Optimal Revenue Adjustment in the Presence of Exogenous Demand Variation

by

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Abstract

We consider optimal adjustments to a regulated firm's revenue when realized revenue diverges from expected revenue. The adjustments we consider include those that arise under two popular forms of incentive regulation – price cap regulation (PCR) and revenue cap regulation (RCR). We show that the optimal revenue adjustment reflects the firm's Lerner Index. The optimal policy differs from both PCR and RCR, but more closely resembles PCR (RCR) when the prevailing fixed charge for the firm's service is large (small).

Keywords: regulatory policy; *ex post* revenue adjustment; demand uncertainty.

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1 Introduction.

Considerable uncertainty prevails about the demand for energy services in the coming years. Governments around the world are encouraging expanded consumption of electricity generated by renewable resources while discouraging the use of fossil fuels. The nature, scope, and impact of the associated government policies are difficult to predict. Consequently, it is presently challenging to predict future demand for electricity and natural gas in many jurisdictions.[1]

The purpose of this article is to examine the optimal design of regulatory policy in settings where the regulated firm faces potentially large variation in the demand for its service (e.g., electricity supply). We consider a class of regulatory policies that includes two popular forms of incentive regulation – price cap regulation (PCR) and revenue cap regulation (RCR).[2] A key difference between PCR and RCR is the extent to which the firm's revenue is adjusted when demand, and thus revenue, diverge from their expected levels. No revenue adjustment is implemented under PCR. In contrast, RCR entails full revenue adjustment in the following sense. If the firm's realized revenue falls below expected revenue in one year, a surcharge that fully offsets the firm's revenue "shortfall" is imposed on consumers in the following year. Alternatively, if realized revenue exceeds expected revenue in some year, the regulated firm is effectively required to rebate the entire "excess" revenue it has collected to its customers in the following year.

In our model, the regulator chooses the fraction of any realized revenue "shortfall" that is awarded to the firm and the corresponding fraction of any "excess" revenue that is returned to consumers. The regulator acts to maximize expected consumer welfare while ensuring the operation of a risk averse firm that faces potentially substantial exogenous demand variation.[3]

[1] See Bovera et al. (2021), Cave (2024), Duma et al. (2024), and Joskow (2024), for example. The New Zealand Commerce Commission (2019, §4.1) observes that "substantial changes are occurring in the electricity sector, driven by an increasing focus on decarbonisation as well as increasing affordability of certain technologies that provide new opportunities to distributors and consumers. However, there is uncertainty as to the extent, timing, and impact of these changes."

[2] In reviewing the Council of European Energy Regulators (2022) report on regulation in 36 European energy networks, London Economics International (2023, p. 25) observes that some form of PCR was employed in nearly 20% of these networks in 2021 and some form of RCR was employed in more than 45% of these networks. London Economics International (2023, p. 26) also documents the use of PCR and RCR in other electricity networks around the world, including those in Australia, Canada, New Zealand, and the United States.

[3] The New Zealand Commerce Commission (2019, p. 3) observes that it implemented "a revenue cap (as opposed to the previous price cap) [in part to] give distributors greater certainty about revenue recovery." Such increased certainty can benefit consumers by reducing the regulated firm's cost of capital. Under the *Hope Standard*, a regulated firm in the U.S. is entitled to a "return ... sufficient to assure confidence in the financial integrity of the enterprise ... and to compensate its investors for the risks assumed." See *Fed. Power Comm'n v. Hope Nat. Gas Co.*, 320 U.S. 591, 603, 605 (1944).

We find that the optimal policy in this setting is neither PCR nor RCR. The optimal policy implements a partial revenue adjustment, where the optimal adjustment reflects the firm's Lerner Index.[4] The optimal adjustment eliminates variation in the firm's profit, which both obviates the need for any risk premium to ensure the firm's operation and reduces the firm's cost of capital. The optimal policy more closely resembles PCR when the fixed charge for the firm's service is large and the firm's per-unit price is close to its marginal cost. The optimal policy more closely resembles RCR when fixed charges are small and the firm's per-unit price substantially exceeds its marginal cost.

One might expect RCR to systematically manage the risk induced by demand variation better than PCR because RCR eliminates variation in the firm's revenue. However, revenue variation is only one component of profit variation, and fully correcting for revenue variation can increase profit variation if the pricing structure itself sufficiently limits the impact of demand variation on profit. Specifically, when the firm's unit price is close to its marginal cost, demand variation induces little profit variation under PCR.

Many studies document drawbacks to RCR. For example, Crew and Kleindorfer (1996) identify conditions under which RCR can induce a regulated enterprise to set a price for its service that exceeds the price an unregulated monopolist would set. Other studies, including Armstrong et al. (1994), Comnes et al. (1995), De Villemeur et al. (2003), Raineri and Giaconi (2005), Decker (2009), and Campbell (2018), demonstrate that RCR can induce a multiproduct firm to set prices that diverge considerably from Ramsey prices,[5] and so can reduce consumer welfare substantially below the level that would arise under PCR.[6] Our analysis differs from these studies in at least two respects. First, rather than examine the performance of RCR or compare the performance of PCR and RCR, we characterize the optimal regulatory policy in a class of policies that includes PCR and RCR. Second, the studies identified above abstract from risk aversion, which plays a central role in our analysis.

Some studies identify potential benefits of RCR. For example, Brennan (2010) demonstrates that policies like RCR can encourage energy suppliers to promote energy conservation. We document a distinct potential benefit of RCR, namely, its ability to reduce variation in the regulated firm's profit under certain conditions. However, we also show that RCR is not the best policy to reduce profit variation.[7] As we explain in Section 3, policies akin to net

[4] See Lerner (1934). Formally, the optimal adjustment is proportional to $\frac{p-c}{p}$, where p denotes the unit price of the firm's service and c denotes the firm's marginal cost.

[5] See Ramsey (1927) and Baumol and Bradford (1970).

[6] Weisman (2023) demonstrates that relative to PCR, RCR often leads to higher prices, lower service quality, and less cost-reducing innovation.

[7] Brennan and Crew (2015) explain how to adjust the parameters of a PCR plan when the demand for a

lost revenue adjustment mechanisms (e.g., Baxter, 1995) can better serve this function.

Our analysis of these issues proceeds as follows. Section 2 describes our basic model, which takes the firm's cost structure to be exogenous and assumes that the regulator and the firm share the same imperfect information about the demand for the firm's service. Section 3 presents our primary findings in this setting. Section 4 extends the analysis to consider endogenous cost structures and asymmetric knowledge of industry demand. Section 5 summarizes our key findings and suggests directions for future research. The Appendix provides the proofs of all formal conclusions.

2 The Model

We consider a setting in which a regulator oversees the operations of a monopoly supplier of a single service (e.g., electricity supply). The regulated firm requires $K > 0$ units of capital to serve its customers. The total cost of this capital is $k\,K$, where $k > 0$ is the firm's unit cost of capital. The firm also incurs a unit production cost, $c > 0$. Consequently, when it supplies $Q > 0$ units of output, the firm incurs total cost $c\,Q + k\,K$.

$p > 0$ is the unit price that the regulated firm charges for its service. Customers also pay a fixed charge, T, for the right to purchase the firm's service at unit price p.[8] $\overline{T} > 0$ is the maximum feasible fixed charge. The upper bound on the fixed charge reflects standard concerns with potentially regressive rate structures: a large fixed charge imposed on all households could compel low-income households to spend an unduly large fraction of their limited income on an essential regulated service.[9]

Consumer demand for the regulated service is stochastic. $\varepsilon \in \{\varepsilon_1, ..., \varepsilon_n\}$ is the realization of an exogenous demand parameter, where $\varepsilon_1 < ... < \varepsilon_n$. Realized demand, $Q(p, \varepsilon)$, increases as ε increases or as p declines.[10] $\phi_i \in (0, 1)$ is the probability that $\varepsilon = \varepsilon_i$ for $i = 1, ..., n$. Therefore, expected demand when the regulated unit price is p is $Q^e(p) = \sum_{i=1}^{n} \phi_i\, Q(p, \varepsilon_i)$.

We consider a class of regulatory policies in which a "revenue adjustment" is imple-

regulated service (e.g., postal delivery) is declining in a known, deterministic manner. (Also see Decker (2016).) In contrast, uncertainty about future demand plays a central role in our analysis.

[8] For simplicity, the number of consumers is normalized to 1.

[9] Low-income households typically consume relatively small amounts of essential regulated services like electricity. Consequently, per-unit charges (as opposed to a common fixed charge for all households) can help to reduce the total bill incurred by low-income households. In principle, targeted subsidies might be employed to help ensure that low-income customers can afford essential services. However, in practice, residential electricity customers in the U.S. generally face per-unit charges that exceed the marginal cost of supplying electricity. See Borenstein and Bushnell (2022) and Borenstein et al. (2022), for example.

[10] Formally, for any $p > 0$, $Q(p, \varepsilon_i) \lesseqgtr Q(p, \varepsilon_h) \Leftrightarrow \varepsilon_i \lesseqgtr \varepsilon_h$ for $i, h \in \{1, ..., n\}$ $(h \neq i)$. Furthermore, for each $\varepsilon \in \{\varepsilon_1, ..., \varepsilon_n\}$, $Q_p(p, \varepsilon) < 0$, where the subscript p denotes the partial derivative with respect to p.

mented if the firm's realized (variable) revenue ($p\,Q(p,\varepsilon_i)$) differs from its expected revenue ($p\,Q^e(p)$). The revenue adjustment is the product of a parameter, $\alpha \geq 0$, and the difference between expected revenue and realized revenue. Formally, when demand parameter ε_i is realized, revenue adjustment $R_i = \alpha\, p\,[\,Q^e(p) - Q(p,\varepsilon_i)\,]$ is collected from consumers and delivered to the firm.[11] When $\alpha = 0$, no revenue adjustment is implemented, as is the case under PCR. When $\alpha = 1$, full revenue adjustment is implemented in the sense that the regulated firm's realized revenue coincides with its expected revenue because: (i) consumers "reimburse" the firm for any shortfall of realized revenue ($p\,Q(p,\varepsilon_i)$) below expected revenue ($p\,Q^e(p)$); and (ii) the firm "reimburses" consumers for any excess of realized revenue above expected revenue. The regulatory policy with full revenue adjustment can be viewed as RCR.[12] For analytic ease, we assume that consumer demand does not vary with T or R_i.[13]

Including any revenue adjustment that is implemented, the firm's profit when the realized demand parameter is ε_i is:

$$\pi_i \;=\; [\,p - c\,]\,Q(p,\varepsilon_i) - k\,K + T + \alpha\, p\,[\,Q^e(p) - Q(p,\varepsilon_i)\,]\,. \tag{1}$$

$U(\pi_i)$ is the utility the firm derives from profit π_i. The firm values profit and is risk averse, so $U(\cdot)$ is a strictly increasing, strictly concave function of π_i (i.e., $U'(\cdot) > 0$ and $U''(\cdot) < 0$). $\overline{U}$ is the firm's reservation level of expected utility, i.e., the minimum expected utility that will induce the firm to operate in the regulated industry.

The timing in the model is the following. The regulator first specifies p, T, and α. Then the demand parameter ε_i is realized. Next, the firm serves all realized demand. Finally, the revenue adjustment is implemented.

The regulator chooses p, T, and α to maximize expected consumer surplus while ensuring that the firm's expected utility is at least $\overline{U}$. $S(p,\varepsilon_i)$ denotes the surplus that consumers secure given p and ε_i, not counting the fixed charge T or the revenue adjustment R_i. The regulator's problem, [RP], is:

$$\underset{p,\,T,\,\alpha}{\text{Maximize}} \quad \sum_{i=1}^{n} \phi_i \;\{\, S(p,\varepsilon_i) - \alpha\, p\,[\,Q^e(p) - Q(p,\varepsilon_i)\,]\,\} - T$$

[11] If $R_i < 0$, the firm delivers a payment to its customers to offset the fraction α of the difference between the firm's realized revenue and its expected revenue.

[12] In practice, RCR adjusts revenue over time, raising (reducing) p in one year to offset the extent to which revenue fell below (exceeded) expected revenue in the preceeding year. Our static model captures this intertemporal adjustment process as a revenue adjustment that occurs after consumers have made their one-time consumption decisions.

[13] Eto et al. (1994) and NARUC (2007) report that the revenue adjustments that arise in practice under policies of the type analyzed here generally constitute a very small fraction of the typical consumer's monthly utility bill.

$$\text{subject to:} \quad \sum_{i=1}^{n} \phi_i \, U(\pi_i) \;\geq\; \overline{U} \quad \text{and} \quad T \;\leq\; \overline{T}\,. \tag{2}$$

3 Primary Findings

To derive the solution to [RP], it is helpful to consider how α affects the manner in which the firm's profit varies with realized demand. Lemma 1 refers to $Q_i \equiv Q(p, \varepsilon_i)$ for $i \in \{1, ..., n\}$.

Lemma 1. *For all $h > i$ $(i, h \in \{1, ..., n\})$: (i) $\pi_h \lesseqgtr \pi_i \;\Leftrightarrow\; \alpha \gtreqless \frac{p-c}{p}$; and (ii) $\frac{\pi_i - \pi_h}{p\,[\,Q_h - Q_i\,]} = \alpha - \frac{p-c}{p}$.*

Conclusion (i) in Lemma 1 implies that the firm's profit does not vary with the realization of the demand parameter (ε_i) if $\alpha = \frac{p-c}{p}$. In this case, the fraction of any revenue "shortfall" $(\,p\,[\,Q^e(p) - Q(p, \varepsilon_i)\,]\,)$ that is awarded to the firm is the firm's proportionate price-cost margin (i.e., the firm's Lerner Index). Conclusion (i) in Lemma 1 also reports that the firm's profit increases as ε_i increases if $\alpha < \frac{p-c}{p}$, as is the case under PCR, for example, where $\alpha = 0$. Furthermore, the firm's profit declines as ε_i increases if $\alpha > \frac{p-c}{p}$, as is the case under RCR, for example, where $\alpha = 1$.

To explain these observations, assume that the firm's price-cost margin is positive (so $p > c$), as is typically the case in practice. First consider the case where α is relatively small (i.e., $\alpha < \frac{p-c}{p}$). Any revenue adjustment that is implemented in this case is relatively small. Consequently, the firm's positive price-cost margin ensures that the firm's profit increases as demand increases. Now consider the case where α is relatively large (i.e., $\alpha > \frac{p-c}{p}$). In this case, the relatively pronounced revenue adjustment ensures that the firm's post-adjustment revenue is relatively close to expected revenue. Consequently, the primary effect of increased demand is to increase the firm's costs, which causes the firm's profit to decline.

Conclusion (ii) in Lemma 1 considers the sensitivity of the firm's profit to demand variation, $V_{ih} \equiv \left| \frac{\pi_i - \pi_h}{Q_h - Q_i} \right|$. As noted above, the firm's profit does not vary with realized demand when $\alpha = \frac{p-c}{p}$, so $V_{ih} = 0$ in this case. Further recall that: (i) the firm's profit increases as demand increases (so $\pi_h > \pi_i$ when $Q_h > Q_i$) if $\alpha < \frac{p-c}{p}$; and (ii) the firm's profit declines as demand increases (so $\pi_h < \pi_i$ when $Q_h > Q_i$) if $\alpha > \frac{p-c}{p}$. Consequently, conclusion (ii) in Lemma 1 implies that V_{ih} is 0 when $\alpha = \frac{p-c}{p}$, V_{ih} increases as α increases when $\alpha > \frac{p-c}{p}$, and V_{ih} declines as α increases when $\alpha < \frac{p-c}{p}$.[14] Therefore, V_{ih} is a convex function of α with a unique minimum at $\alpha = \frac{p-c}{p}$.

[14] Conclusion (ii) in Lemma 1 implies that $V_{ih} = \frac{\pi_h - \pi_i}{Q_h - Q_i} \overset{s}{=} \frac{p-c}{p} - \alpha \;\Rightarrow\; \frac{\partial V_{ih}}{\partial \alpha} = -1$ when $\alpha < \frac{p-c}{p}$. This conclusion also implies that $V_{ih} = \frac{\pi_i - \pi_h}{Q_h - Q_i} \overset{s}{=} \alpha - \frac{p-c}{p} \;\Rightarrow\; \frac{\partial V_{ih}}{\partial \alpha} = 1$ when $\alpha > \frac{p-c}{p}$.

In practice, a firm's cost of capital often increases as the variation in its profit increases, *ceteris paribus*.[15] Consequently, we introduce Assumption 1, which implicitly presumes that the firm's unit cost of capital, k, reflects the sensitivity of the firm's profit to the demand variation it faces.

Assumption 1. k is a convex function of α with a unique minimum at $\alpha = \frac{p-c}{p}$.

We also introduce Assumption 2 to reflect common settings in which $\overline{T}$ is sufficiently small relative to $k\,K$ that the firm's participation constraint (the first constraint in (2)) can only be met if the firm secures some variable profit from sales (i.e., if $p > c$).

Assumption 2. $U(\overline{T} - \underline{k}\,K) < \overline{U}$, where $\underline{k} \equiv \min\limits_{\alpha} k(\alpha)$.[16]

Proposition 1 now identifies the revenue adjustment parameter, α, that the regulator implements at the solution to [RP].

Proposition 1. *Suppose Assumptions 1 and 2 hold. Then* $\alpha = \frac{p-c}{p} \in (0,1)$, $\pi_1 = ... = \pi_n \equiv \overline{\pi}$, *and* $U(\overline{\pi}) = \overline{U}$ *at the solution to [RP].*

Proposition 1 implies that the optimal regulatory policy in the present setting effectively reimburses the regulated firm for the fraction $\frac{p-c}{p} \in (0,1)$ of any revenue "shortfall" it experiences.[17] The policy also effectively reimburses consumers for the fraction $\frac{p-c}{p}$ of any "excess" revenue payment (i.e., revenue payment in excess of expected revenue payment) they make. This "partial" adjustment for revenue variation serves to eliminate the variation in the firm's profit caused by exogenous variation in demand. In other words, after the optimal revenue adjustment is implemented, the firm's realized profit is the same for all demand realizations. This is the case because (1) implies that when $\alpha = \frac{p-c}{p}$:

$$\begin{aligned} \pi_i &= [\,p-c\,]\,Q(p,\varepsilon_i) + T - k\,K + \left[\frac{p\,-\,c}{p}\right] p\,[\,Q^e(p) - Q(p,\varepsilon_i)\,] \\ &= [\,p-c\,]\,Q^e(p) + T - k\,K \;\equiv\; \overline{\pi}\,. \end{aligned} \tag{3}$$

(3) implies that $\pi_1 = ... = \pi_n = \overline{\pi}$ when $\alpha = \frac{p-c}{p}$.

[15] See Pedell (2006) and Biggar (2023), for example.

[16] If the firm were risk neutral (so $U(\overline{T} - \underline{k}\,K) = \overline{T} - \underline{k}\,K$) and if the firm's reservation profit level ($\overline{U}$) were normalized to zero, then Assumption 2 would state that the maximum fixed charge ($\overline{T}$) is always less than the firm's fixed cost. (Recall that the number of customers is normalized to one.)

[17] A revenue "shortfall" is the difference between expected revenue ($p\,Q^e(p)$) and realized revenue ($p\,Q(p,\varepsilon_i)$).

A complete adjustment for realized revenue variation (i.e., $\alpha = 1$, as under RCR) would cause the firm's profit to decline as demand increases (so $\pi_1 > ... > \pi_n$). As noted above, this is the case because the firm's production costs increase as demand increases. Therefore, the firm's profit would decline as demand increases if the firm's revenue did not vary with demand. In contrast, no adjustment for unanticipated revenue variation (i.e., $\alpha = 0$, as under PCR) would cause the firm's profit to increase as demand increases (so $\pi_1 < ... < \pi_n$). The profit increase reflects the firm's positive price-cost margin ($p - c > 0$).[18]

These observations imply that the optimal revenue adjustment policy (i.e., the solution to [RP]) is neither PCR nor RCR. Instead, the optimal policy can be viewed as a type of "net lost revenue adjustment mechanism" that eliminates all variation in the firm's profit.[19] The absence of profit variation minimizes the firm's unit cost of capital and thereby benefits consumers by reducing the smallest value of p and/or T that will ensure the firm's operation.[20]

Proposition 1 implies that the optimal revenue adjustment policy varies with the prevailing rate structure. The optimal policy entails less revenue adjustment (i.e., α declines toward 0), and so more closely resembles PCR, when a high $\overline{T}$ permits a unit price (p) close to marginal cost (c).[21] In contrast, the optimal policy entails more revenue adjustment (i.e., α increases toward 1), and so more closely resembles RCR, when a low $\overline{T}$ requires p to substantially exceed c. Proposition 1 thereby implies that if the regulator were restricted to implementing either RCR or PCR: (i) RCR would better limit the variation in the firm's profit if fixed charges were relatively small, so the firm's price-cost margin was relatively large; whereas (ii) PCR would better limit the variation in the firm's profit if fixed charges

[18] If Assumption 2 did not hold and the regulator set $p = c$, then the optimal revenue adjustment policy would be PCR (because $\alpha = 0$). When $p = c$, the firm's profit does not change as demand changes, so PCR eliminates all variation in profit due to exogenous variation in demand. Of course, a regulator who seeks to promote energy conservation might prefer to set p above c in order to discourage consumption.

[19] Technically, a net lost revenue adjustment mechanism adjusts the firm's profit to eliminate profit variation induced by the firm's demand-side management activities (Hirst and Blank, 1994; Hirst et al., 1994; Baxter, 1995). Here, the revenue adjustment policy with $\alpha = \frac{p-c}{p}$ eliminates profit variation that arises from all sources of (exogenous) demand variation.

[20] The absence of profit variation can eliminate the need for earnings sharing or financial re-openers to avoid particularly high or low earnings due to exogenous demand variation. (A financial re-opener is a re-examination of key elements of a regulatory plan designed to identify the likely causes of particularly high or low earnings. See Alberta Utilities Commission (2012, ¶819) and Ontario Energy Board (2008, §2.7), for example.) A regulator might continue to include earnings sharing or financial re-openers in a regulatory plan if high earnings – even high earnings realized *on the merits* – are politically problematic. However, because the use of these instruments can diminish incentives for cost-reducing innovation, such use may not serve the long-term interests of consumers.

[21] (3) implies that $[p - c] Q^e(p) = \overline{\pi} + k K - T$. Therefore, as long as the firm's expected variable profit ($[p - c] Q^e(p)$) increases as p increases, the price that ensures profit $\overline{\pi}$ for the firm declines as T increases.

were relatively high, so the firm's price-cost margin was relatively small.

Proposition 1 characterizes the optimal regulatory policy when the regulator can choose the rate structure (i.e., p and T) and the revenue adjustment policy (i.e., α) simultaneously. As is apparent from the proof of Proposition 1, the optimal revenue adjustment policy continues to be $\alpha = \frac{p-c}{p}$ if the regulator considers p and T to be exogenous parameters when she determines α. Consequently, in settings where a regulator designs a revenue adjustment policy after the firm's rate structure has been determined, the established rate structure will affect the optimal revenue adjustment policy. To illustrate, suppose a small fixed charge (F) and a relatively high per-unit charge ($p > c$) have been implemented, perhaps in an attempt to achieve affordability objectives. The resulting relatively high value of $\alpha = \frac{p-c}{p}$ implies that the optimal policy will entail a relatively pronounced adjustment for differences between realized and expected revenue, as under RCR, for example.[22]

4 Model Extensions

We now consider two extensions of the streamlined setting considered above.

The Setting with Endogenous Costs

The foregoing analysis treated the firm's marginal cost (c) and capital requirement (K) as exogenous parameters. In practice, a regulated firm may be able to undertake effort designed to reduce its marginal cost and/or reduce the amount of capital required to serve its customers. Indeed, incentive regulation plans like PCR and RCR often are advocated on the grounds that they can motivate the firm to exert non-contractible effort to reduce its costs.

Recall that under the solution to [RP], the partial adjustment for revenue variation ($\alpha = \frac{p-c}{p}$) renders the firm's measured profit invariant to realized demand. One might suspect that this profit invariance would limit the firm's incentive to deliver non-contractible effort that serves to reduce c and/or K. However, this is not the case. Even though the firm's profit does not vary with demand when $\alpha = \frac{p-c}{p}$, the constant level of profit the firm secures increases as its costs decline. Consequently, the firm may find it profitable to deliver cost-reducing effort even when the effort is non-contractible, so the regulator cannot directly compensate the firm for any effort it supplies.

To analyze the firm's incentive to deliver non-contractible cost reducing effort ($r \geq 0$) under the policy that constitutes the solution to [RP], let $D(r)$ denote the personal (and non-contractible) cost the firm incurs when it delivers cost-reducing effort r. This personal

[22] The optimal revenue adjustment policy will not entail the full revenue adjustment that is implemented under RCR because $\frac{p-c}{p} < 1$ whenever $c > 0$.

cost increases with r at an increasing rate, so $D'(r) > 0$ and $D''(r) > 0$ for all $r \geq 0$. The firm's cost-reducing effort serves to reduce the firm's marginal cost and/or reduce the amount of capital required to serve consumers. Formally, let $c(r)$ and $K(r)$, respectively, denote the firm's marginal cost and the firm's capital requirement when it delivers r units of cost-reducing effort. We assume $c'(r) < 0$ and $K'(r) < 0$ for all $r \geq 0$.

(3) implies that when $\alpha = \frac{p-c}{p}$, the firm's profit for each demand realization when it supplies cost-reducing effort r is:

$$\overline{\pi}(r) \;=\; [\, p - c(r)\,]\, Q^e(p) + T - k\, K(r) - D(r)\,. \tag{4}$$

(4) implies that the level of r that maximizes the firm's expected utility when $\alpha = \frac{p-c}{p}$ is:

$$r^* \;=\; \arg\min \{\, c(r)\, Q^e(p) + k\, K(r) + D(r)\,\}. \tag{5}$$

(5) implies that when the firm anticipates that the regulator will implement the solution to [RP], the firm will supply the efficient level of cost-reducing effort because the firm's measured profit increases with r at the same rate that the firm's measured cost declines.[23]

The Setting with Asymmetric Knowledge of Demand

The analysis in Sections 2 and 3 implicitly assumes that the regulator and firm share the same imperfect knowledge of the demand for the firm's service. In practice, a regulated firm often has better information about consumer demand than does the regulator.[24] To determine the optimal regulatory policy under such circumstances, consider the following setting with asymmetric knowledge of demand.

In this setting, prior to the start of its interaction with the regulator, the regulated firm (privately) observes an informative signal, $s \in \{s_1, ..., s_N\}$, about the level of demand that will ultimately arise. The larger is the realization of s, the greater is expected demand. To state this normalization formally, let $\phi_{ij} \in (0,1)$ denote the conditional probability that $\varepsilon = \varepsilon_i$ when $s = s_j$, for $i \in \{1, ..., n\}$ and $j \in \{1, ..., N\}$. Then $Q^e(p, s_j) = \sum_{i=1}^{n} \phi_{ij}\, Q(p, \varepsilon_i)$ is the expected demand for the regulated firm's service when the unit price is p and the signal is s_j. Consequently, the maintained normalization is that $Q^e(p, s_j) > Q^e(p, s_l)$ for all $p > 0$, for any $s_j > s_l$. All elements of the setting with asymmetric knowledge of demand other than the firm's initial private observation of the signal $s \in \{s_1, ..., s_N\}$ are as specified in Section 2.

[23] The efficient level of r is the level that minimizes total expected cost, which is the sum of capital cost, expected production cost, and effort cost. (5) implies that when r^* is strictly positive and finite, it is determined by $D'(r^*) = -\,[\, c'(r^*)\, Q^e(p) + k\, K'(r^*)\,]$. (4) implies that the level of r that maximizes the firm's profit is determined by $D'(r) = -\,[\, c'(r)\, Q^e(p) + k\, K'(r)\,]$.

[24] See Lewis and Sappington (1988), for example.

Because the firm observes the signal before the start of its interaction with the regulator, the regulator can tailor the policy that she implements to the firm's report of the signal it has observed. α_j denotes the fraction of any realized revenue shortfall that is awarded to the firm when it reports that it observed signal s_j.[25] p_j and T_j, respectively, denote the unit price and the fixed charge that the regulator implements when the firm claims to have observed signal s_j. The firm's profit when ε_i is realized after the firm reports that it observed signal s_j is:

$$\pi_{ij} \;=\; [\, p_j - c \,]\, Q(p_j, \varepsilon_i) + T_j - k\, K + \alpha_j\, p_j \, [\, Q^e(p_j, s_j) - Q(p_j, \varepsilon_i) \,] \,. \tag{6}$$

Let $\eta_j \in (0, 1)$ denote the probability that $s = s_j$, for $j \in \{1, ..., N\}$. The regulator's problem in this setting with asymmetric knowledge of demand, [RP-A], is:

$$\underset{\alpha_j,\, p_j,\, T_j}{\text{Maximize}} \quad \sum_{j=1}^{N} \eta_j \left[\sum_{i=1}^{n} \phi_{ij} \; \{ \, S(p_j, \varepsilon_i) - \alpha_j\, p_j \, [\, Q^e(p_j, s_j) - Q(p_j, \varepsilon_i) \,] \, \} - T_j \right] \tag{7}$$

subject to, for $j, l \in \{1, ..., N\}$:

$$\sum_{i=1}^{n} \phi_{ij}\, U(\pi_{ij}) \;\geq\; \overline{U}\,; \tag{8}$$

$$\sum_{i=1}^{n} \phi_{ij}\, U(\pi_{ij}) \;\geq\; \sum_{i=1}^{n} \phi_{ij}\, U(\pi_{il}) \;\text{ for } \; l \neq j\,; \; \text{ and} \tag{9}$$

$$T_j \;\leq\; \overline{T}\,. \tag{10}$$

(7) captures expected consumer welfare when the firm truthfully reports the signal it observes.[26] (8) ensures that the firm's expected utility is at least $\overline{U}$ when it truthfully reports the signal it has observed. (9) ensures that the firm (weakly) prefers to truthfully report the signal it has observed than to misrepresent this signal. (10) reflects the upper bounds on feasible fixed charges.[27]

Proposition 2 characterizes the solution to [RP-A].

Proposition 2. *Suppose Assumptions 1 and 2 hold. Then at the solution to [RP-A], for* $j = \{1, ..., N\}$*: (i)* $\alpha_j \,=\, \frac{p_j - c}{p_j} > 0$*; (ii)* $\pi_{1j} = ... = \pi_{nj} \equiv \overline{\pi}_j$*; and (iii)* $U(\overline{\pi}_j) \,=\, \overline{U}$ *.*

[25] When the firm claims to have observed signal s_j, the unit price is p_j, and the realized demand parameter is ε_i, the revenue shortfall is $Q^e(p_j, s_j) - Q(p_j, \varepsilon_i)$.

[26] Expected consumer welfare here reflects the expectation of the regulator, who does not know *ex ante* which signal the firm has observed.

[27] The Revelation Principle (e.g., Myerson, 1979) ensures that this formulation of the regulator's problem is without loss of generality.

Proposition 2 implies that the key conclusions reported in Proposition 1 persist in the setting with asymmetric knowledge of demand. Proposition 2 indicates that, for any report about expected demand that the firm might make ($s_j \in \{ s_1, ..., s_N \}$), the regulator optimally implements the values of p, T, and α, that she would implement if she were certain that the firm had reported s_j accurately. These values eliminate all variation in the firm's realized profit because (6) implies that when $\alpha_j = \frac{p_j - c}{p_j}$:

$$\begin{aligned} \pi_{ij} &= [\, p_j - c \,]\, Q(p_j, \varepsilon_i) + T_j - k\,K + \left[\frac{p_j - c}{p_j} \right] p_j \,[\, Q^e(p_j, s_j) - Q(p_j, \varepsilon_i) \,] \\ &= [\, p_j - c \,]\, Q^e(p) + T - k\,K \;\equiv\; \overline{\pi}_j \,. \end{aligned} \tag{11}$$

The values of p_j, T_j, and α_j that the regulator implements also ensure that the firm secures exactly its reservation level of expected utility for each demand realization, i.e.:

$$U(\pi_{ij}) \;=\; U(\overline{\pi}_j) \;=\; \overline{U} \;\text{ for all }\; i \in \{1, ..., n\}\,,\text{ for each }\; j \in \{1, ..., N\}. \tag{12}$$

The firm has no incentive to misrepresent the level of expected demand when the regulator pursues this strategy. To explain this conclusion, suppose the firm observes signal s_j. (11) implies that if the firm reports this signal truthfully, the firm's profit is $\overline{\pi}_j$ for every demand (ε_i) realization. Therefore, (12) implies that the firm's expected utility is $U(\overline{\pi}_j) = \overline{U}$. Now suppose the firm misrepresents expected demand by claiming to have observed signal $s_l \neq s_j$. (11) implies that the firm's profit would be $\overline{\pi}_l$ for each demand realization. Consequently, (12) implies that the firm's expected utility would be $U(\overline{\pi}_l) = \overline{U}$. Therefore, the firm cannot increase its expected utility by misrepresenting its private information about expected demand.

One might suspect that the firm could gain by understating expected demand to induce the regulator to set a relatively high unit price in order to ensure the firm's operation. This would be the case in the absence of a revenue adjustment. However, the revenue adjustment that the regulator optimally implements ensures that the firm's profit does not vary with the demand realization. Furthermore, for each demand realization, the firm secures exactly its reservation level of expected utility ($\overline{U}$), regardless of the regulator's initial belief about the ultimate demand realization.

5 Conclusions

We have examined the optimal design of regulatory policy in the presence of potentially substantial variation in the firm's revenue induced by exogenous variation in the demand for the regulated firm's service. We have shown that the optimal policy typically entails a partial adjustment for revenue variation. This partial adjustment differs from the full adjustment that is implemented under RCR. It also differs from the absence of adjustment that prevails under PCR. The optimal revenue adjustment is proportional to the firm's Lerner Index, $\frac{p-c}{p}$. This finding implies that the optimal revenue adjustment varies with the regulated firm's rate structure. Specifically, the optimal policy more closely resembles PCR when the prevailing fixed charge is large and the unit price is close to marginal cost. In contrast, the optimal policy more closely resembles RCR when the prevailing fixed charge is small and the unit price substantially exceeds marginal cost.

We have considered settings in which the prevailing demand variation is exogenous. The optimal policy in this setting delivers the same profit to the regulated firm for all demand realizations. Consequently, the firm has no strict incentive to increase or reduce demand. If the regulator wished to induce the firm to encourage its customers to reduce their consumption of the firm's service (e.g., to undertake energy conservation), the regulator would increase the revenue adjustment parameter, α, above $\frac{p-c}{p}$, thereby implementing a policy that more closely resembles RCR.[28] Alternatively, if the regulator wished to encourage the firm to increase the level of demand-enhancing quality it supplies, the regulator would reduce α below $\frac{p-c}{p}$, *ceteris paribus*.[29]

To facilitate analytic tractability, we have abstracted from any potential impact of revenue adjustment on the demand for the regulated service. In principle, a customer who is well informed about aggregate industry demand might understand how her consumption decision affects the firm's revenue and thereby affects the magnitude of the aggregate revenue adjustment, a portion of which accrues to, or is borne by, the customer. In this event, increased revenue adjustment (i.e., a larger value of α) could lead consumers to increase their consumption of the regulated service because increased consumption increases the firm's revenue, which either increases the "excess" revenue that the firm returns to consumers or reduces the revenue "shortfall" that consumers deliver to the firm. If the increased demand sufficiently enhances consumer welfare, then the regulator might optimally impose some profit risk on the risk averse firm by increasing α above $\frac{p-c}{p}$.

We have also abstracted from consumer risk aversion in order to focus on the effects of firm risk aversion and to facilitate analytic tractability. Risk sharing considerations arise if the firm and its customers are both risk averse. In such settings, the optimal regulatory

[28] Recall from Lemma 1 that when $\alpha > \frac{p-c}{p}$, the firm's profit (after implementing the revenue adjustment) increases as demand declines.

[29] The regulator faces a trade-off in this regard. Although a higher value of α will enhance the regulated firm's incentive to promote energy conservation, it will also reduce the firm's incentive to supply demand-enhancing service quality. In practice, this trade-off might be alleviated to some extent by employing additional policy instruments (e.g., explicit financial penalties or rewards for particularly low or high levels of realized service quality).

policy will expose the firm to some profit variation if doing so reduces the risk that customers experience. The precise impact of customer risk aversion on the optimal revenue adjustment will vary with the nature and the extent of consumer aversion to risk.

In addition to analyzing these extensions of our model in detail, future research might consider a broader class of regulatory policies and examine the effects of consumer heterogeneity (e.g., income variation) and the associated distributional considerations.[30] The key qualitative effects that arise in our streamlined model seem likely to persist more generally, although the details of the optimal policy may differ.

[30] Future research might also consider non-constant marginal cost and allow the regulated firm's capital investment to affect its marginal cost.

Appendix

This Appendix provides the proofs of the formal conclusions in the text.

Proof of Lemma 1. Define $Q_i \equiv Q(p, \varepsilon_i)$ for $i = 1, ..., n$. Then (1) implies:

$$\begin{aligned}
\pi_i - \pi_j &= p\,[\,1 - \alpha\,]\,Q_i - c\,Q_i - (\,p\,[\,1 - \alpha\,]\,Q_j - c\,Q_j\,) \\
&= [\,p\,(1 - \alpha) - c\,]\,[\,Q_i - Q_j\,] \;=\; [\,p\,\alpha - p + c\,]\,[\,Q_j - Q_i\,] \\
&= p\left[\alpha - \frac{p - c}{p}\right][\,Q_j - Q_i\,] \\
\Rightarrow \quad \frac{\pi_i - \pi_j}{p\,[\,Q_j - Q_i\,]} &= \alpha - \frac{p - c}{p}\,. \qquad (13)
\end{aligned}$$

Suppose $j > i$, without loss of generality. Then (13) implies that because $Q_j > Q_i$, $\pi_j \lesseqgtr \pi_i$ $\Leftrightarrow$ $\alpha \gtreqless \frac{p-c}{p}$. ■

Proof of Proposition 1. (1) implies:

$$\begin{aligned}
\frac{\partial \pi_i}{\partial \alpha} &= p\,[\,Q^e(p) - Q(p, \varepsilon_i)\,] - k'(\alpha)\,K\,; \qquad \frac{\partial \pi_i}{\partial T} \;=\; 1\,; \\
\frac{\partial \pi_i}{\partial p} &= \alpha\,Q^e(p) + [\,1 - \alpha\,]\,Q(p, \varepsilon_i) \\
&\quad + \; p\left[\alpha\,\frac{\partial Q^e(p)}{\partial p} + (1 - \alpha)\,\frac{\partial Q(p, \varepsilon_i)}{\partial p}\right] - c\,\frac{\partial Q(p, \varepsilon_i)}{\partial p}\,. \qquad (14)
\end{aligned}$$

Let $\lambda \geq 0$ and $\gamma \geq 0$ denote the Lagrange multipliers associated with the first and second constraints in (2), respectively. Then because $Q^e(p) = \sum_{i=1}^{n} \phi_i\, Q(p, \varepsilon_i)$, (2) implies that the Lagrangian function associated with [RP] is:

$$\mathcal{L} \;=\; \sum_{i=1}^{n} \phi_i\, S(p, \varepsilon_i) - T + \lambda\left[\,\sum_{i=1}^{n} \phi_i\, U(\pi_i) - \overline{U}\,\right] + \gamma\,[\,\overline{T} - T\,]\,. \qquad (15)$$

(15) implies that the necessary conditions for a solution to [RP] include:

$$\frac{\partial \mathcal{L}}{\partial T} \;=\; -\,1 + \lambda \sum_{i=1}^{n} \phi_i\, U'(\pi_i) - \gamma \;=\; 0\,; \qquad (16)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha} \;=\; \lambda \sum_{i=1}^{n} \phi_i\, U'(\pi_i)\,\frac{\partial \pi_i}{\partial \alpha} \;=\; 0\,; \qquad (17)$$

$$\frac{\partial \mathcal{L}}{\partial p} \;=\; \sum_{i=1}^{n} \phi_i\,\frac{\partial S(p, \varepsilon_i)}{\partial p} + \lambda \sum_{i=1}^{n} \phi_i\, U'(\pi_i)\,\frac{\partial \pi_i}{\partial p} \;=\; 0\,. \qquad (18)$$

(16) reflects the fact that $\frac{\partial \pi_i}{\partial T} = 1$, from (14).

(16) implies that $\lambda > 0$. Therefore, (14) and (17) imply:

$$\sum_{i=1}^{n} \phi_i \, U'(\pi_i) \; \{ \, p \, [\, Q^e - Q_i \,] - k'(\alpha) \, K \, \} \; = \; 0 \tag{19}$$

(13) implies:

$$\text{If } Q_i \, > \, Q_j, \text{ then } \pi_i \, > \, \pi_j \;\; \Leftrightarrow \;\; p \, [\, 1 - \alpha \,] \, > \, c \, ; \text{ and}$$

$$\text{If } Q_i \, < \, Q_j, \text{ then } \pi_i \, > \, \pi_j \;\; \Leftrightarrow \;\; p \, [\, 1 - \alpha \,] \, < \, c \, . \tag{20}$$

First suppose that $p \, [\, 1 - \alpha \,] > \, c$. Then (20) implies that because $Q_1 < ... < Q_n$, it must be the case that $\pi_1 < ... < \pi_n$. Consequently, because $U^{''}(\pi_i) < 0$:

$$U'(\pi_1) \, > \, ... \, > \, U'(\pi_n) \, . \tag{21}$$

$\sum_{i=1}^{n} \phi_i \, [\, Q^e - Q_i \,] = 0$ and $Q^e - Q_i$ is strictly decreasing in i. Therefore, (21) implies:

$$\sum_{i=1}^{n} \phi_i \, U'(\pi_i) \; p \, [\, Q^e - Q_i \,] \, > \, 0. \tag{22}$$

(19) and (22) imply that $k'(\alpha) > 0$. Consequently, Assumption 1 implies:

$$\alpha \, > \, \frac{p - c}{p} \;\; \Rightarrow \;\; \alpha \, p \, > \, p - c \;\; \Rightarrow \;\; p \, [\, 1 - \alpha \,] \, < \, c \, .$$

This contradiction implies that $p \, [\, 1 - \alpha \,] \leq \, c$.

Now suppose that $p \, [\, 1 - \alpha \,] < \, c$. Then (20) implies that because $Q_1 < ... < Q_n$, it must be the case that $\pi_1 > ... > \pi_n$. Consequently, because $U^{''}(\pi_i) < 0$:

$$U'(\pi_1) \, < \, ... \, < \, U'(\pi_n) \, . \tag{23}$$

$\sum_{i=1}^{n} \phi_i \, [\, Q^e - Q_i \,] = 0$ and $Q^e - Q_i$ is strictly decreasing in i. Therefore, (23) implies:

$$\sum_{i=1}^{n} \phi_i \, U'(\pi_i) \; p \, [\, Q^e - Q_i \,] \, < \, 0. \tag{24}$$

(19) and (24) imply that $k'(\alpha) < 0$. Consequently, Assumption 1 implies:

$$\alpha \, < \, \frac{p - c}{p} \;\; \Rightarrow \;\; \alpha \, p \, < \, p - c \;\; \Rightarrow \;\; p \, [\, 1 - \alpha \,] \, > \, c \, .$$

This contradiction implies that $p \, [\, 1 - \alpha \,] \geq \, c$.

Because $p \, [\, 1 - \alpha \,] \leq \, c$ and $p \, [\, 1 - \alpha \,] \geq \, c$, it follows that:

$$p \, [\, 1 - \alpha \,] \, = \, c \;\; \Rightarrow \;\; 1 - \alpha \, = \, \frac{c}{p} \;\; \Rightarrow \;\; \alpha \, = \, 1 - \frac{c}{p} \;\; \Rightarrow \;\; \alpha \, = \, \frac{p - c}{p} \, . \tag{25}$$

(1) and (25) imply that $\pi_1 = ... \pi_n \, \equiv \, \overline{\pi}$. Therefore, $U(\overline{\pi}) \, = \, \overline{U}$ because $\lambda > 0$.

Suppose $p \leq c$. Then (1) and (25) imply:

$$\begin{aligned}\pi_i &= p\,[\,\alpha\,Q^e + (1-\alpha)\,Q_i\,] + T - c\,Q_i - k\,K \\ &= [\,p - c\,]\,Q^e + c\,Q_i + T - c\,Q_i - k\,K \\ &= [\,p - c\,]\,Q^e + T - k\,K \;\leq\; T - \underline{k}\,K\,. \end{aligned} \tag{26}$$

The inequality in (26) holds because $p \leq c$, by assumption, and because $k(\alpha) \geq \underline{k}$, by definition. Assumption 2, the second constraint in (2), and (26) imply:

$$\sum_{i=1}^{n} \phi_i\, U(\pi_i) \;\leq\; U(T - \underline{k}\,K) \;\leq\; U(\overline{T} - \underline{k}\,K) \;<\; \overline{U}. \tag{27}$$

(27) violates the first constraint in (2). Therefore, by contradiction, $p > c$. ■

Lemma 2 is employed in the proof of Proposition 2**.**

Lemma 2. *For each $j \in \{1, ..., N\}$, for all $h > i$ $(i, h \in \{1, ..., n\})$, $\pi_{hj} \lesseqgtr \pi_{ij} \;\Leftrightarrow\; \alpha_j \gtreqless \frac{p_j - c}{p_j}$.*

Proof. Define $Q_{ij} \equiv Q(p_j, \varepsilon_i)$ for $i = 1, ..., n$ and $j \in \{1, .., N\}$. Then (6) implies:

$$\begin{aligned}\pi_{ij} - \pi_{hj} &= p_j\,[\,1 - \alpha_j\,]\,Q_{ij} - c\,Q_{ij} - (\,p_j\,[\,1 - \alpha_j\,]\,Q_{hj} - c\,Q_{hj}\,) \\ &= [\,p_j\,(1 - \alpha_j) - c\,]\,[\,Q_{ij} - Q_{hj}\,] \;=\; [\,p_j\,\alpha_j - p_j + c\,]\,[\,Q_{hj} - Q_{ij}\,] \\ &= p_j \left[\alpha_j - \frac{p_j - c}{p_j}\right][\,Q_{hj} - Q_{ij}\,] \\ \Rightarrow\quad \frac{\pi_{ij} - \pi_{lj}}{p_j\,[\,Q_{hj} - Q_{ij}\,]} &= \alpha_j - \frac{p_j - c}{p_j}\,. \end{aligned} \tag{28}$$

Suppose $h > i$, without loss of generality. Then (28) implies that because $Q_{hj} > Q_{ij}$, $\pi_{hj} \lesseqgtr \pi_{ij} \;\Leftrightarrow\; \alpha_j \gtreqless \frac{p_j - c}{p_j}$. ■

Proof of Proposition 2. To characterize the solution to [RP-A], we will characterize the solution to the following relaxed problem, [RP-A]$'$.

$$\underset{\alpha_j,\, p_j,\, T_j}{\text{Maximize}} \quad \sum_{j=1}^{N} \eta_j \left[\sum_{i=1}^{n} \phi_{ij}\,\{\,S(p_j, \varepsilon_i) - \alpha_j\,p_j\,[\,Q^e(p_j, s_j) - Q(p_j, \varepsilon_i)\,]\,\} - T_j\right] \tag{29}$$

$$\text{subject to, for } j \in \{1, ..., N\}\text{:} \quad \sum_{i=1}^{n} \phi_{ij}\,U(\pi_{ij}) \;\geq\; \overline{U}\,; \tag{30}$$

$$T_j \;\leq\; \overline{T}\,. \tag{31}$$

We will show that the solution to [RP-A]$'$ satisfies all of the constraints in [RP-A], and so constitutes a solution to [RP-A].

(6) implies:

$$\frac{\partial \pi_{ij}}{\partial \alpha_j} \;=\; p_j \, [\, Q^e(p_j, s_j) - Q(p_j, \varepsilon_i) \,] - k'(\alpha_j) \, K \, ; \qquad \frac{\partial \pi_{ij}}{\partial T_j} \;=\; 1 \, ;$$

$$\begin{aligned} \frac{\partial \pi_{ij}}{\partial p_j} \;&=\; \alpha_j \, Q^e(p_j, s_j) + [\, 1 - \alpha_j \,] \, Q(p_j, \varepsilon_i) \\ &+\; p_j \left[\alpha_j \, \frac{Q^e(p_j, s_j)}{\partial p_j} + (1 - \alpha_j) \, \frac{\partial Q(p_j, \varepsilon_i)}{\partial p_j} \right] - c \, \frac{\partial Q(p_j, \varepsilon_i)}{\partial p_j} \, . \end{aligned} \tag{32}$$

Let $\lambda_j \geq 0$ denote the Lagrange multiplier associated with constraint (30). Also let $\gamma_j \geq 0$ denote the Lagrange multiplier- associated with constraint (31). Then because $\sum_{i=1}^{n} \phi_{ij} \, Q^e(p_j, s_j) = Q^e(p_j, s_j) = \sum_{i=1}^{n} \phi_i \, Q(p_j, \varepsilon_i)$, (29) – (31) imply that the Lagrangian function associated with [RP-A]$'$ is:

$$\begin{aligned} \mathcal{L} \;=\; \sum_{j=1}^{N} \eta_j \left[\sum_{i=1}^{n} \phi_{ij} \, S(p_j, \varepsilon_i) - T_j \right] + \sum_{j=1}^{N} \lambda_j \left[\sum_{i=1}^{n} \phi_{ij} \, U(\pi_{ij}) - \overline{U} \right] \\ + \sum_{j=1}^{N} \gamma_j \, [\, \overline{T} - T_j \,] \, . \end{aligned} \tag{33}$$

(33) implies that the necessary conditions for a solution to [RP-A]$'$ include, for $j \in \{1, ..., N\}$:

$$\frac{\partial \mathcal{L}}{\partial T_j} \;=\; -\,\eta_j + \lambda_j \sum_{i=1}^{n} \phi_{ij} \, U'(\pi_{ij}) - \gamma_j \;=\; 0 \, ; \tag{34}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_j} \;=\; \lambda_j \sum_{i=1}^{n} \phi_{ij} \, U'(\pi_{ij}) \, \frac{\partial \pi_{ij}}{\partial \alpha_j} \;=\; 0 \, ; \tag{35}$$

$$\frac{\partial \mathcal{L}}{\partial p_j} \;=\; \eta_j \, \sum_{i=1}^{n} \phi_{ij} \, \frac{\partial S(p_j, \varepsilon_i)}{\partial p_j} + \lambda_j \, \sum_{i=1}^{n} \phi_{ij} \, U'(\pi_{ij}) \, \frac{\partial \pi_{ij}}{\partial p_j} \;=\; 0 \, . \tag{36}$$

(34) reflects the fact that $\frac{\partial \pi_{ij}}{\partial T_j} = 1$ for $j \in \{1, ..., N\}$, from (32).

Because $\eta_j > 0$ and $\gamma_j \geq 0$ for $j \in \{1, ..., N\}$, (34) implies that $\lambda_j > 0$ for $j \in \{1, ..., N\}$. Therefore:

$$\sum_{i=1}^{n} \phi_{ij} \, U(\pi_{ij}) \;=\; \overline{U} \;\; \text{for all} \;\; j \in \{1, ..., N\} \, . \tag{37}$$

Furthermore, (32) and (35) imply:

$$\sum_{i=1}^{n} \phi_{ij}\, U'(\pi_{ij}) \{ p_j [Q^e(p_j, s_j) - Q(p_j, \varepsilon_i)] - k'(\alpha_j) K \} \;=\; 0 \;\text{ for all }\; j \in \{1, ..., N\}. \quad (38)$$

(28) implies that for $i, h \in \{1, ..., n\}$ and $j \in \{1, ..., N\}$:

$$\text{If } Q_{ij} > Q_{hj}, \text{ then } \pi_{ij} > \pi_{hj} \;\Leftrightarrow\; \alpha_j < \frac{p_j - c}{p_j} \;\Leftrightarrow\; p_j [1 - \alpha_j] > c\,; \text{ and}$$

$$\text{If } Q_{ij} < Q_{hj}, \text{ then } \pi_{ij} > \pi_{hj} \;\Leftrightarrow\; \alpha_j > \frac{p_j - c}{p_j} \;\Leftrightarrow\; p_j [1 - \alpha_j] < c\,. \quad (39)$$

First suppose that $p_j [1 - \alpha_j] > c$ for some $j \in \{1, ..., N\}$. Then (39) implies that because $Q_{1j} < ... < Q_{nj}$, it must be the case that $\pi_{1j} < ... < \pi_{nj}$. Consequently, because $U''(\pi_{ij}) < 0$:

$$U'(\pi_{1j}) \;>\; ... \;>\; U'(\pi_{nj})\,. \quad (40)$$

$\sum_{i=1}^{n} \phi_{ij} [Q_j^e - Q_{ij}] = 0$ and $Q_j^e - Q_{ij}$ is strictly decreasing in i, where $Q_j^e \equiv \sum_{i=1}^{n} \phi_{ij} Q_{ij}$ for $j \in \{1, ..., N\}$. Therefore, (40) implies:

$$\sum_{i=1}^{n} \phi_{ij}\, U'(\pi_{ij})\, p_j [Q_j^e - Q_{ij}] \;>\; 0. \quad (41)$$

(38) and (41) imply that $k'(\alpha_j) > 0$. Consequently, Assumption 1 implies:

$$\alpha_j \;>\; \frac{p_j - c}{p_j} \;\Rightarrow\; \alpha_j\, p_j \;>\; p_j - c \;\Rightarrow\; p_j [1 - \alpha_j] \;<\; c\,.$$

This contradiction implies that $p_j [1 - \alpha_j] \leq c$.

Now suppose that $p_j [1 - \alpha_j] < c$. Then (39) implies that because $Q_{1j} < ... < Q_{nj}$, it must be the case that $\pi_{1j} > ... > \pi_{nj}$. Consequently, because $U''(\pi_{ij}) < 0$:

$$U'(\pi_{1j}) \;<\; ... \;<\; U'(\pi_{nj})\,. \quad (42)$$

$\sum_{i=1}^{n} \phi_{ij} [Q_j^e - Q_{ij}] = 0$ and $Q_j^e - Q_{ij}$ is strictly decreasing in i, for $j \in \{1, ..., N\}$. Therefore, (42) implies:

$$\sum_{i=1}^{n} \phi_{ij}\, U'(\pi_{ij})\, p_j [Q_j^e - Q_{ij}] \;<\; 0. \quad (43)$$

(38) and (43) imply that $k'(\alpha_j) < 0$. Consequently, Assumption 1 implies:

$$\alpha_j \;<\; \frac{p_j - c}{p_j} \;\Rightarrow\; \alpha_j\, p_j \;<\; p_j - c \;\Rightarrow\; p_j [1 - \alpha_j] \;>\; c\,.$$

This contradiction implies that $p_j [1 - \alpha_j] \geq c$.

Because $p_j [1 - \alpha_j] \leq c$ and $p_j [1 - \alpha_j] \geq c$, it follows that for $j \in \{1, ..., N\}$:

$$p_j [1 - \alpha_j] \;=\; c \;\Rightarrow\; 1 - \alpha_j \;=\; \frac{c}{p_j} \;\Rightarrow\; \alpha_j \;=\; 1 - \frac{c}{p_j} \;\Rightarrow\; \alpha_j \;=\; \frac{p_j - c}{p_j}\,. \quad (44)$$

(44) and Assumption 1 imply:

$$k(\alpha_j) \;=\; \underline{k} \;\text{ for all }\; j \in \{1, ..., N\}\,. \tag{45}$$

(6), (44), and (45) imply:

$$\begin{aligned} \pi_{ij} \;&=\; [\,p_j - c\,]\,Q_{ij} + T_j - k(\alpha_j)\,K + [\,p_j - c\,]\,\big[\,Q_j^e - Q_{ij}\,\big] \\ &=\; [\,p_j - c\,]\,Q_j^e + T_j - \underline{k}\,K \;\equiv\; \overline{\pi}_j\,. \end{aligned} \tag{46}$$

(37) and (46) imply that for $j \in \{1, ..., N\}$:

$$\sum_{i=1}^{n} \phi_{ij}\,U(\pi_{ij}) \;=\; U(\overline{\pi}_j) \;=\; \overline{U} \quad \Rightarrow \quad \overline{\pi}_j \;=\; U^{-1}(\overline{U})\,. \tag{47}$$

Suppose $p_j \leq c$. Then (6), (44), and (45) imply:

$$\begin{aligned} \pi_{ij} \;&=\; p_j\,\big[\,\alpha_j\,Q_j^e + (1-\alpha_j)\,Q_{ij}\,\big] + T_j - c\,Q_{ij} - k(\alpha_j)\,K \\ &=\; [\,p_j - c\,]\,Q_j^e + c\,Q_{ij} + T_j - c\,Q_{ij} - \underline{k}\,K \\ &=\; [\,p_j - c\,]\,Q_j^e + T_j - \underline{k}\,K \;\leq\; T_j - \underline{k}\,K\,. \end{aligned} \tag{48}$$

The inequality in (48) holds because $p_j \leq c$, by assumption. (31), (48), and Assumption 2 imply that for $j \in \{1, ..., N\}$:

$$\sum_{i=1}^{n} \phi_{ij}\,U(\pi_{ij}) \;\leq\; U(T_j - \underline{k}\,K) \;\leq\; U(\overline{T} - \underline{k}\,K) \;<\; \overline{U}\,. \tag{49}$$

(49) violates the constraint in (30). Therefore, by contradiction, $p_j > c$ for $j \in \{1, ..., N\}$.

It remains to verify that the constraints in (9) are satisfied at the solution to [RP-A]$'$. (47) implies that for $j, l \in \{1, ..., N\}$ $(l \neq j)$:

$$\sum_{i=1}^{n} \phi_{ij}\,U(\pi_{il}) \;=\; \sum_{i=1}^{n} \phi_{ij}\,U(\overline{\pi}_l) \;=\; U(\overline{\pi}_l) \;=\; U(U^{-1}(\overline{U})) \;=\; \overline{U}\,. \tag{50}$$

(37) and (50) imply that the constraints in (9) are satisfied at the solution to [RP-A]$'$ because:

$$\sum_{i=1}^{n} \phi_{ij}\,U(\pi_{il}) \;=\; \overline{U} \;=\; \sum_{i=1}^{n} \phi_{ij}\,U(\pi_{il}) \;\text{ for }\; j, l \in \{1, ..., N\}\;(l \neq j)\,. \quad \blacksquare$$

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