,b^I\,(\,2\,r_{0i}-c\,)-a^I-\overline{Q}_j\,\right] + \frac{3\,\gamma_i\,\overline{Q}_i}{A_{Bi}\,b^I\,[\,1+\alpha_i\,\gamma_i\,]^2\,(\,\overline{\varepsilon}\,)^2}\,. \tag{48}$$

(48), (122), and (124) imply that the buyers' equilibrium forward contracting positions are determined by:

$$F_i \;=\; \Psi_i - \left[\frac{\alpha_i\,\gamma_i}{1+\alpha_i\,\gamma_i}\right]F_j \quad\Leftrightarrow\quad F_i \;=\; \Psi_i - \frac{\alpha_i\,\gamma_i}{1+\alpha_i\,\gamma_i}\left[\Psi_j - \frac{\alpha_j\,\gamma_j}{1+\alpha_j\,\gamma_j}\,F_i\right]$$

$$\Leftrightarrow\quad F_i\left[1-\left(\frac{\alpha_i\,\gamma_i}{1+\alpha_i\,\gamma_i}\right)\left(\frac{\alpha_j\,\gamma_j}{1+\alpha_j\,\gamma_j}\right)\right] \;=\; \frac{1}{1+\alpha_i\,\gamma_i}\,[\,\Psi_i\,(1+\alpha_i\,\gamma_i)-\alpha_i\,\gamma_i\,\Psi_j\,]$$

$$\Leftrightarrow\quad F_i\,\frac{[\,1+\alpha_i\,\gamma_i\,]\,[\,1+\alpha_j\,\gamma_j\,]-\alpha_i\,\gamma_i\,\alpha_j\,\gamma_j}{[\,1+\alpha_i\,\gamma_i\,]\,[\,1+\alpha_j\,\gamma_j\,]} \;=\; \frac{1}{1+\alpha_i\,\gamma_i}\,[\,\Psi_i\,(1+\alpha_i\,\gamma_i)-\alpha_i\,\gamma_i\,\Psi_j\,]$$

$$\Leftrightarrow\quad F_i\left[\frac{1+\alpha_i\,\gamma_i+\alpha_j\,\gamma_j}{1+\alpha_j\,\gamma_j}\right] \;=\; \Psi_i\,[\,1+\alpha_i\,\gamma_i\,]-\alpha_i\,\gamma_i\,\Psi_j$$

$$\Leftrightarrow\quad F_i \;=\; \frac{1+\alpha_j\,\gamma_j}{1+\alpha_i\,\gamma_i+\alpha_j\,\gamma_j}\,[\,\Psi_i\,(\,1+\alpha_i\,\gamma_i\,)-\alpha_i\,\gamma_i\,\Psi_j\,] \;=\; F_{BiDB}\,. \tag{49}$$

(48) implies that when $B1$ and $B2$ are symmetric:

$$\Psi_i \;=\; \frac{1}{2}\,\overline{Q} + \frac{\frac{1}{2}\,\gamma}{1+\frac{1}{2}\,\gamma}\left[\,\frac{1}{2}\,\overline{Q} + a^I - b^I\,(\,2\,r_0 - c\,)\right] + \frac{3\,\gamma\left[\,\frac{1}{2}\,\right]\,\overline{Q}}{A_B\,b^I\left[\,1+\frac{1}{2}\,\gamma\,\right]^2(\,\overline{\varepsilon}\,)^2}$$

$$=\; \frac{1}{2}\,\overline{Q} + \frac{\gamma}{2+\gamma}\left[\,\frac{1}{2}\,\overline{Q} + a^I - b^I\,(\,2\,r_0 - c\,)\right] + \frac{3\,\gamma\,\overline{Q}}{2\,A_B\,b^I\left[\,\frac{2+\gamma}{2}\,\right]^2(\,\overline{\varepsilon}\,)^2}$$

$$=\; \frac{1}{2}\,\overline{Q} + \frac{\gamma}{2\,[\,2+\gamma\,]}\,\overline{Q} + \frac{\gamma}{2+\gamma}\left[\,a^I - b^I\,(\,2\,r_0 - c\,)\,\right] + \frac{12\,\gamma\,\overline{Q}}{2\,A_B\,b^I\,[\,2+\gamma\,]^2\,(\,\overline{\varepsilon}\,)^2}$$

$$=\; \left[\frac{1+\gamma}{2+\gamma}\right]\overline{Q} + \frac{\gamma}{2+\gamma}\left[\,a^I - b^I\,(\,2\,r_0 - c\,)\,\right] + \frac{6\,\gamma\,\overline{Q}}{A_B\,b^I\,[\,2+\gamma\,]^2\,(\,\overline{\varepsilon}\,)^2}\,. \tag{50}$$

(49) implies that in this case:

$$F_{BiDB} \;=\; \frac{1+\frac{1}{2}\,\gamma}{1+\frac{1}{2}\,\gamma+\frac{1}{2}\,\gamma}\,[\,\Psi_i\,] \;=\; \frac{1+\frac{1}{2}\,\gamma}{1+\gamma}\,[\,\Psi_i\,] \;=\; \frac{2+\gamma}{2\,[\,1+\gamma\,]}\,[\,\Psi_i\,]\,. \tag{51}$$

(50) and (51) imply:

$$F_{BiDB} \;=\; \frac{\overline{Q}}{2} + \frac{\gamma\left[\,a^I - b^I\,(\,2\,r_0 - c\,)\right]}{2\,[\,1+\gamma\,]} + \frac{3\,\gamma\,\overline{Q}}{A_B\,b^I\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\overline{\varepsilon}\,)^2}\,. \tag{52}$$

Define $F_{BDB} \equiv F_{B1DB} + F_{B2DB}$. Then (52) implies that when $B1$ and $B2$ are symmetric:

$$F_{BDB} \;=\; \overline{Q} + \frac{\gamma\left[\,a^I - b^I\,(\,2\,r_0 - c\,)\right]}{1+\gamma} + \frac{6\,\gamma\,\overline{Q}}{A_B\,b^I\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\overline{\varepsilon}\,)^2}\,. \tag{53}$$

(38), (40), and (53) imply:

$$F_{BDB} \gtreqless F_{BDM} \;\Leftrightarrow\; \overline{Q} + \frac{\gamma}{1+\gamma}\left[\,a^I - b^I\,(\,2\,r_0 - c\,)\,\right] + \left[\frac{\gamma}{1+\gamma}\right]\frac{6\,\overline{Q}}{A_B\,b^I\,[\,2+\gamma\,]\,(\,\overline{\varepsilon}\,)^2}$$

$$\gtreqless \overline{Q} + \frac{\gamma}{1+\gamma}\left[\,a^I - b^I\,(\,2\,r_0 - c\,)\,\right] + \left[\frac{\gamma}{1+\gamma}\right]\frac{3\,\overline{Q}}{A_B\,b^I\,[\,1+\gamma\,]\,(\,\overline{\varepsilon}\,)^2}\,. \tag{54}$$

(54) implies that if $\gamma = 0$:

$$F_{BDB} \gtreqless F_{BDM} \;\Leftrightarrow\; \overline{Q} \gtreqless \overline{Q} \;\Rightarrow\; F^*_{BDB} \;=\; F_{BDM}.$$

(54) implies that if $\gamma > 0$:

$$F_{BDB} \gtreqless F_{BDM} \;\Leftrightarrow\; \frac{6}{2+\gamma} \gtreqless \frac{3}{1+\gamma} \;\Leftrightarrow\; 6 + 6\,\gamma \gtreqless 6 + 3\,\gamma$$

$$\Leftrightarrow\; 3\,\gamma \gtreqless 0 \;\Rightarrow\; F_{BDB} \;>\; F_{BDM}\,. \;\blacksquare$$

Proposition 7. *Suppose $\overline{Q}_1 + \overline{Q}_2 = \overline{Q}$ and the generators choose the levels of forward contracting. Then Gi ($i \in \{1,2\}$) implements the same number of forward contracts in equilibrium in the dual duopoly setting and in the duopoly generator setting.*

Proof. Let F_{ij} denote the number of forward contracts that Bi signs with Gj for $i, j \in \{1, 2\}$. Also define $F_{\cdot i} \equiv F_{1i} + F_{2i}$. Gi's problem in the dual duopoly setting after ε is realized is:

$$\underset{q_i \geq 0}{Maximize} \;\; \pi^{Gi}(\varepsilon) \; = \; w(\varepsilon)\,[\,q_i - F_{\cdot i}\,] - c_i\, q_i + p^F\, F_{\cdot i}\,. \tag{55}$$

(12) and (55) imply that at an interior optimum:

$$\frac{\partial \pi^{Gi}(\varepsilon)}{\partial q_i} \; = \; w(\varepsilon) + [\,q_i - F_{\cdot i}\,]\,\frac{\partial w(\cdot)}{\partial Q} - c_i \; = \; 0 \tag{56}$$

$$\Rightarrow \;\; a + \varepsilon - b\,[\,q_i + q_j\,] - b\,[\,q_i - F_{\cdot i}\,] - c_i \; = \; 0$$

$$\Rightarrow \;\; 2\,b\,q_i \; = \; a + \varepsilon - b\,q_j + b\,F_{\cdot i} - c_i$$

$$\Rightarrow \;\; q_i \; = \; \frac{1}{2\,b}\,[\,a + \varepsilon - c_i + b\,F_{\cdot i}\,] - \frac{1}{2}\,q_j\,. \tag{57}$$

(57) implies that in equilibrium in this case:

$$q_i \; = \; \frac{1}{2\,b}\,[\,a + \varepsilon - c_i + b\,F_{\cdot i}\,] - \frac{1}{2}\left\{ \frac{1}{2\,b}\,[\,a + \varepsilon - c_j + b\,F_{\cdot j}\,] - \frac{1}{2}\,q_i \right\}$$

$$\Rightarrow \;\; \frac{3}{4}\,q_i \; = \; \frac{1}{4\,b}\,[\,2\,a + 2\,\varepsilon - 2\,c_i + 2\,b\,F_{\cdot i} - a - \varepsilon + c_j - b\,F_{\cdot j}\,]$$

$$\Rightarrow \;\; q_i(\varepsilon) \; = \; \frac{1}{3\,b}\,[\,a + \varepsilon - 2\,c_i + c_j + b\,(\,2\,F_{\cdot i} - F_{\cdot j}\,)\,] \tag{58}$$

$$\Rightarrow \;\; Q(\varepsilon) \; = \; q_1(\varepsilon) + q_2(\varepsilon) \; = \; \frac{1}{3\,b}\,[\,2\,a + 2\,\varepsilon - c_1 - c_2 + b\,(\,F_{\cdot 1} + F_{\cdot 2}\,)\,]\,. \tag{59}$$

(12) and (59) imply:

$$w(\varepsilon) \; = \; a + \varepsilon - \frac{1}{3}\,[\,2\,a + 2\,\varepsilon - c_1 - c_2 + b\,(\,F_{\cdot 1} + F_{\cdot 2}\,)\,]$$

$$= \; \frac{1}{3}\,[\,a + \varepsilon + c_1 + c_2 - b\,(\,F_{\cdot 1} + F_{\cdot 2}\,)\,] \tag{60}$$

$$\Rightarrow \;\; p^F \; = \; E\,\{\,w^*(\varepsilon)\,\} \; = \; \frac{1}{3}\,[\,a + E\,\{\varepsilon\} + c_1 + c_2 - b\,(\,F_{\cdot 1} + F_{\cdot 2}\,)\,]\,. \tag{61}$$

(58), (60), and (61) imply that Gi's equilibrium profit when ε is realized is:

$$\pi^{Gi}(\varepsilon) \; = \; [\,w(\varepsilon) - c_i\,]\,q_i(\varepsilon) + \left[\,p^F - w(\varepsilon)\,\right]F_{\cdot i}$$

$$= \frac{1}{3}\left[a + \varepsilon - 2\,c_i + c_j - b\,(\,F_{\cdot i} + F_{\cdot j}\,)\right] \frac{1}{3\,b}\left[a + \varepsilon - 2\,c_i + c_j + b\,(\,2\,F_{\cdot i} - F_{\cdot j}\,)\right]$$

$$+ \frac{1}{3}\,\{\,a + E\,\{\varepsilon\} + c_1 + c_2 - b\,(\,F_{\cdot 1} + F_{\cdot 2}\,) - [\,a + \varepsilon + c_1 + c_2 - b\,(\,F_{\cdot 1} + F_{\cdot 2}\,)\,]\,\}\,F_{\cdot i}$$

$$= \frac{1}{9\,b}\left[a + \varepsilon - 2\,c_i + c_j - b\,(\,F_{\cdot i} + F_{\cdot j}\,)\right]\left[a + \varepsilon - 2\,c_i + c_j + b\,(\,2\,F_{\cdot i} - F_{\cdot j}\,)\right]$$

$$+ \frac{1}{3}\left[E\,\{\varepsilon\} - \varepsilon\right]\,F_{\cdot i}\,. \tag{62}$$

(62) implies:

$$E\left\{\pi^{Gi}(\varepsilon)\right\} = \frac{1}{9\,b}\{\,[\,a - 2\,c_i + c_j - b\,(\,F_{\cdot i} + F_{\cdot j}\,)\,]\,[\,a - 2\,c_i + c_j + b\,(\,2\,F_{\cdot i} - F_{\cdot j}\,)\,]$$

$$+ \; E\{\,\varepsilon\,\}\,[\,a - 2\,c_i + c_j + b\,(\,2\,F_{\cdot i} - F_{\cdot j}\,) + a - 2\,c_i + c_j - b\,(\,F_{\cdot i} + F_{\cdot j}\,)\,]$$

$$+ \; E\left\{\varepsilon^2\right\}\,\} + \frac{1}{3}\left[E\,\{\varepsilon\} - E\,\{\varepsilon\}\right]\,F_{\cdot i}$$

$$= \frac{1}{9\,b}\left[a - 2\,c_i + c_j - b\,(\,F_{\cdot i} + F_{\cdot j}\,)\right]\left[a - 2\,c_i + c_j + b\,(\,2\,F_{\cdot i} - F_{\cdot j}\,)\right] + \frac{1}{9\,b}\,E\left\{\varepsilon^2\right\}. \tag{63}$$

(26), (62), and (63) imply:

$$\pi^{Gi}(\varepsilon) - E\left\{\pi^{Gi}(\varepsilon)\right\}$$

$$= \frac{1}{9\,b}\left[a + \varepsilon - 2\,c_i + c_j - b\,(\,F_{\cdot i} + F_{\cdot j}\,)\right]\left[a + \varepsilon - 2\,c_i + c_j + b\,(\,2\,F_{\cdot i} - F_{\cdot j}\,)\right]$$

$$+ \frac{1}{3}\left[E\,\{\varepsilon\} - \varepsilon\right]\,F_{\cdot i}$$

$$- \frac{1}{9\,b}\left[a - 2\,c_i + c_j - b\,(\,F_{\cdot i} + F_{\cdot j}\,)\right]\left[a - 2\,c_i + c_j + b\,(\,2\,F_{\cdot i} - F_{\cdot j}\,)\right] - \frac{(\overline{\varepsilon})^2}{27\,b}$$

$$= \frac{\varepsilon}{9\,b}\left[a - 2\,c_i + c_j - b\,(\,F_{\cdot i} + F_{\cdot j}\,) + a - 2\,c_i + c_j + b\,(\,2\,F_{\cdot i} - F_{\cdot j}\,)\right]$$

$$+ \frac{1}{9\,b}\,\varepsilon^2 - \frac{1}{3}\,\varepsilon\,F_{\cdot i} - \frac{(\overline{\varepsilon})^2}{27\,b}$$

$$= \frac{3\,\varepsilon}{27\,b}\left[\,2\,(\,a - 2\,c_i + c_j\,) + b\,(\,F_{\cdot i} - 2\,F_{\cdot j}\,)\,\right] + \frac{1}{27\,b}\left[\,3\,\varepsilon^2 - 9\,b\,F_{\cdot i}\,\varepsilon - (\overline{\varepsilon})^2\,\right]$$

$$= \frac{1}{27\,b}\left[\,3\,\varepsilon^2 + 3\,Z_i\,\varepsilon - (\overline{\varepsilon})^2\,\right] \tag{64}$$

where $Z_i \;=\; 2\,[\,a - 2\,c_i + c_j\,] + b\,[\,F_{\cdot i} - 2\,F_{\cdot j}\,] - 3\,b\,F_{\cdot i}$

$$=\; 2\,[\,a - 2\,c_i + c_j - b\,(F_{\cdot i} + F_{\cdot j})\,] \;=\; X_i\,. \tag{65}$$

(64), (87), and (88) imply that the variance of Gi's equilibrium profit is:

$$V_{Gi} \;=\; \frac{(\,\bar{\varepsilon}\,)^2}{729\,b^2}\left[\,\frac{14}{5}\,(\,\bar{\varepsilon}\,)^2 + 3\,(X_i)^2 - 2\,(\,\bar{\varepsilon}\,)^2\,\right]. \tag{66}$$

(63), (65), and (66) imply that Gi's expected utility is:

$$\begin{aligned}
EU^{Gi} \;&=\; \frac{1}{9\,b}\,[\,a - 2\,c_i + c_j - b\,(\,F_{\cdot i} + F_{\cdot j}\,)\,]\,[\,a - 2\,c_i + c_j + b\,(\,2\,F_{\cdot i} - F_{\cdot j}\,)\,] + \frac{(\,\bar{\varepsilon}\,)^2}{27\,b} \\
&\quad - \frac{A_{Gi}\,(\,\bar{\varepsilon}\,)^2}{729\,b^2}\left[\,\frac{14}{5}\,(\,\bar{\varepsilon}\,)^2 + 3\,(X_i)^2 - 2\,(\,\bar{\varepsilon}\,)^2\,\right]
\end{aligned}$$

$$\begin{aligned}
\Rightarrow\;\; \frac{\partial EU^{Gi}}{\partial F_{\cdot i}} \;&=\; \frac{1}{9\,b}\,\{\,2\,b\,[\,a - 2\,c_i + c_j - b\,(\,F_{\cdot i} + F_{\cdot j}\,)\,] - b\,[\,a - 2\,c_i + c_j + b\,(\,2\,F_{\cdot i} - F_{\cdot j}\,)\,]\,\} \\
&\quad - \frac{A_{Gi}\,(\,\bar{\varepsilon}\,)^2\;6\,X_i}{729\,b^2}\;\frac{\partial X_i}{\partial F_{\cdot i}} \\
&= \frac{1}{9\,b}\,\left[\,b\,(\,a - 2\,c_i + c_j)\; - \;4\,b^2\,F_{\cdot i} - b^2\,F_{\cdot j}\,\right] - \frac{A_{Gi}\,(\,\bar{\varepsilon}\,)^2\;6\,X_i}{729\,b^2}\;[\,-\,2\,b\,] \\
&= \frac{1}{9}\,[\,a - 2\,c_i + c_j\,] - \frac{4}{9}\,b\,F_{\cdot i} - \frac{1}{9}\,b\,F_{\cdot j} + \frac{24\,A_{Gi}\,(\,\bar{\varepsilon}\,)^2}{729\,b}\;[\,a - 2\,c_i + c_j - b\,(\,F_{\cdot i} + F_{\cdot j}\,)\,] \\
&= \frac{24\,A_{Gi}\,(\,\bar{\varepsilon}\,)^2 + 81\,b}{729\,b}\;[\,a - 2\,c_i + c_j\,] - b\,F_{\cdot i}\left[\,\frac{4}{9} + \frac{24\,A_{Gi}\,(\,\bar{\varepsilon}\,)^2}{729\,b}\,\right] - b\,F_{\cdot j}\left[\,\frac{1}{9} + \frac{24\,A_{Gi}\,(\,\bar{\varepsilon}\,)^2}{729\,b}\,\right] \\
&= \frac{24\,A_{Gi}\,(\,\bar{\varepsilon}\,)^2 + 81\,b}{729\,b}\;[\,a - 2\,c_i + c_j\,] - b\left[\,\frac{24\,A_{Gi}\,(\,\bar{\varepsilon}\,)^2 + 324\,b}{729\,b}\,\right]F_{\cdot i} - b\left[\,\frac{24\,A_{Gi}\,(\,\bar{\varepsilon}\,)^2 + 81\,b}{729\,b}\,\right]F_{\cdot j} \\
&= \frac{1}{729\,b}\,\{\,\left[\,24\,A_{Gi}\,(\,\bar{\varepsilon}\,)^2 + 81\,b\,\right][\,a - 2\,c_i + c_j\,] - b\left[\,24\,A_{Gi}\,(\,\bar{\varepsilon}\,)^2 + 324\,b\,\right]F_{\cdot i} \\
&\qquad - \,b\left[\,24\,A_{Gi}\,(\,\bar{\varepsilon}\,)^2 + 81\,b\,\right]F_{\cdot j}\,\}\,.
\end{aligned} \tag{67}$$

(10), (11), and (67) imply that EU^{Gi} is a strictly concave function of $F_{\cdot i}$, so the level of $F_{\cdot i}$ that maximizes Gi's expected utility is given by:

$$b\left[\,24\,A_{Gi}\,(\,\bar{\varepsilon}\,)^2 + 324\,b\,\right]F_{\cdot i} \;=\; \left[\,24\,A_{Gi}\,(\,\bar{\varepsilon}\,)^2 + 81\,b\,\right][\,a - 2\,c_i + c_j\,] - b\left[\,24\,A_{Gi}\,(\,\bar{\varepsilon}\,)^2 + 81\,b\,\right]F_{\cdot j}$$

$$\begin{aligned}
\Rightarrow\;\; F_{\cdot i} \;&=\; \frac{\left[\,24\,A_{Gi}\,(\,\bar{\varepsilon}\,)^2 + 81\,b\,\right][\,a - 2\,c_i + c_j\,] - b\left[\,24\,A_{Gi}\,(\,\bar{\varepsilon}\,)^2 + 81\,b\,\right]F_{\cdot j}}{b\left[\,24\,A_{Gi}\,(\,\bar{\varepsilon}\,)^2 + 324\,b\,\right]} \\
&= \delta_i - \beta_i\,F_{\cdot j}\,.
\end{aligned} \tag{68}$$

By symmetry:

$$F_{\cdot j} = \frac{\left[24\, A_{Gj}\, (\overline{\varepsilon})^2 + 81\, b \right] [\, a - 2\, c_j + c_i \,] - b \left[24\, A_{Gj}\, (\overline{\varepsilon})^2 + 81\, b \right] F_{\cdot i}}{b \left[24\, A_{Gj}\, (\overline{\varepsilon})^2 + 324\, b \right]}$$

$$= \delta_j - \beta_j\, F_{\cdot i}\,. \tag{69}$$

(68) and (69) imply that in equilibrium:

$$F_{\cdot i} = \delta_i - \beta_i\, F_{\cdot j} \quad \text{and} \quad F_{\cdot j} = \delta_j - \beta_j\, F_{\cdot i}$$

$$\Rightarrow \quad F_{\cdot i} = \delta_i - \beta_i \left[\delta_j - \beta_j\, F_{\cdot i} \right] \quad \Rightarrow \quad F_{\cdot iDD} = \frac{\delta_i - \beta_i\, \delta_j}{1 - \beta_i\, \beta_j}$$

$$\Rightarrow \quad F_{\cdot jDD} = \delta_j - \beta_j \left[\frac{\delta_i - \beta_i\, \delta_j}{1 - \beta_i\, \beta_j} \right] = \frac{\delta_j \left[1 - \beta_i\, \beta_j \right] - \beta_j \left[\delta_i - \beta_i\, \delta_j \right]}{1 - \beta_i\, \beta_j}$$

$$= \frac{\delta_j - \beta_j\, \delta_i}{1 - \beta_i\, \beta_j}\,. \tag{70}$$

The conclusion in the proposition follows directly from (70) and (93) (below). ■

Lemma 5. *Suppose $c_1 = c_2 = c$. Then the rate at which a buyer's expected profit increases with its forward contracting in the dual duopoly setting is less than one half of the corresponding rate in the dual monopoly setting.*

<u>Proof</u>. (5) implies that under the specified conditions in the dual monopoly setting:

$$\frac{\partial E\left\{ \pi^B \right\}}{\partial F} = \frac{\gamma}{2\, b^I}\, \overline{Q}\,. \tag{71}$$

(9) implies that under the specified conditions in the dual duopoly setting:

$$\frac{\partial E\left\{ \pi^B \right\}}{\partial F_i} = \frac{\gamma}{6\, b^I}\, \overline{Q}\,. \tag{72}$$

The conclusion follows from (71) and (72) because $\frac{1}{6} < \frac{1}{2}\left[\frac{1}{2} \right]$. ■

Proposition 8. *Suppose $B1$ and $B2$ are symmetric, $c_1 = c_2 = c$, $\gamma > 0$, and the buyers choose the levels of forward contracting. Then the equilibrium number of forward contracts is lower in the dual duopoly setting than in the dual monopoly setting if and only if:*

$$r_0 > c + \left[\frac{1 + 2\,\gamma}{(\,1+\gamma\,)\,(\,2+\gamma\,)} \right] \frac{3\, \overline{Q}}{A_B\, (b^I)^2\, (\overline{\varepsilon})^2}\,. \tag{73}$$

Proof. Define:

$$\widehat{\Psi}_i \;\equiv\; \overline{Q}_i + \frac{\alpha_i\,\gamma_i}{1+\alpha_i\,\gamma_i}\,[\,\overline{Q}_j + a^I - b^I\,(\,3\,r_{0i} - c_1 - c_2\,)\,] + \frac{9\,\gamma_i\,\overline{Q}_i}{2\,A_{Bi}\,b^I\,[\,1+\alpha_i\,\gamma_i\,]^2\,(\,\overline{\varepsilon}\,)^2}\,. \tag{74}$$

(74) implies that when $B1$ and $B2$ are symmetric:

$$\begin{aligned}
\widehat{\Psi}_i \;&=\; \frac{1}{2}\,\overline{Q} + \frac{\frac{1}{2}\,\gamma}{1+\frac{1}{2}\,\gamma}\left[\,\frac{1}{2}\,\overline{Q} + a^I - b^I\,(\,3\,r_0 - c_1 - c_2\,)\right] + \frac{9\,\gamma\,[\,\frac{1}{2}\,]\,\overline{Q}}{2\,A_B\,b^I\,\left[\,1+\frac{1}{2}\,\gamma\,\right]^2\,(\,\overline{\varepsilon}\,)^2} \\
&=\; \frac{1}{2}\,\overline{Q} + \frac{\gamma}{2+\gamma}\left[\,\frac{1}{2}\,\overline{Q} + a^I - b^I\,(\,3\,r_0 - c_1 - c_2\,)\right] + \frac{9\,\gamma\,\overline{Q}}{4\,A_B\,b^I\,[\,\frac{2+\gamma}{2}\,]^2\,(\,\overline{\varepsilon}\,)^2} \\
&=\; \frac{1}{2}\,\overline{Q} + \frac{\gamma}{2\,[\,2+\gamma\,]}\,\overline{Q} + \frac{\gamma}{2+\gamma}\,[\,a^I - b^I\,(\,3\,r_0 - c_1 - c_2\,)\,] + \frac{9\,\gamma\,\overline{Q}}{A_B\,b^I\,[\,2+\gamma\,]^2\,(\,\overline{\varepsilon}\,)^2} \\
&=\; \left[\frac{1+\gamma}{2+\gamma}\right]\overline{Q} + \frac{\gamma}{2+\gamma}\,[\,a^I - b^I\,(\,3\,r_0 - c_1 - c_2\,)\,] + \frac{9\,\gamma\,\overline{Q}}{A_B\,b^I\,[\,2+\gamma\,]^2\,(\,\overline{\varepsilon}\,)^2}\,.
\end{aligned} \tag{75}$$

(141) (below) implies that in this case:

$$F_{Bi\cdot DD} \;=\; \frac{1+\frac{1}{2}\,\gamma}{1+\frac{1}{2}\,\gamma+\frac{1}{2}\,\gamma}\left[\,\widehat{\Psi}_i\,\right] \;=\; \frac{1+\frac{1}{2}\,\gamma}{1+\gamma}\left[\,\widehat{\Psi}_i\,\right] \;=\; \frac{2+\gamma}{2\,[\,1+\gamma\,]}\left[\,\widehat{\Psi}_i\,\right]. \tag{76}$$

(75) and (76) imply:

$$F_{Bi\cdot DD} \;=\; \frac{\overline{Q}}{2} + \frac{\gamma\,[\,a^I - b^I\,(\,3\,r_0 - c_1 - c_2\,)\,]}{2\,[\,1+\gamma\,]} + \frac{9\,\gamma\,\overline{Q}}{2\,A_B\,b^I\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\overline{\varepsilon}\,)^2}\,. \tag{77}$$

(77) implies:

$$\begin{aligned}
F_{BDD} \;=\; F_{B1\cdot DD} + F_{B2\cdot DD} \;=\; \overline{Q} \;&+\; \frac{\gamma\,[\,a^I - b^I\,(\,3\,r_0 - c_1 - c_2\,)\,]}{1+\gamma} \\
&+\; \frac{9\,\gamma\,\overline{Q}}{A_B\,b^I\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\overline{\varepsilon}\,)^2}\,.
\end{aligned} \tag{78}$$

(38), (40), and (78) imply that $F_{BDM} > F_{BDD}$ under the specified conditions if and only if:

$$\begin{aligned}
&\overline{Q} + \frac{\gamma}{1+\gamma}\,[\,a^I - b^I\,(\,2\,r_0 - c\,)\,] + \left[\frac{\gamma}{1+\gamma}\right]\frac{3\,\overline{Q}}{b^I A_B[\,1+\gamma\,]\,(\,\overline{\varepsilon}\,)^2} \\
&\quad > \;\overline{Q} + \left[\frac{\gamma}{1+\gamma}\right][\,a^I - b^I\,(\,3\,r_0 - 2\,c\,)\,] + \left[\frac{\gamma}{1+\gamma}\right]\frac{9\,\overline{Q}}{A_B\,b^I\,[\,2+\gamma\,]\,(\,\overline{\varepsilon}\,)^2}
\end{aligned}$$

$$\Leftrightarrow \;\; a^I - b^I\,[\,2\,r_0 - c\,] + \frac{3\,\overline{Q}}{b^I A_B[\,1+\gamma\,]\,(\,\overline{\varepsilon}\,)^2}$$

$$> \; a^I - b^I\,[\,3\,r_0 - 2\,c\,] + \frac{9\,\overline{Q}}{A_B\,b^I\,[\,2+\gamma\,]\,(\,\overline{\varepsilon}\,)^2}$$

$$\Leftrightarrow \;\; b^I\,[\,3\,r_0 - 2\,c - (2\,r_0 - c\,)\,] \; > \; \left[\frac{9}{2+\gamma} - \frac{3}{1+\gamma}\right] \frac{\overline{Q}}{A_B\,b^I\,(\,\overline{\varepsilon}\,)^2}$$

$$\Leftrightarrow \;\; b^I\,[\,r_0 - c\,] \; > \; \left[\frac{9\,(1+\gamma) - 3\,(2+\gamma)}{(1+\gamma)\,(2+\gamma)}\right] \frac{\overline{Q}}{A_B\,b^I\,(\,\overline{\varepsilon}\,)^2} \; = \; \left[\frac{3+6\,\gamma}{(1+\gamma)\,(2+\gamma)}\right] \frac{\overline{Q}}{A_B\,b^I\,(\,\overline{\varepsilon}\,)^2}$$

$$\Leftrightarrow \;\; b^I\,[\,r_0 - c\,] \; > \; \left[\frac{1+2\,\gamma}{1+\gamma}\right] \frac{3\,\overline{Q}}{A_B\,b^I\,[\,2+\gamma\,]\,(\,\overline{\varepsilon}\,)^2}$$

$$\Leftrightarrow \;\; r_0 \; > \; c + \left[\frac{1+2\,\gamma}{1+\gamma}\right] \frac{3\,\overline{Q}}{A_B\,[\,2+\gamma\,]\,(b^I)^2\,(\,\overline{\varepsilon}\,)^2}\,. \quad \blacksquare$$

B. Detailed Proofs of Formal Conclusions in Appendix B.

Lemma B1. In equilibrium in the duopoly generator setting, for $i, j \in \{1, 2\}$ $(j \neq i)$:

$$w(\varepsilon) \; = \; \frac{1}{3\,b^I}\left[\,a^I + \overline{Q} - (\,F_1 + F_2\,) + b^I\,(\,c_1 + c_2 + \varepsilon\,)\,\right];$$

$$p^f \; = \; \frac{1}{3\,b^I}\left[\,a^I + \overline{Q} - (\,F_1 + F_2\,) + b^I\,(\,c_1 + c_2\,)\,\right];$$

$$q_i(\varepsilon) \; = \; \frac{1}{3}\left[\,a^I \, + \overline{Q} + 2\,F_i - F_j \, - b^I\,(\,2\,c_i - c_j - \varepsilon\,)\,\right]; \text{ and}$$

$$E\left\{\pi^{Gi}(\varepsilon)\right\} \; = \; \frac{1}{9\,b^I}\left[\,a^I + \overline{Q} - (\,F_i + F_j\,) - b^I\,(\,2\,c_i - c_j\,)\,\right]$$
$$\cdot \left[\,a^I + \overline{Q} + 2\,F_i - F_j \, - b^I\,(\,2\,c_i - c_j\,)\,\right] + \frac{b^I\,(\,\overline{\varepsilon}\,)^2}{27}\,.$$

<u>Proof</u>. When ε is realized, Gi's problem is:

$$\underset{q_i \geq 0}{Maximize} \;\; \pi^{Gi}(\varepsilon) \; = \; w(\varepsilon)\,[\,q_i - F_i\,] - c_i\,q_i + p^F\,F_i\,. \tag{79}$$

(2) and (79) imply that Gi's profit-maximizing choice of $q_i > 0$ is determined by:

$$\frac{\partial \pi^{Gi}(\varepsilon)}{\partial q_i} \; = \; w(\varepsilon) + [\,q_i - F_i\,]\,\frac{\partial w(\cdot)}{\partial Q} - c_i \; = \; a + \varepsilon - b\,[\,q_i + q_j\,] - b\,[\,q_i - F_i\,] - c_i \; = \; 0$$

$$\Rightarrow \;\; q_i \; = \; \frac{1}{2\,b}\,[\,a + \varepsilon - c_i + b\,F_i\,] - \frac{1}{2}\,q_j\,. \tag{80}$$

(2) and (80) imply that in equilibrium:

$$
\begin{aligned}
q_i &= \frac{1}{2\,b}\,[\,a+\varepsilon-c_i+b\,F_i\,]-\frac{1}{2}\left[\frac{1}{2\,b}\,(\,a+\varepsilon-c_j+b\,F_j\,)-\frac{1}{2}\,q_i\right] \\
\Rightarrow \quad \frac{3}{4}\,q_i &= \frac{1}{4\,b}\,[\,2\,a+2\,\varepsilon-2\,c_i+2\,b\,F_i-a-\varepsilon+c_j-b\,F_j\,] \\
\Rightarrow \quad q_i(\varepsilon) &= \frac{1}{3\,b}\,[\,a+\varepsilon-2\,c_i+c_j+b\,(\,2\,F_i-F_j\,)\,] \\
&= \frac{b^I}{3}\left[\frac{a^I+\overline{Q}}{b^I}+\varepsilon-2\,c_i+c_j+b\,(\,2\,F_i-F_j\,)\right] \\
&= \frac{1}{3}\left[\,a^I+\overline{Q}+2\,F_i-F_j-b^I\,(\,2\,c_i-c_j-\varepsilon\,)\,\right].
\end{aligned} \tag{81}
$$

(81) implies:

$$
Q(\varepsilon) = q_1(\varepsilon)+q_2(\varepsilon) = \frac{1}{3}\left[\,2\left(a^I+\overline{Q}\right)+F_1+F_2-b^I\,(\,c_1+c_2-2\,\varepsilon\,)\,\right]. \tag{82}
$$

(2) and (82) imply:

$$
\begin{aligned}
w(\varepsilon) &= \frac{a^I+\overline{Q}+b^I\varepsilon}{b^I}-\frac{1}{3\,b^I}\left[\,2\left(a^I+\overline{Q}\right)+F_1+F_2-b^I\,(\,c_1+c_2-2\,\varepsilon\,)\,\right] \\
&= \frac{1}{3\,b^I}\left[\,a^I+\overline{Q}-(\,F_1+F_2\,)+b^I\,(\,c_1+c_2+\varepsilon\,)\,\right] \\
\Rightarrow \quad p^f &= E\,\{\,w(\varepsilon)\,\} = \frac{1}{3\,b^I}\left[\,a^I+\overline{Q}-(\,F_1+F_2\,)+b^I\,(\,c_1+c_2\,)\,\right].
\end{aligned} \tag{83}
$$

(79), (81), and (83) imply that Gi's equilibrium profit when ε is realized is:

$$
\begin{aligned}
\pi^{Gi}(\varepsilon) &= [\,w(\varepsilon)-c_i\,]\,q_i(\varepsilon)+\left[\,p^F-w(\varepsilon)\,\right]F_i \\
&= \frac{1}{9\,b^I}\left[\,a^I+\overline{Q}-(\,F_1+F_2\,)-b^I\,(\,2\,c_i-c_j-\varepsilon\,)\,\right] \\
&\quad \cdot\left[\,a^I+\overline{Q}+2\,F_i-F_j-b^I\,(\,2\,c_i-c_j-\varepsilon\,)\,\right]-\frac{1}{3\,b^I}\,\varepsilon\,F_i\,.
\end{aligned} \tag{84}
$$

(26) and (84) imply:

$$
\begin{aligned}
E\left\{\pi^{Gi}(\varepsilon)\right\} &= \frac{1}{9\,b^I}\left[\,a^I+\overline{Q}-(\,F_1+F_2\,)-b^I\,(\,2\,c_i-c_j\,)\,\right] \\
&\quad \cdot\left[\,a^I+\overline{Q}+2\,F_i-F_j-b^I\,(\,2\,c_i-c_j\,)\,\right]+\frac{1}{9\,b^I}\,\left(b^I\right)^2\,E\left\{\varepsilon^2\right\} \\
&= \frac{1}{9\,b^I}\left[\,a^I+\overline{Q}-(\,F_1+F_2\,)-b^I\,(\,2\,c_i-c_j\,)\,\right]
\end{aligned}
$$

$$\cdot \left[a^I + \overline{Q} + 2\,F_i - F_j \; - b^I \left(2\,c_i - c_j \right) \right] + \frac{b^I}{27} \left(\overline{\varepsilon} \right)^2 . \quad \blacksquare \tag{85}$$

Lemma B2. $V_{Gi} = \frac{\left(b^I\right)^2 (\overline{\varepsilon})^2}{729} \left[\frac{14}{5} (\overline{\varepsilon})^2 + 3\,(X_i)^2 - 2\,(\overline{\varepsilon})^2 \right]$, so $\frac{\partial V_{Gi}}{\partial F_i} < 0$ for all $F_i < E\{\, q_i(\varepsilon) \,\}^2$ in the duopoly generator setting.

Proof. (2), (84), and (85) imply:

$$\pi^{Gi}(\varepsilon) - E\left\{ \pi^{Gi}(\varepsilon) \right\}$$

$$= \frac{1}{9\,b} \left[a + \varepsilon - 2\,c_i + c_j - b\left(F_i + F_j \right) \right] \left[a + \varepsilon - 2\,c_i + c_j + b\left(2\,F_i - F_j \right) \right] - \frac{1}{3}\,\varepsilon\,F_i$$

$$- \frac{1}{9\,b} \left[a - 2\,c_i + c_j - b\left(F_1 + F_2 \right) \right] \left[a - 2\,c_i + c_j + b\left(2\,F_i - F_j \right) \right] - \frac{(\overline{\varepsilon})^2}{27\,b}$$

$$= \frac{\varepsilon}{9\,b} \left[a - 2\,c_i + c_j - b\left(F_i + F_j \right) + a - 2\,c_i + c_j + b\left(2\,F_i - F_j \right) \right]$$

$$+ \frac{1}{9\,b}\,\varepsilon^2 - \frac{1}{3}\,\varepsilon\,F_i - \frac{(\overline{\varepsilon})^2}{27\,b}$$

$$= \frac{3\,\varepsilon}{27\,b} \left[2\left(a - 2\,c_i + c_j \right) + b\left(F_i - 2\,F_j \right) \right] + \frac{1}{27\,b} \left[3\,\varepsilon^2 - 9\,b\,F_i\,\varepsilon - (\overline{\varepsilon})^2 \right]$$

$$= \frac{1}{27\,b} \left[3\,\varepsilon^2 + 3\,Z_i\,\varepsilon - (\overline{\varepsilon})^2 \right] \tag{86}$$

where $Z_i = 2\left[a - 2\,c_i + c_j \right] + b\left[F_i - 2\,F_j \right] - 3\,b\,F_i = X_i$

because $X_i = \frac{2}{b^I} \left[a^I + \overline{Q} - \left(F_i + F_j \right) - b^I \left(2\,c_i - c_j \right) \right] = 2\left[a - 2\,c_i + c_j - b\left(F_i + F_j \right) \right]$.

Observe that:

$$\left[3\,\varepsilon^2 + 3\,X_i\,\varepsilon - (\overline{\varepsilon})^2 \right]^2$$

$$= 9\,\varepsilon^4 + 18\,X_i\,\varepsilon^3 + 3\left[3\,(X_i)^2 - 2\,(\overline{\varepsilon})^2 \right] \varepsilon^2 - 6\,X_i\,(\overline{\varepsilon})^2\,\varepsilon + (\overline{\varepsilon})^4 . \tag{87}$$

(28) and (87) imply:

$$E\left\{ \left[3\,\varepsilon^2 + 3\,X_i\,\varepsilon - (\overline{\varepsilon})^2 \right]^2 \right\}$$

$$= 9\,E\left\{\varepsilon^4\right\} + 18\,X_i\,E\left\{\varepsilon^3\right\} + 3\left[3\,(X_i)^2 - 2\,(\overline{\varepsilon})^2 \right] E\left\{\varepsilon^2\right\} + (\overline{\varepsilon})^4$$

[2] Lemma B1 implies that $F_i < E\{\, q_i(\varepsilon) \,\} \;\Leftrightarrow\; F_i < \frac{1}{3} \left[a^I + \overline{Q} + 2\,F_i - F_j - b^I \left(2\,c_i - c_j \right) \right] \;\Leftrightarrow\; \frac{1}{3} \left[a^I + \overline{Q} - \left(F_i + F_j \right) - b^I \left(2\,c_i - c_j \right) \right] > 0$.

$$= \frac{9}{10\,\overline{\varepsilon}}\left[\,(\overline{\varepsilon})^5 - (-\overline{\varepsilon})^5\,\right] + \frac{18\,X_i}{8\,\overline{\varepsilon}}\left[\,(\overline{\varepsilon})^4 - (-\overline{\varepsilon})^4\,\right]$$

$$+ \frac{1}{6\,\overline{\varepsilon}}\,3\left[\,3\,(X_i)^2 - 2\,(\overline{\varepsilon})^2\,\right]\left[\,(\overline{\varepsilon})^3 - (-\overline{\varepsilon})^3\,\right] + (\overline{\varepsilon})^4$$

$$= \frac{9\,(\overline{\varepsilon})^5}{5\,\overline{\varepsilon}} + \frac{\left[\,3\,(X_i)^2 - 2\,(\overline{\varepsilon})^2\,\right]2\,(\overline{\varepsilon})^3}{2\,\overline{\varepsilon}} + (\overline{\varepsilon})^4$$

$$= \frac{14}{5}\,(\overline{\varepsilon})^4 + \left[\,3\,(X_i)^2 - 2\,(\overline{\varepsilon})^2\,\right](\overline{\varepsilon})^2 \;=\; (\overline{\varepsilon})^2\left[\,\frac{4}{5}\,(\overline{\varepsilon})^4 + 3\,(X_i)^2\,\right]. \tag{88}$$

Because $V_{Gi} = E\{\,[\,\pi^{Gi}(\varepsilon) - E\,\{\,\pi^{Gi}(\varepsilon)\}\,]^2\,\}$, (86) and (88) imply that the variance of π^{Gi} is:

$$V_{Gi} \;=\; \frac{\left(b^I\right)^2(\overline{\varepsilon})^2}{729}\left[\,\frac{4}{5}\,(\overline{\varepsilon})^2 + 3\,(X_i)^2\,\right]$$

$$\Rightarrow\quad \frac{\partial V_{Gi}(\,\pi^{Gi}\,)}{\partial F_i} \;\overset{s}{=}\; X_i\,\frac{\partial X_i}{\partial F_i} \;=\; -\,\frac{2\,X_i}{b^I}\,.\;\blacksquare \tag{89}$$

Proposition B1. Suppose the generators choose the levels of forward contracting in the duopoly generator setting. Then in equilibrium, for $i,j \in \{1,2\}$ ($j \neq i$), $F_i = \max\{\,0, \frac{\delta_i - \beta_i\,\delta_j}{1-\beta_1\,\beta_2}\,\}$ and $F_i > 0$ if $c_1 = c_2$ and $A_{G1} = A_{G2}$. Furthermore, if $A_{G1} = A_{G2} = 0$, then $F_i = \max\{\,\frac{1}{5}\left[\,a^I + \overline{Q} - b^I\,(3\,c_i - 2\,c_j\,)\,\right], 0\,\}$. If $A_{G1} = A_{G2} \equiv A_G > 0$, then $F_1 + F_2$ *is: (i) increasing in* A_G, $\overline{\varepsilon}$, a^I, b^I, and $\overline{Q}$*; and (ii) decreasing in* c_1 *and* c_2*.*

Proof. (2), (8), (85), and (89) imply that Gi's expected utility is:

$$EU^{Gi} \;=\; \frac{1}{9\,b}\left[\,a - 2\,c_i + c_j - b\,(\,F_i + F_j\,)\right]\left[\,a - 2\,c_i + c_j + b\,(\,2\,F_i - F_j\,)\right] + \frac{(\overline{\varepsilon})^2}{27\,b}$$

$$-\;\frac{A_{Gi}\,(\overline{\varepsilon})^2}{729\,b^2}\left[\,\frac{14}{5}\,(\overline{\varepsilon})^2 + 3\,(X_i)^2 - 2\,(\overline{\varepsilon})^2\,\right]$$

$$\Rightarrow\quad \frac{\partial EU^{Gi}}{\partial F_i} \;=\; \frac{1}{9\,b}\,\{\,2\,b\left[\,a - 2\,c_i + c_j - b\,(\,F_i + F_j\,)\right] - b\left[\,a - 2\,c_i + c_j + b\,(\,2\,F_i - F_j\,)\right]\}$$

$$-\;\frac{A_{Gi}\,(\overline{\varepsilon})^2\;6\,X_i}{729\,b^2}\;\frac{\partial X_i}{\partial F_i}$$

$$= \frac{1}{9\,b}\left[\,b\,(\,a - 2\,c_i + c_j) \;-\; 4\,b^2\,F_i - b^2\,F_j\,\right] - \frac{A_{Gi}\,(\overline{\varepsilon})^2\;6\,X_i}{729\,b^2}\left[\,-\,2\,b\,\right]$$

$$= \frac{1}{9}\left[\,a - 2\,c_i + c_j\,\right] - \frac{4}{9}\,b\,F_i - \frac{1}{9}\,b\,F_j + \frac{24\,A_{Gi}\,(\overline{\varepsilon})^2}{729\,b}\left[\,a - 2\,c_i + c_j - b\,(\,F_i + F_j\,)\right]$$

$$= \frac{24\,A_{Gi}\,(\overline{\varepsilon}\,)^2 + 81\,b}{729\,b}\,[\,a - 2\,c_i + c_j\,] - b\,F_i \left[\frac{4}{9} + \frac{24\,A_{Gi}\,(\overline{\varepsilon}\,)^2}{729\,b} \right] - b\,F_j \left[\frac{1}{9} + \frac{24\,A_{Gi}\,(\overline{\varepsilon}\,)^2}{729\,b} \right]$$

$$= \frac{24\,A_{Gi}\,(\overline{\varepsilon}\,)^2 + 81\,b}{729\,b}\,[\,a - 2\,c_i + c_j\,] - b \left[\frac{24\,A_{Gi}\,(\overline{\varepsilon}\,)^2 + 324\,b}{729\,b} \right] F_i - b \left[\frac{24\,A_{Gi}\,(\overline{\varepsilon}\,)^2 + 81\,b}{729\,b} \right] F_j$$

$$= \frac{1}{729\,b} \{ \left[\,24\,A_{Gi}\,(\overline{\varepsilon}\,)^2 + 81\,b\,\right] [\,a - 2\,c_i + c_j\,] - b \left[\,24\,A_{Gi}\,(\overline{\varepsilon}\,)^2 + 324\,b\,\right] F_i$$

$$- \, b \left[\,24\,A_{Gi}\,(\overline{\varepsilon}\,)^2 + 81\,b\,\right] F_j \,\} \;\; \Rightarrow \;\; \frac{\partial E U^{Gi}}{\partial F_i} \; < \; 0 \, . \tag{90}$$

(2), (10), (11), and (90) imply that the level of F_i that maximizes Gi's expected utility is given by:

$$b \left[\,24\,A_{Gi}\,(\overline{\varepsilon}\,)^2 + 324\,b\,\right] F_i$$

$$= \left[\,24\,A_{Gi}\,(\overline{\varepsilon}\,)^2 + 81\,b\,\right] [\,a - 2\,c_i + c_j\,] - b \left[\,24\,A_{Gi}\,(\overline{\varepsilon}\,)^2 + 81\,b\,\right] F_j$$

$$\Rightarrow \;\; F_i \; = \; \frac{\left[\,24\,A_{Gi}\,(\overline{\varepsilon}\,)^2 + 81\,b\,\right] [\,a - 2\,c_i + c_j\,] - b \left[\,24\,A_{Gi}\,(\overline{\varepsilon}\,)^2 + 81\,b\,\right] F_j}{b \left[\,24\,A_{Gi}\,(\overline{\varepsilon}\,)^2 + 324\,b\,\right]}$$

$$= \frac{\left[\,24\,b^I A_{Gi}\,(\overline{\varepsilon}\,)^2 + 81\,\right] \left[\,a^I + \overline{Q} - b^I\,(\,2\,c_i - c_j\,)\,\right] - \left[\,24\,b^I A_{Gi}\,(\overline{\varepsilon}\,)^2 + 81\,\right] F_j}{24\,b^I A_{Gi}\,(\overline{\varepsilon}\,)^2 + 324}$$

$$= \; \delta_i - \beta_i\,F_j \, . \tag{91}$$

(91) reflects the facts that, from (2):

$$b \left[\,24\,A_{Gi}\,(\overline{\varepsilon}\,)^2 + 324\,b\,\right] \; = \; \frac{1}{(b^I)^2} \left[\,24\,b^I A_{Gi}\,(\overline{\varepsilon}\,)^2 + 324\,\right] \quad \text{and}$$

$$\left[\,24\,A_{Gi}\,(\overline{\varepsilon}\,)^2 + 81\,b\,\right] [\,a - 2\,c_i + c_j\,] - b \left[\,24\,A_{Gi}\,(\overline{\varepsilon}\,)^2 + 81\,b\,\right] F_j$$

$$= \; \frac{1}{(b^I)^2} \left[\,24\,b^I A_{Gi}\,(\overline{\varepsilon}\,)^2 + 81\,\right] \left[\,a^I + \overline{Q} - b^I\,(\,2\,c_i - c_j\,)\,\right] - \frac{1}{(b^I)^2} \left[\,24\,b^I A_{Gi}\,(\overline{\varepsilon}\,)^2 + 81\,\right] F_j \, .$$

By symmetry with (91):

$$F_j \; = \; \frac{\left[\,24\,A_{Gj}\,(\overline{\varepsilon}\,)^2 + 81\,b\,\right] [\,a - 2\,c_j + c_i\,] - b \left[\,24\,A_{Gj}\,(\overline{\varepsilon}\,)^2 + 81\,b\,\right] F_i}{b \left[\,24\,A_{Gj}\,(\overline{\varepsilon}\,)^2 + 324\,b\,\right]} \; = \; \delta_j - \beta_j\,F_i \, . \tag{92}$$

(91) and (92) imply:

$$F_i \; = \; \delta_i - \beta_i \left[\,\delta_j - \beta_j\,F_i\,\right] \;\; \Rightarrow \;\; F_i \left[\,1 - \beta_i\,\beta_j\,\right] \; = \; \delta_i - \beta_i\,\delta_j \;\; \Rightarrow \;\; F_i \; = \; \frac{\delta_i - \beta_i\,\delta_j}{1 - \beta_i\,\beta_j}$$

$$\Rightarrow \quad F_j \; = \; \delta_j - \beta_j \left[\frac{\delta_i - \beta_i \, \delta_j}{1 - \beta_i \, \beta_j} \right] \; = \; \frac{\delta_j \left[1 - \beta_i \, \beta_j \right] - \beta_j \left[\delta_i - \beta_i \, \delta_j \right]}{1 - \beta_i \, \beta_j} \; = \; \frac{\delta_j - \beta_j \, \delta_i}{1 - \beta_i \, \beta_j} \, . \tag{93}$$

Using (11), define $\beta \equiv \frac{24\, A_G (\bar{\varepsilon})^2 + 81\, b}{24\, A_G (\bar{\varepsilon})^2 + 324\, b}$. Then (93) implies that when $A_{G1} = A_{G2} = A_G$:

$$F_1 + F_2 \; = \; \frac{1}{1 - \beta^2} \left[\gamma_1 - \beta \, \gamma_2 + \gamma_2 - \beta \, \gamma_1 \right] \; = \; \frac{1 - \beta}{1 - \beta^2} \left[\gamma_1 + \gamma_2 \right] \; = \; \frac{\gamma_1 + \gamma_2}{1 + \beta} \, . \tag{94}$$

(10) implies that in this case:

$$\delta_1 + \delta_2 \; = \; \frac{\left[24\, A_G \, (\bar{\varepsilon})^2 + 81\, b \right] \left[a - 2\, c_1 + c_2 + a - 2\, c_2 + c_1 \right]}{b \left[24 \; A_G \, (\bar{\varepsilon})^2 + 324\, b \right]}$$

$$= \; \frac{\left[24\, A_G \, (\bar{\varepsilon})^2 + 81\, b \right] \left[2\, a - c_1 - c_2 \right]}{b \left[24\, A_G \, (\bar{\varepsilon})^2 + 324\, b \right]} \, ; \; \text{ and} \tag{95}$$

$$1 + \beta \; = \; \frac{24\, A_G \, (\bar{\varepsilon})^2 + 324\, b + 24\, A_G \, (\bar{\varepsilon})^2 + 81\, b}{24\, A_G \, (\bar{\varepsilon})^2 + 324\, b} \; = \; \frac{48\, A_G \, (\bar{\varepsilon})^2 + 405\, b}{24\, A_G \, (\bar{\varepsilon})^2 + 324\, b} \, . \tag{96}$$

(2) and (94) – (96) imply:

$$F_1 + F_2 \; = \; \left[\frac{24\, A_G \, (\bar{\varepsilon})^2 + 324\, b}{48\, A_G \, (\bar{\varepsilon})^2 + 405\, b} \right] \frac{\left[24\, A_G \, (\bar{\varepsilon})^2 + 81\, b \right] \left[2\, a - c_1 - c_2 \right]}{b \left[24 \; A_G \, (\bar{\varepsilon})^2 + 324\, b \right]}$$

$$= \; \frac{\left[24\, A_G \, (\bar{\varepsilon})^2 + 81\, b \right] \left[2\, a - c_1 - c_2 \right]}{b \left[48\, A_G \, (\bar{\varepsilon})^2 + 405\, b \right]} \, . \tag{97}$$

It is apparent from (97) that $F_1 + F_2$ is increasing in a (and thus in a^I and $\overline{Q}$) and decreasing in c_1 and c_2. Furthermore, because $2\, a > c_1 + c_2$ by assumption, (97) implies:

$$\frac{\partial \left(F_1 + F_2 \right)}{\partial A_G} \; \stackrel{s}{=} \; 24 \, (\bar{\varepsilon})^2 \left[48\, A_G \, (\bar{\varepsilon})^2 + 405\, b \right] - 48 \, (\bar{\varepsilon})^2 \left[24\, A_G \, (\bar{\varepsilon})^2 + 81\, b \right]$$

$$\stackrel{s}{=} \; 48\, A_G \, (\bar{\varepsilon})^2 + 405\, b - 2 \left[24\, A_G \, (\bar{\varepsilon})^2 + 81\, b \right] \; = \; 243\, b \; > \; 0 \, ;$$

$$\frac{\partial \left(F_1 + F_2 \right)}{\partial \bar{\varepsilon}} \; \stackrel{s}{=} \; 48\, A_G \, (\bar{\varepsilon}) \left[48\, A_G \, (\bar{\varepsilon})^2 + 405\, b \right] - 96\, A_G \, (\bar{\varepsilon}) \left[24\, A_G \, (\bar{\varepsilon})^2 + 81\, b \right]$$

$$\stackrel{s}{=} \; 48\, A_G \, (\bar{\varepsilon})^2 + 405\, b - 2 \left[24\, A_G \, (\bar{\varepsilon})^2 + 81\, b \right] \; = \; 243\, b \; > \; 0 \, ;$$

$$\frac{\partial \left(F_1 + F_2 \right)}{\partial b} \; \stackrel{s}{=} \; 81\, b \left[48\, A_G \, (\bar{\varepsilon})^2 + 405\, b \right] - \left[48\, A_G \, (\bar{\varepsilon})^2 + 810\, b \right] \left[24\, A_G \, (\bar{\varepsilon})^2 + 81\, b \right]$$

$$= \; 48 \, (81) \, A_G \, (\bar{\varepsilon})^2 \, b + 81 \, (405) \, b^2 - \; 48 \, (24) \, (A_G)^2 \, (\bar{\varepsilon})^4 - 48 \, (81) \, A_G \, (\bar{\varepsilon})^2 \, b$$

$$- \, 24\,(810)\, A_G\,(\bar{\varepsilon})^2\, b - 81\,(810)\, b^2$$

$$= \; [\, 48\,(81) - 48\,(81) - 24\,(810)\,]\, A_G\,(\bar{\varepsilon})^2\, b - 81\,[\,810 - 405\,]\, b^2 - 24\,(48)\,(A_G)^2\,(\bar{\varepsilon})^4$$

$$= \; -\,19{,}440\, A_G\,(\bar{\varepsilon})^2\, b - 32{,}805\, b^2 - 1{,}152\,(A_G)^2\,(\bar{\varepsilon})^4 \; < \; 0 \quad \Rightarrow \quad \frac{\partial\,(F_1 + F_2)}{\partial b^I} \; > \; 0\,.$$

When $A_{G1} = A_{G2} = 0$, (10) and (11) imply:

$$\beta_1 \; = \; \beta_2 \; = \; \frac{81}{324} \; = \; \frac{1}{4} \quad \Rightarrow \quad \beta_1\,\beta_2 \; = \; \frac{1}{16} \quad \Rightarrow \quad 1 - \beta_1\,\beta_2 \; = \; \frac{15}{16}\,,$$

$$\delta_i \; = \; \frac{81\,[\, a - 2\,c_i + c_j\,]}{324\, b} \; = \; \frac{1}{4\, b}\,[\, a - 2\,c_i + c_j\,]\,, \;\text{ and }\; \delta_j \; = \; \frac{1}{4\, b}\,[\, a - 2\,c_j + c_i\,]\,. \tag{98}$$

(2), (93), and (98) imply:

$$F_i \; = \; \frac{16}{15}\left[\frac{1}{4\,b}\right]\left[\, a - 2\,c_i + c_j - \frac{1}{4}\,(\, a - 2\,c_j + c_i\,)\right] \; = \; \frac{4}{15\,b}\left[\,\frac{3}{4}\, a - \frac{9}{4}\, c_i + \frac{6}{4}\, c_j\right]$$

$$= \; \frac{1}{15\,b}\,[\, 3\,a - 9\,c_i + 6\,c_j\,] \; = \; \frac{1}{5\,b}\,[\, a - 3\,c_i + 2\,c_j\,] \; = \; \frac{1}{5}\left[\, a^I + \overline{Q} - b^I\,(3\,c_i - 2\,c_j\,)\right].$$

Finally, because $a > \max\{c_1, c_2\}$ by assumption, (97) implies that $F_1 + F_2 > 0$ (and so $F_1 > 0$ and $F_2 > 0$) when the generators are symmetric. ■

Proposition B2. In the duopoly generator setting: (i) V_B is a strictly convex function of F_i for $i \in \{1, 2\}$; (ii) $\frac{\partial V_B}{\partial F_i} \lesseqgtr 0 \Leftrightarrow F_1 + F_2 \lesseqgtr \underline{F}_{DG} \equiv \overline{Q} - \left[\frac{\gamma}{1+\gamma}\right]\left[\, b^I\,(\,3\,r_0 - c_1 - c_2\,) - a^I\,\right] < \overline{Q}$ when $\gamma > 0$; (iii) $\underline{F}_{DG}$ is increasing in $\overline{Q}$; and, if $\gamma > 0$, (iv) $\underline{F}_{DG}$ is increasing in a^I, c_1, and c_2, and deceasing in r_0, b^I, and γ.

Proof. Let $F = F_1 + F_2$. Then (3) implies that B's profit in the duopoly generator setting is:

$$\pi^B(\varepsilon) \; = \; [\,\gamma\, r_0 + (\,1 - \gamma\,)\, E\{\, w(\varepsilon)\,\}\,]\left[\,\overline{Q} + b^I\,\varepsilon\,\right] - w(\varepsilon)\left[\,\overline{Q} + b^I\,\varepsilon - F\,\right] - p^F\, F\,. \tag{99}$$

(99) implies that because $p^F = E\,\{\, w(\varepsilon)\,\}$:

$$E\left\{\,\pi^B(\varepsilon)\,\right\} \; = \; [\,\gamma\, r_0 + (\,1 - \gamma\,)\, E\{\, w(\varepsilon)\,\}\,]\,\overline{Q} - E\left\{\, w(\varepsilon)\left[\,\overline{Q} + b^I\,\varepsilon\,\right]\right\} + F\left[\, E\{\, w(\varepsilon)\,\} - p^F\,\right]$$

$$= \; [\,\gamma\, r_0 + (\,1 - \gamma\,)\, E\{\, w(\varepsilon)\,\}\,]\,\overline{Q} - E\,\{\, w(\varepsilon)\,\}\,\overline{Q} - b^I\, E\,\{\,\varepsilon\, w(\varepsilon)\,\}\,. \tag{100}$$

(99) and (100) imply:

$$\pi^B(\varepsilon) - E\left\{\,\pi^B(\varepsilon)\,\right\} \; = \; [\,\gamma\, r_0 + (\,1 - \gamma\,)\, E\{\, w(\varepsilon)\,\}\,]\, b^I\,\varepsilon \, - w(\varepsilon)\left[\,\overline{Q} + b^I\,\varepsilon - F\,\right]$$

$$- \;\; p^F\, F + E\,\{\, w(\varepsilon)\,\}\, \overline{Q} + b^I\, E\,\{\, \varepsilon\, w(\varepsilon)\,\}$$

$$= \;\; [\, \gamma\, r_0 + (\, 1 - \gamma\,)\, E\{\, w(\varepsilon)\,\}\,]\, b^I\, \varepsilon \; - [\, w(\varepsilon) - E\,\{\, w(\varepsilon)\,\}\,]\, \overline{Q}$$

$$- \;\; b^I\, [\, \varepsilon\, w(\varepsilon) - E\,\{\, \varepsilon\, w(\varepsilon)\,\}\,] + [\, w(\varepsilon) - E\,\{\, w(\varepsilon)\,\}\,]\, F\,. \qquad (101)$$

Lemma B1 implies:

$$E\,\{\, w(\varepsilon)\}\;\; = \;\; \frac{1}{3}\,[\, a + c_1 + c_2 - b\,(\, F_1 + F_2\,)\,] \quad \Rightarrow \quad \frac{\partial E\,\{w(\varepsilon)\}}{\partial F_i} \;\; = \;\; -\,\frac{b}{3}\,; \qquad (102)$$

$$E\,\{\, \varepsilon\, w(\varepsilon)\}\;\; = \;\; E\left\{\, \frac{1}{3}\,[\, a + c_1 + c_2 - b\,(\, F_1 + F_2\,)\,]\, \varepsilon + \frac{1}{3}\, \varepsilon^2\,\right\}$$

$$= \;\; \frac{1}{3}\, E\left\{\, \varepsilon^2\,\right\} \;\; = \;\; \frac{(\,\overline{\varepsilon}\,)^2}{9} \quad \Rightarrow \quad \frac{\partial E\,\{\, \varepsilon\, w(\varepsilon)\}}{\partial F_i} \;\; = \;\; 0\,. \qquad (103)$$

Because $b\, b^I = 1$, (83) and (101) – (103) imply that in the duopoly generator setting:

$$\frac{\partial\left(\, \pi^B(\varepsilon) - E\left\{\, \pi^B(\varepsilon)\,\right\}\right)}{\partial F_i} \;\; = \;\; w(\varepsilon) - E\,\{\, w(\varepsilon)\,\} + [\, 1 - \gamma\,]\, b^I\, \varepsilon\; \frac{\partial E\{\, w(\varepsilon)\,\}}{\partial F_i}$$

$$- \;\left[\, \frac{\partial w(\varepsilon)}{\partial F_i} - \frac{\partial E\{\, w(\varepsilon)\,\}}{\partial F_i}\,\right] [\, \overline{Q} - F\,] - b^I \left[\, \varepsilon\; \frac{\partial w(\varepsilon)}{\partial F_i} - \frac{\partial\, E\,\{\, \varepsilon\, w(\varepsilon)\,\}}{\partial F_i}\,\right]$$

$$= \;\; w(\varepsilon) - E\{\, w(\varepsilon)\,\} + [\, 1 - \gamma\,]\, b^I\, \varepsilon \left[\, -\,\frac{b}{3}\,\right] - \left[\left(-\,\frac{b}{3}\right) - \left(-\,\frac{b}{3}\right)\right] [\, \overline{Q} - F\,]$$

$$- \;\; b^I \left[\left(-\,\varepsilon\; \frac{b}{3}\right) - 0\,\right] \;\; = \;\; w(\varepsilon) - E\{\, w(\varepsilon)\,\} - \frac{1}{3}\,[\, 1 - \gamma\,]\, \varepsilon + \frac{1}{3}\, \varepsilon$$

$$= \;\; \frac{\gamma}{3}\; \varepsilon + \; w(\varepsilon) - E\{\, w(\varepsilon)\,\} \;\; = \;\; \frac{\gamma}{3}\; \varepsilon + \frac{1}{3}\; \varepsilon \;\; = \;\; \left[\, \frac{1 + \gamma}{3}\,\right] \varepsilon\,. \qquad (104)$$

(103) and Lemma B1 imply:

$$\varepsilon\; w(\varepsilon) - E\,\{\, \varepsilon\; w(\varepsilon)\,\} \;\; = \;\; \frac{\varepsilon}{3}\,[\, a + \varepsilon + c_1 + c_2 - b\, F\,\,] - E\left\{\, \frac{\varepsilon}{3}\,[\, a + \varepsilon + c_1 + c_2 - b\, F\,\,]\,\right\}$$

$$= \;\; \frac{\varepsilon}{3}\,[\, a + \varepsilon + c_1 + c_2 - b\, F\,\,] - \frac{1}{3}\, E\left\{\, \varepsilon^2\,\right\}$$

$$= \;\; \frac{\varepsilon}{3}\,[\, a + \varepsilon + c_1 + c_2 - b\, F\,\,] - \frac{(\,\overline{\varepsilon}\,)^2}{9}\,. \qquad (105)$$

(102), (103), and (105) imply that (101) can be written as:

$$\pi^B(\varepsilon) - E\{\pi^B(\varepsilon)\} \;=\; \left[\gamma\, r_0 + (\,1 - \gamma\,)\, \frac{1}{3}\, (\,a + c_1 + c_2 - b\, F\,) \right] b^I\, \varepsilon$$

$$-\;\frac{1}{3}\, \varepsilon\, \overline{Q} - b^I \left[\frac{\varepsilon}{3}\, (\,a + \varepsilon + c_1 + c_2 - b\, F\,) - \frac{(\,\overline{\varepsilon}\,)^2}{9} \right] + \frac{1}{3}\, \varepsilon\, F\,. \quad (106)$$

(4) implies that B's expected utility is:

$$E\left\{\, U_B(\,\pi^B\,) \,\right\} \;=\; E\left\{\, \pi^B \,\right\} - A_B\, V_B \;\; \text{where} \;\; A_B \;\geq\; 0 \quad (107)$$

and where V_B, the variance of B's profit, is:

$$V_B \;=\; E\{\, [\, \pi^B(\varepsilon) - E\,\{\, \pi^B(\varepsilon)\}\,]^2\,\}\,. \quad (108)$$

(101), (102), (104), (106), and (108) imply:

$$\frac{\partial V_B}{\partial F_i} \;=\; \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} 2\,[\, \pi^B(\varepsilon) - E\{\pi^B(\varepsilon)\}\,]\; \frac{\partial\left(\, \pi^B(\varepsilon) - E\{\pi^B(\varepsilon)\}\,\right)}{\partial F}\, dH(\varepsilon)$$

$$= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} 2 \left\{ \left[\gamma\, r_0 + (\,1 - \gamma\,)\, \frac{1}{3}\, (\,a + c_1 + c_2 - b\, F\,) \right] b^I\, \varepsilon \right.$$

$$\left. -\;\frac{1}{3}\, \varepsilon\, \overline{Q} - b^I \left[\frac{\varepsilon}{3}\, (\,a + \varepsilon + c_1 + c_2 - b\, F\,) - \frac{(\,\overline{\varepsilon}\,)^2}{9} \right] + \frac{1}{3}\, \varepsilon\, F \right\} \left[\frac{1 + \gamma}{3} \right] \varepsilon\; dH(\varepsilon)$$

$$= \;2 \left[\frac{1 + \gamma}{3} \right] \left[\gamma\, r_0 + (\,1 - \gamma\,)\, \frac{1}{3}\, (\,a + c_1 + c_2 - b\, F\,) \right] b^I \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon^2\, dH(\varepsilon)$$

$$-\;\frac{2}{3}\, \overline{Q} \left[\frac{1 + \gamma}{3} \right] \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon^2\, d\,H(\varepsilon) + \frac{2}{3}\, F \left[\frac{1 + \gamma}{3} \right] \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon^2\, dH(\varepsilon)$$

$$-\;2 \left[\frac{1 + \gamma}{3} \right] b^I \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \left[\frac{\varepsilon^2}{3}\, (\,a + \varepsilon + c_1 + c_2 - b\, F\,) - \frac{(\,\overline{\varepsilon}\,)^2}{9}\, \varepsilon \right] dH(\varepsilon)$$

$$= \;\frac{2}{3}\, [\, 1 + \gamma\,] \left[\gamma\, r_0 + (\,1 - \gamma\,)\, \frac{1}{3}\, (\,a + c_1 + c_2 - b\, F\,) \right] b^I\, E\{\, \varepsilon^2\,\} - \frac{2}{3}\, \overline{Q} \left[\frac{1 + \gamma}{3} \right] E\{\, \varepsilon^2\,\}$$

$$+\;\frac{2}{3}\, F \left[\frac{1 + \gamma}{3} \right] E\{\, \varepsilon^2\,\} - \frac{2}{3} \left[\frac{1 + \gamma}{3} \right] b^I\, [\, a + c_1 + c_2 - b\, F\,]\; E\{\, \varepsilon^2\,\}$$

$$-\;\frac{2}{3} \left[\frac{1 + \gamma}{3} \right] b^I\, E\{\, \varepsilon^3\,\} + 2 \left[\frac{1 + \gamma}{27} \right] b^I (\,\overline{\varepsilon}\,)^2\, E\{\, \varepsilon\,\}$$

$$= \;\frac{2}{9}\, [\, 1 + \gamma\,]\, E\{\, \varepsilon^2\,\} \left\{ b^I\, [\, 3\, \gamma\, r_0 + (\,1 - \gamma\,)\, (\,a + c_1 + c_2 - b\, F\,)\,] \right.$$

$$+ F - \overline{Q} - b^I [a + c_1 + c_2 - b F] \Big\}$$

$$= \frac{2}{9} [1 + \gamma] E\{\varepsilon^2\} b^I \left[3 \gamma r_0 - \gamma (a + c_1 + c_2 - b F) + b \left(F - \overline{Q}\right)\right]. \tag{109}$$

Because $b\, b^I = 1$, (109) implies:

$$\frac{\partial^2 V_B}{\partial (F_i)^2} = \frac{2}{9} [1 + \gamma] E\{\varepsilon^2\} [\gamma + 1] > 0. \tag{110}$$

(2), (109), and (110) imply that the variance-minimizing forward quantity ($F = F_1 + F_2$) is determined by:

$$3 \gamma r_0 - \gamma [a + c_1 + c_2] - b \overline{Q} + b [1 + \gamma] F = 0$$

$$\Rightarrow F = \frac{1}{1+\gamma} \left[\overline{Q} + b^I \gamma (a + c_1 + c_2) - 3 \gamma b^I r_0\right]$$

$$= \frac{1}{1+\gamma} \left[\overline{Q} + b^I \gamma (c_1 + c_2 - 3 r_0) + \gamma b^I \left(\frac{a^I + \overline{Q}}{b^I}\right)\right]$$

$$= \frac{1}{1+\gamma} \left[\overline{Q} + \gamma \overline{Q} + \gamma \left(a^I - b^I [3 r_0 - c_1 - c_2]\right)\right]$$

$$= \overline{Q} + \left[\frac{\gamma}{1+\gamma}\right] \left[a^I - b^I (3 r_0 - c_1 - c_2)\right] = \underline{F}_{DG}. \tag{111}$$

(111) implies that $\underline{F}_{DG} < \overline{Q}$ when $\gamma > 0$ because Lemma B1 implies that $r_0 > E\{w(\varepsilon)\}$ for all $F \geq 0$ if and only if:

$$r_0 - \frac{1}{3 b^I} \left[a^I + \overline{Q} + b^I (c_1 + c_2)\right] > 0 \Leftrightarrow a^I + \overline{Q} + b^I (c_1 + c_2) < 3 b^I r_0$$

$$\Leftrightarrow a^I - b^I [3 r_0 - c_1 - c_2] < -\overline{Q} \Rightarrow a^I - b^I [3 r_0 - c_1 - c_2] < 0. \tag{112}$$

It is apparent from (111) that $\frac{\partial \underline{F}_{DG}}{\partial \overline{Q}} > 0$ and that $\frac{\partial \underline{F}_{DG}}{\partial a^I} > 0$, $\frac{\partial \underline{F}_{DG}}{\partial c_1} > 0$, $\frac{\partial \underline{F}_{DG}}{\partial c_2} > 0$, and $\frac{\partial \underline{F}_{DG}}{\partial r_0} < 0$ if $\gamma > 0$. (111) also implies that when $\gamma > 0$: (i) $\frac{\partial \underline{F}_{DG}}{\partial b^I} \overset{s}{=} -(3 r_0 - c_1 - c_2) < 0$ (because $r_0 > \max\{c_1, c_2\}$); and, from (112), (ii) $\frac{\partial \underline{F}_{DM}}{\partial \gamma} \overset{s}{=} -\frac{\partial}{\partial \gamma}\left(\frac{\gamma}{1+\gamma}\right) = -\frac{1}{[1+\gamma]^2} < 0$. ■

Proposition B3. In the duopoly generator setting, EU^B does not vary with F_i if $\gamma = A_B = 0$. If $\gamma > 0$, then: (i) EU^B is strictly increasing in $F = F_1 + F_2$ if $A_B = 0$; whereas if $A_B > 0$, (ii) EU^B is maximized at $F_{BDG} = \underline{F}_{DG} + \frac{\gamma}{[1+\gamma]^2}\left[\frac{9\,\overline{Q}}{2\, b^I A_B (\overline{\varepsilon})^2}\right] > \underline{F}_{DG}$; and (iii) F_{BDG} is increasing in $\overline{Q}$, a^I, c_1, and c_2, and decreasing in r_0, b^I, A_B, and $\overline{\varepsilon}$.

Proof. (2), (9), (107), and (109) imply that if $A_B > 0$, then $E\{U_B(\pi^B)\}$ is maximized where:

$$\frac{\partial E\{\,U_B(\pi^B)\,\}}{\partial F_i} \;=\; \frac{\partial E\{\,\pi^B(\varepsilon)\,\}}{\partial F_i} - A_B\,\frac{\partial V_B}{\partial F_i} \;=\; 0$$

$$\Leftrightarrow\;\; \gamma\,\frac{b}{3}\,\overline{Q} - A_B\,\frac{2}{9}\,[\,1+\gamma\,]\,E\{\,\varepsilon^2\,\}\,b^I\left[\,3\,\gamma\,r_0 - \gamma\,(\,a + c_1 + c_2 - b\,F\,) + b\left(\,F - \overline{Q}\,\right)\right] \;=\; 0$$

$$\Leftrightarrow\;\; \gamma\,\frac{b}{3}\,\overline{Q} - A_B\,\frac{2}{9}\,[\,1+\gamma\,]\,E\{\,\varepsilon^2\,\}\left[\,3\,b^I\,\gamma\,r_0 - \,b^I\gamma\,(\,a + c_1 + c_2\,) - \overline{Q}\,\right]$$

$$=\; A_B\,\frac{2}{9}\,[\,1+\gamma\,]\,E\{\,\varepsilon^2\,\}\,[\,1+\gamma\,]\,F$$

$$\Leftrightarrow\;\; F\,2\,A_B\,[\,1+\gamma\,]^2\,E\{\,\varepsilon^2\,\}$$

$$=\; 3\,b\,\gamma\,\overline{Q} \;-\; 2\,A_B\,[\,1+\gamma\,]\,E\{\,\varepsilon^2\,\}\,3\,b^I\,\gamma\,r_0 - \,b^I\gamma\,(\,a + c_1 + c_2\,) - \overline{Q}$$

$$\Leftrightarrow\;\; F \;=\; \frac{3\,b\,\gamma\,\overline{Q}}{2\,A_B\,[\,1+\gamma\,]^2\,E\{\,\varepsilon^2\,\}} - \frac{3\,b^I\,\gamma\,r_0 - \,b^I\gamma\,(c_1 + c_2\,) - \,b^I\gamma\left[\,\frac{a^I + \overline{Q}}{b^I}\,\right] - \overline{Q}}{1+\gamma}$$

$$=\; \frac{9\,b\,\gamma\,\overline{Q}}{2\,A_B\,[\,1+\gamma\,]^2\,(\,\overline{\varepsilon}\,)^2} + \frac{\overline{Q}\,[\,1+\gamma\,] + \gamma\left[\,a^I - b^I\,(\,3\,r_0 - c_1 - c_2\,)\,\right]}{1+\gamma}$$

$$=\; \overline{Q} + \frac{\gamma}{1+\gamma}\,\left[\,a^I - b^I\,(\,3\,r_0 - c_1 - c_2\,)\,\right] + \frac{\gamma}{[\,1+\gamma\,]^2}\left[\,\frac{9\,\overline{Q}}{2\,b^I A_B\,(\,\overline{\varepsilon}\,)^2}\,\right] \;=\; F_{BDG}\,. \qquad (113)$$

It is apparent from (113) that $\frac{\partial F_{BDG}}{\partial \overline{Q}} > 0$, and that $\frac{\partial F_{BDG}}{\partial a^I} > 0$, $\frac{\partial F_{BDG}}{\partial c_1} > 0$, $\frac{\partial F_{BDG}}{\partial c_2} > 0$, $\frac{\partial F_{BDG}}{\partial r_0} < 0$, $\frac{\partial F_{BDG}}{\partial A_B} < 0$, and $\frac{\partial F_{BDG}}{\partial \overline{\varepsilon}} < 0$ if $\gamma > 0$. (113) also implies that $\frac{\partial F_{BDG}}{\partial b^I} < 0$ because $r_0 > \max\{c_1, c_2\}$, by assumption. ■

Proposition B4. In the duopoly buyer setting for $i, j \in \{1,2\}$ ($j \neq i$): (i) $\frac{\partial V_{Bi}}{\partial F_i} \lesseqgtr 0$ $\Leftrightarrow$ $F_i \lesseqgtr \underline{F}_{iDB}$, where $\underline{F}_{iDB} \equiv \overline{Q}_i - \left[\,\frac{\alpha_i\,\gamma_i}{1+\alpha_i\,\gamma_i}\,\right] F_j - \frac{\alpha_i\,\gamma_i}{1+\alpha_i\,\gamma_i}\left[\,b^I\,(\,2\,r_{0i} - c\,) - a^I - \overline{Q}_j\,\right]$. If $\alpha_i\,\gamma_i > 0$, then $\underline{F}_{iDB}$ is: (i) strictly less than $\overline{Q}_i$; (ii) decreasing in F_j, r_{0i}, b^I, and γ_i; and (iii) increasing in $\overline{Q}_j$, a^I, and c.

<u>Proof</u>. (30) implies that Bi's profit in the duopoly buyer setting, given F_i and $p^F = E\{\,w(\varepsilon)\,\}$, is:

$$\pi^{Bi}(\varepsilon) \;=\; [\,\gamma_i\,r_{0i} + (\,1-\gamma_i\,)\,E\{\,w(\varepsilon)\,\}\,]\,[\,\overline{Q}_i + \alpha_i\,b^I\,\varepsilon\,]$$

$$-\; w(\varepsilon)\,[\,\overline{Q}_i + \alpha_i\,b^I\,\varepsilon - F_i\,] - p^F\,F_i\,. \qquad (114)$$

$$\Rightarrow \quad E\left\{ \pi^{Bi}(\varepsilon) \right\} \;=\; [\, \gamma_i \, r_{0i} + (\, 1 - \gamma_i \,) \, E\{ \, w(\varepsilon) \, \} \,] \, \overline{Q}_i - E\left\{ \, w(\varepsilon) \left[\, \overline{Q}_i + \alpha_i \, b^I \, \varepsilon \, \right] \right\}$$

$$+ \; F_i \left[\, E\{ \, w(\varepsilon) \, \} - p^F \, \right]$$

$$= \; [\, \gamma_i \, r_{0i} + (\, 1 - \gamma_i \,) \, E\{ \, w(\varepsilon) \, \} \,] \, \overline{Q}_i - E\,\{ \, w(\varepsilon) \, \} \, \overline{Q}_i - \alpha_i \, b^I \, E\,\{ \, \varepsilon \; w(\varepsilon) \, \} \, . \tag{115}$$

(114) and (115) imply:

$$\pi^{Bi}(\varepsilon) - E\left\{ \pi^{Bi}(\varepsilon) \right\} \;=\; [\, \gamma_i \, r_{0i} + (\, 1 - \gamma_i \,) \, E\{ \, w(\varepsilon) \, \} \,] \, \alpha_i \, b^I \, \varepsilon \; - w(\varepsilon) \left[\, \overline{Q}_i + \alpha_i \, b^I \, \varepsilon - F_i \, \right]$$

$$- \;\; p^F \, F_i + E\,\{ \, w(\varepsilon) \, \} \, \overline{Q}_i + \alpha_i \, b^I \, E\,\{ \, \varepsilon \; w(\varepsilon) \, \}$$

$$= \; [\, \gamma_i \, r_{0i} + (\, 1 - \gamma_i \,) \, E\{ \, w(\varepsilon) \, \} \,] \, \alpha_i \, b^I \, \varepsilon \; - [\, w(\varepsilon) - E\,\{ \, w(\varepsilon) \, \} \,] \, \overline{Q}_i$$

$$- \; \alpha_i \, b^I \, [\, \varepsilon \, w(\varepsilon) - E\,\{ \, \varepsilon \; w(\varepsilon) \, \} \,] + [\, w(\varepsilon) - E\,\{ \, w(\varepsilon) \, \} \,] \, F_i \, . \tag{116}$$

Because $b \, b^I = 1$, (26), (27), (29), and (116) imply that in the present setting:

$$\frac{\partial \left(\pi^{Bi}(\varepsilon) - E\left\{ \pi^{Bi}(\varepsilon) \right\} \right)}{\partial F_i} \;=\; w(\varepsilon) - E\,\{ \, w(\varepsilon) \, \} + [\, 1 - \gamma_i \,] \, \alpha_i \, b^I \, \varepsilon \; \frac{\partial E\{ \, w(\varepsilon) \, \}}{\partial F_i}$$

$$- \left[\frac{\partial w(\varepsilon)}{\partial F_i} - \frac{\partial E\{ \, w(\varepsilon) \, \}}{\partial F_i} \right] [\, \overline{Q}_i - F_i \,]$$

$$- \;\; \alpha_i \, b^I \left[\varepsilon \; \frac{\partial w(\varepsilon)}{\partial F_i} - \frac{\partial \, E\,\{ \, \varepsilon \, w(\varepsilon) \, \}}{\partial F_i} \right]$$

$$= \; w(\varepsilon) - E\{ \, w(\varepsilon) \, \} + [\, 1 - \gamma_i \,] \, \alpha_i \, b^I \, \varepsilon \left[- \frac{b}{2} \right] - \left[\left(- \frac{b}{2} \right) - \left(- \frac{b}{2} \right) \right] [\, \overline{Q}_i - F_i \,]$$

$$- \; \alpha_i \, b^I \left[\left(- \varepsilon \; \frac{b}{2} \right) - 0 \right] \;=\; w(\varepsilon) - E\{ \, w(\varepsilon) \, \} - \; \frac{1}{2} \, [\, 1 - \gamma_i \,] \, \alpha_i \, \varepsilon + \alpha_i \; \frac{1}{2} \, \varepsilon$$

$$= \; \frac{\gamma_i}{2} \; \alpha_i \, \varepsilon + \; w(\varepsilon) - E\{ \, w(\varepsilon) \, \} \;\; = \;\; \frac{\gamma_i}{2} \; \alpha_i \, \varepsilon + \frac{1}{2} \; \varepsilon \;\; = \;\; \left[\frac{1 + \alpha_i \, \gamma_i}{2} \right] \varepsilon \, . \tag{117}$$

(26) and (27) imply that (116) can be written as:

$$\pi^{Bi}(\varepsilon) - E\{\pi^{Bi}(\varepsilon)\} \;=\; \left[\gamma_i \, r_{0i} + (\, 1 - \gamma_i \,) \, \frac{1}{2} \, (\, a + c - b \, F \,) \right] \alpha_i \, b^I \, \varepsilon$$

$$- \;\; \frac{1}{2} \, \varepsilon \, \overline{Q}_i - \alpha_i \, b^I \left[\frac{\varepsilon}{2} \, (\, a + \varepsilon + c - b \, F \,) - \frac{(\, \overline{\varepsilon} \,)^2}{6} \right] + \frac{1}{2} \, \varepsilon \, F_i \tag{118}$$

where $F = F_1 + F_2$.

(26), (117), and (118) imply:

$$\frac{\partial V_{Bi}}{\partial F_i} = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} 2\left[\pi_i^B(\varepsilon) - E\{\pi_i^B(\varepsilon)\}\right] \frac{\partial\left(\pi^{Bi}(\varepsilon) - E\{\pi^{Bi}(\varepsilon)\}\right)}{\partial F_i} dH(\varepsilon) \tag{119}$$

$$= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} 2\Bigg\{ \left[\gamma_i r_{0i} + (1-\gamma_i)\frac{1}{2}(a+c-bF)\right]\alpha_i b^I \varepsilon$$

$$- \frac{1}{2}\varepsilon \overline{Q}_i - \alpha_i b^I\left[\frac{\varepsilon}{2}(a+\varepsilon+c-bF) - \frac{(\overline{\varepsilon})^2}{6}\right]$$

$$+ \frac{1}{2}\varepsilon F_i\Bigg\}\left[\frac{1+\alpha_i\gamma_i}{2}\right]\varepsilon\, dH(\varepsilon)$$

$$= 2\left[\frac{1+\alpha_i\gamma_i}{2}\right]\left[\gamma_i r_{0i} + (1-\gamma_i)\frac{1}{2}(a+c-bF)\right]\alpha_i b^I \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon^2 dH(\varepsilon)$$

$$- \overline{Q}_i\left[\frac{1+\alpha_i\gamma_i}{2}\right]\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon^2 d\,H(\varepsilon) + F_i\left[\frac{1+\alpha_i\gamma_i}{2}\right]\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon^2 dH(\varepsilon)$$

$$- 2\left[\frac{1+\alpha_i\gamma_i}{2}\right]\alpha_i b^I \int_{\underline{\varepsilon}}^{\overline{\varepsilon}}\left[\frac{\varepsilon^2}{2}(a+\varepsilon+c-bF) - \frac{(\overline{\varepsilon})^2}{6}\varepsilon\right] dH(\varepsilon)$$

$$= [1+\alpha_i\gamma_i]\left[\gamma_i r_{0i} + (1-\gamma_i)\frac{1}{2}(a+c-bF)\right]\alpha_i b^I E\{\varepsilon^2\}$$

$$- \overline{Q}_i\left[\frac{1+\alpha_i\gamma_i}{2}\right]E\{\varepsilon^2\}$$

$$+ F_i\left[\frac{1+\alpha_i\gamma_i}{2}\right]E\{\varepsilon^2\} - \left[\frac{1+\alpha_i\gamma_i}{2}\right]\alpha_i b^I [a+c-bF]\, E\{\varepsilon^2\}$$

$$- \left[\frac{1+\alpha_i\gamma_i}{2}\right]\alpha_i b^I E\{\varepsilon^3\} + [1+\alpha_i\gamma_i]\,\alpha_i b^I \frac{(\overline{\varepsilon})^2}{6} E\{\varepsilon\}$$

$$= [1+\alpha_i\gamma_i]\left[\gamma_i r_{0i} + (1-\gamma_i)\frac{1}{2}(a+c-bF)\right]\alpha_i b^I E\{\varepsilon^2\}$$

$$- \overline{Q}_i\left[\frac{1+\alpha_i\gamma_i}{2}\right]E\{\varepsilon^2\} + F_i\left[\frac{1+\alpha_i\gamma_i}{2}\right]E\{\varepsilon^2\}$$

$$- \left[\frac{1+\alpha_i\gamma_i}{2}\right]\alpha_i b^I [a+c-bF]\, E\{\varepsilon^2\}$$

$$= \alpha_i b^I\left[\frac{1+\alpha_i\gamma_i}{2}\right]E\{\varepsilon^2\}\,\{2\gamma_i r_{0i} + [1-\gamma_i][a+c-bF] + \frac{b}{\alpha_i}[F_i - \overline{Q}_i]$$

$$- \left[a + c - b F \right] \}$$

$$= \alpha_i b^I \left[\frac{1 + \alpha_i \gamma_i}{2} \right] E\{ \varepsilon^2 \} \left[2 \gamma_i r_{0i} - \gamma_i (a + c - b F) + \frac{b}{\alpha_i} \left(F_i - \overline{Q}_i \right) \right] \tag{120}$$

$$\Rightarrow \frac{\partial^2 V_{Bi}}{\partial (F_i)^2} = \alpha_i b^I \left[\frac{1 + \alpha_i \gamma_i}{2} \right] E\left\{ \varepsilon^2 \right\} b \left[\gamma_i + \frac{1}{\alpha_i} \right] > 0 \, . \tag{121}$$

(12), (120), and (121) imply that the variance-minimizing forward quantity is determined by:

$$2 \gamma_i r_{0i} - \gamma_i [a + c - b F] + \frac{b}{\alpha_i} [F_i - \overline{Q}_i] = 0$$

$$\Leftrightarrow \quad 2 \alpha_i \gamma_i r_{0i} - \alpha_i \gamma_i [a + c - b F] + b F_i - b \overline{Q}_i = 0$$

$$\Leftrightarrow \quad 2 \alpha_i \gamma_i r_{0i} - \alpha_i \gamma_i \left[\frac{a^I + \overline{Q}_i + \overline{Q}_j}{b^I} + c \right] + \alpha_i \gamma_i b [F_i + F_j] + b F_i - b \overline{Q}_i = 0$$

$$\Leftrightarrow \quad 2 \alpha_i \gamma_i b^I r_{0i} - \alpha_i \gamma_i \left[a^I + \overline{Q}_i + \overline{Q}_j + b^I c \right] + [1 + \alpha_i \gamma_i] F_i + \alpha_i \gamma_i F_j - \overline{Q}_i = 0$$

$$\Leftrightarrow \quad [1 + \alpha_i \gamma_i] F_i = [1 + \alpha_i \gamma_i] \overline{Q}_i + \alpha_i \gamma_i \overline{Q}_j + \alpha_i \gamma_i a^I + \alpha_i \gamma_i b^I c$$
$$- 2 \alpha_i \gamma_i b^I r_{0i} - \alpha_i \gamma_i F_j$$

$$\Leftrightarrow \quad F_i = \overline{Q}_i - \left[\frac{\alpha_i \gamma_i}{1 + \alpha_i \gamma_i} \right] F_j + \frac{\alpha_i \gamma_i}{1 + \alpha_i \gamma_i} \left[\overline{Q}_j + a^I - b^I (2 r_{0i} - c) \right] \equiv \underline{F}_{iDB} \, . \tag{122}$$

It is apparent from (122) that $\underline{F}_{iDB}$ is increasing in $\overline{Q}_i$. It is also apparent that if $\alpha_i \gamma_i > 0$, then $\underline{F}_{iDB}$ is increasing in $\overline{Q}_j$, a^I, and c, and decreasing in F_j and r_{0i}. (122) also implies that when $\alpha_i \gamma_i > 0$:

$$\frac{\partial \underline{F}_{iDB}}{\partial b^I} = - \frac{\alpha_i \gamma_i}{1 + \alpha_i \gamma_i} [2 r_{0i} - c] < 0 \text{ because } r_{0i} > c \, .$$

In addition, (122) implies that $\underline{F}_{iDB}$ is decreasing in γ_i when $\alpha_i \gamma_i > 0$ because: (i) $F_j \geq 0$; (ii) $r_{0i} > E\{ w(\varepsilon) \}$ for all $F_1 \geq 0$ and $F_2 \geq 0$, so Lemma 1 implies that $\overline{Q}_j + a^I - b^I [2 r_{0i} - c] < 0$; and (iii) $\frac{\partial}{\partial x} \left(\frac{x}{1+x} \right) \overset{s}{=} 1 + x - x = 1 > 0$. ■

Proposition B5. In the duopoly buyer setting, EU^{Bi} does not vary with F_i if $\alpha_i \gamma_i = A_{Bi} = 0$. If $\alpha_i \gamma_i > 0$, then: (i) EU^B is strictly increasing in F_i if $A_{Bi} = 0$; whereas if $A_{Bi} > 0$, (ii) EU^{Bi} is maximized at $\widetilde{F}_{BiDB} = \underline{F}_{iDB} + \frac{3 \gamma_i \overline{Q}_i}{A_{Bi} b^I [1 + \alpha_i \gamma_i]^2 (\overline{\varepsilon})^2}$; and (iii) $\widetilde{F}_{BiDB}$ is increasing in $\overline{Q}_i$, $\overline{Q}_j$, a^I, and c, and decreasing in F_j, r_{0i}, b^I, A_{Bi}, and $\overline{\varepsilon}$.

Proof. (29) and (115) imply:

$$\frac{\partial E\left\{\pi^{Bi}(\varepsilon)\right\}}{\partial F_i} = [\,1-\gamma_i\,]\,\overline{Q}_i\;\frac{\partial E\{\,w(\varepsilon)\,\}}{\partial F_i} - \overline{Q}_i\;\frac{\partial E\{\,w(\varepsilon)\,\}}{\partial F_i} - \alpha_i\, b^I\;\frac{\partial E\{\,\varepsilon\, w(\varepsilon)\,\}}{\partial F_i}$$

$$= \;-\,\gamma_i\,\overline{Q}_i\left[-\frac{b}{2}\right] \;=\; \gamma_i\,\frac{b}{2}\,\overline{Q}_i \;\geq\; 0\,. \tag{123}$$

(12), (26), and (120) – (123) imply that if $A_{Bi} > 0$, then $E\{\,U_{Bi}(\pi^{Bi})\,\}$ is maximized where:

$$\frac{\partial E\{\,U_{Bi}(\pi^{Bi})\,\}}{\partial F_i} = \frac{\partial E\{\,\pi^{Bi}\,\}}{\partial F_i} - A_{Bi}\,\frac{\partial V_{Bi}}{\partial F_i} = 0$$

$$\Leftrightarrow\;\; \gamma_i\,\frac{b}{2}\,\overline{Q}_i - A_{Bi}\,\alpha_i\, b^I\left[\frac{1+\alpha_i\,\gamma_i}{2}\right] E\{\,\varepsilon^2\,\}$$

$$\cdot\left[2\,\gamma_i\, r_{0i} - \gamma_i\,(\,a+c-b\,F\,) + \frac{b}{\alpha_i}\left(F_i - \overline{Q}_i\right)\right] = 0$$

$$\Leftrightarrow\;\; \gamma_i\,\frac{b}{2}\,\overline{Q}_i - \frac{A_{Bi}}{2}\,\alpha_i\,[\,1+\alpha_i\,\gamma_i\,]\,E\{\,\varepsilon^2\,\}$$

$$\cdot\left[2\,\gamma_i\, b^I\, r_{0i} - \gamma_i\, b^I\,(\,a+c-b\,F\,) + \frac{1}{\alpha_i}\left(F_i - \overline{Q}_i\right)\right] = 0$$

$$\Leftrightarrow\;\; \gamma_i\,\frac{b}{2}\,\overline{Q}_i - \frac{A_{Bi}}{2}\,[\,1+\alpha_i\,\gamma_i\,]\,E\{\,\varepsilon^2\,\}$$

$$\cdot\left[\,2\,\alpha_i\,\gamma_i\, b^I\, r_{0i} - \alpha_i\,\gamma_i\, b^I\,(\,a+c-b\,F\,) + F_i - \overline{Q}_i\,\right] = 0$$

$$\Leftrightarrow\;\; \gamma_i\,\frac{b}{2}\,\overline{Q}_i - \frac{A_{Bi}}{2}\,[\,1+\alpha_i\,\gamma_i\,]\,E\{\,\varepsilon^2\,\}$$

$$\cdot\left[\,2\,\alpha_i\,\gamma_i\, b^I\, r_{0i} - \alpha_i\,\gamma_i\, b^I\,(\,a+c\,) - \overline{Q}_i + \alpha_i\,\gamma_i\, F_j + (\,1+\alpha_i\,\gamma_i\,)\,F_i\right] = 0$$

$$\Leftrightarrow\;\; F_i\,\frac{A_{Bi}}{2}\,[\,1+\alpha_i\,\gamma_i\,]^2\;E\{\,\varepsilon^2\,\}$$

$$= \gamma_i\,\frac{b}{2}\,\overline{Q}_i + \frac{A_{Bi}}{2}\,[\,1+\alpha_i\,\gamma_i\,]\,E\{\,\varepsilon^2\,\}\left[\,\alpha_i\,\gamma_i\, b^I\,(\,a+c-2\,r_{0i}\,) + \overline{Q}_i - \alpha_i\,\gamma_i\, F_j\,\right]$$

$$\Leftrightarrow\;\; F_i\;A_{Bi}\,[\,1+\alpha_i\,\gamma_i\,]^2\;E\{\,\varepsilon^2\,\}$$

$$= A_{Bi}\,[\,1+\alpha_i\,\gamma_i\,]\,E\{\,\varepsilon^2\,\}\left[\,\alpha_i\,\gamma_i\, b^I\,(\,a+c-2\,r_{0i}\,) + \overline{Q}_i - \alpha_i\,\gamma_i\, F_j\,\right] + \gamma_i\; b\;\overline{Q}_i$$

$$\Leftrightarrow\;\; F_i = \frac{1}{1+\alpha_i\,\gamma_i}\left[\,\alpha_i\,\gamma_i\, b^I\,(\,a+c-2\,r_{0i}\,) + \overline{Q}_i - \alpha_i\,\gamma_i\, F_j\,\right]$$

$$+\;\frac{\gamma_i\;b\;\overline{Q}_i}{A_{Bi}\,[\,1+\alpha_i\,\gamma_i\,]^2\;E\{\,\varepsilon^2\,\}}$$

$$= \frac{1}{1+\alpha_i\,\gamma_i}\left[\alpha_i\,\gamma_i\,b^I\left(\frac{a^I+\overline{Q}_i+\overline{Q}_j}{b^I}+c-2\,r_{0i}\right)+\overline{Q}_i-\alpha_i\,\gamma_i\,F_j\right]$$

$$+\frac{3\,\gamma_i\;b\;\overline{Q}_i}{A_{Bi}\,[\,1+\alpha_i\,\gamma_i\,]^2\,(\,\overline{\varepsilon}\,)^2}$$

$$= \frac{1}{1+\alpha_i\,\gamma_i}\left[\,\alpha_i\,\gamma_i\left(\,a^I+\overline{Q}_i+\overline{Q}_j+b^I\,[\,c-2\,r_{0i}\,]\,\right)+\overline{Q}_i-\alpha_i\,\gamma_i\,F_j\,\right]$$

$$+\frac{3\,\gamma_i\;b\;\overline{Q}_i}{A_{Bi}\,[\,1+\alpha_i\,\gamma_i\,]^2\,(\,\overline{\varepsilon}\,)^2}$$

$$= \overline{Q}_i+\frac{\alpha_i\,\gamma_i\,[\,\overline{Q}_j+a^I-b^I\,(\,2\,r_{0i}-c\,)-F_j\,]}{1+\alpha_i\,\gamma_i}+\frac{3\,\gamma_i\;\overline{Q}_i}{A_{Bi}\,b^I\,[\,1+\alpha_i\,\gamma_i\,]^2\,(\,\overline{\varepsilon}\,)^2}$$

$$= \underline{F}_{iDB}+\frac{3\,\gamma_i\;\overline{Q}_i}{A_{Bi}\,b^I\,[\,1+\alpha_i\,\gamma_i\,]^2\,(\,\overline{\varepsilon}\,)^2} \;=\; \widetilde{F}_{BiDB}\,. \tag{124}$$

It is apparent from (122) and (124) that $\widetilde{F}_{BiDB}$ is: (i) increasing in $\overline{Q}_i$; and, if $\alpha_i\,\gamma_i > 0$ (ii) increasing in $\overline{Q}_j$, a^I, and c, and decreasing in F_j, r_{0i}, b^I, A_{Bi}, and $\overline{\varepsilon}$. ■

C. Additional Formal Conclusions.

Lemma C1. *Suppose $c_1 = c_2 = c$ and $F_1 = F_2 \equiv \frac{F}{2}$, where F_i (F) is the number of forward contracts that Gi (G) signs in the duopoly generator (dual monopoly) setting. Also suppose $F < a^I + \overline{Q} - b^I c$. Then the variance of a generator's profit declines more rapidly as its forward quantity increases in the dual monopoly setting than in the duopoly generator setting.*

Proof. (2) and (23) imply that in the dual monopoly setting:

$$\frac{\partial V_G}{\partial F} \;=\; \frac{1}{144\,b^2}\;2\,(\,\overline{\varepsilon}\,)^2\;12\,X\,\frac{\partial X}{\partial F} \;=\; -\,\frac{1}{6\,b}\;(\,\overline{\varepsilon}\,)^2\,[\,a-c-b\,F\,]\,. \tag{125}$$

(2) and (89) imply that in the duopoly generator setting:

$$\frac{\partial V_{Gi}}{\partial F} \;=\; \frac{1}{729\,b^2}\,(\,\overline{\varepsilon}\,)^2\;6\,X_i\,\frac{\partial X_i}{\partial F_i} \;=\; -\,\frac{8}{243\,b}\,(\,\overline{\varepsilon}\,)^2\;[\,a-c-b\,(\,F_1+F_2\,)\,]\,. \tag{126}$$

(125) and (126) imply that when $F_1+F_2 = F$ and $F < a^I+\overline{Q}-b^I c = \frac{a-c}{b}$:

$$\left|\,\frac{\partial V_G}{\partial F}\,\right| \;>\; \left|\,\frac{\partial V_G}{\partial F}\,\right| \;\Leftrightarrow\; \frac{1}{6\,b}\,(\,\overline{\varepsilon}\,)^2\,[\,a-c-b\,F\,] \;>\; \frac{8}{243\,b}\,(\,\overline{\varepsilon}\,)^2\,[\,a-c-b\,F\,]$$

$$\Leftrightarrow \quad \frac{1}{6} > \frac{8}{243} \quad \Leftrightarrow \quad \frac{65}{486} > 0 . \quad \blacksquare$$

Proposition C1. *In the dual duopoly setting,* $\frac{\partial V_{Bi}}{\partial F_i} \lesseqgtr 0 \Leftrightarrow F_1 + F_2 \lesseqgtr \underline{F}_{iDD}$, where $\underline{F}_{iDD} \equiv \overline{Q}_i - \left[\frac{\alpha_i \gamma_i}{1+\alpha_i \gamma_i} \right] F_{j\cdot} + \frac{\alpha_i \gamma_i}{1+\alpha_i \gamma_i} \left[\overline{Q}_j + a^I - b^I (3 r_{0i} - c_1 - c_2) \right]$.

Proof. Lemma B1 implies that in equilibrium in the dual duopoly setting:

$$w(\varepsilon) = \frac{1}{3} [a + \varepsilon + c_1 + c_2 - b (F_{\cdot 1} + F_{\cdot 2})] ,$$

$$q_i(\varepsilon) = \frac{1}{3 b} [a + \varepsilon - 2 c_i + c_j + b (2 F_{\cdot i} - F_{\cdot j})] \quad \text{and}$$

$$p^F = E\{ w(\varepsilon) \} = \frac{1}{3} [a + c_1 + c_2 - b (F_{\cdot 1} + F_{\cdot 2})] . \tag{127}$$

(26) and (127) imply:

$$E \{ w(\varepsilon) \} = \frac{1}{3} [a + c_1 + c_2 - b (F_{\cdot 1} + F_{\cdot 2})] \Rightarrow \frac{\partial E \{ w(\varepsilon) \}}{\partial F_{\cdot i}} - \frac{b}{3} ;$$

$$E \{ \varepsilon \, w(\varepsilon) \} = \frac{(\overline{\varepsilon})^2}{9} \Rightarrow \frac{\partial E \{ \varepsilon \, w(\varepsilon) \}}{\partial F_{i\cdot}} = 0 . \tag{128}$$

Bi's profit, given $F_{i\cdot}$ and ε, in the dual duopoly setting is:

$$\pi^{Bi}(\varepsilon) = [\gamma_i r_{0i} + (1 - \gamma_i) E\{ w(\varepsilon) \}] \left[\overline{Q}_i + \alpha_i b^I \varepsilon \right] - w(\varepsilon) \left[\overline{Q}_i + \alpha_i b^I \varepsilon - F_{i\cdot} \right] - p^F F_{i\cdot} \tag{129}$$

$$\Rightarrow E \left\{ \pi^{Bi}(\varepsilon) \right\} = [\gamma_i r_{0i} + (1 - \gamma_i) E\{ w(\varepsilon) \}] \overline{Q}_i - E \left\{ w(\varepsilon) \left[\overline{Q}_i + \alpha_i b^I \varepsilon \right] \right\} + F_{i\cdot} \left[E\{ w(\varepsilon) \} - p^F \right]$$

$$= [\gamma_i r_{0i} + (1 - \gamma_i) E\{ w(\varepsilon) \}] \overline{Q}_i - E \{ w(\varepsilon) \} \overline{Q}_i - \alpha_i b^I E \{ \varepsilon \, w(\varepsilon) \} . \tag{130}$$

(129) and (130) imply:

$$\pi^{Bi}(\varepsilon) - E \left\{ \pi^{Bi}(\varepsilon) \right\} = [\gamma_i r_{0i} + (1 - \gamma_i) E\{ w(\varepsilon) \}] \alpha_i b^I \varepsilon - w(\varepsilon) \left[\overline{Q}_i + \alpha_i b^I \varepsilon - F_{i\cdot} \right] - p^F F_{i\cdot} + E \{ w(\varepsilon) \} \overline{Q}_i + \alpha_i b^I E \{ \varepsilon \, w(\varepsilon) \}$$

$$= [\gamma_i r_{0i} + (1 - \gamma_i) E\{ w(\varepsilon) \}] \alpha_i b^I \varepsilon - [w(\varepsilon) - E \{ w(\varepsilon) \}] \overline{Q}_i - \alpha_i b^I [\varepsilon \, w(\varepsilon) - E \{ \varepsilon \, w(\varepsilon) \}] + [w(\varepsilon) - E \{ w(\varepsilon) \}] F_{i\cdot} . \tag{131}$$

Because $b\,b^I = 1$, (26), (127), (128) and (131) imply:

$$\frac{\partial\left(\pi^{Bi}(\varepsilon) - E\{\pi^{Bi}(\varepsilon)\}\right)}{\partial F_{i\cdot}} = w(\varepsilon) - E\{w(\varepsilon)\} + [1-\gamma_i]\,\alpha_i\,b^I\,\varepsilon\,\frac{\partial E\{w(\varepsilon)\}}{\partial F_{i\cdot}}$$

$$- \left[\frac{\partial w(\varepsilon)}{\partial F_{i\cdot}} - \frac{\partial E\{w(\varepsilon)\}}{\partial F_{i\cdot}}\right][\,\overline{Q}_i - F_{i\cdot}\,]$$

$$- \alpha_i\,b^I\left[\varepsilon\,\frac{\partial w(\varepsilon)}{\partial F_{i\cdot}} - \frac{\partial\,E\{\varepsilon\,w(\varepsilon)\}}{\partial F_{i\cdot}}\right]$$

$$= w(\varepsilon) - E\{w(\varepsilon)\} + [1-\gamma_i]\,\alpha_i\,b^I\,\varepsilon\left[-\frac{b}{3}\right] - \left[\left(-\frac{b}{3}\right) - \left(-\frac{b}{3}\right)\right][\,\overline{Q}_i - F_{i\cdot}\,]$$

$$- \alpha_i\,b^I\left[\left(-\varepsilon\,\frac{b}{3}\right) - 0\right] = w(\varepsilon) - E\{w(\varepsilon)\} - \frac{1}{3}[1-\gamma_i]\,\alpha_i\,\varepsilon + \alpha_i\,\frac{1}{3}\,\varepsilon$$

$$= \frac{\gamma_i}{3}\,\alpha_i\,\varepsilon + w(\varepsilon) - E\{w(\varepsilon)\} = \frac{\gamma_i}{3}\,\alpha_i\,\varepsilon + \frac{1}{3}\,\varepsilon = \left[\frac{1+\alpha_i\,\gamma_i}{3}\right]\varepsilon\,. \tag{132}$$

(127) and (128) imply that (131) can be written as:

$$\pi^{Bi}(\varepsilon) - E\{\pi^{Bi}(\varepsilon)\} = \left[\gamma_i\,r_{0i} + (1-\gamma_i)\,\frac{1}{3}\,(a + c_1 + c_2 - b\,F)\right]\alpha_i\,b^I\,\varepsilon - \frac{1}{3}\,\varepsilon\,\overline{Q}_i$$

$$- \alpha_i\,b^I\left[\frac{\varepsilon}{3}\,(a + \varepsilon + c_1 + c_2 - b\,F) - \frac{(\overline{\varepsilon})^2}{9}\right] + \frac{1}{3}\,\varepsilon\,F_{i\cdot} \tag{133}$$

where $F = F_{1\cdot} + F_{2\cdot}$.

(26), (132), and (133) imply:

$$\frac{\partial V_{Bi}}{\partial F_{i\cdot}} = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} 2\left[\pi^{Bi}(\varepsilon) - E\{\pi^{Bi}(\varepsilon)\}\right]\frac{\partial\left(\pi^{Bi}(\varepsilon) - E\{\pi^{Bi}(\varepsilon)\}\right)}{\partial F_{i\cdot}}\,dH(\varepsilon) \tag{134}$$

$$= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} 2\left\{\left[\gamma_i\,r_{0i} + (1-\gamma_i)\,\frac{1}{3}\,(a + c_1 + c_2 - b\,F)\right]\alpha_i\,b^I\,\varepsilon - \frac{1}{3}\,\varepsilon\,\overline{Q}_i\right.$$

$$- \alpha_i\,b^I\left[\frac{\varepsilon}{3}\,(a + \varepsilon + c_1 + c_2 - b\,F) - \frac{(\overline{\varepsilon})^2}{9}\right]$$

$$\left. + \frac{1}{3}\,\varepsilon\,F_{i\cdot}\right\}\left[\frac{1+\alpha_i\,\gamma_i}{3}\right]\varepsilon\,dH(\varepsilon)$$

$$= 2\left[\frac{1+\alpha_i\,\gamma_i}{3}\right]\left[\gamma_i\,r_{0i} + (1-\gamma_i)\,\frac{1}{3}\,(a + c_1 + c_2 - b\,F)\right]\alpha_i\,b^I\int_{\underline{\varepsilon}}^{\overline{\varepsilon}}\varepsilon^2\,dH(\varepsilon)$$

$$- \frac{2}{3}\,\overline{Q}_i\,\left[\frac{1+\alpha_i\,\gamma_i}{3}\right]\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon^2\,d\,H(\varepsilon) + \frac{2}{3}\,F_{i\cdot}\,\left[\frac{1+\alpha_i\,\gamma_i}{3}\right]\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon^2\,dH(\varepsilon)$$

$$- 2\left[\frac{1+\alpha_i\,\gamma_i}{3}\right]\,\alpha_i\,b^I \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \left[\frac{\varepsilon^2}{3}\,(\,a+\varepsilon+c_1+c_2-b\,F\,) - \frac{(\,\overline{\varepsilon}\,)^2}{9}\,\varepsilon\right]\,dH(\varepsilon)$$

$$= \frac{2}{3}\,[\,1+\alpha_i\,\gamma_i\,]\left[\gamma_i\,r_{0i} + (\,1-\gamma_i\,)\,\frac{1}{3}\,(\,a+c_1+c_2-b\,F\,)\right]\alpha_i\,b^I\,E\{\,\varepsilon^2\,\}$$

$$- \frac{2}{3}\,\overline{Q}_i\,\left[\frac{1+\alpha_i\,\gamma_i}{3}\right]\,E\{\,\varepsilon^2\,\}$$

$$+ \frac{2}{3}\,F_{i\cdot}\,\left[\frac{1+\alpha_i\,\gamma_i}{3}\right]\,E\{\,\varepsilon^2\,\} - \frac{2}{3}\left[\frac{1+\alpha_i\,\gamma_i}{3}\right]\,\alpha_i\,b^I\,[\,a+c_1+c_2-b\,F\,]\,E\{\,\varepsilon^2\,\}$$

$$- \frac{2}{3}\left[\frac{1+\alpha_i\,\gamma_i}{3}\right]\,\alpha_i\,b^I\,E\{\,\varepsilon^3\,\} + \frac{2}{3}\,[\,1+\alpha_i\,\gamma_i\,]\,\alpha_i\,b^I\,\frac{(\,\overline{\varepsilon}\,)^2}{9}\,E\{\,\varepsilon\,\}$$

$$= \frac{2}{3}\,[\,1+\alpha_i\,\gamma_i\,]\left[\gamma_i\,r_{0i} + (\,1-\gamma_i\,)\,\frac{1}{3}\,(\,a+c_1+c_2-b\,F\,)\right]\alpha_i\,b^I\,E\{\,\varepsilon^2\,\}$$

$$- \frac{2}{3}\,\overline{Q}_i\,\left[\frac{1+\alpha_i\,\gamma_i}{3}\right]\,E\{\,\varepsilon^2\,\} + \frac{2}{3}\,F_{i\cdot}\,\left[\frac{1+\alpha_i\,\gamma_i}{3}\right]\,E\{\,\varepsilon^2\,\}$$

$$- \frac{2}{3}\left[\frac{1+\alpha_i\,\gamma_i}{3}\right]\,\alpha_i\,b^I\,[\,a+c_1+c_2-b\,F\,]\,E\{\,\varepsilon^2\,\}$$

$$= \frac{2}{9}\,\alpha_i\,b^I\,[\,1+\alpha_i\,\gamma_i\,]\,E\{\,\varepsilon^2\,\}\,\{\,3\,\gamma_i\,r_{0i} + [\,1-\gamma_i\,]\,[\,a+c_1+c_2-b\,F\,] + \frac{b}{\alpha_i}\,[\,F_{i\cdot} - \overline{Q}_i\,]$$
$$- [\,a+c_1+c_2-b\,F\,]\,\}$$

$$= \frac{2}{9}\,\alpha_i\,b^I\,[\,1+\alpha_i\,\gamma_i\,]\,E\{\,\varepsilon^2\,\}\left[3\,\gamma_i\,r_{0i} - \gamma_i\,(\,a+c_1+c_2-b\,F\,) + \frac{b}{\alpha_i}\,\big(\,F_{i\cdot} - \overline{Q}_i\,\big)\right] \tag{135}$$

$$\Rightarrow\ \frac{\partial^2 V_{Bi}}{\partial\,(F_{i\cdot})^2} \;=\; \frac{2}{9}\,\alpha_i\,b^I\,[\,1+\alpha_i\,\gamma_i\,]\,E\left\{\varepsilon^2\right\}\,b\left[\gamma_i + \frac{1}{\alpha_i}\right] \;>\; 0\,. \tag{136}$$

(12) and (135) imply that the value of $F_{i\cdot}$ at which V_{Bi} is minimized is determined by:

$$3\,\gamma_i\,r_{0i} - \gamma_i\,[\,a+c_1+c_2-b\,F\,] + \frac{b}{\alpha_i}\,[\,F_{i\cdot} - \overline{Q}_i\,] \;=\; 0$$

$$\Leftrightarrow\ 3\,\alpha_i\,\gamma_i\,r_{0i} - \alpha_i\,\gamma_i\,[\,a+c_1+c_2-b\,F\,] + b\,F_{i\cdot} - b\,\overline{Q}_i \;=\; 0$$

$$\Leftrightarrow\ 3\,\alpha_i\,\gamma_i\,r_{0i} - \alpha_i\,\gamma_i\left[\frac{a^I+\overline{Q}_i+\overline{Q}_j}{b^I} + c_1 + c_2\right] + \alpha_i\,\gamma_i\,b\,[\,F_{i\cdot}+F_{j\cdot}\,] + b\,F_{i\cdot} - b\,\overline{Q}_i \;=\; 0$$

$$\Leftrightarrow \;\; 3\,\alpha_i\,\gamma_i\,b^I\,r_{0i} - \alpha_i\,\gamma_i\left[\,a^I + \overline{Q}_i + \overline{Q}_j + b^I\,(c_1 + c_2)\,\right] + [\,1 + \alpha_i\,\gamma_i\,]\,F_{i\cdot} + \alpha_i\,\gamma_i\,F_{j\cdot} - \overline{Q}_i \;=\; 0$$

$$\Leftrightarrow \;\; [\,1 + \alpha_i\,\gamma_i\,]\,F_{i\cdot} \;=\; [\,1 + \alpha_i\,\gamma_i\,]\,\overline{Q}_i + \alpha_i\,\gamma_i\,\overline{Q}_j + \alpha_i\,\gamma_i\,a^I + \alpha_i\,\gamma_i\,b^I\,[\,c_1 + c_2\,]$$

$$- \; 3\,\alpha_i\,\gamma_i\,b^I\,r_{0i} - \alpha_i\,\gamma_i\,F_{j\cdot}\,.$$

$$\Leftrightarrow \;\; F_{i\cdot} \;=\; \overline{Q}_i - \left[\frac{\alpha_i\,\gamma_i}{1 + \alpha_i\,\gamma_i}\right] F_{j\cdot}$$

$$+ \;\; \frac{\alpha_i\,\gamma_i}{1 + \alpha_i\,\gamma_i}\left[\,\overline{Q}_j + a^I - b^I\,(\,3\,r_{0i} - c_1 - c_2\,)\,\right] \;\equiv\; \underline{F}_{iDD}\,. \;\blacksquare \tag{137}$$

Lemma C2. *Suppose B1 and B2 are symmetric. Then the level of forward contracting that minimizes the variance of the buyer's profit in the dual monopoly setting exceeds the sum of the levels of forward contracting that minimize the variance of each buyer's profit in the dual duopoly setting.*

Proof. (137) implies that when $B1$ and $B2$ are symmetric and $F_{j\cdot} = \underline{F}_{Bi\cdot}$ in the dual duopoly setting:

$$\underline{F}_{Bi\cdot} \;=\; \overline{Q}_i - \left[\frac{\frac{1}{2}\,\gamma}{1 + \frac{1}{2}\,\gamma}\right]\underline{F}_{Bi\cdot} + Z_{DDi}$$

$$\Leftrightarrow \;\; \underline{F}_{Bi\cdot}\left[1 + \frac{\frac{1}{2}\,\gamma}{\frac{1}{2}\,(2+\gamma)}\right] \;=\; \overline{Q}_i + \frac{\frac{1}{2}\,\gamma}{1 + \frac{1}{2}\,\gamma}\left[\,\overline{Q}_j + a^I - b^I\,(\,3\,r_0 - 2\,c\,)\,\right]$$

$$\Leftrightarrow \;\; 2\,\underline{F}_{Bi\cdot}\left[\frac{1+\gamma}{2+\gamma}\right] \;=\; \frac{1}{2}\,\overline{Q}\left[1 + \frac{\gamma}{2+\gamma}\right] + \frac{\gamma}{2+\gamma}\left[\,a^I - b^I\,(\,3\,r_0 - 2\,c\,)\,\right]$$

$$\Leftrightarrow \;\; 2\,\underline{F}_{Bi\cdot}\left[\frac{1+\gamma}{2+\gamma}\right] \;=\; \overline{Q}\left[\frac{1+\gamma}{2+\gamma}\right] + \frac{\gamma}{2+\gamma}\left[\,a^I - b^I\,(\,3\,r_0 - 2\,c\,)\,\right]$$

$$\Leftrightarrow \;\; \underline{F}_{Bi\cdot} \;=\; \frac{1}{2}\,\overline{Q} + \frac{1}{2}\left[\frac{\gamma}{1+\gamma}\right]\left[\,a^I - b^I\,(\,3\,r_0 - 2\,c\,)\,\right]. \tag{138}$$

The conclusion follows from (38) and (138) because:

$$\underline{F}_B \;>\; \underline{F}_{B1\cdot} + \underline{F}_{B2\cdot}$$

$$\Leftrightarrow \;\; \overline{Q} + \frac{\gamma}{1+\gamma}\left[\,a^I - b^I\,(\,2\,r_0 - c\,)\,\right] \;>\; \overline{Q} + \frac{\gamma}{1+\gamma}\left[\,a^I - b^I\,(\,3\,r_0 - 2\,c\,)\,\right]$$

$$\Leftrightarrow \;\; a^I - b^I\,[\,2\,r_0 - c\,] \;>\; a^I - b^I\,[\,3\,r_0 - 2\,c\,] \quad \Leftrightarrow \;\; b^I\,[\,3\,r_0 - 2\,c - 2\,r_0 + c\,] \;>\; 0$$

$$\Leftrightarrow \;\; b^I\,[\,r_0 - c\,] \;>\; 0\,. \;\blacksquare$$

Proposition C2. *When the buyers choose the levels of forward contracting in the dual duopoly setting, Bi's equilibrium level of forward contracting is, for* $i,\, j \in \{1,2\}$ *(*$j \neq i$*):*

$$F_{BiDD} \;=\; \frac{1+\alpha_j\,\gamma_j}{1+\alpha_i\,\gamma_i+\alpha_j\,\gamma_j}\,\left[\,\widehat{\Psi}_i\,(\,1+\alpha_i\,\gamma_i\,)-\alpha_i\,\gamma_i\,\widehat{\Psi}_j\,\right].$$

$F_{Bi\cdot DD}$ is increasing in $\overline{Q}_i$, A_{Bj}, and r_{0j}, and decreasing in A_{Bi} and r_{0i}.

Proof. (128) and (130) imply:

$$\begin{aligned}\frac{\partial E\left\{\,\pi^{Bi}(\varepsilon)\,\right\}}{\partial F_{i\cdot}} \;&=\; [\,1-\gamma_i\,]\,\overline{Q}_i\,\frac{\partial E\{\,w(\varepsilon)\,\}}{\partial F_{i\cdot}}-\overline{Q}_i\,\frac{\partial E\{\,w(\varepsilon)\,\}}{\partial F_{i\cdot}}-\alpha_i\,b^I\,\frac{\partial E\{\,\varepsilon\,w(\varepsilon)\,\}}{\partial F_{i\cdot}}\\ &=\; -\,\gamma_i\,\overline{Q}_i\left[-\frac{b}{3}\right] \;=\; \gamma_i\,\frac{b}{3}\,\overline{Q}_i \;\geq\; 0\,. \qquad (139)\end{aligned}$$

(12), (26), and (135) – (139) imply that if $A_{Bi} > 0$, then $E\{\,U_{Bi}(\pi^{Bi})\,\}$ is maximized where:

$$\frac{\partial E\{\,U_{Bi}(\pi^{Bi})\,\}}{\partial F_{i\cdot}} \;=\; \frac{\partial E\{\,\pi^{Bi}\,\}}{\partial F_{i\cdot}}-A_{Bi}\,\frac{\partial V_{Bi}}{\partial F_{i\cdot}} \;=\; 0$$

$$\Leftrightarrow\;\; \gamma_i\,\frac{b}{3}\,\overline{Q}_i-A_{Bi}\,\frac{2}{9}\,\alpha_i\,b^I\,[\,1+\alpha_i\,\gamma_i\,]\,E\{\,\varepsilon^2\,\}$$
$$\cdot\left[\,3\,\gamma_i\,r_{0i}-\gamma_i\,(\,a+c_1+c_2-b\,F\,)+\frac{b}{\alpha_i}\left(\,F_{i\cdot}-\overline{Q}_i\,\right)\right] \;=\; 0$$

$$\Leftrightarrow\;\; \gamma_i\,\frac{b}{3}\,\overline{Q}_i-\frac{2}{9}\,A_{Bi}\,\alpha_i\,[\,1+\alpha_i\,\gamma_i\,]\,E\{\,\varepsilon^2\,\}$$
$$\cdot\left[\,3\,\gamma_i\,b^I\,r_{0i}-\gamma_i\,b^I\,(\,a+c_1+c_2-b\,F\,)+\frac{1}{\alpha_i}\left(\,F_{i\cdot}-\overline{Q}_i\,\right)\right] \;=\; 0$$

$$\Leftrightarrow\;\; \gamma_i\,\frac{b}{3}\,\overline{Q}_i-\frac{2}{9}\,A_{Bi}\,[\,1+\alpha_i\,\gamma_i\,]\,E\{\,\varepsilon^2\,\}$$
$$\cdot\left[\,3\,\alpha_i\,\gamma_i\,b^I\,r_{0i}-\alpha_i\,\gamma_i\,b^I\,(\,a+c_1+c_2-b\,F\,)+F_{i\cdot}-\overline{Q}_i\,\right] \;=\; 0$$

$$\Leftrightarrow\;\; \gamma_i\,\frac{b}{3}\,\overline{Q}_i-\frac{2}{9}\,A_{Bi}\,[\,1+\alpha_i\,\gamma_i\,]\,E\{\,\varepsilon^2\,\}$$
$$\cdot\left[\,3\,\alpha_i\,\gamma_i\,b^I\,r_{0i}-\alpha_i\,\gamma_i\,b^I\,(\,a+c_1+c_2\,)-\overline{Q}_i+\alpha_i\,\gamma_i\,F_{j\cdot}+(\,1+\alpha_i\,\gamma_i\,)\,F_{i\cdot}\,\right] \;=\; 0$$

$$\Leftrightarrow\;\; F_{i\cdot}\,\frac{2}{9}\,A_{Bi}\,[\,1+\alpha_i\,\gamma_i\,]^2\;E\{\,\varepsilon^2\,\}$$
$$=\; \gamma_i\,\frac{b}{3}\,\overline{Q}_i+\frac{2}{9}\,A_{Bi}\,[\,1+\alpha_i\,\gamma_i\,]\,E\{\,\varepsilon^2\,\}\,\left[\,\alpha_i\,\gamma_i\,b^I\,(\,a+c_1+c_2-3\,r_{0i}\,)+\overline{Q}_i-\alpha_i\,\gamma_i\,F_{j\cdot}\,\right]$$

$$\Leftrightarrow \quad F_{i\cdot}\, A_{Bi}\,[\,1+\alpha_i\,\gamma_i\,]^2\;E\{\,\varepsilon^2\,\}$$

$$= \; A_{Bi}\,[\,1+\alpha_i\,\gamma_i\,]\,E\{\,\varepsilon^2\,\}\left[\,\alpha_i\,\gamma_i\,b^I\,(\,a+c_1+c_2-3\,r_{0i}\,)+\,\overline{Q}_i-\alpha_i\,\gamma_i\,F_{j\cdot}\,\right]+\frac{3}{2}\,\gamma_i\;b\;\overline{Q}_i$$

$$\Leftrightarrow \quad F_{i\cdot} \; = \; \frac{1}{1+\alpha_i\,\gamma_i}\left[\,\alpha_i\,\gamma_i\,b^I\,(\,a+c_1+c_2-3\,r_{0i}\,)+\,\overline{Q}_i-\alpha_i\,\gamma_i\,F_{j\cdot}\,\right]$$

$$+ \; \frac{3\,\gamma_i\;b\;\overline{Q}_i}{2\,A_{Bi}\,[\,1+\alpha_i\,\gamma_i\,]^2\;E\{\,\varepsilon^2\,\}}$$

$$= \; \frac{1}{1+\alpha_i\,\gamma_i}\left[\alpha_i\,\gamma_i\,b^I\left(\frac{a^I+\overline{Q}_i+\overline{Q}_j}{b^I}+c_1+c_2-3\,r_{0i}\right)+\,\overline{Q}_i-\alpha_i\,\gamma_i\,F_{j\cdot}\right]$$

$$+ \; \frac{9\,\gamma_i\;b\;\overline{Q}_i}{2\,A_{Bi}\,[\,1+\alpha_i\,\gamma_i\,]^2\,(\,\overline{\varepsilon}\,)^2}$$

$$= \; \frac{1}{1+\alpha_i\,\gamma_i}\left[\,\alpha_i\,\gamma_i\left(\,a^I+\overline{Q}_i+\overline{Q}_j+b^I\,[\,c_1+c_2-3\,r_{0i}\,]\,\right)+\,\overline{Q}_i-\alpha_i\,\gamma_i\,F_{j\cdot}\,\right]$$

$$+ \; \frac{9\,\gamma_i\;b\;\overline{Q}_i}{2\,A_{Bi}\,[\,1+\alpha_i\,\gamma_i\,]^2\,(\,\overline{\varepsilon}\,)^2}$$

$$= \; \overline{Q}_i+\frac{\alpha_i\,\gamma_i\left[\,\overline{Q}_j+a^I-b^I\,(\,3\,r_{0i}-c_1-c_2\,)-F_{j\cdot}\,\right]}{1+\alpha_i\,\gamma_i}+\frac{9\,\gamma_i\;b\;\overline{Q}_i}{2\,A_{Bi}\,[\,1+\alpha_i\,\gamma_i\,]^2\,(\,\overline{\varepsilon}\,)^2}$$

$$= \; \underline{F}_{iDD}+\frac{9\,\gamma_i\,\overline{Q}_i}{2\,b^I\,A_{Bi}\,[\,1+\alpha_i\,\gamma_i\,]^2\,(\,\overline{\varepsilon}\,)^2}\,. \tag{140}$$

(74), (137), and (140) imply that the buyers' equilibrium forward contracting positions are determined by:

$$F_i \; = \; \widehat{\Psi}_i-\left[\frac{\alpha_i\,\gamma_i}{1+\alpha_i\,\gamma_i}\right]F_j \quad\Leftrightarrow\quad F_i \; = \; \widehat{\Psi}_i-\frac{\alpha_i\,\gamma_i}{1+\alpha_i\,\gamma_i}\left[\widehat{\Psi}_j-\frac{\alpha_j\,\gamma_j}{1+\alpha_j\,\gamma_j}\,F_i\right]$$

$$\Leftrightarrow \quad F_i\left[1-\left(\frac{\alpha_i\,\gamma_i}{1+\alpha_i\,\gamma_i}\right)\left(\frac{\alpha_j\,\gamma_j}{1+\alpha_j\,\gamma_j}\right)\right] \; = \; \frac{1}{1+\alpha_i\,\gamma_i}\left[\widehat{\Psi}_i\;(1+\alpha_i\,\gamma_i)-\alpha_i\,\gamma_i\,\widehat{\Psi}_j\right]$$

$$\Leftrightarrow \quad F_i\;\frac{[\,1+\alpha_i\,\gamma_i\,]\left[\,1+\alpha_j\,\gamma_j\,\right]-\alpha_i\,\gamma_i\,\alpha_j\,\gamma_j}{[\,1+\alpha_i\,\gamma_i\,]\left[\,1+\alpha_j\,\gamma_j\,\right]} \; = \; \frac{1}{1+\alpha_i\,\gamma_i}\left[\widehat{\Psi}_i\;(1+\alpha_i\,\gamma_i)-\alpha_i\,\gamma_i\,\widehat{\Psi}_j\right]$$

$$\Leftrightarrow \quad F_i\left[\frac{1+\alpha_i\,\gamma_i+\alpha_j\,\gamma_j}{1+\alpha_j\,\gamma_j}\right] \; = \; \widehat{\Psi}_i\,[\,1+\alpha_i\,\gamma_i\,]-\alpha_i\,\gamma_i\,\widehat{\Psi}_j$$

$$\Leftrightarrow \quad F_i \; = \; \frac{1+\alpha_j\,\gamma_j}{1+\alpha_i\,\gamma_i+\alpha_j\,\gamma_j}\left[\,\widehat{\Psi}_i\,(\,1+\alpha_i\,\gamma_i\,)-\alpha_i\,\gamma_i\,\widehat{\Psi}_j\,\right] \; = \; F_{Bi\cdot DD}. \tag{141}$$

(74) and (141) imply:

$$\frac{\partial F_{Bi \cdot DD}}{\partial A_{Bi}} = \frac{[1+\alpha_i\gamma_i][1+\alpha_j\gamma_j]}{1+\alpha_i\gamma_i+\alpha_j\gamma_j}\frac{\partial\widehat{\Psi}_i}{\partial A_{Bi}} \stackrel{s}{=} \frac{\partial\widehat{\Psi}_i}{\partial A_{Bi}} < 0;$$

$$\frac{\partial F_{Bi \cdot DD}}{\partial r_{0i}} = \frac{[1+\alpha_i\gamma_i][1+\alpha_j\gamma_j]}{1+\alpha_i\gamma_i+\alpha_j\gamma_j}\frac{\partial\widehat{\Psi}_i}{\partial r_{0i}} \stackrel{s}{=} \frac{\partial\widehat{\Psi}_i}{\partial r_{0i}} < 0;$$

$$\frac{\partial F_{Bi \cdot DD}}{\partial A_{Bj}} = -\frac{\alpha_i\gamma_i[1+\alpha_j\gamma_j]}{1+\alpha_i\gamma_i+\alpha_j\gamma_j}\frac{\partial\widehat{\Psi}_j}{\partial A_{Bj}} \stackrel{s}{=} -\frac{\partial\widehat{\Psi}_j}{\partial A_{Bj}} > 0;$$

$$\frac{\partial F_{Bi \cdot DD}}{\partial r_{0j}} = -\frac{\alpha_i\gamma_i[1+\alpha_j\gamma_j]}{1+\alpha_i\gamma_i+\alpha_j\gamma_j}\frac{\partial\widehat{\Psi}_j}{\partial r_{0j}} \stackrel{s}{=} -\frac{\partial\widehat{\Psi}_j}{\partial r_{0j}} > 0;$$

$$\begin{aligned}\frac{\partial F_{Bi \cdot DD}}{\partial \overline{Q}_i} &= \frac{[1+\alpha_i\gamma_i][1+\alpha_j\gamma_j]}{1+\alpha_i\gamma_i+\alpha_j\gamma_j}\frac{\partial\widehat{\Psi}_i}{\partial \overline{Q}_i} - \frac{\alpha_i\gamma_i[1+\alpha_j\gamma_j]}{1+\alpha_i\gamma_i+\alpha_j\gamma_j}\frac{\partial\widehat{\Psi}_j}{\partial \overline{Q}_i} \\ &\stackrel{s}{=} [1+\alpha_i\gamma_i]\left[1+\frac{9\gamma_i}{2A_{Bi}b^I[1+\alpha_i\gamma_i]^2(\overline{\varepsilon})^2}\right] - \alpha_i\gamma_i\left[\frac{\alpha_j\gamma_j}{1+\alpha_j\gamma_j}\right] \\ &\geq 1+\alpha_i\gamma_i - \alpha_i\gamma_i\left[\frac{\alpha_j\gamma_j}{1+\alpha_j\gamma_j}\right] \stackrel{s}{=} [1+\alpha_i\gamma_i][1+\alpha_j\gamma_j] - \alpha_i\gamma_i\alpha_j\gamma_j \\ &= 1+\alpha_i\gamma_i+\alpha_j\gamma_j > 0. \quad \blacksquare\end{aligned}$$

Let F_{GDD} denote the aggregate equilibrium level of forward contracting in the dual duopoly setting when the generators set the levels of forward contracting.

Proposition C3. *Suppose $a^I \geq b^I c$, the buyers are symmetric, the generators are symmetric, $A_{B1} = A_{B2} = A_{G1} = A_{G2} \equiv A$, and $\gamma > 0$. Then there exists an $\widetilde{A} > 0$* such that *$F_{BDD} \gtreqless F_{GDD} \Leftrightarrow A \lesseqgtr \widetilde{A}$.*

<u>Proof</u>. (97) and Proposition 7 imply that under the specified conditions:

$$\begin{aligned}F_{GDD} &= \frac{2[24A_G(\overline{\varepsilon})^2+81b][a-c]}{b[48A_G(\overline{\varepsilon})^2+405b]} = \frac{2b^I[24A_G(\overline{\varepsilon})^2+81b]\left[\frac{a^I+\overline{Q}}{b^I}-c\right]}{48A_G(\overline{\varepsilon})^2+405b} \\ &= \frac{2[24A_G(\overline{\varepsilon})^2+81b][\overline{Q}+a^I-b^Ic]}{48A_G(\overline{\varepsilon})^2+405/b^I} \\ &= \frac{2[24b^IA_G(\overline{\varepsilon})^2+81][\overline{Q}+a^I-b^Ic]}{48b^IA_G(\overline{\varepsilon})^2+405}. \qquad (142)\end{aligned}$$

(78) and (142) imply that under the specified conditions:

$$F_{BDD} \gtreqless F_{GDD} \;\Leftrightarrow\; \overline{Q} + \frac{\gamma \left[\, a^I - b^I \left(3\, r_0 - 2\, c \right) \right]}{1 + \gamma} + \frac{9\, \gamma\, \overline{Q}}{A\, b^I \left[1 + \gamma \right] \left[2 + \gamma \right] \left(\overline{\varepsilon} \right)^2}$$

$$\gtreqless \frac{2 \left[24\, b^I A \left(\overline{\varepsilon} \right)^2 + 81 \right] \left[\, \overline{Q} + a^I - b^I c \,\right]}{48\, b^I A \left(\overline{\varepsilon} \right)^2 + 405}$$

$$\Leftrightarrow \quad \overline{Q} + \frac{\gamma \left[\, a^I - b^I \left(3\, r_0 - 2\, c \right) \right]}{1 + \gamma} \gtreqless \left[\frac{1}{\left(48\, b^I A \left(\overline{\varepsilon} \right)^2 + 405 \right) \left(A\, b^I \left[1 + \gamma \right] \left[2 + \gamma \right] \left(\overline{\varepsilon} \right)^2 \right)} \right]$$

$$\times \left\{ 2 \left[24\, b^I A \left(\overline{\varepsilon} \right)^2 + 81 \right] \left[\, \overline{Q} + a^I - b^I c \,\right] A\, b^I \left[1 + \gamma \right] \left[2 + \gamma \right] \left(\overline{\varepsilon} \right)^2 \right.$$

$$\left. - \; 9\, \gamma\, \overline{Q} \left[48\, b^I A \left(\overline{\varepsilon} \right)^2 + 405 \right] \right\}$$

$$\Leftrightarrow \quad \overline{Q} + \frac{\gamma \left[\, a^I - b^I \left(3\, r_0 - 2\, c \right) \right]}{1 + \gamma} \gtreqless \left[\frac{1}{\left(48\, b^I A \left(\overline{\varepsilon} \right)^2 + 405 \right) \left(A\, b^I \left[1 + \gamma \right] \left[2 + \gamma \right] \left(\overline{\varepsilon} \right)^2 \right)} \right]$$

$$\times \left\{ 2 \left[1 + \gamma \right] \left[2 + \gamma \right] \left(\overline{\varepsilon} \right)^2 \left[24 \left(b^I \right)^2 A^2 \left(\overline{\varepsilon} \right)^2 + 81\, b^I A \right] \left[\, \overline{Q} + a^I - b^I c \,\right] \right.$$

$$\left. - \; 9\, \gamma\, \overline{Q} \left[48\, b^I A \left(\overline{\varepsilon} \right)^2 + 405 \right] \right\}. \qquad (143)$$

Observe that:

$$2 \left[1 + \gamma \right] \left[2 + \gamma \right] \left(\overline{\varepsilon} \right)^2 \left[24 \left(b^I \right)^2 A^2 \left(\overline{\varepsilon} \right)^2 + 81\, b^I A \right] \left[\, \overline{Q} + a^I - b^I c \,\right]$$

$$- \; 9\, \gamma\, \overline{Q} \left[48\, b^I A \left(\overline{\varepsilon} \right)^2 + 405 \right]$$

$$= \; 2 \left[1 + \gamma \right] \left[2 + \gamma \right] \left(\overline{\varepsilon} \right)^2 \left[24 \left(b^I \right)^2 A^2 \left(\overline{\varepsilon} \right)^2 + 81\, b^I A \right] \overline{Q}$$

$$+ \; 2 \left[1 + \gamma \right] \left[2 + \gamma \right] \left(\overline{\varepsilon} \right)^2 \left[24 \left(b^I \right)^2 A^2 \left(\overline{\varepsilon} \right)^2 + 81\, b^I A \right] \left[\, a^I - b^I c \,\right]$$

$$- \; 9\, \gamma\, \overline{Q} \left[48\, b^I A \left(\overline{\varepsilon} \right)^2 + 405 \right]$$

$$= \; \overline{Q} \left\{ 48 \left[1 + \gamma \right] \left[2 + \gamma \right] \left(b^I \right)^2 A^2 \left(\overline{\varepsilon} \right)^4 + 162 \left[1 + \gamma \right] \left[2 + \gamma \right] b^I A \left(\overline{\varepsilon} \right)^2 \right.$$

$$\left. - \; 432\, \gamma\, b^I A \left(\overline{\varepsilon} \right)^2 - 3,645\, \gamma \right\}$$

$$+ \; 2 \left[1 + \gamma \right] \left[2 + \gamma \right] \left(\overline{\varepsilon} \right)^2 \left[24 \left(b^I \right)^2 A^2 \left(\overline{\varepsilon} \right)^2 + 81\, b^I A \right] \left[\, a^I - b^I c \,\right]$$

$$= \; \overline{Q} \left\{ 48 \left[1 + \gamma \right] \left[2 + \gamma \right] \left(b^I \right)^2 A^2 \left(\overline{\varepsilon} \right)^4 \right.$$

$$\left. + \; b^I A \left(\overline{\varepsilon} \right)^2 \left[162 \left(1 + \gamma \right) \left(2 + \gamma \right) - 432\, \gamma \right] - 3,645\, \gamma \right\}$$

$$+\,2\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\bar\varepsilon\,)^2\left[\,24\,(b^I)^2\,A^2\,(\,\bar\varepsilon\,)^2+81\,b^I\,A\,\right]\left[\,a^I-b^I\,c\,\right]$$

$$=\;\overline{Q}\,\Big\{\,48\,[\,1+\gamma\,]\,[\,2+\gamma\,]\;(b^I)^2\,A^2\,(\,\bar\varepsilon\,)^4$$

$$+\;b^I\,A\,(\,\bar\varepsilon\,)^2\left[\,162\left(\,2+3\gamma+\gamma^2\,\right)-432\,\gamma\,\right]-3,645\,\gamma\,\Big\}$$

$$+\;2\;[\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\bar\varepsilon\,)^2\left[\,24\,(b^I)^2\,A^2\,(\,\bar\varepsilon\,)^2+81\,b^I\,A\,\right]\left[\,a^I-b^I\,c\,\right]$$

$$=\;\overline{Q}\,\left\{\,48\;[\,1+\gamma\,]\,[\,2+\gamma\,]\;(b^I)^2\,A^2\,(\,\bar\varepsilon\,)^4+\;b^I\,A\,(\,\bar\varepsilon\,)^2\left[\,162\left(\,2+\gamma^2\,\right)+54\,\gamma\,\right]-3,645\,\gamma\,\right\}$$

$$+\;2\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\bar\varepsilon\,)^2\left[\,24\,(b^I)^2\,A^2\,(\,\bar\varepsilon\,)^2+81\,b^I\,A\,\right]\left[\,a^I-b^I\,c\,\right]. \tag{144}$$

(143) and (144) imply:

$$F_{BDD}\;\gtreqless\;F_{GDD}\quad\Leftrightarrow\quad\overline{Q}+\frac{\gamma\left[\,a^I-b^I\,(\,3\,r_0-2\,c\,)\,\right]}{1+\gamma}\;\gtreqless$$

$$\frac{1}{\left[\,48\,b^I\,A\,(\,\bar\varepsilon\,)^2+405\,\right]\left[\,A\,b^I\,(\,1+\gamma\,)\,(\,2+\gamma\,)\,(\,\bar\varepsilon\,)^2\,\right]}$$

$$\times\Big\{\,\overline{Q}\,\Big(\,48\;[\,1+\gamma\,]\,[\,2+\gamma\,]\,\left(b^I\right)^2\,A^2\,(\,\bar\varepsilon\,)^4$$

$$+\;b^I\,A\,(\,\bar\varepsilon\,)^2\left[\,162\left(\,2+\gamma^2\,\right)+54\,\gamma\,\right]-3,645\,\gamma\,\Big)$$

$$+\;2\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\bar\varepsilon\,)^2\left[24\left(b^I\right)^2A^2\,(\,\bar\varepsilon\,)^2+81\,b^I A\right]\left[\,a^I-b^I\,c\,\right]\;\Big\}$$

$$\Leftrightarrow\quad\overline{Q}+\frac{\gamma\left[\,a^I-b^I\,(\,3\,r_0-2\,c\,)\,\right]}{1+\gamma}\;\gtreqless\;\left(\frac{1}{\left[\,48\,b^I\,A\,(\,\bar\varepsilon\,)^2+405\,\right]b^I\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\bar\varepsilon\,)^2}\right)$$

$$\times\,\frac{1}{A}\,\Big\{\,\overline{Q}\,\Big(\,48\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,\left(b^I\right)^2\,A^2\,(\,\bar\varepsilon\,)^4$$

$$+\;b^I\,A\,(\,\bar\varepsilon\,)^2\left[\,162\left(\,2+\gamma^2\,\right)+54\,\gamma\,\right]-3,645\,\gamma\,\Big)$$

$$+\;2\;[\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\bar\varepsilon\,)^2\left[24\left(b^I\right)^2A^2\,(\,\bar\varepsilon\,)^2+81\,b^I\,A\right]\left[\,a^I-b^I\,c\,\right]\;\Big\}$$

$$\Leftrightarrow\quad\overline{Q}+\frac{\gamma\left[\,a^I-b^I\,(\,3\,r_0-2\,c\,)\,\right]}{1+\gamma}\;\gtreqless\;\beta(A) \tag{145}$$

where

$$\beta(A)\;\equiv\;\frac{1}{\left[\,48\,b^I\,A\,(\,\bar\varepsilon\,)^2+405\,\right]b^I\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\bar\varepsilon\,)^2}$$

$$\times \left\{ \overline{Q} \left(48\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,\left(b^I\right)^2 A\,(\,\overline{\varepsilon}\,)^4 \right.\right.$$

$$\left. + \, b^I\,(\,\overline{\varepsilon}\,)^2\,[\,162\,(\,2+\gamma^2\,)+54\,\gamma\,] - 3,645\,\frac{\gamma}{A} \right)$$

$$\left. + \, 2\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\overline{\varepsilon}\,)^2 \left[24\left(b^I\right)^2 A\,(\,\overline{\varepsilon}\,)^2 + 81\,b^I \right] [\,a^I - b^I\,c\,] \right\}. \quad (146)$$

(146) implies:

$$\lim_{A\to 0} \beta(A) \;=\; \lim_{A\to 0} \left(\frac{1}{[\,48\,b^I A\,(\,\overline{\varepsilon}\,)^2 + 405\,]\,b^I\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\overline{\varepsilon}\,)^2} \right)$$

$$\times \left\{ \overline{Q} \left(48\;[\,1+\gamma\,]\,[\,2+\gamma\,]\,\left(b^I\right)^2 A\,(\,\overline{\varepsilon}\,)^4 \right.\right.$$

$$\left. + \, b^I\,(\,\overline{\varepsilon}\,)^2\,[\,162\,(\,2+\gamma^2\,)+54\,\gamma\,] \right)$$

$$\left. + \, 2\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\overline{\varepsilon}\,)^2 \left[24\left(b^I\right)^2 A\,(\,\overline{\varepsilon}\,)^2 + 81\,b^I \right] [\,a^I - b^I\,c\,] \right\}$$

$$- \lim_{A\to 0}\; 3,645\,\frac{\gamma}{A}\,\overline{Q} \left[\frac{1}{(\,48\,b^I A\,(\,\overline{\varepsilon}\,)^2 + 405\,)\,b^I\,(\,1+\gamma\,)\,(\,2+\gamma\,)\,(\,\overline{\varepsilon}\,)^2} \right]$$

$$= \; \frac{1}{405\,b^I\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\overline{\varepsilon}\,)^2} \left\{ \overline{Q}\; b^I\,(\,\overline{\varepsilon}\,)^2\,[\,162\,(\,2+\gamma^2\,)+54\,\gamma\,] \right.$$

$$\left. + \, 162\;[\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\overline{\varepsilon}\,)^2\,b^I\,[\,a^I - b^I\,c\,] \right\}$$

$$- \, 3,645\,\gamma\,\overline{Q} \left[\lim_{A\to 0} \left(\frac{1}{A\,[\,48\,b^I A\,(\,\overline{\varepsilon}\,)^2 + 405\,]\,b^I\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\overline{\varepsilon}\,)^2} \right) \right]$$

$$= \; \frac{1}{405\;b^I\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\overline{\varepsilon}\,)^2} \left\{ \overline{Q}\; b^I\,(\,\overline{\varepsilon}\,)^2\,[\,162\,(\,2+\gamma^2\,)+54\,\gamma\,] \right.$$

$$\left. + \, 162\;[\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\overline{\varepsilon}\,)^2\;b^I\,[\,a^I - b^I\,c\,] \right\}$$

$$- \, 3,645\,\gamma\,\overline{Q}\;[\,\infty\,] \;=\; -\,\infty\,. \quad (147)$$

(146) also implies:

$$\lim_{A\to \infty} \beta(A) \;=\; \frac{1}{48\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,(b^I)^2\,(\,\overline{\varepsilon}\,)^4} \;\{\; \overline{Q}\;48\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,\left(b^I\right)^2(\,\overline{\varepsilon}\,)^4$$

$$+ \; 48\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,\left(b^I\right)^2(\,\overline{\varepsilon}\,)^4\,[\,a^I - b^I\,c\,]\,\}$$

$$= \overline{Q} + a^I - b^I c > \overline{Q} + \frac{\gamma \left[a^I - b^I (3 r_0 - 2 c) \right]}{1 + \gamma} . \tag{148}$$

The inequality in (148) holds because:

$$\overline{Q} + a^I - b^I c > \overline{Q} + \frac{\gamma \left[a^I - b^I (3 r_0 - 2 c) \right]}{1 + \gamma}$$

$$\Leftrightarrow [1 + \gamma] \left[a^I - b^I c \right] > \gamma \left[a^I - b^I (3 r_0 - 2 c) \right]$$

$$\Leftrightarrow a^I - b^I c + 3 \gamma b^I [r_0 - c] > 0 .$$

This inequality holds because $a^I \geq b^I c$ and $r_0 > c$, by assumption.

(146) further implies:

$$\beta'(A) \overset{s}{=} \left[48 b^I A (\bar{\varepsilon})^2 + 405 \right] b^I [1 + \gamma] [2 + \gamma] (\bar{\varepsilon})^2$$

$$\times \left\{ 48 \overline{Q} [1 + \gamma] [2 + \gamma] \left(b^I\right)^2 (\bar{\varepsilon})^4 + 3,645 \overline{Q} \frac{\gamma}{A^2} + 48 [1 + \gamma] [2 + \gamma] (\bar{\varepsilon})^4 \left(b^I\right)^2 \left[a^I - b^I c \right] \right\}$$

$$- 48 \left(b^I\right)^2 [1 + \gamma] [2 + \gamma] (\bar{\varepsilon})^4 \left\{ \overline{Q} \left(48 [1 + \gamma] [2 + \gamma] \left(b^I\right)^2 A (\bar{\varepsilon})^4 \right.\right.$$

$$\left. + b^I (\bar{\varepsilon})^2 \left[162 \left(2 + \gamma^2 \right) + 54 \gamma \right] - 3,645 \frac{\gamma}{A} \right)$$

$$\left. + 2 [1 + \gamma] [2 + \gamma] (\bar{\varepsilon})^2 \left[24 \left(b^I\right)^2 A (\bar{\varepsilon})^2 + 81 b^I \right] \left[a^I - b^I c \right] \right\} > 0$$

$$\Leftrightarrow \left[48 b^I A (\bar{\varepsilon})^2 + 405 \right] \left\{ 48 \overline{Q} [1 + \gamma] [2 + \gamma] \left(b^I\right)^2 (\bar{\varepsilon})^4 + 3,645 \overline{Q} \frac{\gamma}{A^2} \right.$$

$$\left. + 48 [1 + \gamma] [2 + \gamma] (\bar{\varepsilon})^4 \left(b^I\right)^2 \left[a^I - b^I c \right] \right\}$$

$$- 48 b^I (\bar{\varepsilon})^2 \left\{ \overline{Q} \left(48 [1 + \gamma] [2 + \gamma] \left(b^I\right)^2 A (\bar{\varepsilon})^4 \right.\right.$$

$$\left. + b^I (\bar{\varepsilon})^2 \left[162 \left(2 + \gamma^2 \right) + 54 \gamma \right] - 3,645 \frac{\gamma}{A} \right)$$

$$\left. + 2 [1 + \gamma] [2 + \gamma] (\bar{\varepsilon})^2 \left[24 \left(b^I\right)^2 A (\bar{\varepsilon})^2 + 81 b^I \right] \left[a^I - b^I c \right] \right\} > 0$$

$$\Leftrightarrow 2,304 A \overline{Q} [1 + \gamma] [2 + \gamma] \left(b^I\right)^3 (\bar{\varepsilon})^6 + 174,960 b^I (\bar{\varepsilon})^2 \overline{Q} \frac{\gamma}{A}$$

$$+ 2,304 A [1 + \gamma] [2 + \gamma] (\bar{\varepsilon})^6 \left(b^I\right)^3 \left[a^I - b^I c \right]$$

$$+ 19,440 \overline{Q} [1 + \gamma] [2 + \gamma] \left(b^I\right)^2 (\bar{\varepsilon})^4 + 1,476,225 \overline{Q} \frac{\gamma}{A^2}$$

$$+\ 19,440\ [\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\overline{\varepsilon}\,)^4\left(b^I\right)^2\left[\,a^I - b^I\,c\,\right]$$

$$-\ 2,304\,A\,\overline{Q}\ [\,1+\gamma\,]\,[\,2+\gamma\,]\left(b^I\right)^3(\,\overline{\varepsilon}\,)^6$$

$$-\ 48\left(b^I\right)^2(\,\overline{\varepsilon}\,)^4\ \overline{Q}\ \left[\,162\left(\,2+\gamma^2\,\right)+54\,\gamma\,\right]+\ 174,960\,b^I\,(\,\overline{\varepsilon}\,)^2\,\overline{Q}\ \frac{\gamma}{A}$$

$$-\ 96\,b^I\,(\,\overline{\varepsilon}\,)^4\,[\,1+\gamma\,]\,[\,2+\gamma\,]\left[24\left(b^I\right)^2A\,(\,\overline{\varepsilon}\,)^2+81\,b^I\right]\left[\,a^I - b^I\,c\,\right]\ >\ 0$$

$$\Leftrightarrow\quad 2\,[174,960\,]\,b^I\,(\,\overline{\varepsilon}\,)^2\,\overline{Q}\ \frac{1}{A}\,\gamma+2,304\,A\,[\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\overline{\varepsilon}\,)^6\left(b^I\right)^3\left[\,a^I - b^I\,c\,\right]$$

$$+\ 1,476,225\ \overline{Q}\ \frac{1}{A^2}\,\gamma+\ 19,440\ \overline{Q}\,[\,1+\gamma\,]\,[\,2+\gamma\,]\left(b^I\right)^2(\,\overline{\varepsilon}\,)^4$$

$$-\ 48\left(b^I\right)^2(\,\overline{\varepsilon}\,)^4\ \overline{Q}\ \left[\,162\left(\,2+\gamma^2\,\right)+54\,\gamma\,\right]$$

$$+\ 19,440\ [\,1+\gamma\,]\,[\,2+\gamma\,]\,(\,\overline{\varepsilon}\,)^4\left(b^I\right)^2\left[\,a^I - b^I\,c\,\right]$$

$$-\ 96\,b^I\,(\,\overline{\varepsilon}\,)^4\,[\,1+\gamma\,]\,[\,2+\gamma\,]\left[24\left(b^I\right)^2A\,(\,\overline{\varepsilon}\,)^2+81\,b^I\right]\left[\,a^I - b^I\,c\,\right]\ >\ 0$$

$$\Leftrightarrow\quad 2\,[\,174,960\,]\,b^I\,(\,\overline{\varepsilon}\,)^2\,\overline{Q}\ \frac{\gamma}{A}+1,476,225\ \overline{Q}\ \frac{\gamma}{A^2}$$

$$+\ \left(b^I\right)^2(\,\overline{\varepsilon}\,)^4\,\overline{Q}\ \left[\,19,440\,(\,1+\gamma\,)\,(\,2+\gamma\,)-48\left(\,162\left[\,2+\gamma^2\,\right]+54\,\gamma\,\right)\,\right]$$

$$+\ [\,1+\gamma\,]\,[\,2+\gamma\,]\left[\,a^I - b^I\,c\right][\,2,304\ A\ (\,\overline{\varepsilon}\,)^6\left(b^I\right)^3+19,440\ (\,\overline{\varepsilon}\,)^4\ (b^I)^2$$

$$-\ 96\,b^I\,(\,\overline{\varepsilon}\,)^4\left(\,24\,(b^I)^2A\,(\,\overline{\varepsilon}\,)^2+81\,b^I\,\right)\,]\ >\ 0$$

$$\Leftrightarrow\quad 2\,[\,174,960\,]\,b^I\,(\,\overline{\varepsilon}\,)^2\,\overline{Q}\ \frac{\gamma}{A}+1,476,225\ \overline{Q}\ \frac{\gamma}{A^2}$$

$$+\ \left(b^I\right)^2(\,\overline{\varepsilon}\,)^4\,\overline{Q}\ \left[\,19,440\left(\,2+3\gamma+\gamma^2\,\right)-7,776\left(\,2+\gamma^2\,\right)-2,592\,\gamma\,\right]$$

$$+\ [\,1+\gamma\,]\,[\,2+\gamma\,]\left[\,a^I - b^I\,c\right]\ [\ 2,304\,A\ (\,\overline{\varepsilon}\,)^6\left(b^I\right)^3+19,440\ (\,\overline{\varepsilon}\,)^4\left(b^I\right)^2$$

$$-\ 2,304\left(b^I\right)^3A\,(\,\overline{\varepsilon}\,)^6-7,776\left(b^I\right)^2(\,\overline{\varepsilon}\,)^4\,]\ >\ 0$$

$$\Leftrightarrow\quad 2\,[\,174,960\,]\,b^I\,(\,\overline{\varepsilon}\,)^2\,\overline{Q}\ \frac{\gamma}{A}+1,476,225\ \overline{Q}\ \frac{\gamma}{A^2}$$

$$+\ \left(b^I\right)^2(\,\overline{\varepsilon}\,)^4\,\overline{Q}\ \left[\,23,328+55,728\,\gamma+11,664\,\gamma^2\,\right]$$

$$+\ [\,1+\gamma\,]\,[\,2+\gamma\,]\left[\,a^I - b^I\,c\right]\left[\,11,664\ (\,\overline{\varepsilon}\,)^4\left(b^I\right)^2\right]\ >\ 0\,. \tag{149}$$

The conclusion follow from (145), (147), (148), and (149). ■

Proposition C4. *Suppose the two generators are symmetric and a generator will only operate if doing so ensures it earns nonnegative profit. Further suppose the buyers are symmetric and each buyer signs the same number of forward contracts with each generator. Then the largest total number of forward contracts that ensures each generator secures nonnegative profit for all* $\varepsilon \in [-\overline{\varepsilon}, \overline{\varepsilon}]$ *in the dual duopoly setting is:*

$$F = \frac{1}{b}\left[\sqrt{9\,(a-c)^2 + 30\,(a-c)\,\overline{\varepsilon} + 57\,\overline{\varepsilon}^2} - (\,a - c + 7\,\overline{\varepsilon}\,)\right].$$

Proof. Let F denote the number of forward contracts each generator signs. Then Lemma B1 implies that a generator's profit in the dual duopoly setting under the specified conditions is:

$$\begin{aligned}
\pi_G(\varepsilon) &= [\,w(\varepsilon) - c\,]\,q(\varepsilon) + \left[\,p^F - w(\varepsilon)\,\right] F \\
&= \frac{1}{3}[\,a + \varepsilon + 2\,c - b\,F - 3\,c\,]\,\frac{1}{3\,b}\left[a + \varepsilon - c + \frac{b}{2}\,F\right] \\
&\quad + \frac{1}{3}[\,a + 2\,c - b\,F - (\,a + \varepsilon + 2\,c - b\,F\,)\,]\,F \\
&= \frac{1}{9\,b}[\,a - c + \varepsilon - b\,F\,]\left[a - c + \varepsilon + \frac{b}{2}\,F\right] - \frac{1}{3}\,\varepsilon\,F \\
&= \frac{1}{9\,b}\left\{[\,a - c + \varepsilon\,]^2 + \frac{b}{2}\,F\,[\,a - c + \varepsilon\,] - b\,F\,[\,a - c + \varepsilon\,] - \frac{b^2}{2}\,F^2\right\} - \frac{1}{3}\,\varepsilon\,F \\
&= \frac{1}{9\,b}\left\{[\,a - c + \varepsilon\,]^2 - \frac{b}{2}\,F\,[\,a - c + \varepsilon\,] - \frac{b^2}{2}\,F^2\right\} - \frac{1}{3}\,\varepsilon\,F \\
&= \frac{1}{9\,b}\,[\,a - c + \varepsilon\,]^2 - \frac{1}{18}\,F\,[\,a - c + \varepsilon\,] - \frac{b}{18}\,F^2 - \frac{1}{3}\,\varepsilon\,F \\
&= \frac{1}{9\,b}\,[\,a - c + \varepsilon\,]^2 - \frac{1}{18}\,F\,[\,a - c + 7\,\varepsilon\,] - \frac{b}{18}\,F^2\,. \qquad (150)
\end{aligned}$$

(150) implies:

$$\begin{aligned}
\pi_G(\varepsilon) \;\geq\; 0 \;\;&\Leftrightarrow\;\; 2\,[\,a - c + \varepsilon\,]^2 - b\,F\,[\,a - c + 7\,\varepsilon\,] - b^2\,F^2 \;\geq\; 0 \\
&\Leftrightarrow\;\; b^2\,F^2 + b\,[\,a - c + 7\,\varepsilon\,]\,F - 2\,[\,a - c + \varepsilon\,]^2 \;\leq\; 0\,. \qquad (151)
\end{aligned}$$

(151) holds with equality when:

$$F = \frac{1}{2b^2}\left[-b(a-c+7\varepsilon) \pm \sqrt{b^2(a-c+7\varepsilon)^2 + 8b^2(a-c+\varepsilon)^2} \right]$$
$$= \frac{1}{2b}\left[-(a-c+7\varepsilon) \pm \sqrt{(a-c+7\varepsilon)^2 + 8(a-c+\varepsilon)^2} \right]. \tag{152}$$

(152) implies that the only positive root of the equality in (151) is:

$$F_+ = \frac{1}{2b}\left[\sqrt{(a-c+7\varepsilon)^2 + 8(a-c+\varepsilon)^2} - (a-c+7\varepsilon) \right]. \tag{153}$$

(150) implies that $\pi_G(\cdot)$ is a strictly concave function of F because:

$$\frac{\partial \pi_G(\cdot)}{\partial F} = -\frac{1}{18}[a-c+7\varepsilon] - \frac{b}{9}F \;\Rightarrow\; \frac{\partial \pi_G^2(\cdot)}{\partial F^2} = -\frac{b}{9} < 0. \tag{154}$$

(153) and (154) imply:

$$\left.\frac{\partial \pi_G(\cdot)}{\partial F}\right|_{F=F_+} = -\frac{1}{18}[a-c+7\varepsilon] - \frac{b}{9}F_+$$
$$= -\frac{1}{18}[a-c+7\varepsilon] - \frac{1}{18}\left[\sqrt{[a-c+7\varepsilon]^2 + 8[a-c+\varepsilon]^2} - (a-c+7\varepsilon) \right]$$
$$= -\frac{1}{18}\sqrt{[a-c+7\varepsilon]^2 + 8[a-c+\varepsilon]^2} < 0. \tag{155}$$

(153) – (155) imply that for each ε, the generator's profit is negative for all $F > F^+$.

(150) implies that $\pi_G(\cdot)$ is a strictly convex function of ε because:

$$\frac{\partial \pi_G(\cdot)}{\partial \varepsilon} = \frac{2}{9b}[a-c+\varepsilon] - \frac{7}{18}F \;\Rightarrow\; \frac{\partial^2 \pi_G(\cdot)}{\partial \varepsilon^2} = \frac{2}{9b} > 0. \tag{156}$$

(153) and (156) imply:

$$\left.\frac{\partial \pi_G(\cdot)}{\partial \varepsilon}\right|_{F=F^+} = \frac{2}{9b}[a-c+\varepsilon] - \frac{7}{18}F^+$$
$$= \frac{2}{9b}[a-c+\varepsilon]$$
$$- \frac{7}{18}\left[\frac{1}{2b}\right]\left[\sqrt{[a-c+7\varepsilon]^2 + 8[a-c+\varepsilon]^2} - (a-c+7\varepsilon) \right]$$

$$\stackrel{s}{=}\; 2\,[\,a - c + \varepsilon\,] - \frac{7}{4}\left[\sqrt{[\,a - c + 7\,\varepsilon\,]^2 + 8\,[\,a - c + \varepsilon\,]^2} - (\,a - c + 7\,\varepsilon\,)\right]$$

$$\stackrel{s}{=}\; 8\,[\,a - c + \varepsilon\,] + 7\,[\,a - c + 7\,\varepsilon\,] - 7\sqrt{[\,a - c + 7\,\varepsilon\,]^2 + 8\,[\,a - c + \varepsilon\,]^2}$$

$$=\; 15\,[\,a - c\,] + 49\,\varepsilon - 7\sqrt{[\,a - c + 7\,\varepsilon\,]^2 + 8\,[\,a - c + \varepsilon\,]^2}\,. \tag{157}$$

(157) implies:

$$\left.\frac{\partial \pi_G(\cdot)}{\partial \varepsilon}\right|_{F = F^+} \;<\; 0 \quad\Leftrightarrow\quad 15\,[\,a - c\,] + 49\,\varepsilon \;<\; 7\sqrt{[\,a - c + 7\,\varepsilon\,]^2 + 8\,[\,a - c + \varepsilon\,]^2}$$

$$\Leftrightarrow\quad [\,15\,(\,a - c\,) + 49\,\varepsilon\,]^2 \;<\; 49\,[\,a - c + 7\,\varepsilon\,]^2 + 392\,[\,a - c + \varepsilon\,]^2$$

$$\Leftrightarrow\quad 225\,[\,a - c\,]^2 + 1,470\,[\,a - c\,]\,\varepsilon + 2,401\,\varepsilon^2$$

$$<\; 49\,[\,a - c\,]^2 + 686\,[\,a - c\,]\,\varepsilon + 2,401\,\varepsilon^2$$

$$+\; 392\,[\,a - c\,]^2 + 784\,[\,a - c\,]\,\varepsilon + 392\,\varepsilon^2$$

$$\Leftrightarrow\quad -\,392\,\varepsilon^2 - 216\,[\,a - c\,]^2 \;<\; 0\,. \tag{158}$$

(156) and (158) imply that to ensure $\pi_G(\cdot) \geq 0$ for all $\varepsilon \in [\,-\overline{\varepsilon}, \overline{\varepsilon}\,]$, it must be the case that $F \,\leq\, F_+\Big|_{\varepsilon = \overline{\varepsilon}}$. (153) implies:

$$F_+\Big|_{\varepsilon = \overline{\varepsilon}} \;=\; \frac{1}{2\,b}\left[\sqrt{(\,a - c + 7\,\overline{\varepsilon}\,)^2 + 8\,(\,a - c + \overline{\varepsilon}\,)^2} - (\,a - c + 7\,\overline{\varepsilon}\,)\right]$$

$$=\; \frac{1}{2\,b}\left[\sqrt{9\,(a - c)^2 + 30\,(a - c)\,\overline{\varepsilon} + 57\,\overline{\varepsilon}^2} - (\,a - c + 7\,\overline{\varepsilon}\,)\right]. \tag{159}$$

(159) implies that the largest total number of forward contracts that ensures each of two symmetric generators always secures nonnegative profit is:

$$2\,F_+\Big|_{\varepsilon = \overline{\varepsilon}} \;=\; \frac{1}{b}\left[\sqrt{9\,(a - c)^2 + 30\,(a - c)\,\overline{\varepsilon} + 57\,\overline{\varepsilon}^2} - (\,a - c + 7\,\overline{\varepsilon}\,)\right]. \quad ■$$

D. Extending the Analysis in Section 3.5 of the Paper.

Suppose expected welfare (W) in the dual monopoly setting is a weighted sum of the expected utility of the buyer ($E\{U^B\}$), the expected utility of the generator ($E\{U^G\}$), the expected utility of retail customers ($E\{U^R\}$), and the expected utility of industrial customers ($E\{U^I\}$). Formally, suppose:

$$W \;=\; \xi_R \, E\{U^R\} + \xi_I \, E\{U^I\} + \xi_B \, E\{U^B\} + \xi_G \, E\{U^G\} \, . \tag{160}$$

The foregoing analysis derives expressions for $E\{U^B\}$, $E\{U^G\}$, $\frac{\partial E\{U^B\}}{\partial F}$, and $\frac{\partial E\{U^G\}}{\partial F}$, where F denotes the level of forward contracting. To identify the value of F that maximizes W, it remains to derive expressions for $E\{U^R\}$, $E\{U^I\}$, $\frac{\partial E\{U^R\}}{\partial F}$, and $\frac{\partial E\{U^I\}}{\partial F}$. The ensuing analysis derives these expressions in the setting where ε has a uniform density on $[\,-\overline{\varepsilon}, \overline{\varepsilon}\,]$, so:

$$E\{\varepsilon\} \;=\; 0, \quad E\{\varepsilon^2\} \;=\; \frac{1}{3}\,(\,\overline{\varepsilon}\,)^2 , \quad E\{\varepsilon^3\} \;=\; 0, \text{ and } \; E\{\varepsilon^4\} \;=\; \frac{1}{5}\,(\,\overline{\varepsilon}\,)^4 . \tag{161}$$

To derive expressions for $E\{U^R\}$ and $\frac{\partial E\{U^R\}}{\partial F}$, suppose each retail customer derives value v from each unit of electricity. Then the payoff (welfare) of retail customers, given ε, is:

$$P^R \;=\; [\,v - r\,]\,[\,\overline{Q} + b^I \varepsilon\,] \quad \Rightarrow \quad E\{P^R\} \;=\; [\,v - r\,]\,\overline{Q} \, . \tag{162}$$

(162) implies:

$$P^R - E\{P^R\} \;=\; [\,v - r\,]\,b^I \varepsilon \quad \Rightarrow \quad \left[\,P^R - E\{P^R\}\,\right]^2 \;=\; \left(b^I\right)^2 [\,v - r\,]^2 \, \varepsilon^2 \, . \tag{163}$$

(163) implies that the variance of P^R is:

$$V_R \;=\; E\left\{\,\left[\,P^R - E\{P^R\}\,\right]^2\,\right\} \;=\; \left(b^I\right)^2 [\,v - r\,]^2 \; E\{\varepsilon^2\}$$

$$=\; \frac{1}{3}\,(\,\overline{\varepsilon}\,)^2 \left(b^I\right)^2 [\,v - r\,]^2 \, . \tag{164}$$

(162) and (164) imply that if retail consumers have mean-variance preferences, their expected utility can be written as:

$$E\{U^R\} \;=\; E\{P^R\} - A_R \, V_R \;=\; [\,v - r\,]\,\overline{Q} \;-\; \frac{1}{3}\,A_R\,(\,\overline{\varepsilon}\,)^2 \left(b^I\right)^2 [\,v - r\,]^2 \, . \tag{165}$$

(165) implies that if $r \;=\; \gamma\, r_0 + [\,1 - \gamma\,]\,E\{w\}$, then:

$$\frac{\partial E\{U^R\}}{\partial F} \;=\; \left[\,\frac{2}{3}\,A_R\,(\,\overline{\varepsilon}\,)^2 \left(b^I\right)^2 (\,v - r\,) - \;\overline{Q}\,\right] \frac{\partial r}{\partial F}$$

$$=\; [\,1 - \gamma\,]\,\frac{\partial E\{w\}}{\partial F} \left[\,\frac{2}{3}\,A_R\,(\,\overline{\varepsilon}\,)^2 \left(b^I\right)^2 (\,v - r\,) - \;\overline{Q}\,\right] .$$

To derive expressions for $E\{U^I\}$ and $\frac{\partial E\{U^I\}}{\partial F}$, recall that the demand curve of industrial customers is:

$$Q^I(w) \;=\; a^I - b^I\, w\,. \tag{166}$$

(166) implies that the payoff (surplus) of industrial customers given wholesale price w is:

$$P^I \;=\; \frac{1}{2}\, Q^I(w) \left[\frac{a^I}{b^I} - w \right] \;=\; \frac{1}{2} \left[\, a^I - b^I\, w \,\right] \left[\frac{a^I}{b^I} - w \right] \;=\; \frac{\left[\, a^I - b^I\, w \,\right]^2}{2\, b^I}$$

$$=\; \frac{1}{2\, b^I} \left[\left(a^I\right)^2 - 2\, a^I\, b^I\, w + \left(b^I\right)^2\, w^2 \right]. \tag{167}$$

(167) implies:

$$E\{P^I\} \;=\; \frac{1}{2\, b^I} \left[\left(a^I\right)^2 - 2\, a^I\, b^I\, E\{w) + \left(b^I\right)^2\, E\{w^2\} \right]. \tag{168}$$

(167) and (168) imply:

$$P^I - E\{P^I\} \;=\; \frac{1}{2\, b^I} \left\{ 2\, a^I\, b^I \left[\, E\{w\} - w \,\right] + \left(b^I\right)^2 \left[\, w^2 - E\{w^2\} \,\right] \right\}$$

$$=\; \frac{b^I}{2} \left[\, w^2 - E\{w^2\} \,\right] + a^I \left[\, E\{w\} - w \,\right]. \tag{169}$$

(169) implies:

$$\left[\, P^I - E\{P^I\} \,\right]^2 \;=\; \frac{\left(b^I\right)^2}{4} \left[\, w^2 - E\{w^2\} \,\right]^2 + a^I\, b^I \left[\, E\{w\} - w \,\right] \left[\, w^2 - E\{w^2\} \,\right]$$

$$+\; \left(a^I\right)^2 \left[\, E\{w\} - w \,\right]^2$$

$$=\; \frac{\left(b^I\right)^2}{4} \left[w^4 - 2\, w^2\, E\{w^2\} + \left(\, E\{w^2\} \,\right)^2 \right]$$

$$+\;\; a^I\, b^I \left[\, w^2\, E\{w\} - \; E\{w\}\, E\{w^2\} - w^3 + w\, E\{w^2\} \,\right]$$

$$+\; \left(a^I\right)^2 \left[\, (\, E\{w\} \,)^2 - 2\, w\, E\{w\} + w^2 \,\right]. \tag{170}$$

(170) implies that the variance of S^I is:

$$V_I \;=\; E \left\{ \left[\, P^I - E\{P^I\} \,\right]^2 \right\}$$

$$=\; \frac{\left(b^I\right)^2}{4} \left[E\{w^4\} - 2\, E\{w^2\}\, E\{w^2\} + \left(\, E\{w^2\} \,\right)^2 \right]$$

$$+\;\; a^I\, b^I \left[\, E\{w^2\}\, E\{w\} - \; E\{w\}\, E\{w^2\} - E\{w^3\} + E\{w\}\, E\{w^2\} \,\right]$$

$$+\; \left(a^I\right)^2 \left[\, (\, E\{w\} \,)^2 - 2\, E\{w\}\, E\{w\} + E\{w^2\} \,\right]$$

$$= \frac{(b^I)^2}{4}\left[E\{w^4\} - \left(E\{w^2\}\right)^2\right] + a^I\, b^I\left[E\{w\}\, E\{w^2\} - E\{w^3\}\right]$$

$$+ \left(a^I\right)^2\left[E\{w^2\} - (E\{w\})^2\right]. \tag{171}$$

(168) and (171) imply that if industrial consumers have mean-variance preferences, their expected utility can be written as:

$$\begin{aligned} E\{U^I\} &= E\{P^I\} - A_I\, V_I \\ &= \frac{1}{2\, b^I}\left[\left(a^I\right)^2 - 2\, a^I\, b^I\, E\{w\} + \left(b^I\right)^2 E\{w^2\}\right] \\ &\quad - \frac{A_I\,(b^I)^2}{4}\left[E\{w^4\} - \left(E\{w^2\}\right)^2\right] - A_I\, a^I\, b^I\left[E\{w\}\, E\{w^2\} - E\{w^3\}\right] \\ &\quad - A_I\left(a^I\right)^2\left[E\{w^2\} - (E\{w\})^2\right]. \end{aligned} \tag{172}$$

(172) implies:

$$\begin{aligned} \frac{\partial E\{U^I\}}{\partial F} &= -\, a^I\, \frac{\partial E\{w\}}{\partial F} + \frac{b^I}{2}\, \frac{\partial E\{w^2\}}{\partial F} \\ &\quad - \frac{A_I\,(b^I)^2}{4}\left[\frac{\partial E\{w^4\}}{\partial F} - 2\, E\{w^2\}\, \frac{\partial E\{w^2\}}{\partial F}\right] \\ &\quad - A_I\, a^I\, b^I\left[E\{w\}\, \frac{\partial E\{w^2\}}{\partial F} + E\{w^2\}\, \frac{\partial E\{w\}}{\partial F} - \frac{\partial E\{w^3\}}{\partial F}\right] \\ &\quad - A_I\left(a^I\right)^2\left[\frac{\partial E\{w^2\}}{\partial F} - 2\, E\{w\}\, \frac{\partial E\{w\}}{\partial F}\right]. \end{aligned} \tag{173}$$

It remains to derive expressions for the terms in (172) and (173). To do so, define $Y \equiv a + c - b\, F$. Then in the dual monopoly setting:

$$w = \frac{1}{2}\left[a + \varepsilon + c - b\, F\right] = \frac{1}{2}\left[Y + \varepsilon\right] \quad \Rightarrow \quad E\{w\} = \frac{1}{2}\, Y. \tag{174}$$

(161) and (174) imply:

$$w^2 = \frac{1}{4}\left[Y + \varepsilon\right]^2 = \frac{1}{4}\left[Y^2 + 2\,\varepsilon\, Y + \varepsilon^2\right]$$

$$\Rightarrow \quad E\{w^2\} = \frac{1}{4}\left[Y^2 + E\{\varepsilon^2\}\right] = \frac{1}{12}\left[3\, Y^2 + (\overline{\varepsilon})^2\right]. \tag{175}$$

(161), (174), and (175) imply:

$$\begin{aligned} w^3 &= \frac{1}{8}\left[Y^2 + 2\,\varepsilon\,Y + \varepsilon^2 \right] [\, Y + \varepsilon \,] \\ &= \frac{1}{8}\left[Y^3 + 2\,\varepsilon\,Y^2 + \varepsilon^2\,Y + \varepsilon\,Y^2 + 2\,\varepsilon^2\,Y + \varepsilon^3 \right] \\ &= \frac{1}{8}\left[Y^3 + 3\,\varepsilon\,Y^2 + 3\,\varepsilon^2\,Y + \varepsilon^3 \right] \\ \Rightarrow \quad E\{w^3\} &= \frac{1}{8}\left[Y^3 + 3\,Y\,E\{\varepsilon^2\} + E\{\varepsilon^3\} \right] \;=\; \frac{Y}{8}\left[Y^2 + (\overline{\varepsilon})^2 \right]. \end{aligned} \tag{176}$$

(161), (174), and (176) imply:

$$\begin{aligned} w^4 &= \frac{1}{16}\left[Y^3 + 3\,\varepsilon\,Y^2 + 3\,\varepsilon^2\,Y + \varepsilon^3 \right] [\, Y + \varepsilon \,] \\ &= \frac{1}{16}[\, Y^4 + 3\,\varepsilon\,Y^3 + 3\,\varepsilon^2\,Y^2 + \varepsilon^3\,Y \\ &\qquad\quad + \varepsilon\,Y^3 + 3\,\varepsilon^2\,Y^2 + 3\,\varepsilon^3\,Y + \varepsilon^4 \,] \\ &= \frac{1}{16}\left[Y^4 + 4\,\varepsilon\,Y^3 + 6\,\varepsilon^2\,Y^2 + 4\,\varepsilon^3\,Y + \varepsilon^4 \right] \\ \Rightarrow \quad E\{w^4\} &= \frac{1}{16}\left[Y^4 + 2\,Y^2\,(\overline{\varepsilon})^2 + \frac{1}{5}\,(\overline{\varepsilon})^4 \right]. \end{aligned} \tag{177}$$

(161) and (174) – (177) imply:

$$\begin{aligned} \frac{\partial E\{w\}}{\partial F} &= \frac{1}{2}\,\frac{\partial Y}{\partial F} \;=\; -\,\frac{b}{2}\,; \\ \frac{\partial E\{w^2\}}{\partial F} &= \frac{1}{2}\,Y\,\frac{\partial Y}{\partial F} \;=\; -\,\frac{b}{2}\,Y\,; \\ \frac{\partial E\{w^3\}}{\partial F} &= \frac{1}{8}\left[3\,Y^2 + (\overline{\varepsilon})^2 \right] \frac{\partial Y}{\partial F} \;=\; -\,\frac{b}{8}\left[3\,Y^2 + (\overline{\varepsilon})^2 \right]; \\ \frac{\partial E\{w^4\}}{\partial F} &= \frac{1}{16}\left[4\,Y^3 + 4\,Y\,(\overline{\varepsilon})^2 \right] \frac{\partial Y}{\partial F} \;=\; -\,\frac{b\,Y}{4}\left[Y^2 + (\overline{\varepsilon})^2 \right]. \end{aligned} \tag{178}$$

We now employ the foregoing analysis to calculate: (i) F_{GDM}, the level of F that maximizes $E\{U^G\}$; (ii) F_{BDM}, the level of F that maximizes $E\{U^B\}$; and (iii) F_{WDM}, the level of F that maximizes W, all in the dual monopoly setting. We first calculate these values for the baseline parameter values described in the text. These parameter values are reproduced in Table T1.

a^I	$\overline{Q}$	b^I	$\overline{\eta}$	c	v	r_0	A_R	A_{-R}	γ	ξ_i
5,326.52	3,348.48	5	837.12	25	10,000	1,000	0.0000005	0.00001	1	1

Table T1. Baseline Parameter Values

The corresponding outcomes in the dual monopoly setting reported in Table 2 in the text are reproduced in Table T2.

	F	$E\{w\}$	$E\{q\}$	$E\{U^G\}$	$E\{U^B\}$	$E\{U^R\}$	$E\{U^I\}$	W
$F = 0$	0	880	4,275	1,857	266	1,013	71	3,207
F_{GDM}	2,723	608	5,636	1,857	1,032	1,013	407	4,885
F_{BDM}	2,866	593	5,708	1,857	1,034	1,013	432	4,911
F_{WDM}	3,407	539	5,979	1,857	1,006	1,013	536	4,961

Table T2. Equilibrium Outcomes for the Baseline Parameters

Tables T3 – T13 record the corresponding equilibrium outcomes that arise in the dual monopoly setting when each of the first nine parameters in Table T1 is reduced by 50% below its baseline level, holding all other parameters at their baseline levels.[3]

	F	$E\{w\}$	$E\{q\}$	$E\{U^G\}$	$E\{U^B\}$	$E\{U^R\}$	$E\{U^I\}$	W
$F = 0$	0	880	4,275	2,812	266	20,676	71	23,826
F_{GDM}	1,619	718	5,085	2,974	888	20,676	237	24,775
F_{BDM}	2,866	593	5,708	2,878	1,034	20,676	432	25,020
F_{WDM}	2,965	583	5,758	2,862	1,033	20,676	450	25,022

Table T3. Equilibrium Outcomes when $A_G = 0.000005$

	F	$E\{w\}$	$E\{q\}$	$E\{U^G\}$	$E\{U^B\}$	$E\{U^R\}$	$E\{U^I\}$	W
$F = 0$	0	614	2,943	934	1,260	20,676	18	22,888
F_{GDM}	1,874	426	5,085	1,192	1,469	20,676	27	23,364
F_{BDM}	1,534	460	5,708	1,183	1,480	20,676	15	23,354
F_{WDM}	1,797	434	5,758	1,191	1,473	20,676	24	23,364

Table T4. Equilibrium Outcomes when $A_I = 0.000005$

[3] Corresponding variation in γ and ξ_i is considered in Part D of the Appendix in the paper. Each entry in Tables T3 – T13 is rounded to the nearest whole number.

	F	$E\{w\}$	$E\{q\}$	$E\{U^G\}$	$E\{U^B\}$	$E\{U^R\}$	$E\{U^I\}$	W
$F=0$	0	880	4,275	2,812	266	20,676	71	23,826
F_{GDM}	1,619	718	5,085	2,974	888	20,676	237	24,775
F_{BDM}	2,866	593	5,708	2,878	1,034	20,676	432	25,020
F_{WDM}	2,965	583	5,758	2,862	1,033	20,676	450	25,022

Table T5. Equilibrium Outcomes when $A_B = 0.000005$

	F	$E\{w\}$	$E\{q\}$	$E\{U^G\}$	$E\{U^B\}$	$E\{U^R\}$	$E\{U^I\}$	W
$F=0$	0	880	4,275	2,812	266	20,676	71	23,826
F_{GDM}	1,619	718	5,085	2,974	888	20,676	237	24,775
F_{BDM}	2,866	593	5,708	2,878	1,034	20,676	432	25,020
F_{WDM}	2,965	583	5,758	2,862	1,033	20,676	450	25,022

Table T6. Equilibrium Outcomes when $A_R = 0.00000025$

	F	$E\{w\}$	$E\{q\}$	$E\{U^G\}$	$E\{U^B\}$	$E\{U^R\}$	$E\{U^I\}$	W
$F=0$	0	712	3,438	1,270	420	5,608	244	7,542
F_{GDM}	2,189	494	4,533	1,622	93	5,608	632	7,954
F_{BDM}	296	683	3,586	1,359	428	5,608	286	7,680
F_{WDM}	1,733	539	4,304	1,606	235	5,608	536	7,985

Table T7. Equilibrium Outcomes when $\overline{Q} = 1,674.24$

	F	$E\{w\}$	$E\{q\}$	$E\{U^G\}$	$E\{U^B\}$	$E\{U^R\}$	$E\{U^I\}$	W
$F=0$	0	614	2,943	934	1,260	20,676	18	22,888
F_{GDM}	1,874	426	3.881	1,912	1,469	20,676	27	23,364
F_{BDM}	1,534	460	3,711	1,183	1,480	20,676	16	23,354
F_{WDM}	1,797	434	3,842	1,191	1,473	20,676	24	23,365

Table T8. Equilibrium Outcomes when $a^I = 2,663.26$

	F	$E\{w\}$	$E\{q\}$	$E\{U^G\}$	$E\{U^B\}$	$E\{U^R\}$	$E\{U^I\}$	W
$F=0$	0	880	4,275	3,231	369	27,771	82	31,453
F_{GDM}	894	791	4,722	3,276	694	27,771	179	31,920
F_{BDM}	8,242	56	8,396	261	1,955	27,771	2,400	32,388
F_{WDM}	5,140	366	6,845	2,269	1,731	27,771	1,153	32,924

Table T9. Equilibrium Outcomes when $\overline{\eta} = 418.56$

	F	$E\{w\}$	$E\{q\}$	$E\{U^G\}$	$E\{U^B\}$	$E\{U^R\}$	$E\{U^I\}$	W
$F=0$	0	1,748	4,306	5,690	−3,689	27,771	165	29,938
F_{GDM}	1,631	1,421	5,122	6,019	−1,765	27,771	558	32,583
F_{BDM}	7,127	322	7,870	2,293	1,057	27,771	3,613	34,734
F_{WDM}	5,513	645	7,063	4,160	814	27,771	2,439	35,184

Table T10. Equilibrium Outcomes when $b^I = 2.5$

	F	$E\{w\}$	$E\{q\}$	$E\{U^G\}$	$E\{U^B\}$	$E\{U^R\}$	$E\{U^I\}$	W
$F=0$	0	880	4,275	1,958	266	11,525	71	13,821
F_{GDM}	2,723	608	5,636	2,502	1,032	11,525	407	15,466
F_{BDM}	2,866	593	5,708	2,500	1,034	11,525	432	15,492
F_{WDM}	3,407	539	5,979	2,467	1,006	11,525	536	15,535

Table T11. Equilibrium Outcomes when $v = 5,000$

	F	$E\{w\}$	$E\{q\}$	$E\{U^G\}$	$E\{U^B\}$	$E\{U^R\}$	$E\{U^I\}$	W
$F=0$	0	880	4,275	1,958	−2,494	21,270	71	20,805
F_{GDM}	2,723	608	5,636	2,502	−456	21,270	407	23,722
F_{BDM}	5,366	343	6,958	1,989	197	21,270	1,004	24,460
F_{WDM}	4,989	381	6,770	2,125	183	21,270	903	24,481

Table T12. Equilibrium Outcomes when $r_0 = 500$

	F	$E\{w\}$	$E\{q\}$	$E\{U^G\}$	$E\{U^B\}$	$E\{U^R\}$	$E\{U^I\}$	W
$F=0$	0	874	4,306	1,987	293	20,676	76	23,032
F_{GDM}	2,742	600	5,677	2,538	1,043	20,676	421	24,679
F_{BDM}	2,835	590	5,724	2,538	1,044	20,676	438	24,696
F_{WDM}	3,405	533	6,009	2,506	1,014	20,676	548	24,744

Table T13. Equilibrium Outcomes when $c = 12.5$

Finally, Table T14 identifies settings where the levels of forward contracting that arise in the dual monopoly setting (F_{GDM} and F_{BDM}) differ from the levels that arise when: (i) one party makes a one-time offer of a level of contracting at unit price $p^f = E\{w\}$; and (ii) the other party either accepts or rejects the offer. If the offer is rejected, no forward contracting is implemented. $\widehat{F}_{GDM}$ ($\widehat{F}_{BDM}$) will denote this "bargaining" level of forward contracting when G (B) makes the offer. The first column in Table T14 identifies the parameter value for which $F_{GDM} \neq \widehat{F}_{GDM}$ or $F_{BDM} \neq \widehat{F}_{BDM}$ when all other parameter values are as specified in Table T1.[4,5]

	F_{GDM}	$\widehat{F}_{GDM}$	F_{BDM}	$\widehat{F}_{BDM}$
$\overline{Q} = 1,674.24$	2,189	591	296	296
$b^I = 7.5$	3,487	0	0	0
$b^I = 2.5$	1,631	1,631	7,127	3,261
$\overline{\eta} = 1,255.68$	4,382	3,741	1,871	1,871
$\overline{\eta} = 418.56$	894	894	8,242	1,788
$r_0 = 1,500$	2,723	732	366	366

Table T14. Preferred and Bargained Levels of Forward Contracting

Table 14 indicates that when preferred levels of forward contracting differ sufficiently, the party that proposes the level of contracting may have to propose a level other than its most preferred level to ensure the other party does not reject the proposal.

[4] The entries in the last four columns of Table T14 are rounded to the nearest whole number.

[5] $F_{GDM} = \widehat{F}_{GDM}$ and $F_{BDM} = \widehat{F}_{BDM}$ for all parameter values that are: (i) identified either in one of Tables D1 – D14 in the text or in one of Tables T3 – T13 above; but (ii) not listed in the first column of Table T14.