Hotelling Competition with Avoidable Horizontal Product Differentiation: Choosing Between Online Privacy and Disclosure

by

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Abstract

We extend the standard model of Hotelling competition to allow consumers to change the horizontal quality dimension of the product they purchase. Specifically, a consumer can incur a personal cost (K) to change the default privacy/disclosure setting on the smartphone she purchases. A unilateral increase in K can increase a supplier's equilibrium profit in some settings, despite rendering the supplier's phone less attractive to many potential customers. In other settings, an increase in K can reduce or have no effect on a supplier's equilibrium profit. Furthermore, a small increase in K can reduce a supplier's profit whereas a large increase in K can increase a supplier's profit.

Keywords: privacy choice; avoidable horizontal product differentiation.

JEL Codes: D43, D60, L13

November 2023

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1 Introduction.

There are presently more than 310 million smartphones in operation in the United States (Statista, 2023). 56.9% of these phones employ the iOS operating system. 42.6% employ the Android operating system (Statcounter, 2023). Apple installs the iOS operating system in the iPhone. Several device manufacturers, including Samsung, LG, HTC, and Sony, use the Android operating system in their smartphones, a system developed primarily by Google. Both operating systems can collect data about users' online activities and/or allow other applications installed on user devices to collect these data. Relevant data include a user's real-time location, her information queries, the websites she visits, and the products she purchases online. Access to such information can help potential advertisers better assess the likely efficacy of targeted advertisements.

Some consumers are reluctant to share information about their online activities, in part because the information might be (mis)construed to reflect their personal beliefs and preferences.[1] Such consumers may prefer to share little, if any, information about their online activities. Other consumers may prefer to share information about some or all of their online activities with potential advertisers because doing so can increase the likelihood of receiving targeted, customized information that helps the consumers make informed purchasing decisions.[2]

Applications that run on an iPhone must obtain the user's permission to collect personal data that might be employed to inform targeted advertising.[3] Thus, the default "privacy/disclosure" (PD) setting on the iPhone is "privacy," which means that personal data cannot be shared with advertisers without the user's explicit permission. In contrast, the default PD setting on many Android smartphones is "disclosure," which means that the user's personal data can be shared with advertisers unless the user explicitly prohibits such sharing.[4]

We employ a modified Hotelling model to analyze competition between two suppliers of smartphones with distinct default PD settings. In contrast to the standard model of Hotelling competition,[5] we allow consumers to change the horizontal dimension of quality on the product they purchase. Specifically, by incurring personal cost K_1, a consumer can

[1] Some consumers may prefer not to share information about their online activities so as to limit the number of unsolicited and undesired messages they receive from advertisers.

[2] See Tucker (2012), Taylor and Wagman (2014), and Acquisti et al. (2016), for example.

[3] https://developer.apple.com/app-store/user-privacy-and-data-use.

[4] See Grant and *Bloomberg* (2021), for example.

[5] For selective views of the literature on Hotelling competition, see Graitson (1982), Gabszewicz and Thisse (1992), and Biscaia and Mota (2013), for example.

change the default “privacy” setting on Firm 1’s phone to “disclosure.” Similarly, by incurring personal cost K_2, a consumer can change the default “disclosure” setting on Firm 2’s phone to “privacy.”

This ability to change the default PD setting implies that a consumer can, at personal cost, effectively eliminate the horizontal product differentiation that she otherwise perceives. When default-switching costs are sufficiently low, the substantial horizontal product homogeneity that effectively prevails can fundamentally change the nature of the competition between suppliers. The firms can find it more profitable to focus on attracting all potential customers with a relatively low price than on attracting only “close” customers with a relatively high price.[6]

When relatively low default-switching costs induce a “market dominant” (MD) equilibrium in which all consumers purchase a phone from the same supplier, the effects of default-switching costs vary across suppliers. When Firm 1 attracts all consumers in equilibrium, its profit declines as its default-switching cost increases. The profit reduction arises in part because Firm 1 must reduce the price of its phone to continue to attract distant consumers who must now incur a higher cost to ensure their preferred PD setting. In contrast, when Firm 2 attracts all consumers in equilibrium, its profit can increase as its default-switching cost (K_2) increases. This is the case because when K_2 increases, fewer customers change Firm 2’s default “disclosure” setting. Consequently, Firm 2 secures higher payments from advertisers, who pay a premium for access to the detailed personal information that is revealed under the “disclosure” setting.

Furthermore, for reasons that are explained in detail below, an increase in K_2 does not affect Firm 1’s profit when Firm 1 serves all consumers in equilibrium. In contrast, when Firm 2 serves all consumers in equilibrium, its profit increases as K_1 increases. When both firms serve consumers in equilibrium, neither firm’s profit is affected by changes in its own default-switching cost or in the rival’s corresponding cost.

Even though a higher default-switching cost reduces the attraction of a firm’s phone to some consumers, a unilateral increase in a firm’s default-switching cost can enhance the firm’s equilibrium profit. This is the case, for example, when the increased cost alters the nature of the prevailing equilibrium. As noted above, a higher default-switching cost can encourage a firm to focus on attracting (only) relatively close customers and charging them a relatively high price.[7] Because prices are strategic complements under Hotelling competition, the

[6] “Close” consumers are those with a relatively strong affinity for the firm’s default PD setting. “Distant” consumers are those with a relatively strong affinity for the rival’s default PD setting.

[7] A higher default-switching cost can also increase Firm 2’s profit by inducing more customers to retain Firm 2’s default “disclosure” setting, which ensures that Firm 2 secures higher payments from advertisers.

ensuing relaxed price competition can enhance the profits of both industry suppliers in the "market sharing" (MS) equilibrium that prevails.[8] We find that a higher default-switching cost is particularly likely to increase a firm's equilibrium profit when the firm's competitive advantage is limited and consumer transportation costs are relatively large (so the horizontal dimension of product quality is relatively important to consumers).

MD and MS equilibria can both arise, regardless of whether default switching costs (K_1 and K_2) are endogenous or exogenous. However, certain equilibria that exist when K_1 and K_2 are exogenous do not arise when K_1 and K_2 are endogenous. To illustrate, there are conditions under which Firm 1 will not set $K_1 > 0$ in the MD equilibrium where it serves all consumers because, by setting $K_1 = 0$, Firm 1 could both increase the payments it receives from advertisers (by inducing expanded switching of Firm 1's default "privacy" setting) and allow Firm 1 to attract all consumers with a higher price (by reducing the cost that distant customers must incur to secure their preferred PD setting). Furthermore, there are conditions under which Firm 2 will not implement small values of K_2 because a higher default-switching cost would increase Firm 2's payment from advertisers by more than it would dissuade distant customers from purchasing Firm 2's phone.

This research contributes to the literature on Hotelling competition primarily by characterizing the changes that arise when consumers can incur a personal cost to eliminate the horizontal product differentiation that they otherwise perceive. In part because of its relevance and practical importance in today's "information age," the particular horizontal product differentiation that we consider pertains to the default PD setting on smartphones.[9] However, our analysis and findings are relevant more generally.[10] Because our model differs from its predecessors primarily by incorporating default-switching costs, we emphasize how changes in these costs affect both the nature and the details of equilibrium outcomes. As noted above, we show that an increase in a firm's default-switching cost can increase, reduce, or have no effect on the firm's equilibrium profit.[11]

In our model, an increase in a firm's default-switching cost reduces the attraction of the

[8] Because a firm (Firm i) can benefit from an increase in its default-switching cost (K_i) that alters the nature of the prevailing equilibrium, a substantial increase in K_i can increase Firm i's equilibrium profit even though a marginal increase in K_i can reduce its equilibrium profit.

[9] Rhodes and Zhou (2022) consider endogenous data sharing that can allow suppliers to practice perfect price discrimination. The authors show that the welfare effects of data sharing can differ with the extent of equilibrium market coverage. They also show that data sharing in excess of the welfare-maximizing level can arise in equilibrium.

[10] For example, a consumer might, in principle, pay a detailing company to change the color of an automobile that she has purchased.

[11] Furthermore, an increase in one firm's default-switching cost can either increase or have no effect on the rival's equilibrium profit.

firm's product to distant consumers by a fixed amount. Other authors (e.g., von Ungern-Sternberg, 1988; Hendel and de Figueiredo, 1997; Troncoso-Valverde and Robert, 2004; Hou et al., 2013) have examined the impact of an increase in consumer transportation costs, which reduces the attraction of a firm's product to all consumers. In these models, the extent of the reduced attraction increases linearly with the distance between a consumer's location and the relevant firm's location on the Hotelling line. These differences generate different conclusions about the conditions under which higher costs increase profits. However, in these models and in our model, this profit-enhancing effect of higher costs arises because the increased cost induces a firm to increase its focus on attracting particularly close customers.

We develop and further explain our findings as follows. Section 2 describes our model. Section 3 characterizes equilibrium outcomes in two benchmark settings: one where default PD settings cannot be changed and one where consumers can change the default settings costlessly. Section 4 characterizes the distinct equilibria that can arise in the setting of primary interest, where default-switching costs are intermediate in magnitude. Section 5 examines the impacts of default-switching costs on equilibrium profits. Section 6 identifies conditions under which an increase in one supplier's default-switching cost increases the equilibrium profit of both suppliers. Section 7 examines equilibrium outcomes when default-switching costs are endogenous. Section 8 summarizes our key findings and suggests directions for future research. The Appendix provides the proofs of all formal conclusions in the text.

2 The Model

We analyze competition between two sellers of smartphones. A phone can operate under either a "privacy" setting or a "disclosure" setting. When a consumer uses a phone to access the internet under the "privacy" setting, the consumer's online activities (e.g., the sites she visits) are not revealed to potential advertisers. In contrast, the consumer's online activities are revealed to advertisers when her phone operates under the "disclosure" setting.

For the reasons discussed above, consumers' preferences for "privacy" and "disclosure" differ. These preferences are captured by a consumer's location on the unit interval. The consumer located at 0 has the strongest preference for privacy. The further is a consumer from 0, the more amenable is the consumer to disclosure. The consumer located at 1 has the strongest preference for disclosure. Potential consumers are distributed uniformly on the unit interval. The total mass of consumers is normalized to 1.

In standard Hotelling fashion, the two suppliers of smartphones are located at opposite ends of the unit interval: Firm 1 is located at 0, Firm 2 is located at 1. The default "privacy/disclosure" (PD) setting on Firm 1's phone is "privacy." The default PD setting on Firm 2's phone is "disclosure."[12] If a consumer located at $x \in [0,1]$ purchases a phone from Firm 1 (respectively, Firm 2) and does not change the default PD setting on the phone she purchases, the consumer incurs "transportation" cost $t\,x$ (respectively, $t\,[1-x]$). $t > 0$ is a parameter that reflects the diversity and intensity of consumers' preferences for privacy vs. disclosure.

In contrast to the standard Hotelling analysis, we allow consumers to change the default PD setting on the phones they purchase. $K_i \geq 0$ is the personal cost that a customer must incur to change the default PD setting on a phone she purchases from Firm $i \in \{1, 2\}$. This cost might reflect, for example, the time and effort required to learn how to change the phone's default setting, and then implement the change.[13]

The default PD setting on a phone reflects a horizontal dimension of service quality. The phones that Firms 1 and 2 sell also can differ on vertical dimensions of quality (e.g., processor speed, memory, battery life, photographic capabilities, and ease of use). G_i is the gross value that each customer derives from a phone supplied by Firm $i \in \{1, 2\}$, a value that primarily reflects the phone's vertical quality dimensions. The utility that a consumer derives from a phone purchased from Firm i is G_i, less any transportation and default-switching costs the consumer incurs, less p_i, which is the price that Firm i charges for its phone.

Firm i incurs unit production cost c_i. In addition to the revenue it collects from customers, each firm receives payments from advertisers. Firm i receives payment $r_H > 0$ (respectively, $r_L \geq 0$) from advertisers for each phone it sells when the purchaser employs the phone under the disclosure (respectively, the privacy) PD setting. $\Delta \equiv r_H - r_L$ is strictly positive because advertisers are willing to pay a premium for detailed information about a consumer's preferences, as revealed by her online activities.

Each customer buys at most one phone. We further assume that G_1 and G_2 are sufficiently large relative to c_1, c_2, and t that every potential consumer purchases a phone in equilibrium. It will be convenient to refer to a firm's "competitive advantage" in the ensuing analysis. Formally, we define $A \equiv \frac{1}{3}\,[\,G_2 - c_2 + r_H - (G_1 - c_1 + r_L)\,]$ to be Firm 2's competitive advantage, and $|\,A\,|$ to be Firm 1's competitive advantage. Firm 2 is the *advantaged firm* when $A > 0$. Firm 1 is the advantaged firm when $A < 0$.

The timing in the model is as follows. After G_i, c_i, K_i ($i \in \{1, 2\}$), t, r_L, r_H, and default PD settings are determined exogenously, the two suppliers set their prices simultaneously

[12] We take the firms' distinct PD choices to be exogenous. See Gal-Or et al. (2018), for example, for an analysis of how competing platforms choose the levels of privacy that they provide to their users.

[13] In practice, firms can influence the costs that consumers incur to change the default PD setting in part by choosing the nature and the number of steps that must be undertaken to implement the change. See, for example, Athey et al. (2017).

and noncooperatively. Consumers then decide whether to purchase a phone from Firm 1 or from Firm 2. Next, each customer decides whether to retain or change the default PD setting on the phone she has purchased. Finally, the firms collect the stipulated payments from advertisers.[14]

Each consumer purchases the phone that ensures her the highest utility (gross value, less price, less relevant transportation and default-switching costs). The consumer located at $x \in [0, 1]$ purchases a phone from Firm 1 rather than from Firm 2 if:

$$G_1 - p_1 - \min\{ t x, t[1-x] + K_1 \} > G_2 - p_2 - \min\{ t[1-x], t x + K_2 \}. \quad (1)$$

The consumer located at $x \in [0, 1]$ will purchase a phone from Firm 2 rather than from Firm 1 if the inequality in (1) is reversed.[15] (1) reflects the fact that, after purchasing a phone from Firm 1, the consumer located at x will change the default PD setting on the phone ("privacy") if and only if the sum of the default-switching cost on Firm 1's phone and the consumer's transportation cost associated with the "disclosure" setting is less than the consumer's transportation associated with the "privacy" setting, i.e.:

$$t[1-x] + K_1 < t x \quad \Leftrightarrow \quad x > \frac{1}{2} + \frac{K_1}{2t}. \quad (2)$$

(2) implies that if $K_1 < t$, the (only) consumers who will change the default PD setting on a phone they purchase from Firm 1 are those located closest to Firm 2, i.e., those located in $(\frac{1}{2} + \frac{K_1}{2t}, 1]$.

Similarly, after purchasing a phone from Firm 2, the consumer located at x will change the default PD setting on the phone ("disclosure") if and only if:[16]

$$t x + K_2 < t[1-x] \quad \Leftrightarrow \quad x < \frac{1}{2} - \frac{K_2}{2t}. \quad (3)$$

Most of the ensuing analysis will consider settings in which $K_i \in (0, t)$ for $i \in \{1, 2\}$ to avoid the relatively uninteresting case in which default-switching costs exceed the transportation cost associated with traversing the entire unit interval. In this case, no consumer would ever change the default PD setting on a phone she purchased.

For expositional ease, we restrict attention to undominated strategies for Firms 1 and

[14] Because our model is static and each consumer purchases at most one phone, we do not consider how a firm might "poach" the customers of a rival supplier or set different prices for "new" and "existing" customers in a setting with customer switching costs (e.g., Gehrig and Stenbacka, 2004; Arie and Grieco, 2014; Rhodes, 2014).

[15] We assume that when a consumer is indifferent between purchasing a phone from Firm 1 and from Firm 2, she purchases a phone from each firm with probability $\frac{1}{2}$.

[16] For expositional ease, we assume that when a consumer is indifferent between retaining and changing the default PD setting on the phone she purchases, she retains the default setting.

2. Formally, we assume that no firm ever sets a price that would never allow it to secure nonnegative profit, regardless of the price set by the rival supplier.

3 Benchmark Settings

In this section, we briefly characterize equilibrium outcomes in two benchmark settings. The first benchmark reflects the standard Hotelling model in which all product characteristics (including the default PD setting) are immutable. The second benchmark reflects the other extreme in which each consumer can costlessly change the default PD setting on the phone she purchases.

Lemma 1 characterizes outcomes in the setting where the default PD setting cannot be changed. The lemma refers to π_i, which denotes the profit of Firm $i \in \{1, 2\}$.

Lemma 1. *Suppose the default PD setting cannot be changed and $t > |A|$. Then in equilibrium: (i) the consumer located at $x_0 \equiv \frac{1}{2} - \frac{A}{2t} \in (0, 1)$ is indifferent between buying a phone from Firm 1 and from Firm 2; (ii) all consumers located in $[0, x_0)$ buy a phone from Firm 1; and (iii) all consumers located in $(x_0, 1]$ buy a phone from Firm 2. Furthermore: $p_1 = c_1 - r_L + t - A$; $p_2 = c_2 - r_H + t + A$; $\pi_1 = \frac{1}{2t}[t - A]^2$; and $\pi_2 = \frac{1}{2t}[t + A]^2$.*

Lemma 1 considers settings in which consumers' preferences for privacy vs. disclosure are relatively pronounced in the sense that the unit transportation cost exceeds each firm's competitive advantage (i.e., $t > |A|$). When all product characteristics are immutable in this setting, those consumers with the strongest preference for privacy (respectively, disclosure) purchase a phone from Firm 1 (respectively, Firm 2). The advantaged firm (i.e., Firm 2 when $A > 0$ and Firm 1 when $A < 0$) sells more phones and secures greater profit than its rival.[17] Furthermore, equilibrium prices increase with own production costs ($\frac{\partial p_i}{\partial c_i} > 0$ for $i \in \{1, 2\}$), with transportation costs ($\frac{\partial p_i}{\partial t} > 0$), and with a firm's competitive advantage ($\frac{\partial p_1}{\partial |A|} > 0$ when $A < 0$ and $\frac{\partial p_2}{\partial A} > 0$ when $A > 0$). Furthermore, prices decline as payments from advertisers increase ($\frac{\partial p_1}{\partial r_L} < 0$ and $\frac{\partial p_2}{\partial r_H} < 0$) because these payments enhance each firm's incentive to expand its sales by reducing its price.

Lemma 2. *Suppose $K_1 = K_2 = 0$. Then in equilibrium, all consumers purchase a phone from Firm 1 if $G_1 - c_1 > G_2 - c_2$. In contrast, all consumers purchase a phone from Firm 2 if $G_2 - c_2 > G_1 - c_1$. Consumers located in $[0, \frac{1}{2})$ implement the privacy setting on the phone*

[17] This outcome reflects the relatively high gross value that consumers derive from the advantaged firm's phone, the advantaged firm's relatively low production cost, and/or the relatively high advertising revenue associated with the firm's default PD setting.

they purchase, whereas consumers located in $(\frac{1}{2}, 1]$ *implement the disclosure setting.*[18] *When all consumers purchase a phone from Firm* i*, the firm's profit is (nearly)* $G_i - c_i - (G_j - c_j)$ *for* $i, j \in \{1, 2\}$ $(j \neq i)$*.*

When consumers can costlessly change the default PD setting on the phone they purchase, they effectively perceive the two phones to have the same horizontal quality. Each consumer simply implements her preferred PD setting on the phone she purchases, so the two phones are equally effective at satisfying each consumer's PD preference.[19] The effective absence of horizontal quality differentiation leads to intense "winner-take-all" price competition. When $G_1 - c_1 > G_2 - c_2$, for example, Firm 1 reduces its price to the highest level that allows Firm 1 to attract all consumers when Firm 2 sets the lowest price at which it could profitably serve all consumers (i.e., $p_2 = c_2 - \frac{1}{2}[r_L + r_H]$). (1) implies that this price for Firm 1 is determined by $G_1 - p_1 = G_2 - p_2$. Therefore, $p_1 = G_1 - G_2 + c_2 - \frac{1}{2}[r_L + r_H]$, which generates profit $p_1 - c_1 + \frac{1}{2}[r_L + r_H] = G_1 - c_1 - (G_2 - c_2)$ for Firm 1.

When $K_1 = K_2 = 0$, equilibrium prices and profits do not vary with payments from advertisers. This is the case because when each firm effectively competes to serve all consumers and when every consumer implements her preferred PD setting, each firm anticipates the same total payment from advertisers $(\frac{1}{2}[r_L + r_H])$. The relatively intense competition between suppliers of a product with no horizontal quality differentiation effectively compels the firms to pass along all payments from advertisers to consumers.

4 Characterizing Equilibrium Outcomes

We now characterize equilibrium outcomes in the setting of primary interest where default-switching costs are intermediate in magnitude. Specifically, $K_1 \in (0, t)$ and $K_2 \in (0, t)$, so: (i) it is costly for consumers to change the default PD setting on the phone they purchase; and (ii) default-switching costs are less than the transportation cost of traversing the entire unit interval.

Proposition 1 identifies conditions under which a *market-sharing equilibrium* (i.e., an equilibrium in which both firms sell a strictly positive number of phones) arises. These conditions include the following:

[18] The consumer located at $\frac{1}{2}$ is indifferent between the two PD settings. Consequently, by assumption, this consumer retains the default PD setting on the phone she purchases.

[19] For consumers located in $(0, 1)$, neither phone delivers the consumer's ideal PD level. However, when it is costless to change each phone's default PD setting, every consumer perceives the two phones to be equally effective at meeting her PD preference.

Condition 1A. $t \geq \max\{r_H - c_2 - A,\, r_L - c_1 + A\}$.

Condition 1B. $\frac{1}{2t}\,[t - A]^2 > -2A + \frac{t - K_1}{2t}\,[2t + r_H - r_L]$ if $A \leq 0$;

$$\frac{1}{2t}\,[t + A]^2 > 2A + \frac{t - K_2}{2t}\,[2t - (r_H - r_L)] \text{ if } A \geq 0.$$

Condition 1A helps to ensure that prices are positive in the market-sharing equilibrium. Condition 1B ensures that each firm earns more profit in the market-sharing equilibrium identified in Proposition 1 than it could secure by unilaterally lowering its price to the level required to secure the patronage of all consumers.[20]

Proposition 1. *Suppose: (i)* $t > |A|$*; (ii)* $K_2 \in (A, t)$ *if* $A > 0$*; and (iii)* $K_1 \in (|A|, t)$ *if* $A < 0$*. Further suppose that Conditions 1A and 1B hold. Then an equilibrium exists in which the outcomes identified in Lemma 1 all prevail and no consumer changes the default setting on the phone she purchases.*

Proposition 1 indicates that once the default-switching cost of the advantaged firm is sufficiently pronounced, the firm finds it more profitable to set a relatively high price and only attract "close" consumers than to set the lower price required to attract all consumers.[21] It typically becomes less profitable for a firm to attract distant consumers as default-switching costs increases. This is the case because the firm must reduce its price to convince distant consumers to either accept their less-preferred PD setting or incur the higher default-switching cost required to implement their preferred PD setting. Once its default-switching cost increases above a critical level, the advantaged firm effectively cedes distant consumers to the rival firm by setting a relatively high price that is attractive only to close consumers. The close consumers that the advantaged firm serves in equilibrium are either: (i) those that prefer the firm's default PD setting; or (ii) those whose preference for the rival's default PD setting is sufficiently mild that they choose not to change the default PD setting on the phone they purchase. Consequently, no consumer changes the default PD setting on the phone she purchases, even though such default switching is not prohibitively costly (i.e., even though $K_1 < t$ and $K_2 < t$). Therefore, prices, outputs, and profits do not vary with default-switching costs in this equilibrium.

In contrast, equilibrium profits do vary with default-switching costs in *market-dominant equilibria*, which are equilibria in which all consumers purchase a phone from the same firm.

[20] It can be shown that the first inequality in Condition 1B holds if and only if $[t - K_1][2t + r_H - r_L] \leq [t + A]^2$. The second inequality in Condition 1B holds if and only if $[t - K_2][2t - r_H + r_L] \leq [t - A]^2$.

[21] Close consumers are those wih the strongest preferences for the firm's default PD setting. We will refer to a firm's "distant" consumers as those with the strongest preferences for the rival's default PD setting.

Market-dominant equilibria arise when default-switching costs are sufficiently small that the advantaged firm finds it more profitable to set the relatively low price required to attract all consumers than to set a higher price that would only attract close consumers. Proposition 2 characterizes outcomes in the market-dominant equilibrium in which all consumers purchase a phone from Firm 1. The proposition refers to the following conditions.

Condition 2A. $c_2 > r_H$. **Condition 2B**. $\Omega_1 \equiv \frac{1}{2}[r_H - r_L] - 3A - \frac{1}{8t}[t - A]^2 > 0$.

Condition 2C. $K_1 \leq \frac{2t}{2t + r_H - r_L}\Omega_1$. **Condition 2D**. $K_1 < c_1 - r_L - 3A$.

Condition 2A ensures that $c_2 - r_H$, the minimum price that Firm 2 can profitably charge when it serves all consumers and no consumer changes the default PD setting, is positive. Conditions 2B and 2C ensure that Firm 1 secures positive profit when it attracts all consumers by setting p_1 marginally below $p_2 + G_1 - G_2 - K_1$ when Firm 2 sets $p_2 = c_2 - r_H$,[22] the lowest price that Firm 2 can profitably charge for its phone when every consumer that purchases a phone from Firm 2 retains the phone's default PD setting. Condition 2D ensures that the highest value of p_1 that allows Firm 1 to attract all consumers when Firm 2 sets $p_2 = c_2 - r_H$ is positive.[23]

Proposition 2. *Suppose $K_1 \in (0, t)$, $K_2 \in (0, t)$, $A < 0$, and Conditions 2A – 2D hold. Then an equilibrium exists in which all consumers purchase a phone from Firm 1. At this equilibrium, all consumers located in $(\frac{1}{2} + \frac{K_1}{2t}, 1]$ (and only these consumers) change the default setting on the phone they purchase. Furthermore, p_1 is marginally below $c_1 - r_L - 3A - K_1 > 0$; $p_2 = c_2 - r_H > 0$; $\pi_1 \approx \frac{t - K_1}{2t}[r_H - r_L] - 3A - K_1 > 0$; and $\pi_2 = 0$.*

To explain Proposition 2, it is helpful to identify two critical levels of advertising revenue:

$$r_1 \equiv \frac{1}{2}[r_L + r_H] - \frac{K_1}{2t}[r_H - r_L]; \quad r_2 \equiv \frac{1}{2}[r_L + r_H] + \frac{K_2}{2t}[r_H - r_L] > r_1. \tag{4}$$

r_i is the revenue that Firm $i \in \{1, 2\}$ receives from advertisers when $K_i < t$ and all consumers purchase a phone from Firm i. Recall from (2) that a consumer who purchases a phone from Firm 1 will change the default setting on the phone if and only if she is located in $(\frac{1}{2} + \frac{K_1}{2t}, 1]$. Therefore, Firm 1's revenue from advertisers when $K_1 < t$ and all consumers buy a phone from Firm 1 is $r_L\left[\frac{1}{2} + \frac{K_1}{2t}\right] + r_H\left[1 - \left(\frac{1}{2} + \frac{K_1}{2t}\right)\right] = r_1$. Analogous considerations explain the expression for r_2 in (4).

[22] (1) and (2) imply that when $K_1 < t$, the consumer located at 1 (and thus all consumers) will purchase a phone from Firm 1 rather than from Firm 2 if $G_1 - p_1 - K_1 > G_2 - p_2 \Leftrightarrow p_1 < p_2 + G_1 - G_2 - K_1$.

[23] This value of p_1 is just below $c_2 - r_H + G_1 - G_2 - K_1 = c_1 - r_L + G_1 - c_1 + r_L - (G_2 - c_2 + r_H) - K_1 = c_1 - r_L - 3A - K_1$.

Firm 2 must charge $p_2 \geq c_2 - r_2$ to profitably serve all consumers. Under the conditions specified in Proposition 2, Firm 1 can profitably attract all consumers by marginally undercutting any $p_2 \geq c_2 - r_2$. Firm 2 can profitably reduce p_2 below $c_2 - r_2$ if it does not serve consumers located in $[0, \frac{1}{2} - \frac{K_2}{2t}]$. In particular, Firm 2 can profitably reduce p_2 as low as $c_2 - r_H$ if it only serves consumers in $\left[\frac{1}{2} - \frac{K_2}{2t}, 1\right]$. (Recall from (3) that these consumers who do not change the default PD setting on a phone they purchase from Firm 2.) Therefore, Firm 2 will reduce p_2 to $c_2 - r_H$ to counteract an attempt by Firm 1 to attract all consumers by setting p_1 marginally below $p_2 + G_1 - G_2 - K_1$. In the unique equilibrium in which Firm 1 serves all consumers, Firm 2 sets $p_2 = c_2 - r_H$ and Firm 1 sets p_1 marginally below $p_2 + G_1 - G_2 - K_1$.[24]

Proposition 3 characterizes the outcomes that arise in the market-dominant equilibria in which all consumers purchase a phone from Firm 2. The proposition refers to the following conditions.

Condition 3A. $c_1 > r_1$. **Condition 3B**. $K_2 < G_2 - G_1 + c_1 - r_1$.

Condition 3C. $K_2 \left[\frac{2t - r_H + r_L}{2t} \right] < \Omega_2(p_1)$

for all $p_1 \in [c_1 - r_1, \, min\, \{ c_1 - r_L, \, c_1 - r_1 + K_1 + K_2 \}]$,

where $\Omega_2(p_1) \equiv p_1 + G_2 - G_1 - c_2 + \frac{1}{2} [r_H + r_L] - x_2(p_1)$, and

$x_2(p_1) \equiv \frac{1}{8t} [t + G_2 - G_1 - c_2 + r_H + p_1]^2 \geq 0.$

Condition 3A ensures that $c_1 - r_1$, the minimum price that Firm 1 can profitably charge when $K_1 < t$ and all consumers purchase a phone from Firm 1, is positive. Condition 3B ensures that the highest p_2 that induces all consumers to purchase a phone from Firm 2 when Firm 1 sets $p_1 = c_1 - r_1$ is positive.[25] Condition 3C ensures that Firm 2 secures positive profit when it successfully attracts all consumers by setting p_2 marginally below $p_1 + G_2 - G_1 - K_2$ when $p_1 = c_1 - r_L$.

Proposition 3. *Suppose $K_1 \in (0, t)$, $K_2 \in (0, t)$, $A > 0$, and Conditions 3A – 3C hold. Then a family of equilibria exist in which all consumers purchase a phone from Firm 2. In each of these equilibria, all consumers located in $[0, \frac{1}{2} - \frac{K_2}{2t})$ (and only these consumers) change the default setting on the phone they purchase. Furthermore, $p_1 \in [c_1 - r_1, \, min\, \{ c_1 -$*

[24] Recall that we restrict attention to undominated strategies for Firms 1 and 2. When identifying equilibrium strategies, we also assume that a firm will not change its price if it has no strict incentive to do so.

[25] This highest price is $c_1 - r_1 + G_2 - G_1 - K_2$. This is the case because (3) implies that when $K_2 < t$, the consumer located at 0 (and thus all consumers) will prefer to purchase a phone from Firm 2 rather than from Firm 1 if $G_2 - p_2 - K_2 > G_1 - p_1 \Leftrightarrow p_2 < p_1 + G_2 - G_1 - K_2$.

$r_L,\, c_1 - r_1 + K_1 + K_2 \}\,] > 0$; p_2 *is marginally below* $p_1 + G_2 - G_1 - K_2 > 0$; $\pi_1 = 0$; *and* $\pi_2 \approx p_1 - c_2 + G_2 - G_1 - K_2 + \frac{1}{2}[\, r_H + r_L \,] + \frac{K_2}{2t}[\, r_H - r_L \,] > 0$.

The multiplicity of equilibria identified in Proposition 3 reflect the following considerations. Suppose Firm 1 initially sets price $\widehat{p}_1 \in [\, c_1 - r_1 \,,\, c_1 - r_L \,]$. Then Firm 2 can set p_2 marginally below $\widehat{p}_2 = \widehat{p}_1 + G_2 - G_1 - K_2$, and thereby ensure that all consumers purchase a phone from Firm 2. In response, Firm 1 could conceivably reduce p_1 marginally below $\widehat{p}_2$ in an attempt to attract consumers. However, doing so would only attract consumers in $[\, 0, \frac{1}{2} - \frac{K_2}{2t} \,]$, who generate the lowest advertising payment, r_L. Firm 1 cannot profitably serve these consumers with a price below $c_1 - r_L$.

Conceivably, Firm 1 might consider a more pronounced reduction in p_1 in an attempt to attract all consumers, including those who generate the highest advertising revenue, r_H. Firm 1 would have to set $p_1 \leq p_2 + G_1 - G_2 - K_1$ to do so.[26] However, such low prices are not profitable for Firm 1 when $p_2 + G_1 - G_2 - K_1 < c_1 - r_1$.

In summary, when $p_1 = \widehat{p}_1 \in (\, c_1 - r_1 \,,\, c_1 - r_L \,]$ and Firm 2 sets p_2 marginally below $\widehat{p}_2 = \widehat{p}_1 + G_2 - G_1 - K_2$, Firm 2 will attract all consumers. Firm 1 cannot profitably reduce p_1 marginally below $\widehat{p}_2$ because doing so would primarily attract consumers who only generate advertising revenue r_L. Furthermore, Firm 1 cannot profitably reduce p_1 by an amount sufficient to attract all consumers when $p_2 + G_1 - G_2 - K_1 < c_1 - r_1$. Therefore, Firm 1 has no strict incentive to change $\widehat{p}_1 \in (\, c_1 - r_1 \,,\, c_1 - r_L \,]$ under these conditions. Consequently, multiple equilibria arise in which all consumers purchase a phone from Firm 2.[27]

5 The Effects of Default-Switching Costs on Profits

Having characterized the key features of market-sharing (MS) and market-dominant (MD) equilibria and the conditions under which they arise, we now examine how default-switching costs affect firms' profits in these equilibria. Proposition 4 reports that the impact of default-switching costs on profits can vary across firms and across equilibria. The proposition refers to: (i) $\pi_j^{Di}(K_1, K_2)$, which is Firm j's profit, given K_1 and K_2, in a MDi equilibrium, which is a MD equilibrium in which all consumers purchase a phone from Firm $i \in \{1, 2\}$; (ii) $\underline{\pi}_j^{Di}(K_1, K_2)$, which is the minimum value of $\pi_j^{Di}(K_1, K_2)$; and (iii) π_i^S, which is Firm i's profit in the MS equilibrium characterized in Proposition 1.

[26] Recall that (1) and (2) imply that a consumer located at 1 (and thus all consumers) will purchase a phone from Firm 1 when $K_1 < t$ if $G_1 - p_1 - K_1 > G_2 - p_2 \Leftrightarrow p_1 < p_2 + G_1 - G_2 - K_1$.

[27] In contrast, a unique equilibrium arises in the setting of Proposition 2, where Firm 1 is the advantaged firm. In this case, Firm 2 find it profitable to marginally undercut any p_1 above $c_2 - r_H$ at which Firm 1 is initially serving all cosnumers. This is the case because a marginal reduction in p_2 below p_1 enables Firm 2 to profitably attract close consumers who generate the high level of advertising revenue, r_H.

Proposition 4. *(i)* $\frac{\partial \pi_1^{D1}(\cdot)}{\partial K_1} < 0$ *and* $\frac{\partial \pi_1^{D1}(\cdot)}{\partial K_2} = 0$ *in the MD1 equilibrium characterized in Proposition 2; (ii)* $\frac{\partial \pi_2^{D2}(\cdot)}{\partial K_2} \gtreqless 0 \Leftrightarrow r_H - r_L \gtreqless 2t$ *and* $\frac{\partial \pi_2^{D2}(\cdot)}{\partial K_1} > 0$ *in the MD2 equilibria characterized in Proposition 3; and (iii)* $\frac{\partial \pi_1^{S}(\cdot)}{\partial K_1} = \frac{\partial \pi_1^{S}(\cdot)}{\partial K_2} = \frac{\partial \pi_2^{S}(\cdot)}{\partial K_2} = \frac{\partial \pi_2^{S}(\cdot)}{\partial K_1} = 0$ *in the MS equilibrium characterized in Proposition 1.*

Conclusion (i) in Proposition 4 indicates that when Firm 1 is the advantaged firm, its profit declines as its default-switching cost (K_1) increases in a MD1 equilibrium for two reasons. First, to continue to attract all relatively distant consumers (i.e., those located in $(\frac{1}{2} + \frac{K_1}{2t}, 1])$ who change the default PD setting on the phone they purchase from Firm 1, Firm 1 must reduce p_1 sufficiently to offset the higher default-switching cost that these customers experience. Second, the increase in K_1 induces more customers to retain Firm 1's default PD setting rather than switch to their preferred PD setting. Firm 1 receives a lower payment from advertisers (r_L rather than r_H) for every customer that retains Firm 1's default "privacy" setting rather than switching to the "disclosure" setting.

Conclusion (i) in Proposition 4 also indicates that Firm 1's profit is unaffected by changes in Firm 2's default-switching cost in the MD1 equilibrium identified in Proposition 2. This is the case because Firm 2 sets $p_2 = c_2 - r_H$ in this equilibrium, which is the lowest price that allows Firm 2 to secure nonnegative profit when it serves close customers who do not change the default PD setting on Firm 2's phone. Because this price does not vary with K_2, the price that Firm 1 must set to attract all consumers does not vary with K_2.[28]

In contrast, conclusion (ii) in Proposition 4 reports that Firm 2's profit can increase as the rival's default-switching cost (K_1) increases in the MD2 equilibria characterized in Proposition 3. This is the case because the lowest price that allows Firm 1 to secure nonnegative profit when it serves all consumers is $c_1 - r_1$. Recall that r_1 is the average per-customer advertising revenue that Firm 1 secures when it serves all consumers. This average advertising revenue declines as K_1 increases because more consumers retain Firm 1's default "privacy" setting (which generates the smaller advertising revenue, r_L, for Firm 1) as K_1 increases.[29] The reduction in its advertising revenue compels Firm 1 to increase p_1 to avoid negative profit. The increase in the lowest price that Firm 1 will charge enables Firm 2 to continue to attract all consumers with a higher price, which increases Firm 2's equilibrium profit.[30]

[28] Recall from Proposition 2 that this price is approximately $p_1 = p_2 + G_1 - G_2 - K_1 = c_2 - r_H + G_1 - G_2 - K_1 = c_1 - r_L - 3A - K_1$.

[29] Formally, (4) implies that $\frac{\partial r_1}{\partial K_1} = \frac{\partial}{\partial K_1}\left(\frac{1}{2}[r_L + r_H] - \frac{K_1}{2t}[r_H - r_L]\right) = -\frac{r_H - r_L}{2t} < 0$.

[30] Firm 2's maximum profit in the MD equilibria characterized in Proposition 3 also increases as K_1 increases if $c_1 - r_1 + K_1 + K_2 < c_1 - r_L$. Firm 2's profit increases with K_1 in this case in part because the higher default-switching costs that distant purchasers of Firm 1's phone would incur enable Firm 2 to attract close consumers with a higher price.

Conclusion (ii) in Proposition 4 also reports that an increase in K_2 can either increase or reduce Firm 2's minimum profit in the MD2 equilibria characterized in Proposition 3. This is the case because an increase in K_2 has two countervailing effects on $\underline{\pi}_2^{D2}(\cdot)$. First, an increase in K_2 reduces the utility that consumers located in $[\,0, \frac{1}{2} - \frac{K_2}{2\,t}\,)$ derive from Firm 2's phone because these consumers now incur a higher cost when they change the default PD setting on the phone they purchase from Firm 2 to their preferred "privacy" setting. This reduced utility requires a reduction in p_2 to ensure the continued patronage of these consumers, which reduces Firm 2's profit, *ceteris paribus*.

Second, the reduced switching away from Firm 2's default "disclosure" setting increases the payment that Firm 2 receives from advertisers. As conclusion (ii) in Proposition 4 reports, if the increase in per-customer advertising payment is sufficiently large (i.e., if $r_H - r_L > 2\,t$), then the reduced default-switching induced by an increase in K_2 increases Firm 2's minimum profit in the MD2 equilibria characterized in Proposition 3.

Conclusion (iii) in Proposition 4 indicates that changes in default-switching costs do not affect profits in the MS equilibrium characterized in Proposition 1. This conclusion reflects the fact that in this equilibrium, each firm sets a relatively high price and thereby secures a relatively high profit margin on close customers, while effectively ceding distant customers to the rival supplier. In particular, the consumer who is indifferent between purchasing a phone from Firm 1 and Firm 2 strictly prefers to retain the default PD setting on the phone she purchases.[31] Consequently, a marginal reduction in default-switching costs would not induce any consumer to change the default PD setting on the phone she purchases in this equilibrium. Therefore, the firms' profits do not change as default-switching costs change in this equilibrium.

6 Comparing Profits Across Equilibria

Having examined how profits vary with switching costs in MD and MS equilibria, we now compare the firms' profits in MD and MS equilibria. Doing so will allow us to identify conditions under which the advantaged firm would benefit from a unilateral increase in its default-switching cost. The ensuing analysis refers to the following specific magnitudes of firms' advantages and transportation costs.

$$|A|_{1L} \;\equiv\; 2\,t - \sqrt{3\,t^2 + 2\,t\,\Delta}\,. \qquad |A|_{1H} \;\equiv\; 2\,t + \sqrt{3\,t^2 + 2\,t\,\Delta}\,.$$

[31] Recall that $x_0 = \frac{1}{2} - \frac{A}{2\,t}$, $K_2 > A$, and $K_1 > |\,A\,|$ in the MS equilibrium characterized in Proposition 1. Consequently, $x_0 \in (\,\frac{1}{2} - \frac{K_2}{2\,t}, \frac{1}{2} + \frac{K_1}{2\,t}\,)$, so the consumer who is indifferent between purchasing a phone from Firm 1 and Firm 2 is located in the region where no consumer will change the default PD setting on the phone she purchases.

$$A_{2L} \equiv 2t - \sqrt{3t^2 - 2t\Delta}\,. \qquad A_{2H} \equiv 2t + \sqrt{3t^2 - 2t\Delta}\,.$$

$$t_{1L} \equiv \Delta + 2|A| - \sqrt{3|A|^2 + 4|A|\,\Delta + \Delta^2}\,. \; t_{1H} \equiv \Delta + 2|A| + \sqrt{3|A|^2 + 4|A|\,\Delta + \Delta^2}\,.$$

$$t_{2L} \equiv 2A - \Delta - \sqrt{3A^2 - 4A\Delta + \Delta^2}\,. \; t_{2H} \equiv 2A - \Delta + \sqrt{3A^2 - 4A\Delta + \Delta^2}\,. \tag{5}$$

Lemma 3. *Suppose the conditions in Proposition 1 hold and* $A < 0$. *Then* $\pi_1^S > \pi_1^{D1}(0,0)$ *if* $t > t_{1H}$, *whereas* $\pi_1^S < \pi_1^{D1}(0,0)$ *if* $t \in (K_1, t_{1H})$. *Furthermore, if* $t > 2\Delta$, *then* $\pi_1^S > \pi_1^{D1}(0,0)$ *if* $|A| < |A|_{1L}$, *whereas* $\pi_1^S < \pi_1^{D1}(0,0)$ *if* $|A| \in (|A|_{1L}, t)$. *If* $t \leq 2\Delta$, *then* $\pi_1^S < \pi_1^{D1}(0,0)$ *for all* $|A| \in (0, t)$.

Lemma 4. *Suppose the conditions in Proposition 1 hold and* $A > \Delta$. *Then: (i)* $\pi_2^S > \pi_2^{D2}(0,0)$ *if* $t > t_{2H}$ *or* $A < A_{2L}$; *and (ii)* $\pi_2^S < \pi_2^{D2}(0,0)$ *if* $t \in (K_2, t_{2H})$ *or* $A \in (A_{2L}, t)$.

Lemmas 3 and 4 identify conditions under which the advantaged firm secures more profit in the MS equilibrium that entails positive default-switching costs for the advantaged firm than in the MD equilibrium with no default-switching costs. The lemmas report that the advantaged firm benefits from higher switching costs when its advantage ($|A|$) is sufficiently limited or the extent of horizontal product differentiation (t) is sufficiently pronounced.

These conclusions reflect the fact that when a firm enjoys only a limited advantage over its rival, the firm must set a relatively low price to attract all consumers, which limits the profit the firm secures in the MD equilibrium. The firm with a limited advantage can earn more profit in the MS equilibrium when a relatively high default-switching cost makes it relatively unprofitable for the firm to attract distant consumers, so the firm sets a relatively high price and serves only close consumers.[32]

When the extent of horizontal product differentiation (t) is relatively pronounced and default-switching costs are relatively large (i.e., $K_1 > |A|$ and $K_2 > A$, as in Proposition 1), consumers are willing to pay a relatively high price for a phone with the consumer's preferred default PD setting. Consequently, when t is relatively large, the advantaged firm can secure more profit by setting a relatively high price and attracting only close customers than by setting the lower price required to attract all consumers.[33]

[32] When a relatively high default-switching cost prevails, a distant consumer will only purchase a phone with her less-preferred default PD setting if the price of the phone is relatively low. This is the case because the consumer recognizes before she purchases the phone that she will ultimately incur the relatively high default-switching cost to secure her preferred PD setting.

[33] The Appendix presents four corollaries to Lemmas 3 and 4. The corollaries explain how the range of

Lemmas 3 and 4 help to identify conditions under which, if default-switching costs were endogenous, a firm could enhance its profit by unilaterally increasing its default-switching cost, even in the absence of any cost-savings from doing so. In this sense, the firm could benefit from engaging in "self-sabotage" that renders its product less attractive to some consumers without increasing the product's appeal to any consumer. Proposition 5 specifies conditions under which Firm 2 can benefit from such self-sabotage when it is the advantaged firm and default-switching costs are initially 0.

Proposition 5. *Suppose: (i)* $G_2 - c_2 > G_1 - c_1$*; (ii)* $\max\{\Delta, r_H - c_2 - t\} < A < \min\{t, t + c_1 - r_L\}$*;*[34] *(iii)* $t > t_{2H}$*; and (iv)* $K_1 = 0$*. Then Firm 1 and Firm 2 both secure strictly greater profit in equilibrium when* $K_2 \in (A, t)$ *than when* $K_2 = 0$*.*

To explain in more detail why an increase in the default-switching cost of the advantaged firm (Firm 2) can increase the equilibrium profit of both industry suppliers, recall that when a consumer can costlessly change the default PD setting on the phone she purchases, the consumer perceives the phones to exhibit no horizontal product differentiation. In such a setting, the firms compete relatively aggressively to serve all consumers. This intense price competition can keep Firm 2's equilibrium profit relatively low, even though $G_2 - c_2 > G_1 - c_1$.[35]

By increasing K_2, Firm 2 ensures that consumers located in $(\frac{1}{2} - \frac{K_2}{2t}, \frac{1}{2})$ consider the products of Firms 1 and 2 to be horizontally differentiated. This is the case because the switching cost, $K_2 > 0$, deters these consumers from switching to their preferred PD setting ("privacy") if they buy a phone from Firm 2. Consequently, by increasing K_2, Firm 2 expands the region in which its competition with Firm 1 is effectively "softened" by perpetuating the default horizontal product differentiation.

The "softened" competition reflects in part the following consideration. By increasing K_2, Firm 2 reduces the utility that consumers in $(\frac{1}{2} - \frac{K_2}{2t}, \frac{1}{2})$ derive from purchasing Firm 2's phone (because these consumers will no longer switch to their preferred privacy setting if they purchase a phone from Firm 2). The associated reduced willingness to pay for Firm 2's phone implies that Firm 2 must reduce its price to attract these consumers. Firm 2 may find it unprofitable to do so because the price reduction applies to all consumers that Firm 2 attracts, not only those in $(\frac{1}{2} - \frac{K_2}{2t}, \frac{1}{2})$. Consequently, the increase in K_2 can effectively

settings in which default-switching costs increase the profit of the advantaged firm by inducing a MS equilibrium varies with model parameters.

[34] Recall that $\Delta \equiv r_H - r_L > 0$.

[35] Recall from Lemma 2 that when $K_1 = K_2 = 0$ and Firm 2 is the advantaged firm, Firm 2's equilibrium profit is $G_2 - c_2 - (G_1 - c_1)$. This profit approaches 0 as $G_2 - c_2$ approaches $G_1 - c_1$.

endow Firm 2 with a commitment to set a relatively high price that only attracts close customers, but generates a relatively high profit margin for Firm 2 on each phone that it sells. Because prices are strategic complements, Firm 2's relatively high price induces Firm 1 to set a relatively high price. This reduced pricing aggression can increase the equilibrium profit of both firms.

An increase in K_2 can deliver an additional benefit to Firm 2. As K_2 increases, fewer consumers who purchase Firm 2's phone change the default PD setting on their phone. Consequently, Firm 2 secures more advertising revenue (r_H rather than r_L) for each phone that it sells to a consumer who no longer changes Firm 2's default PD setting.

Conditions (ii) and (iii) in Proposition 5 require t to be sufficiently large. When t is large, the "softening" of competition in $(\frac{1}{2} - \frac{K_2}{2t}, \frac{1}{2})$ is relatively pronounced because each firm now has substantial market power over close consumers. Consequently, it becomes relatively unprofitable for a firm to attract distant consumers. The higher equilibrium prices that arise under softened competition increase Firm 2's profit.[36]

Proposition 6 provides a corresponding conclusion in the setting where Firm 1 is the advantaged firm.

Proposition 6. *Suppose: (i) $A < 0$; (ii) $r_H - c_2 - t < A < t + c_1 - r_L$; (iii) $t > |A|$; (iv) $t > t_{1H}$; and (v) $K_2 = 0$. Then Firm 1 and Firm 2 both secure strictly greater profit in equilibrium when $K_1 \in (|A|, t)$ than when $K_1 = 0$.*

The explanation for Proposition 6 parallels the explanation for Proposition 5 with one exception. In addition to reducing the attraction of its phone to relatively distant customers who switch the default PD setting, an increase in K_1 that expands the set of customers who do not switch the default PD setting reduces Firm 1's advertising revenue (because $r_L < r_H$). This consideration implies that Firm 1 is less likely to benefit from a unilateral increase in its default-switching cost than Firm 2 in the sense that condition (iv) in Proposition 6 is more restrictive than condition (iii) in Proposition 5.[37]

In summary, an increase in K_i has three key effects. First, it reduces the number of the most distant potential customers who will switch the default PD setting on a phone purchased from Firm i. Second, it increases the number of the less distant customers who will retain the default PD setting on a phone purchased from Firm i.[38] Third, it reduces the amount that

[36] Firm 1's profit also increases because the disadvantaged firm secures no profit in a MD equilibrium.

[37] It is readily verified that $t_{1H} > t_{2H}$ when the $|A|$ in t_{1H} and the A in t_{2H} have the same magnitude.

[38] Distant customers for Firm 1 here are those located in $(\frac{1}{2}, 1]$. The most distant of these customers are those located in $(\frac{1}{2} + \frac{K_1}{2t}, 1]$. The less distant of these customers are those located in $(\frac{1}{2}, \frac{1}{2} + \frac{K_1}{2t}]$.

the most distant consumers are willing to pay for Firm i's phone (because they anticipate incurring the (increased) default-switching cost). These three effects imply that an increase in K_i reduces the attraction to Firm i of competing for the most distant potential customers (given their reduced number and their reduced willingness to pay for Firm i's phone).[39] This reduced attraction can serve as a credible commitment not to compete aggressively for the entire market (and for the most distant customers, in particular), which can induce the rival supplier to compete less aggressively in return.

7 The Setting with Endogenous K

We now extend the foregoing analysis by allowing default-switching costs to be endogenous. Formally, we examine equilibrium outcomes in the *setting with endogenous* K, where the interaction between Firm 1 and Firm 2 proceeds in two stages. In the first stage, Firm 1 chooses K_1 and Firm 2 chooses K_2, simultaneously and noncooperatively. In the second stage, Firm 1 chooses p_1 and Firm 2 chooses p_2, simultaneously and noncooperatively. For simplicity, we abstract from any costs that a firm might incur to change the default-switching cost on its phone.

Proposition 7 identifies conditions under which the market-sharing (MS) equilibrium characterized in Proposition 1 persists in the setting with endogenous K. The conditions include t being sufficiently large relative to $|A|$. The relatively large transport cost implies that a firm must set a relatively low price to secure the patronage of all consumers, even after reducing its default-switching cost to 0. Consequently, rather than eliminating its default-switching cost and competing to attract all consumers, each firm prefers to implement a relatively high default-switching cost and charge a relatively high price that serves to attract (only) relatively close consumers in a MS equilibrium.

Proposition 7. *Suppose: (i)* (K_1^*, K_2^*) *are such that conditions (ii) and (iii) in Proposition 1 hold; (ii)* $t > |A|$*; (iii) Condition 1A holds; (iv) Condition 1B holds when* $K_1 = K_2 = 0$*; and (v)* $\Delta < 2t$*. Then* (K_1^*, K_2^*)*, along with* $p_1 = c_1 - r_L + t - A$ *and* $p_2 = c_2 - r_H + t + A$*, constitute a MS equilibrium in the setting with endogenous* K*.*

Propositions 8 and 9 identify conditions under which market-dominant equilibria with 0 default-switching costs arise in the setting with endogenous K. When a firm's advantage is sufficiently pronounced relative to t, the firm can secure greater profit by eliminating its default-switching cost and competing to attract all consumers than by adopting a default-

[39] The reduced attraction is particularly pronounced for Firm 2 because its most distant potential customers switch away from the relatively lucrative default PD setting if they purchase a phone from Firm 2.

switching cost that gives rise to a MS equilibrium. When $\Delta < 2t$, the advertising revenue that Firm 2 foregoes by allowing customers to costlessly switch the default PD setting on its phone is less than the extra profit the firm secures from the higher price that distant consumers will pay for Firm 2's phone when they can costlessly implement their preferred PD setting on the phone.[40]

Proposition 8. *Suppose* $A < 0$, $G_1 - c_1 - (G_2 - c_2) > \max\{\frac{\Delta}{2}, \frac{1}{2t}[t + |A|]^2\}$, $c_2 > \frac{1}{2}[r_L + r_H]$, *and* $c_1 > r_L$. *Then in the setting with endogenous* K, *there exists a MD1 equilibrium in which: (i)* $K_1 = K_2 = 0$; *(ii)* $p_2 = c_2 - \frac{1}{2}[r_L + r_H]$; *and (iii)* p_1 *is marginally below* $c_2 - r_H + G_1 - G_2$.

Proposition 9. *Suppose* $A > 0$, $G_2 - c_2 - (G_1 - c_1) > \frac{1}{2t}[t + A]^2$, $c_2 > r_H$, $c_1 > \frac{1}{2}[r_L + r_H]$, *and* $\Delta < 2t$. *Then in the setting with endogenous* K, *there exists a MD2 equilibrium in which: (i)* $K_1 = K_2 = 0$; *(ii)* $p_1 = c_1 - \frac{1}{2}[r_L + r_H]$; *and (iii)* p_2 *is marginally below* $c_1 - \frac{1}{2}[r_L + r_H] + G_2 - G_1$.

Proposition 10 reports that when $\Delta > 2t$, Firm 2 can prefer to compete to attract all consumers by raising K_2 sufficiently to ensure that no customer switches the default PD setting on Firm 2's phone. By doing so, Firm 2 secures a larger increase in advertising revenue than the reduction in sales revenue it incurs when it must reduce p_2 to compensate its distant customers for retaining their less-preferred PD setting on the phone they purchase from Firm 2.

Proposition 10. *Suppose: (i)* $K_1 = 0$ *and* $K_2 = K_2^* \geq t$; *(ii) Condition 3A holds; (iii)* $t < G_2 - G_1 + c_1 - r_1$; *(iv)* $\frac{2t - \Delta}{2} < \Omega_2(p_1)$ *for all* $p_1 \in [c_1 - r_1, \min\{c_1 - r_L, c_1 - r_1 + K_1 + t\}]$;[41] *(v)* $G_2 - c_2 > G_1 - c_1$; *and (vi)* $\Delta > 2t$. *Then in the setting with endogenous* K, $(0, K_2^*)$ *and* (p_1, p_2) *prices such that* $p_1 \in [c_1 - r_1, \min\{c_1 - r_L, c_1 - r_1 + t\}]$ *and* p_2 *is marginally below* $p_1 + G_2 - G_1 - t$ *constitute a MD2 equilibrium.*

Before concluding, we demonstrate that some equilibria that can arise when K_1 and K_2 are exogenous cannot exist in the setting with endogenous K. Proposition 11 identifies conditions under which a MD1 equilibrium in which $K_1 > 0$ does not exist. In any such putative equilibrium, Firm 1 could increase its profit by reducing K_1. The reduction in K_1

[40] Recall from Proposition 4 that Firm 1's profit when it serves all consumers declines with K_1. An increase in K_1 both reduces the attraction of Firm 1's phone to distant consumers and reduces Firm 1's advertising revenue (because $r_L < r_H$).

[41] Recall that Ω_2 is defined by Condition 3C.

would allow Firm 1 to: (i) increase its revenue by attracting distant customers with a higher p_1; and (ii) increase the payments that Firm 1 secures from advertisers by inducing more distant customers to change the default "privacy" setting on the phone they purchase from Firm 1.

Proposition 11. *Suppose (i) $A < 0$; (ii) $t < |A|$; (iii) $G_1 - c_1 - (G_2 - c_2) > \frac{\Delta}{2}$; (iv) Conditions 2A and 2B hold; (v) $t < c_1 - r_L - 3A$; (vi) and (vii) $t \leq \frac{2t}{2t + r_H - r_L}\, \Omega_1$.[42] Then a MD1 equilibrium in which $K_1 > 0$ and $K_2 \geq 0$ does not exist in the setting with endogenous K.*

Proposition 12 identifies conditions under which a MD2 equilibrium in which $K_2 > 0$ does not exist when $\Delta < 2t$. In any such putative equilibrium, Firm 2 could increase its profit by reducing K_2. The smaller default-switching cost would allow Firm 2 to attract distant consumers with a higher p_2. When $\Delta < 2t$, the associated increase in revenue exceeds the reduction in payments that advertisers deliver to Firm 2 due to the increased switching of the default "disclosure" setting on Firm 2's phone.

Proposition 12. *Suppose (i) $G_2 - c_2 - (G_1 - c_1) > \frac{\Delta}{2}$; (ii) $2t > \Delta$; (iii) $c_1 > \frac{1}{2}[r_H + r_L]$; (iv) $t < G_2 - G_1 + c_1 - \frac{1}{2}[r_H + r_L]$; (v) Condition 3C holds if $t < K_2$; and (vi) $\frac{1}{2}[2t - \Delta] < \Omega_2(p_1)$ for all $p_1 \in [c_1 - r_1, \min\{c_1 - r_L, c_1 - r_1 + K_1 + t\}]$ if $t \geq K_2$.[43] Then a MD2 equilibrium in which $K_1 \in [0, t)$, $K_2 > 0$, and $p_1 = c_1 - r_1$ does not exist in the setting with endogenous K.*

Proposition 13 identifies conditions under which a MD2 equilibrium in which $K_2 < t$ does not exist when $\Delta > 2t$. In any such putative equilibrium, Firm 2 could increase its profit by increasing K_2. The higher default-switching cost would require Firm 2 to reduce p_2 to continue to attract distant consumers. However, when $\Delta > 2t$, the associated reduction in revenue is outweighed by the increase in payments that advertisers deliver to Firm 2 due to the reduced switching of the default "disclosure" setting on Firm 2's phone.

Proposition 13. *Suppose (i) $G_2 - c_2 - (G_1 - c_1) > 0$; (ii) $\Delta > 2t$; and (iii) Conditions 3A – 3C hold. Then a MD2 equilibrium in which $K_1 \in [0, t)$, $K_2 \in [0, t)$, and $p_1 = c_1 - r_1$ does not exist in the setting with endogenous K.*

[42] Recall that Ω_1 is defined in Condition 2B.

[43] Recall that $\Omega_2(p_1)$ is defined in Condition 3C.

8 Conclusions

We have analyzed a streamlined model of Hotelling competition between two suppliers of smartphones. Our model differs from the standard model of Hotelling competition primarily by allowing customers to change the default privacy/disclosure (PD) setting on the phone they purchase. When consumers can costlessly change this setting, horizontal product differentiation is effectively eliminated. Relatively intense competition can ensue, giving rise to market dominant equilibria is which all consumers purchase a phone from the same supplier.

Non-trivial default-switching costs restore meaningful horizontal product differentiation, which can enhance supplier profit under certain conditions and can induce market sharing equilibria in which both suppliers serve consumers. The supplier with the default “disclosure” setting can be particularly likely to benefit from higher default-switching costs because the reduced default switching that occurs in equilibrium ensures that the supplier secures higher payments from advertisers.

Our finding that the supplier with the default “disclosure” setting can be relatively likely to benefit from an increase in its default-switching costs is consistent with observed industry practice. Changing the default PD setting is generally thought to be more onerous on Android phones than on iPhones. An iPhone user need only respond to a prompt that appears automatically when the user launches an app. In contrast, an Android user must locate and choose the relevant option in the Settings menu on the phone. The user is not automatically prompted to do so.[44]

Our model was intentionally streamlined to facilitate both a tractable analysis and a focus on the effects of default switching. Future research might consider several extensions of our model. Specifically, additional dimensions of consumer heterogeneity might be admitted. When consumers have different incomes or different innate valuations of smartphone services, for example, market sharing equilibria may arise even in the absence of default-switching costs. Market sharing equilibria may also be relatively likely to arise in the presence of additional dimensions of horizontal product differentiation (e.g., phone size, shape, and color).[45]

Future research might also consider endogenous default PD settings, default-switching costs that vary across customers, supplier costs of reducing customer default-switching costs, and repeat purchases of ever-evolving smartphones. These model extensions can help to as-

[44] See Grant and *Bloomberg* (2021) and O'Flaherty (2021), for example. Athey et al. (2017) observe that small navigation frictions can substantially reduce the likelihood that users change default settings on smartphones.

[45] Market sharing equilibria may also be relatively likely to arise if consumers are concentrated near the center of the $[0,1]$ interval, rather than uniformly distributed in this interval.

sess the robustness of our findings and perhaps provide new insights about equilibrium outcomes when firms compete in settings where consumers can alter default levels of horizontal product differentiation.

Appendix

Part A of this Appendix presents four corollaries to Lemmas 3 and 4. Part B of this Appendix outlines the proofs of the formal conclusions in the text. Chakravorty and Sappington (2023) provides the proofs of the corollaries to Lemmas 3 and 4, along with detailed proofs of the formal conclusions in the text

A. Corollaries to Lemmas 3 and 4.

The following Corollaries to Lemmas 3 and 4 explain how the range of settings in which default-switching costs increase the profit of an advantaged firm by inducing a MS equilibrium varies with model parameters.

Corollary 3.1. $\frac{dt_{1H}}{d|A|} > 0$ and $\frac{dt_{1H}}{d\Delta} > 0$, so *the range of t realizations for which default-switching costs increase Firm 1's profit in the setting of Lemma 3 (i.e., $t > t_{1H}$) contracts as $|A|$ increases or as Δ increases.*

Corollary 3.2. $\frac{d|A|_{1L}}{d\Delta} < 0$. *Furthermore,* $\frac{d|A|_{1L}}{dt} > 0$ *if* $\Delta < \frac{t}{2}$. Therefore, *the range of $|A|$ realizations for which default-switching costs increase Firm 1's profit in the setting of Lemma 3 (i.e., $|A| < |A|_{1L}$): (i) contracts as Δ increases; and (ii) expands as t increases if $\Delta < \frac{t}{2}$.*

Corollary 4.1. $\frac{dt_{2H}}{d\Delta} < 0$ *and* $\frac{dt_{2H}}{dA} > 0$*, so the range of t realizations for which default-switching costs increase Firm 2's profit in the setting of Lemma 4 (i.e., $t > t_{2H}$) expands as Δ increases or as A declines.*

Corollary 4.2. $\frac{dA_{2L}}{dt} > 0$ *and* $\frac{dA_{2L}}{d\Delta} > 0$*, so the range of A realizations for which default-switching costs increase Firm 2's profit in the setting of Lemma 4 (i.e., $A < A_{2L}$) expands as t increases or as Δ increases.*

Corollaries 3.1 and 4.1 report that as a firm's advantage ($|A|$) increases, the range of t realizations ($t > t_{1H}$ or $t > t_{2H}$) in which default-switching costs increase the advantaged firm's profit contracts. This is the case because the increased advantage increases the firm's equilibrium profit more rapidly in the MD equilibrium where it serves all consumers than in the MS equilibrium where it serves fewer consumers.

Corollaries 3.2 and 4.2 report that the range of advantage levels for which default-switching costs increase the advantaged firm's profit expands as t increases if: (i) Firm 2 is the advantaged firm; or (ii) Firm 1 is the advantaged firm and $\Delta < \frac{t}{2}$. The first conclusion arises because an increase in horizontal product differentiation reduces the intensity of price competition in the MS equilibrium, where the firms effectively focus on attracting close consumers. The reduced competitive intensity serves to increase the profit of Firm 2, which receives the large payment from advertisers (r_H) for every phone it sells in the MS equilibrium. The positive impact of reduced competitive intensity outweighs the countervailing effect that Firm 2 must reduce its price to attract customers as t increases, *ceteris paribus*. The same net effect for Firm 1 prevails when Δ is sufficiently small, so the advertising revenue that Firm 1 receives for each phone that it sells in the MS equilibrium is not too much smaller than the corresponding revenue that Firm 2 secures.

Corollaries 4.1 and 4.2 report that as the difference in advertising payments ($\Delta \equiv r_H - r_L$) increases, the range of t realizations ($t > t_{2H}$) and the range of A realizations ($A < A_{2L}$) in which default-switching costs increase Firm 2's profit both expand. This is the case because Firm 2's profit increases relatively rapidly in the MS equilibrium as Δ increases due to an increase in r_H because every consumer that Firm 2 serves in the MS equilibrium generates advertising revenue r_H.

Corollaries 3.1 and 3.2 report that as Δ increases, the range of t realizations ($t > t_{1H}$) and the range of $|A|$ realizations ($|A| < |A|_{1L}$) in which default-switching costs increase Firm 1's profit both contract. This is the case because Firm 1 secures advertising revenue r_L for every phone it sells in the MS equilibrium, and this revenue declines as Δ increases, holding r_H constant.

B. Proofs of Formal Conclusions in the Text.

The following lemmas (Lemmas A1 – A18) are employed to prove the formal conclusions in the text.[46]

Lemma A1. A user who buys a phone from Firm 2 will change the default setting on the phone if and only if the user is located in $[\,0, \frac{1}{2} - \frac{K_2}{2t}\,)$.

Lemma A2. A user who buys a phone from Firm 1 will change the default setting on the phone if and only if the user is located in $(\,\frac{1}{2} + \frac{K_1}{2t}, 1\,]$.

Lemma A3. A user located in $\left[\,\frac{1}{2} - \frac{K_2}{2t}, \frac{1}{2} + \frac{K_1}{2t}\,\right]$ will not change the default setting on the phone she purchases.

Lemma A4. Suppose a user located at $x_0 \in [\,\frac{1}{2} - \frac{K_2}{2t}, \frac{1}{2} + \frac{K_1}{2t}\,]$ is indifferent between buying a phone from Firm 1 and from Firm 2. Then: (i) all users located in $[\,0, x_0\,)$ will buy a phone from Firm 1; and (ii) all users located in $[\,x_0, 1\,]$ will buy a phone from Firm 2.

Lemma A5. Suppose a user located at $x_1 \in [\,0, \frac{1}{2} - \frac{K_2}{2t}\,)$ is indifferent between buying a phone from Firm 1 and from Firm 2. Then: (i) all users located in $[\,0, \frac{1}{2} - \frac{K_2}{2t}\,]$ are similarly indifferent; and (ii) all users located in $(\,\frac{1}{2} - \frac{K_2}{2t}, 1\,]$ will buy a phone from Firm 2.

[46] Chakravorty and Sappington (2023) provides the proofs of Lemmas A1 - A18.

Lemma A6. Suppose a user located at $x_2 \in (\frac{1}{2} + \frac{K_1}{2t}, 1]$ is indifferent between buying a phone from Firm 1 and from Firm 2. Then: (i) all users located in $[\frac{1}{2} + \frac{K_1}{2t}, 1]$ are similarly indifferent; and (ii) all users located in $[0, \frac{1}{2} + \frac{K_1}{2t})$ will buy a phone from Firm 1.

Lemma A7. If $p_1 \geq p_2 + G_1 - G_2 + K_2$, then all users located in $[0, \frac{1}{2} - \frac{K_2}{2t})$ (weakly) prefer to buy a phone from Firm 2 than from Firm 1. The preference is strict if the inequality holds strictly.

Lemma A8. If $p_1 \geq p_2 + G_1 - G_2 + K_2$, then all users located in $[\frac{1}{2} - \frac{K_2}{2t}, 1]$ (weakly) prefer to buy a phone from Firm 2 than from Firm 1. The preference is strict if the inequality holds strictly.

Lemma A9. If $p_2 \geq p_1 + G_2 - G_1 + K_1$, then all users located in $(\frac{1}{2} + \frac{K_1}{2t}, 1]$ (weakly) prefer to buy a phone from Firm 1 than from Firm 2. The preference is strict if the inequality holds strictly.

Lemma A10. If $p_2 \geq p_1 + G_2 - G_1 + K_1$, then all users located in $[0, \frac{1}{2} + \frac{K_1}{2t}]$ (weakly) prefer to buy a phone from Firm 1 than from Firm 2. The preference is strict if the inequality holds strictly.

Assumption 1. $K_1 \in [0, t)$, $K_2 \in [0, t)$, and $(K_1, K_2) \neq (0, 0)$.

Lemma A11. Suppose Assumption 1 holds. Then in any equilibrium in which $p_1 - p_2 \in (G_1 - G_2 - K_1, G_1 - G_2 + K_2)$: (i) all users located in $[0, \frac{1}{2} - \frac{K_2}{2t})$ strictly prefer to buy a phone from Firm 1 than from Firm 2; (ii) all users located in $(\frac{1}{2} + \frac{K_1}{2t}, 1]$ strictly prefer to buy a phone from Firm 2 than from Firm 1; and (iii) some user located in $[\frac{1}{2} - \frac{K_2}{2t}, \frac{1}{2} + \frac{K_1}{2t}]$ is indifferent between buying a phone from Firm 1 and from Firm 2.

Lemma A12. When Assumption 1 holds: (i) $c_1 - r_1$ is the lowest price that Firm 1 can profitably charge when all users buy a phone from Firm 1; and (ii) $c_2 - r_2$ is the lowest price that Firm 2 can profitably set when all users buy a phone from Firm 2.

Lemma A13. Suppose Assumption 1 holds. Then $c_1 - r_L > c_1 - r_1$ and $c_2 - r_H < c_2 - r_2$.

Lemma A14. Suppose Assumption 1 holds. Then: (i) setting $p_1 < c_1 - r_1$ is a weakly dominated strategy for Firm 1; and (ii) setting $p_2 < c_2 - r_H$ is a weakly dominated strategy for Firm 2.

Lemma A15. Suppose Assumption 1 holds. Then an equilibrium does not exist in which $p_1 \geq p_2 + G_1 - G_2 + K_2$ and $p_1 > c_1 - r_L$.

Lemma A16. Suppose Assumption 1 holds and $G_2 - G_1 + c_1 - c_2 - K_2 \neq 0$. Then an equilibrium does not exist in which $p_1 = p_2 + G_1 - G_2 + K_2$ and $p_1 = c_1 - r_L$.

Lemma A17. Suppose Assumption 1 holds. An equilibrium does not exist in which $p_1 = p_2 + G_1 - G_2 + K_2$ and $p_1 \leq c_1 - r_1$.

Lemma A18. Suppose Assumption 1 holds. Then an equilibrium does not exist in which $p_2 \geq p_1 + G_2 - G_1 + K_1$ and $p_2 \neq c_2 - r_H$.

Proof of Lemma 1. The proof follows from the following lemmas (Lemmas A1.1 – A1.6).

Lemma A1.1. *Suppose the default privacy setting cannot be changed. Then: (i) all users buy a phone from Firm 1 if* $p_2 - p_1 > G_2 - G_1 + t$*; and (ii) all users buy a phone from Firm 2 if* $p_2 - p_1 < G_2 - G_1 - t$.

Proof. To prove conclusion (i), observe that all users buy a phone from Firm 1 if, for all $x \in [0,1]$:

$$G_1 - t\,x - p_1 > G_2 - t[1-x] - p_2 \quad \Leftrightarrow \quad x < \frac{1}{2} + \frac{1}{2\,t}[G_1 - G_2 - p_1 + p_2]. \tag{6}$$

(6) holds for all $x \in [0,1]$ if:

$$1 < \frac{1}{2} + \frac{1}{2\,t}[G_1 - G_2 - p_1 + p_2] \quad \Leftrightarrow \quad p_2 - p_1 > G_2 - G_1 + t.$$

The proof of conclusion (ii) is analogous. □

Lemma A1.2. Suppose the default privacy setting cannot be changed and $t > 3\,|A|$. Then no equilibrium exists in which one firm serves all users.

Proof. First suppose Firm 1 serves all users. Then Lemma A1.1 implies that for all p_2 that generate nonnegative profit for Firm 2:

$$p_1 \leq p_2 + G_1 - G_2 - t. \tag{7}$$

(7) holds for all such p_2 if:

$$p_1 \leq c_2 - r_H + G_1 - G_2 - t. \tag{8}$$

Firm 1's profit when it serves all users at a price that satisfies (8) is:

$$\begin{aligned}\pi_1 &= p_1 + r_L - c_1 \leq c_2 - r_H + G_1 - G_2 - t + r_L - c_1 \\ &= G_1 + r_L - c_1 - (G_2 + r_H - c_2) - t < 0 \text{ when } t > 3\,|A|. \end{aligned} \tag{9}$$

(9) implies that no equilibrium exists in which Firm 1 serves all users.

The proof for the case where Firm 2 serves all users is analogous. $\square$

Lemma A1.3. Suppose the default privacy setting cannot be changed and $p_2 - p_1 \in [\, G_2 - G_1 - t,\, G_2 - G_1 + t\,]$. Then: (i) a user located at $x_0 \equiv \frac{1}{2t}[\, t + G_1 - G_2 + p_2 - p_1\,] \in [\,0, 1\,]$ is indifferent between purchasing a phone from Firm 1 and from Firm 2; (ii) if $x_0 > 0$, all users located in $[\,0, x_0\,)$ buy a phone from Firm 1; and (iii) if $x_0 < 1$, all users located in $(\,x_0, 1\,]$ buy a phone from Firm 2.

Proof. A user located at x is indifferent between purchasing a phone from Firm 1 and from Firm 2 if:

$$G_1 - t\,x - p_1 \;=\; G_2 - t\,[\,1 - x\,] - p_2 \;\;\Leftrightarrow\;\; t\,[\,1 - 2\,x\,] \;=\; G_2 - G_1 - p_2 + p_1$$

$$\Leftrightarrow\;\; 1 - 2\,x \;=\; \frac{1}{t}\,[\,G_2 - G_1 - p_2 + p_1\,] \;\;\Leftrightarrow\;\; x \;=\; \frac{1}{2\,t}\,[\,t + G_1 - G_2 + p_2 - p_1\,] \;\equiv\; x_0$$

$$\Rightarrow\;\; x_0 \;\in\; [\,0, 1\,] \;\;\Leftrightarrow\;\; p_2 - p_1 \;\in\; [\,G_2 - G_1 - t,\, G_2 - G_1 + t\,]\,.$$

If $x_0 > 0$, then a user located at $x \in [\,0, x_0\,)$ buys a phone from Firm 1 because:

$$G_1 - t\,x - p_1 \;>\; G_2 - t\,[\,1 - x\,] - p_2 \;\;\Leftrightarrow\;\; x \;<\; \frac{1}{2\,t}\,[\,t + G_1 - G_2 + p_2 - p_1\,] \;=\; x_0\,.$$

If $x_0 < 1$, then a user located at $x \in (\,x_0, 1\,]$ buys a phone from Firm 2 because:

$$G_2 - t\,[\,1 - x\,] - p_2 \;>\; G_1 - t\,x - p_1 \;\;\Leftrightarrow\;\; x \;>\; \frac{1}{2\,t}\,[\,t + G_1 - G_2 + p_2 - p_1\,] \;=\; x_0\,. \;\square$$

Lemma A1.4. Suppose the default privacy setting cannot be changed and $t > 3\,|\,A\,|$. Then in equilibrium, there exists a $x_0 \in [\,0, 1\,]$ such that: (i) a user located at x_0 is indifferent between buying a phone from Firm 1 and from Firm 2; (ii) all users located in $[\,0, x_0\,)$ buy a phone from Firm 1; and (iii) all users located in $(\,x_0, 1\,]$ buy a phone from Firm 2. Furthermore: $p_1 = c_1 - r_L + t - A$; $p_2 = c_2 - r_H + t + A$; $\pi_1 = \frac{1}{2t}[\,t - A\,]^2$; and $\pi_2 = \frac{1}{2t}[\,t + A\,]^2$.

Proof. Lemma A1.2 implies that Firm 1 and Firm 2 both serve some users in equilibrium. Therefore, Lemma A1.1 implies that $p_2 - p_1 \in [\,G_2 - G_1 - t,\, G_2 - G_1 - t\,]$. Consequently, Lemma A1.3 implies that a user located at

$$x_0 \;\equiv\; \frac{1}{2\,t}\,[\,t + G_1 - G_2 + p_2 - p_1\,] \;\in\; [\,0, 1\,] \tag{10}$$

is indifferent between purchasing a phone from Firm 1 and from Firm 2. Furthermore, all users located in $[0, x_0)$ buy a phone from Firm 1, and all users located in $(x_0, 1]$ buy a phone from Firm 2. Therefore, (10) implies that Firm 1's profit is:

$$\pi_1 = [p_1 + r_L - c_1] \frac{1}{2t} [t + G_1 - G_2 + p_2 - p_1]. \tag{11}$$

The unique value of p_1 that maximizes π_1 in (11) is given by:

$$\frac{\partial \pi_1}{\partial p_1} = 0 \Leftrightarrow p_1 = \frac{1}{2} [t + c_1 - r_L + G_1 - G_2 + p_2]. \tag{12}$$

(10) and Lemma A1.3 imply that Firm 2's profit is:

$$\pi_2 = [p_2 + r_H - c_2] \frac{1}{2t} [t + G_2 - G_1 + p_1 - p_2]. \tag{13}$$

The unique value of p_2 that maximizes π_2 in (13) is given by:

$$\frac{\partial \pi_2}{\partial p_2} = 0 \Leftrightarrow p_2 = \frac{1}{2} [t + c_2 - r_H + G_2 - G_1 + p_1]. \tag{14}$$

(12) and (14) imply:

$$p_1 = \frac{1}{2} [t + c_1 - r_L + G_1 - G_2] + \frac{1}{4} [t + c_2 - r_H + G_2 - G_1 + p_1]$$

$$\Rightarrow p_1 = \frac{1}{3} [3t - 3A + 3(c_1 - r_L)] = c_1 - r_L + t - A. \tag{15}$$

(14) and (15) imply:

$$p_2 = \frac{1}{3} [3t + 3A + 3(c_2 - r_H)] = c_2 - r_H + t + A. \tag{16}$$

(15) implies that Firm 1's profit margin is positive because:

$$p_1 + r_L - c_1 = t - A > 0. \tag{17}$$

(16) implies that Firm 2's profit margin is positive because:

$$p_2 + r_H - c_2 = t + A > 0. \tag{18}$$

(15) and (16) imply:

$$p_2 - p_1 = \frac{1}{3} [c_2 - c_1 + 2G_2 - 2G_1 + r_L - r_H]. \tag{19}$$

(15), (19), and Lemma A1.3 imply:

$$\pi_1 = \frac{1}{3} [3t + 2c_1 + c_2 + G_1 - G_2 - 2r_L - r_H + 3r_L - 3c_1]$$

$$\cdot \ \frac{1}{2\,t}\left[\, t + G_1 - G_2 + \frac{1}{3}\,(\, c_2 - c_1 + 2\,G_2 - 2\,G_1 + r_L - r_H \,)\right] \ = \ \frac{1}{2\,t}\,[\, t - A\,]^2\,.$$

(16), (19), and Lemma A1.3 imply:

$$\pi_2 \ = \ \frac{1}{3}\,[\,3\,t + 2\,c_2 + c_1 + G_2 - G_1 - 2\,r_H - r_L + 3\,r_H - 3\,c_2\,]$$

$$\cdot \ \frac{1}{2\,t}\left[\, t + G_2 - G_1 + \frac{1}{3}\,(\, c_1 - c_2 + 2\,G_1 - 2\,G_2 + r_H - r_L \,)\right] \ = \ \frac{1}{2\,t}\,[\, t + A\,]^2\,. \ \square$$

Lemma A1.5. Suppose the default privacy setting cannot be changed, $A < 0$, and $t \in (\,|\,A\,|\,, 3\,|\,A\,|\,)$. Then at the unique equilibrium, both firms sell phones, Firm 1's profit is $\pi_1 = \frac{1}{2\,t}\,[\,t - A\,]^2 > 0$ and Firm 2's profit is $\pi_2 = \frac{1}{2\,t}\,[\,t + A\,]^2 > 0$.

Proof. First suppose that an equilibrium exists in which all users buy a phone from Firm 2. Then because the user located at 0 buys a phone from Firm 2:

$$G_2 - p_2 - t \ > \ G_1 - p_1 \ \ \Leftrightarrow \ \ p_2 \ < \ p_1 + G_2 - G_1 - t\,. \tag{20}$$

(20) must hold for all p_1 for which Firm 1's profit margin is positive. Therefore:

$$p_2 \ < \ c_1 - r_L + G_2 - G_1 - t \ \equiv \ \widehat{p}_2\,. \tag{21}$$

Firm 2's profit when it sets a price marginally below $\widehat{p}_2$ is nearly:

$$\pi_2 \ = \ \widehat{p}_2 + r_H - c_2 \ = \ c_1 - r_L + G_2 - G_1 - t + r_H - c_2$$

$$= \ G_2 + r_H - c_2 - (\,G_1 + r_L - c_1\,) - t \ = \ 3\,A - t \ < \ 0\,. \tag{22}$$

(22) implies that an equilibrium in which all users buy a phone from Firm 2 does not exist under the specified conditions.

Now suppose that an equilibrium exists in which all users buy a phone from Firm 1. Then because the user located at 1 buys a phone from Firm 1:

$$G_1 - p_1 - t \ > \ G_2 - p_2 \ \ \Leftrightarrow \ \ p_1 \ < \ p_2 + G_1 - G_2 - t\,. \tag{23}$$

(23) must hold for all p_2 for which Firm 2's profit margin is positive. Therefore:

$$p_1 \ < \ c_2 - r_H + G_1 - G_2 - t \ \equiv \ \widehat{p}_1\,. \tag{24}$$

Firm 1's profit when it sets a price marginally below $\widehat{p}_1$ is nearly:

$$\pi_1 \ = \ \widehat{p}_1 + r_L - c_1 \ = \ c_2 - r_H + G_1 - G_2 - t + r_L - c_1$$

$$= \ G_1 + r_L - c_1 - (\,G_2 + r_H - c_2\,) - t \ = \ -\,3\,A - t \ > \ 0\,. \tag{25}$$

If a user located at $x \in [\,0, 1\,]$ is indifferent between purchasing a phone from Firm 1 and from Firm 2, then:

$$G_1 - t\,x - p_1 \;=\; G_2 - t\,[\,1 - x\,] - p_2 \quad\Leftrightarrow\quad x \;=\; \frac{1}{2\,t}\,[\,t + G_1 - G_2 + p_2 - p_1\,]\,. \tag{26}$$

(26) implies that when $p_2 = c_2 - r_H$ and $p_1 \in (\,\widehat{p}_1,\, \widehat{p}_1 + 2\,t\,)$, users located in $[\,0,\, \widehat{x}_0\,)$ purchase a phone from Firm 1 and users located in $(\,\widehat{x}_0, 1\,]$ purchase a phone from Firm 2, where:

$$\widehat{x}_0 \;=\; \frac{1}{2\,t}\,[\,t + G_1 - G_2 + c_2 - r_H - p_1\,] \;\in\; (\,0, 1\,)\,.$$

Firm 1's corresponding profit is:

$$\pi_1(p_1) \;=\; [\,p_1 + r_L - c_1\,]\;\frac{1}{2\,t}\;[\,t + G_1 - G_2 + c_2 - r_H - p_1\,]\,. \tag{27}$$

Differentiating (27) provides:

$$\pi_1'(p_1) \;=\; \frac{1}{2\,t}\,[\,t + G_1 - G_2 + c_2 - r_H - r_L + c_1 - 2\,p_1\,] \;\;\Rightarrow\;\; \pi_1''(p_1) \;=\; -\,\frac{1}{t} \;<\; 0\,. \tag{28}$$

(24) and (28) imply:

$$\pi_1'(p_1)\,|_{p_1 = \widehat{p}_1} \;=\; \frac{1}{2\,t}\,[\,3\,t + 3\,A\,] \;=\; \frac{3}{2\,t}\,[\,t + A\,] \;>\; 0\,. \tag{29}$$

(28) and (29) imply that when $p_2 = c_2 - r_H$, Firm 1 will increase p_1 above $\widehat{p}_1$, thereby ensuring that both firms sell phones. Consequently, the analysis in the proof of Lemma A1.4 implies that at the unique equilibrium, both firms sell phones, Firm 1's profit is $\pi_1 = \frac{1}{2\,t}\,[\,t - A\,]^2 > 0$ and Firm 2's profit is $\pi_2 = \frac{1}{2\,t}\,[\,t + A\,]^2 > 0$. □

Lemma A1.6. Suppose the default privacy setting cannot be changed, $A > 0$, and $t \in (\,|\,A\,|\,, 3\,|\,A\,|\,)$. Then at the unique equilibrium, both firms sell phones, Firm 1's profit is $\pi_1 = \frac{1}{2\,t}\,[\,t - A\,]^2 > 0$ and Firm 2's profit is $\pi_2 = \frac{1}{2\,t}\,[\,t + A\,]^2 > 0$.

Proof. The proof is analogous to the proof of Lemma A1.5. □ ■

Proof of Lemma 2. The proof follows directly from the following lemmas (Lemmas A2.1 – A2.4).

Lemma A2.1. Suppose $K_1 = K_2 = 0$. Then: (i) a user located in $[\,0, \frac{1}{2}\,)$ will change the default setting on the phone she purchases if and only if she purchases the phone from Firm 2; (ii) a user located in $(\,\frac{1}{2}, 1\,]$ will change the default setting on the phone she purchases if and only if she purchases the phone from Firm 1; and (iii) a user located at $\frac{1}{2}$ will not change the default setting on the phone she purchases.

Proof. The conclusions follow directly from the proofs of Lemmas A1 – A3. □

Lemma A2.2. Suppose $K_1 = K_2 = 0$. Then: (i) all users buy a phone from Firm 1 if $p_2 > p_1 + G_2 - G_1$; (ii) all users buy a phone from Firm 2 if $p_2 < p_1 + G_2 - G_1$; and (iii) all users are indifferent between buying a phone from Firm 1 and from Firm 2 if $p_2 = p_1 + G_2 - G_1$.

Proof. Lemma A2.1 implies that a user located at $x_1 \in [0, \frac{1}{2})$ will buy a phone from Firm 1 if:

$$G_1 - t\,x_1 - p_1 \;>\; G_2 - t\,x_1 - p_2 \quad\Leftrightarrow\quad p_2 \;>\; p_1 + G_2 - G_1\,.$$

Lemma A2.1 also implies that a user located at $x_2 \in (\frac{1}{2}, 1]$ will buy a phone from Firm 1 if:

$$G_1 - t\,[\,1 - x_2\,] - p_1 \;>\; G_2 - t\,[\,1 - x_2\,] - p_2 \quad\Leftrightarrow\quad p_2 \;>\; p_1 + G_2 - G_1\,.$$

Lemma A2.1 further implies that a user located at $\frac{1}{2}$ will buy a phone from Firm 1 if:

$$G_1 - \frac{1}{2}\,t - p_1 \;>\; G_2 - \frac{1}{2}\,t - p_2 \quad\Leftrightarrow\quad p_2 \;>\; p_1 + G_2 - G_1\,.$$

The proofs of the remaining conclusions are analogous, and so are omitted. $\square$

Lemma A2.3. Suppose $K_1 = K_2 = 0$ and $G_2 - c_2 > G_1 - c_1$. Then in equilibrium, all users purchase a phone from Firm 2 at a price just below $c_1 - \frac{1}{2}[\,r_H + r_L\,] + G_2 - G_1$. Firm 1's profit is 0. Firm 2's profit is (nearly) $G_2 - c_2 - (\,G_1 - c_1\,)$.

Proof. Lemmas A2.1 and A2.2 imply that for $\varepsilon_1 > 0$, Firm 2's expected profit is:

$$\pi_2 \;=\; \begin{cases} 0 & \text{if } p_2 \;>\; p_1 + G_2 - G_1 \\ \frac{1}{2}\left[\,p_1 + \frac{r_L + r_H}{2} + G_2 - G_1 - c_2\,\right] & \text{if } p_2 \;=\; p_1 + G_2 - G_1 \\ p_1 + \frac{r_L + r_H}{2} + G_2 - G_1 - c_2 - \varepsilon_1 & \text{if } p_2 \;=\; p_1 + G_2 - G_1 - \varepsilon_1\,. \end{cases} \tag{30}$$

Firm 1 must secure nonnegative profit in equilibrium. Therefore, in any equilibrium in which all users either strictly prefer to purchase a phone from Firm 1 or are indifferent between purchasing a phone from Firm 1 and Firm 2, it must be the case that $p_1 \ge c_1 - \frac{1}{2}[\,r_H + r_L\,]$. Consequently, in any such equilibrium:

$$p_1 + \frac{r_L + r_H}{2} + G_2 - G_1 - c_2 \;\ge\; G_2 - c_2 - (\,G_1 - c_1\,) \;>\; 0\,. \tag{31}$$

(30) and (31) imply that for ε_1 sufficiently small, Firm 2 secures strictly higher profit by setting $p_2 = p_1 + G_2 - G_1 - \varepsilon_1$ than by setting $p_2 \ge p_1 + G_2 - G_1$. Therefore, in equilibrium, Firm 2 will set p_2 just below $c_1 - \frac{1}{2}[\,r_H + r_L\,] + G_2 - G_1$ to ensure that Firm 1 cannot profitably attract any users. Consequently, Firm 1's profit is 0 and Firm 2's profit is nearly:

$$c_1 - \frac{1}{2}[\,r_H + r_L\,] + G_2 - G_1 + \frac{1}{2}[\,r_H + r_L\,] - c_2 \;=\; G_2 - c_2 - (\,G_1 - c_1\,)\,. \;\square$$

Lemma A2.4. Suppose $K_1 = K_2 = 0$ and $G_1 - c_1 > G_2 - c_2$. Then in equilibrium, all users purchase a phone from Firm 1 at a price just below $p_1 = c_2 - \frac{1}{2}[r_H + r_L] + G_1 - G_2$. Firm 2's profit is 0. Firm 1's profit is (nearly) $G_1 - c_1 - (G_2 - c_2)$.

Proof. The proof is analogous to the proof of Lemma A2.3. □ ■

Proof of Proposition 1. (10) and Lemmas A2 – A4 and A11 imply that in any equilibrium with the identified properties, Firm 1's profit is:

$$\pi_1 = [p_1 + r_L - c_1] \frac{1}{2t} [t + G_1 - G_2 + p_2 - p_1]. \tag{32}$$

The unique value of p_1 that maximizes π_1 in (32) is given by:

$$\frac{\partial \pi_1}{\partial p_1} = 0 \Leftrightarrow p_1 = \frac{1}{2}[t + c_1 - r_L + G_1 - G_2 + p_2]. \tag{33}$$

(10) and Lemmas A1, A3, A4, and A11 imply that in any equilibrium with the identified properties, Firm 2's profit is:

$$\pi_2 = [p_2 + r_H - c_2] \frac{1}{2t} [t + G_2 - G_1 + p_1 - p_2]. \tag{34}$$

The unique value of p_2 that maximizes π_2 in (34) is given by:

$$\frac{\partial \pi_2}{\partial p_2} = 0 \Leftrightarrow p_2 = \frac{1}{2}[t + c_2 - r_H + G_2 - G_1 + p_1]. \tag{35}$$

(33) and (35) imply that in any equilibrium:

$$p_1 = \frac{1}{2}[t + c_1 - r_L + G_1 - G_2] + \frac{1}{4}[t + c_2 - r_H + G_2 - G_1 + p_1]$$

$$\Rightarrow p_1 = \frac{1}{3}[3t - 3A + 3(c_1 - r_L)] = c_1 - r_L + t - A. \tag{36}$$

(35) and (36) imply:

$$p_2 = \frac{1}{2}[t + c_2 - r_H + G_2 - G_1] + \frac{1}{6}[3t + 2c_1 + c_2 + G_1 - G_2 - 2r_L - r_H]$$

$$= \frac{1}{3}[3t + 3A + 3(c_2 - r_H)] = c_2 - r_H + t + A. \tag{37}$$

(36) and the maintained assumptions imply that Firm 1's profit margin is positive because:

$$p_1 + r_L - c_1 = t - A > 0.$$

(37) and the maintained assumptions imply that Firm 2's profit margin is positive because:

$$p_2 + r_H - c_2 \;=\; t + A \;>\; 0\,.$$

(36) and Condition 1A imply that $p_1 \geq 0$ because:

$$p_1 \;\geq\; 0 \quad\Leftrightarrow\quad 3\,t + 2\,c_1 + c_2 + G_1 - G_2 - 2\,r_L - r_H \;\geq\; 0$$

$$\Leftrightarrow \quad A \;=\; \frac{1}{3}\,[\,G_2 + r_H - c_2 - (\,G_1 + r_L - c_1\,)\,] \;\leq\; t - r_L + c_1\,. \tag{38}$$

(37) and Condition 1A imply that $p_2 \geq 0$ because:

$$p_2 \;\geq\; 0 \quad\Leftrightarrow\quad 3\,t + 2\,c_2 + c_1 + G_2 - G_1 - 2\,r_H - r_L \;\geq\; 0$$

$$\Leftrightarrow \quad A \;=\; \frac{1}{3}\,[\,G_2 + r_H - c_2 - (\,G_1 + r_L - c_1\,)\,] \;\geq\; r_H - c_2 - t\,. \tag{39}$$

(36) and (37) imply:

$$p_2 - p_1 \;=\; \frac{1}{3}\,[\,c_2 - c_1 + 2\,G_2 - 2\,G_1 + r_L - r_H\,]\,. \tag{40}$$

(10) and (40) imply that the user who is indifferent between purchasing a phone from Firm 1 and Firm 2 is located at:

$$x_0 \;=\; \frac{1}{2} - \frac{1}{6\,t}\,[\,G_2 + r_H - c_2 - (G_1 + r_L - c_1)\,] \;=\; \frac{1}{2} - \frac{A}{2\,t}\,. \tag{41}$$

(41) and the maintained assumptions imply that $x_0 \in (\,\frac{1}{2} - \frac{K_2}{2\,t}, \frac{1}{2} + \frac{K_1}{2\,t}\,)$, so no user changes the default setting on the phone she purchases (from Lemmas A1 – A4).

(41) and (36) imply:

$$\pi_1 \;=\; [\,p_1 + r_L - c_1\,]\,x_0 \;=\; [\,t - A\,]\left[\frac{t - A}{2\,t}\right] \;=\; \frac{1}{2\,t}\,[\,t - A\,]^2\,. \tag{42}$$

(41) and (37) imply:

$$\pi_2 \;=\; [\,p_2 + r_H - c_2\,]\,[\,1 - x_0\,] \;=\; [\,t + A\,]\left[\frac{t + A}{2\,t}\right] \;=\; \frac{1}{2\,t}\,[\,t + A\,]^2\,. \tag{43}$$

(40) implies:

$$p_1 - p_2 \;>\; G_1 - G_2 - K_1 \quad\Leftrightarrow\quad K_1 \;>\; \frac{1}{3}\,[\,G_1 - G_2 + c_2 - c_1 + r_L - r_H\,] \;=\; -\,A\,. \tag{44}$$

(40) also implies:

$$p_1 - p_2 \;<\; G_1 - G_2 + K_2 \quad\Leftrightarrow\quad K_2 \;>\; \frac{1}{3}\,[\,G_2 - G_1 + c_1 - c_2 + r_H - r_L\,] \;=\; A\,. \tag{45}$$

(44), (45), and the maintained assumptions imply:

$$p_1 - p_2 \;\in\; (\,G_1 - G_2 - K_1, G_1 - G_2 + K_2\,)\,. \tag{46}$$

The foregoing analysis and Lemma A11 imply that the identified putative equilibrium is unique among equilibria in which (46) holds. It remains to verify that neither firm can increase its profit by unilaterally changing its price so that (46) does not hold. We first show this is the case for Firm 1.

Lemmas A7 and A8 imply that if Firm 1 sets $p_1 > p_2 + G_1 - G_2 + K_2$, then no users purchase a phone from Firm 1. Therefore, Firm 1's profit (0) is less than the profit specified in (42).

If Firm 1 sets $p_1 = p_2 + G_1 - G_2 + K_2$ when p_2 is as specified in (37), then:

$$\begin{aligned} p_1 &= p_2 + G_1 - G_2 + K_2 = c_2 - r_H + t + A + G_1 - G_2 + K_2 \\ &= -3A + c_1 - r_L + t + A + K_2 = c_1 - r_L - 2A + t + K_2 \\ &> c_1 - r_L \quad \Leftrightarrow \quad t + K_2 - 2A > 0 . \end{aligned}$$

The last inequality holds here because $t > A$ and $K_2 > A$, by assumption. Because $p_1 = p_2 + G_1 - G_2 + K_2$ and $p_1 > c_1 - r_L$, the proof of Lemma A15 implies that Firm 1 can increase its profit when p_2 is as specified in (37) by choosing p_1 to ensure $p_1 - p_2 \in (G_1 - G_2 - K_1, G_1 - G_2 + K_2)$. Therefore, Firm 1 cannot increase its profit above the profit specified in (42) by setting $p_1 = p_2 + G_1 - G_2 + K_2$ when p_2 is as specified in (37).

Lemmas A9 and A10 imply that if Firm 1 sets $p_1 < p_2 + G_1 - G_2 - K_1$, then all users purchase a phone from Firm 1. (4) implies that the maximum profit Firm 1 can secure by setting such a price when p_2 is as specified in (37) is nearly:

$$\begin{aligned} \pi_{1D} &= p_2 + G_1 - G_2 - K_1 + r_1 - c_1 = c_2 - r_H + t + A + G_1 - G_2 - K_1 + r_1 - c_1 \\ &= G_1 - c_1 - (G_2 + r_H - c_2) + A + t + r_1 - K_1 \\ &= -2A + t - K_1 + \frac{r_H - r_L}{2t}[t - K_1] = -2A + \frac{t - K_1}{2t}[2t + r_H - r_L] . \end{aligned} \tag{47}$$

(42) and (47) imply that Firm 1 cannot increase its profit by setting $p_1 < p_2 + G_1 - G_2 - K_1$ when p_2 is as specified in (37) if Condition 1B holds.

If Firm 1 sets $p_1 = p_2 + G_1 - G_2 - K_1$ when p_2 is as specified in (37), then Lemma A6 implies that: (i) all users located in $[\frac{1}{2} + \frac{K_1}{2t}, 1]$ are indifferent between buying a phone from Firm 1 and from Firm 2; and (ii) all users located in $[0, \frac{1}{2} + \frac{K_1}{2t})$ buy a phone from Firm 1. Therefore, Lemma A2 implies that Firm 1's profit is:

$$\begin{aligned} \pi_1 &= [p_1 + r_L - c_1]\left[\frac{1}{2} + \frac{K_1}{2t}\right] + [p_1 + r_H - c_1]\frac{1}{2}\left[\frac{1}{2} - \frac{K_1}{2t}\right] . \\ &< [p_1 + r_L - c_1]\left[\frac{1}{2} + \frac{K_1}{2t}\right] + [p_1 + r_H - c_1]\left[\frac{1}{2} - \frac{K_1}{2t}\right] \end{aligned} \tag{48}$$

$$= p_1 + r_1 - c_1 \ = \ \pi_{1D} \ < \ \frac{1}{2\,t}\,[\,t - A\,]^2\,. \qquad (49)$$

The first inequality in (49) holds because $p_1 + r_H - c_1$ must be strictly positive if Firm 1 is to secure positive profit in this case. The last inequality in (49) reflects (47) and Condition 1B. (42) and (49) imply that Firm 1 will not set $p_1 = p_2 + G_1 - G_2 - K_1$ when p_2 is as specified in (37).

Now we show that Firm 2 cannot increase its profit by unilaterally changing its price so that (46) does not hold when p_1 is as specified in (36).

Lemmas A9 and A10 imply that if Firm 2 sets $p_2 > p_1 + G_2 - G_1 + K_1$, then no users purchase a phone from Firm 2, so Firm 2's profit (0) is less than the profit specified in (43).

If Firm 2 sets $p_2 = p_1 + G_2 - G_1 + K_1$ when p_1 is as specified in (36), then:

$$\begin{aligned} p_2 \ &= \ p_1 + G_2 - G_1 + K_1 \ = \ c_1 - r_L + t - A + G_2 - G_1 + K_1 \\ &= \ G_2 + r_H - c_2 - G_1 - r_L + c_2 + t - A + K_1 + c_2 - r_H \\ &= \ 3\,A + t - A + K_1 + c_2 - r_H \ = \ 2\,A + t + K_1 + c_2 - r_H \ > \ c_2 - r_H\,. \end{aligned}$$

The last inequality holds here because $K_1 > -A$ and $t > -A$, by assumption. Because $p_2 = p_1 + G_2 - G_1 + K_1 > c_2 - r_H$ when p_1 is as specified in (36), the proof of Lemma A20 implies that Firm 2 can increase its profit by setting p_2 to ensure $p_1 - p_2 \in (G_1 - G_2 - K_1, G_1 - G_2 + K_2)$. Therefore, Firm 2 cannot increase its profit by setting $p_2 = p_1 + G_2 - G_1 + K_1$ when p_1 is as specified in (36).

Lemmas A7 and A8 imply that if Firm 2 sets $p_2 < p_1 + G_2 - G_1 - K_2$, then all users purchase a phone from Firm 2. (4) implies that the maximum profit Firm 2 can secure by setting such a price when p_1 is as specified in (36) is nearly:

$$\begin{aligned} \pi_{2D} \ &= \ p_1 + G_2 - G_1 - K_2 + r_2 - c_2 \ = \ c_1 - r_L + t - A + G_2 - G_1 - K_2 + r_2 - c_2 \\ &= \ G_2 - c_2 - (\,G_1 + r_L - c_1\,) - A + t + r_2 - K_2 \\ &= \ 2\,A + [\,t - K_2\,]\left[1 - \frac{1}{2\,t}\,(\,r_H - r_L\,)\right] \ = \ 2\,A + \frac{t - K_2}{2\,t}\,[\,2\,t - r_H + r_L\,]\,. \qquad (50) \end{aligned}$$

(43) and (50) imply that Firm 2 cannot increase its profit by setting $p_2 < p_1 + G_2 - G_1 - K_2$ when p_1 is as specified in (36) if Condition 1B holds.

If Firm 2 sets $p_2 = p_1 + G_2 - G_1 - K_2$ when p_1 is as specified in (36), Lemma A5 implies that: (i) all users located in $[\,0, \frac{1}{2} - \frac{K_2}{2\,t}\,]$ are indifferent between buying a phone from Firm 1 and from Firm 2; and (ii) all users located in $(\,\frac{1}{2} - \frac{K_2}{2\,t}, 1\,]$ buy a phone from Firm 2. Therefore, Lemmas A1, A3, and A5 imply that Firm 2's profit is:

$$\pi_2 \;=\; [\,p_2 + r_H - c_2\,]\left[\frac{1}{2} + \frac{K_2}{2\,t}\right] + [\,p_2 + r_L - c_2\,]\,\frac{1}{2}\left[\frac{1}{2} - \frac{K_2}{2\,t}\right]. \tag{51}$$

If $p_2 = p_1 + G_2 - G_1 - K_2$ and p_1 is as specified in (36):

$$p_2 + r_H - c_2 \;=\; 3\,A + t - A - K_2 \;=\; 2\,A + t - K_2 \tag{52}$$

$$\Rightarrow \quad p_2 + r_L - c_2 \;=\; p_2 + r_H - c_2 - r_H + r_L \;=\; 2\,A + t - K_2 - r_H + r_L\,. \tag{53}$$

(52) implies that if Firm 2 is to secure positive profit under the presumed deviation, it must be the case that $2\,A + t - K_2 > 0$.

Initially suppose $p_2 + r_L - c_2 \;=\; 2\,A + t - K_2 - r_H + r_L > 0$. Then (51) – (53) imply:

$$\begin{aligned}
\pi_2 \;&=\; [\,p_2 + r_H - c_2\,]\left[\frac{1}{2} + \frac{K_2}{2\,t}\right] + [\,p_2 + r_L - c_2\,]\,\frac{1}{2}\left[\frac{1}{2} - \frac{K_2}{2\,t}\right] \\
&<\; [\,p_2 + r_H - c_2\,]\left[\frac{1}{2} + \frac{K_2}{2\,t}\right] + [\,p_2 + r_L - c_2\,]\left[\frac{1}{2} - \frac{K_2}{2\,t}\right] \\
&=\; 2\,A + t - K_2 - \frac{1}{2\,t}\,[\,r_H - r_L\,]\,[\,t - K_2\,] \;=\; \pi_{2D} \;<\; \frac{1}{2\,t}\,[\,t + A\,]^2 .
\end{aligned} \tag{54}$$

(54) implies that Firm 2 cannot increase its profit by undertaking the proposed deviation in this case.

Now suppose $p_2 + r_L - c_2 \;=\; 2\,A + t - K_2 - r_H + r_L \leq 0$. (51) implies that Firm 2's profit is maximized in this case when $p_2 + r_L - c_2 \;=\; 2\,A + t - K_2 - r_H + r_L = 0$. (51) and (52) imply that Firm 2's maximum profit in this case is:

$$\pi_2 \;=\; [\,p_2 + r_H - c_2\,]\left[\frac{1}{2} + \frac{K_2}{2\,t}\right] \;=\; \frac{1}{2\,t}\,[\,2A + t - K_2\,]\,[\,t + K_2\,]\,.$$

Observe that:

$$\frac{1}{2\,t}\,[\,t + A\,]^2 \;>\; \frac{1}{2\,t}\,[\,2A + t - K_2\,]\,[\,t + K_2\,] \quad\Leftrightarrow\quad [\,A - K_2\,]^2 \;>\; 0\,. \tag{55}$$

(43) and (55) imply that Firm 2 cannot increase its profit by undertaking the proposed deviation in this case. ■

Proof of Proposition 2. We first show that the identified p_1 maximizes Firm 1's profit when $p_2 = c_2 - r_H$.

Lemmas A9 and A10 imply that if $p_2 = c_2 - r_H$, then among all $p_1 \leq p_2 + G_1 - G_2 - K_1$, the profit-maximizing p_1 for Firm 1 is marginally below:

$$c_2 - r_H + G_1 - G_2 - K_1 \ = \ c_1 - r_L - 3\,A - K_1 \ > \ 0\,.$$

The inequality here reflects Condition 2D. Lemma A2 implies that Firm 1's corresponding profit is approximately:

$$\begin{aligned} \pi_1 \ &= \ [\, p_1 + r_L - c_1\,] \left[\frac{1}{2} + \frac{K_1}{2\,t} \right] + [\, p_1 + r_H - c_1\,] \left[\frac{1}{2} - \frac{K_1}{2\,t} \right] \\ &= \ G_1 - G_2 + c_2 - c_1 - \frac{1}{2}\,[\, r_H - r_L\,] - K_1 \left[\frac{2\,t + r_H - r_L}{2\,t} \right] \qquad (56) \\ &= \ \left[\frac{t - K_1}{2\,t} \right] [\, r_H - r_L\,] - \ 3\,A - K_1\,. \end{aligned}$$

The expression in (56) is strictly positive because $K_1 \leq \frac{2\,t}{2\,t + r_H - r_L}\ \Omega_1$ (from Condition 2C), and because Condition 2B implies:

$$G_1 - G_2 + c_2 - c_1 - \frac{1}{2}\,[\, r_H - r_L\,] \ = \ \frac{1}{2}\,[\, r_H - r_L\,] - 3\,A \ \geq \ \Omega_1 \ > \ 0\,.$$

We now show that Firm 1 cannot increase its profit by setting $p_1 \in (\,p_2 + G_1 - G_2 - K_1,$ $p_2 + G_1 - G_2 + K_2\,)$ or $p_1 \geq p_2 + G_1 - G_2 + K_2$ when $p_2 = c_2 - r_H$.

(33) implies that when $p_1 \in (\,p_2 + G_1 - G_2 - K_1,\, p_2 + G_1 - G_2 + K_2\,)$, the price that maximizes Firm 1's profit when $p_2 = c_2 - r_H$ is:

$$p_1 \ = \ \frac{1}{2}\,[\, t + c_1 - r_L + G_1 - G_2 + p_2\,] \ = \ \frac{1}{2}\,[\, t + G_1 - G_2 + c_1 + c_2 - r_H - r_L\,]\,. \qquad (57)$$

(32) and (57) Firm 1's corresponding profit is:

$$\pi_1^{'} \ = \ \frac{1}{8\,t}\,[\, t + G_1 - G_2 + c_2 - c_1 - r_H + r_L\,]^2\,. \qquad (58)$$

(56) and (58) imply that Firm 1 cannot increase its profit by setting $p_1 \in (\,p_2 + G_1 - G_2 - K_1,\, p_2 + G_1 - G_2 + K_2\,)$ because:

$$\pi_1 \ \geq \ \pi_1^{'} \ \ \Leftrightarrow \ \ K_1 \ \leq \ \frac{2\,t}{2\,t + r_H - r_L}\ \Omega_1\,. \qquad (59)$$

Lemmas A7 and A8 imply that if Firm 1 sets price $p_1 > p_2 + G_1 - G_2 + K_2$, it will sell no phones, and so will make 0 profit. Therefore, among all $p_1 \geq p_2 + G_1 - G_2 + K_2$, the price that maximizes Firm 1's profit is $p_1 = p_2 + G_1 - G_2 + K_2$. When $p_2 = c_2 - r_H$, this price is $p_1 = c_2 - r_H + G_1 - G_2 + K_2$. Lemma A5 implies that when $p_1 = p_2 + G_1 - G_2 + K_2$:

(i) all users located in $[\,0, \frac{1}{2} - \frac{K_2}{2\,t}\,]$ are indifferent between buying a phone from Firm 1 and from Firm 2; and (ii) all users located in $(\,\frac{1}{2} - \frac{K_2}{2\,t}, 1\,]$ buy a phone from Firm 2. Therefore, Lemma A2 implies that Firm 1's profit is:

$$\tilde{\tilde{\pi}}_1 \;=\; \frac{1}{2}\,[\,G_1 - G_2 - c_1 + c_2 - r_H + r_L + K_2\,]\left[\frac{1}{2} - \frac{K_2}{2\,t}\right]. \tag{60}$$

If $G_1 - G_2 - c_1 + c_2 - r_H + r_L + K_2 \,\le\, 0$, then $\tilde{\tilde{\pi}}_1 \le 0$. Therefore, Firm 1 will never set $p_1 \,\ge\, p_2 + G_1 - G_2 + K_2$ when $p_2 \,=\, c_2 - r_H$ in this case.

If $G_1 - G_2 - c_1 + c_2 - r_H + r_L + K_2 \,>\, 0$, then $\tilde{\tilde{\pi}}_1 > 0$. In this case, if Firm 1 were to reduce its price to $p_1 \,=\, c_2 - r_H + G_1 - G_2 + K_2 - \varepsilon_2$ where $\varepsilon_2 > 0$, all users located in $[\,0, \frac{1}{2} - \frac{K_2}{2\,t}\,]$ would purchase a phone from Firm 1. Consequently, Firm 1's profit would be at least:

$$\begin{aligned}
\pi_1 \;&=\; [\,p_1 - \varepsilon_2 + r_L - c_1\,]\left[\frac{1}{2} - \frac{K_2}{2\,t}\right] \\
&=\; \tilde{\tilde{\pi}}_1 + \tilde{\tilde{\pi}}_1 - \varepsilon_2\left[\frac{1}{2} - \frac{K_2}{2\,t}\right] \;>\; \tilde{\tilde{\pi}}_1 \quad \text{for sufficiently small } \varepsilon_2.
\end{aligned} \tag{61}$$

(61) implies that Firm 1 could increase its profit by reducing its price marginally below $p_2 + G_1 - G_2 + K_2$ when $p_2 \,=\, c_2 - r_H$. Therefore, Firm 1 will never set $p_1 \,\ge\, p_2 + G_1 - G_2 + K_2$ when $p_2 \,=\, c_2 - r_H$.

In summary, we have established that when $p_2 \,=\, c_2 - r_H$, Firm 1 maximizes its profit by setting p_1 marginally below $c_2 - r_H + G_1 - G_2 - K_1$.

We now show that when Firm 1 sets p_1 marginally below $c_2 - r_H + G_1 - G_2 - K_1$, Firm 2 maximizes its profit by setting $p_2 \,=\, c_2 - r_H$. Lemmas A9 and A10 imply that when Firm 1 sets p_1 marginally below $c_2 - r_H + G_1 - G_2 - K_1$, Firm 2 attracts no users (and so secures no profit) if it sets $p_2 \,=\, c_2 - r_H$. Firm 2 continues to attract no users (and so continues to secure no profit) if it sets $p_2 \,>\, c_2 - r_H$. Firm 2 incurs negative profit if it sets $p_2 \,<\, c_2 - r_H$. Therefore, Firm 2 cannot increase its profit by setting $p_2 \,\neq\, c_2 - r_H$ when Firm 1 sets p_1 marginally below $c_2 - r_H + G_1 - G_2 - K_1$.

Finally, Lemma A2 implies that all users located in the interval $\left[\,\frac{1}{2} + \frac{K_1}{2\,t}, 1\,\right]$ (and only these users) change the default setting on the phone they purchase from Firm 1. ■

Proof of Proposition 3. Lemmas A7 and A8 imply that if p_2 is marginally below $p_1 + G_2 - G_1 - K_2$, then all users buy a phone from Firm 2. Therefore, Firm 1's profit is 0.

Among all values of p_2 below $p_1 + G_2 - G_1 - K_2$, the value of p_2 that is most profitable for Firm 2 is marginally below:

$$p_2 \;=\; p_1 + G_2 - G_1 - K_2\,. \tag{62}$$

We first show that if Firm 2 sets p_2 marginally below $p_1+G_2-G_1-K_2$, then Firm 1 will set $p_1 \in [\, c_1 - r_1,\, min\, \{ c_1 - r_L,\, c_1 - r_1 + K_1 + K_2 \}]$ in equilibrium. We do so by first explaining why it cannot be the case that $p_1 < c_1 - r_1$ or $p_1 > min\, \{ c_1 - r_L,\, c_1 - r_1 + K_1 + K_2 \}$ in equilibrium. Then we explain why, when $p_1 \in [\, c_1 - r_1,\, min\, \{ c_1 - r_L,\, c_1 - r_1 + K_1 + K_2 \}]$ and Firm 2 sets p_2 is marginally below $p_1 + G_2 - G_1 - K_2$, Firm 1 cannot strictly increase its profit by setting a different price.

Lemma A14 implies that setting $p_1 < c_1 - r_1$ is a dominated strategy for Firm 1.

Consider a putative equilibrium in which $p_1 > c_1 - r_1 + K_1 + K_2$ and Firm 2 sets p_2 marginally below the p_2 in (62). (62) implies:

$$p_2 \;>\; c_1 - r_1 + K_1 + K_2 + G_2 - G_1 - K_2 \;=\; G_2 - G_1 + K_1 + c_1 - r_1 \,. \tag{63}$$

Lemmas A9 and A10 imply that if Firm 1 sets p_1 marginally below $p_2 + G_1 - G_2 - K_1$, all users will purchase a phone from Firm 1. Consequently, (62) and (63) imply that Firm 1's profit will be nearly:

$$\begin{aligned} \pi_1 \;&=\; p_1 + r_1 - c_1 \;=\; p_2 + G_1 - G_2 - K_1 + r_1 - c_1 \\ &>\; G_2 - G_1 + K_1 + c_1 - r_1 + G_1 - G_2 - K_1 + r_1 - c_1 \;=\; 0 \,. \end{aligned}$$

Because Firm 1 can secure strictly positive profit by deviating from its strategy in the putative equilibrium, the putative equilibrium cannot be an equilibrium.

Next, consider a putative equilibrium in which $p_1 > c_1 - r_L$ and Firm 2 sets p_2 marginally below the p_2 in (62). (62) implies:

$$p_2 \;>\; c_1 - r_L + G_2 - G_1 - K_2 \,. \tag{64}$$

Lemmas A7 and A8 imply all users buy a phone from Firm 2 when (64) holds. Therefore, Firm 1's profit is 0.

Suppose Firm 1 reduces its price to:

$$p_1' \;=\; p_2 + G_1 - G_2 + K_2 \;>\; c_1 - r_L + G_2 - G_1 - K_2 + G_1 - G_2 + K_2 \;=\; c_1 - r_L .$$

Lemma A5 implies that when Firm 1 sets price p_1': (i) all users located in $[\, 0, \frac{1}{2} - \frac{K_2}{2\,t} \,]$ are indifferent between buying a phone from Firm 1 and from Firm 2; and (ii) all users located in $(\, \frac{1}{2} - \frac{K_2}{2\,t}, 1 \,]$ buy a phone from Firm 2. Therefore, Lemma A2 implies that Firm 1's profit is:

$$\pi_1' \;=\; \frac{1}{2} \,[\, p_1' + r_L - c_1 \,] \left[\frac{1}{2} - \frac{K_2}{2\,t} \right] \;>\; 0 \text{ (because } p_1' > c_1 - r_L).$$

Because Firm 1 can secure strictly positive profit by deviating from its strategy in the putative equilibrium, the putative equilibrium cannot be an equilibrium.

We now show that when $p_1 \in [\, c_1 - r_1,\, c_1 - r_L \,]$ and Firm 2 sets p_2 marginally below the p_2 in (62), Firm 1 cannot increase its profit by setting a price $p_1' \in (\, p_2 + G_1 - G_2 - K_1,\, p_2 + G_1 - G_2 + K_2 \,)$. Lemma A11 implies that when Firm 1 sets such a price: (i) all users located in $[0, \frac{1}{2} - \frac{K_2}{2t})$ strictly prefer to buy a phone from Firm 1 than from Firm 2; (ii) all users located in $(\frac{1}{2} + \frac{K_1}{2t}, 1]$ strictly prefer to buy a phone from Firm 2 than from Firm 1; and (iii) some user located in $[\frac{1}{2} - \frac{K_2}{2t}, \frac{1}{2} + \frac{K_1}{2t}]$ is indifferent between buying a phone from Firm 1 and from Firm 2. Furthermore, Lemmas A3 and A4 imply that Firm 1's profit is:

$$\pi_1 \; = \; [\, p_1' + r_L - c_1 \,]\; x_0 \; \leq \; 0\,, \; \text{ where } x_0 \text{ is given by (10).} \tag{65}$$

The inequality in (65) holds because $p_1' \leq c_1 - r_L$ and because (10) implies that $x_0 > 0$ in the present setting. (65) implies that Firm 1 cannot secure strictly positive profit by setting $p_1' \in (\, p_2 + G_1 - G_2 - K_1,\, p_2 + G_1 - G_2 + K_2\,)$ under the maintained conditions.

Next we show that Firm 1 cannot increase its profit by setting a price $p_1' \leq p_2 + G_1 - G_2 - K_1$. Observe that under the maintained conditions:

$$c_1 - r_1 + K_1 + K_2 \; \geq \; p_1 \; > \; p_2 + G_1 - G_2 + K_2 \tag{66}$$

$$\Rightarrow \quad c_1 - r_1 \; > \; p_2 + G_1 - G_2 - K_1 \tag{67}$$

The weak inequality in (66) holds because $p_1 \in [\, c_1 - r_1,\, min\{\, c_1 - r_L,\, c_1 - r_1 + K_1 + K_2 \,\}\,]$. The strict inequality in (66) holds because p_2 is marginally below $p_1 + G_2 - G_1 - K_2$. (67) implies that $p_1' < c_1 - r_1$ if Firm 1 sets $p_1' \leq p_2 + G_1 - G_2 - K_1$. Lemma A14 implies that this is a dominated strategy for Firm 1.

Now we show that Firm 1 cannot increase its profit by setting a price $p_1' > p_2 + G_1 - G_2 + K_2$. Lemmas A7 and A8 imply that no user will purchase a phone from Firm 1 in this case. Consequently, Firm 1's profit is 0.

It remains to show that Firm 1 cannot increase its profit by setting a price $p_1' = p_2 + G_1 - G_2 + K_2$. Because Firm 2 sets p_2 marginally below the p_2 in (62):

$$p_1' \; \approx \; p_1 + G_2 - G_1 - K_2 + G_1 - G_2 + K_2 \; = \; p_1\,. \tag{68}$$

Lemmas A1, A2, and A5 imply that when Firm 1 sets $p_1' = p_2 + G_1 - G_2 + K_2$, all users located in $[\,0, \frac{1}{2} - \frac{K_2}{2t}\,]$ are indifferent between buying a phone from Firm 1 and from Firm 2, whereas all users located in $(\,\frac{1}{2} - \frac{K_2}{2t},\, 1\,]$ purchase a phone from Firm 2. Consequently, (68) implies that Firm 1's profit in this case is:

$$\pi_1 \; = \; \frac{1}{2}\,[\, p_1' + r_L - c_1\,] \left[\frac{1}{2} - \frac{K_2}{2\,t} \right] \; \approx \; \frac{1}{2}\,[\, p_1 + r_L - c_1\,] \left[\frac{1}{2} - \frac{K_2}{2t} \right] \; \leq \; 0\,. \tag{69}$$

The inequality in (69) holds because $p_1 \leq c_1 - r_L$.

In summary, we have shown that if Firm 2 sets p_2 marginally below $p_1 + G_2 - G_1 - K_2$, then Firm 1 will set $p_1 \in [\, c_1 - r_1,\, min\,\{\, c_1 - r_L,\, c_1 - r_1 + K_1 + K_2 \,\}\,]$ in equilibrium.

We now show that Firm 2 maximizes its profit by setting p_2 marginally below $p_1 + G_2 - G_1 - K_2$ when Firm 1 sets $p_1 \in [\, c_1 - r_1\,,\, min\,\{\, c_1 - r_L,\, c_1 - r_1 + K_1 + K_2 \,\}\,]$. Observe first that this value of p_2 is positive because:

$$p_1 + G_2 - G_1 - K_2 \;\geq\; c_1 - r_1 + G_2 - G_1 - K_2 \;>\; 0\,.$$

The inequality here reflects Condition 3B.

When Firm 2 sets p_2 marginally below $p_1 + G_2 - G_1 - K_2$, all users purchase a phone from Firm 2. Lemma A1 implies that Firm 2's profit is:

$$\begin{aligned}
\pi_2 \;&=\; [\,p_2 + r_H - c_2\,]\left[\frac{1}{2} + \frac{K_2}{2\,t}\right] + [\,p_2 + r_L - c_2\,]\left[\frac{1}{2} - \frac{K_2}{2\,t}\right] \\
&\approx\; p_1 + G_2 - G_1 - K_2 - c_2 + \frac{1}{2}\,[\,r_L + r_H\,] + \frac{K_2}{2\,t}\,[\,r_H - r_L\,]\,. \qquad (70)
\end{aligned}$$

In equilibria in which $p_1 \in [\,c_1 - r_1\,,\, min\,\{\,c_1 - r_L,\, c_1 - r_1 + K_1 + K_2\,\}\,]$ and Firm 2 sets p_2 is marginally below $p_1 + G_2 - G_1 - K_2$, Firm 2 secures the least profit when $p_1 = c_1 - r_1$. Consequently, (4) and (70) imply that Firm 2 earns positive profit in all such equilibria if:

$$\pi_2^{min} \;=\; G_2 - G_1 + c_1 - c_2 + \frac{K_1}{2\,t}\,[\,r_H - r_L\,] - K_2\left[\frac{2\,t - (r_H - r_L)}{2\,t}\right]. \qquad (71)$$

(4) and Condition 3C imply that when $p_1 = c_1 - r_1$:

$$x_2 \;=\; \frac{1}{8\,t}\left[t + G_2 - G_1 + c_1 - c_2 + \left(\frac{1}{2} + \frac{K_1}{2\,t}\right)(\,r_H - r_L\,)\right]^2. \qquad (72)$$

(4) and Condition 3C imply that when $p_1 = c_1 - r_1$:

$$\Omega_2(\cdot) \;=\; G_2 - G_1 + c_1 - c_2 + \frac{K_1}{2\,t}\,[\,r_H - r_L\,] - x_2\,. \qquad (73)$$

(71) – (73) imply that when $p_1 = c_1 - r_1$ and Condition 3C holds:

$$\pi_2^{min} \;>\; x_2 \;\geq\; 0\,. \qquad (74)$$

We now show that Firm 2 cannot increase its profit by setting $p_2 \in (\,p_1 + G_2 - G_1 - K_2,\; p_1 + G_2 - G_1 + K_1\,)$ or $p_2 \geq p_1 + G_2 - G_1 + K_1$ when Firm 1 sets $p_1 \in [\,c_1 - r_1\,,\, min\,\{\,c_1 - r_L,\, c_1 - r_1 + K_1 + K_2\,\}\,]$.

(35) and Lemma A11 imply that when $p_2 \in (\,p_1 + G_2 - G_1 - K_2\,,\, p_1 + G_2 - G_1 + K_1\,)$, the price that maximizes Firm 2's profit is:

$$p_2 \;=\; \frac{1}{2}\,[\,t + c_2 - r_H + G_2 - G_1 + p_1\,]\,. \qquad (75)$$

(34) and (75) imply that Firm 2's corresponding profit is:

$$\pi_2^{'} \;=\; \frac{1}{8\,t}\,[\,t + G_2 - G_1 - c_2 + r_H + p_1\,]^2\,. \qquad (76)$$

(74) and (76) imply that Firm 2 cannot increase its profit by setting $p_2 \in (\,p_1 + G_2 - G_1 - K_2\; p_1 + G_2 - G_1 + K_1\,)$ when Condition 3C holds because:

$$\pi_2 \;\geq\; \pi_2^{'} \quad \Leftrightarrow \quad p_1 + G_2 - G_1 - K_2 - c_2 + \frac{1}{2}\,[\,r_L + r_H\,] + \frac{K_2}{2\,t}\,[\,r_H - r_L\,]$$

$$\geq\; \frac{1}{8\,t}\,[\,t + G_2 - G_1 - c_2 + r_H + p_1\,]^2 \quad \Leftrightarrow \quad K_2\left[\frac{2\,t - r_H + r_L}{2\,t}\right] \;\leq\; \Omega_2\,.$$

Lemmas A9 and A10 imply that if Firm 2 sets $p_2 > p_1 + G_2 - G_1 + K_1$, it will sell no phones and so will secure 0 profit. Therefore, $p_2 = p_1 + G_2 - G_1 + K_1$ is the profit-maximizing price for Firm 2 among all $p_2 \geq p_1 + G_2 - G_1 + K_1$. Lemma A6 implies that when $p_2 = p_1 + G_2 - G_1 + K_1$: (i) all users located in $[\,\frac{1}{2} + \frac{K_1}{2\,t}, 1\,]$ are indifferent between buying a phone from Firm 1 and from Firm 2; and (ii) all users located in $[\,0, \frac{1}{2} + \frac{K_1}{2\,t}\,)$ buy a phone from Firm 1. Therefore, Lemma A1 implies that Firm 2's profit is:

$$\pi_2^{''} \;=\; \frac{1}{2}\,[\,p_1 + G_2 - G_1 + K_1 + r_H - c_2\,]\left[\frac{1}{2} - \frac{K_1}{2\,t}\right] \;>\; 0\,. \tag{77}$$

The inequality in (77) holds because (4) implies that the minimum value of $\pi_2^{''}$, which occurs when $p_1 = c_1 - r_1$, is:

$$\pi_2^{''min} \;=\; \frac{1}{2}\left[G_2 - G_1 + c_1 - c_2 + K_1 + \left(\frac{1}{2} + \frac{K_1}{2\,t}\right)(\,r_H - r_L\,)\right]\left[\frac{1}{2} - \frac{K_1}{2\,t}\right] \;>\; 0\,. \tag{78}$$

The inequality in (78) holds because: (i) $\frac{K_1}{2\,t} < \frac{1}{2}$, by assumption; and (ii) the term in the first square brackets in (78) is positive. (ii) holds because Condition 3B ensures this term exceeds:

$$K_2 + K_1 + \left[\frac{1}{2} + \frac{K_1}{2\,t}\right][\,r_H - r_L\,] \;>\; 0\,.$$

(78) ensures that (77) holds.

If Firm 2 were to reduce its price to $p_2 = p_1 + G_2 - G_1 + K_1 - \varepsilon_3$ where $\varepsilon_3 > 0$, all users located in $[\,\frac{1}{2} + \frac{K_1}{2\,t}, 1\,]$ would purchase a phone from Firm 2. Consequently, (77) implies that Firm 2's profit would be at least:

$$\begin{aligned}
\pi_2 \;&=\; [\,p_2 - \varepsilon_3 + r_H - c_2\,]\left[\frac{1}{2} - \frac{K_1}{2\,t}\right] \\
&=\; [\,p_1 + G_2 - G_1 + K_1 + r_H - c_2\,]\left[\frac{1}{2} - \frac{K_1}{2\,t}\right] - \varepsilon_3\left[\frac{1}{2} - \frac{K_1}{2\,t}\right] \\
&=\; \pi_2^{''} + \pi_2^{''} - \varepsilon_3\left[\frac{1}{2} - \frac{K_1}{2\,t}\right] \;>\; \pi_2^{''} \quad \text{for sufficiently small } \varepsilon_3\,.
\end{aligned} \tag{79}$$

(79) implies that Firm 2 could increase its profit by reducing its price marginally below $p_1 + G_2 - G_1 + K_1$. Therefore, Firm 2 will never set $p_2 \geq p_1 + G_2 - G_1 + K_1$ when $p_1 \in [\,c_1 - r_1\,,\, min\,\{\,c_1 - r_L,\, c_1 - r_1 + K_1 + K_2\,\}\,]$.

In summary, we have shown that when $p_1 \in [\,c_1 - r_1\,,\, min\,\{\,c_1 - r_L,\, c_1 - r_1 + K_1 + K_2\,\}\,]$, Firm 2 maximizes its profit by setting p_2 marginally below $p_1 + G_2 - G_1 - K_2$.

Finally, Lemma A1 implies that all users located in the interval $[0, \frac{1}{2} - \frac{K_2}{2t})$ (and only these users) change the default setting on the phone they purchase from Firm 2. ■

Proof of Proposition 4. Conclusion (i) follows from Proposition 2, which implies that:

$$\pi_1^{D1}(\cdot) \approx G_1 - G_2 + c_2 - c_1 - \frac{1}{2}[r_H - r_L] - K_1 \left[\frac{2t + r_H - r_L}{2t}\right]$$

$$\Rightarrow \frac{\partial \pi_1^{D1}(\cdot)}{\partial K_1} = -1 - \left[\frac{r_H - r_L}{2t}\right] < 0 \text{ and } \frac{\partial \pi_1^{D1}(\cdot)}{\partial K_2} = 0. \tag{80}$$

Conclusion (ii) follows from Proposition 3, which states that:

$$\underline{\pi}_2^{D2}(\cdot) \approx c_1 - r_1 - c_2 + G_2 - G_1 - K_2 + \frac{1}{2}[r_L + r_H] + \frac{K_2}{2t}[r_H - r_L]$$

$$\Rightarrow \frac{\partial \underline{\pi}_2^{D2}(\cdot)}{\partial K_2} = -1 + \frac{r_H - r_L}{2t} \gtreqless 0 \Leftrightarrow r_H - r_L \gtreqless 2t; \text{ and} \tag{81}$$

$$\frac{\partial \underline{\pi}_2^{D2}(\cdot)}{\partial K_1} = -\frac{\partial r_1}{\partial K_1} = \frac{1}{2t}[r_H - r_L] > 0. \tag{82}$$

Conclusion (iii) follows directly from Lemma 1 and Proposition 1. ■

Proof of Lemma 3. Observe that $A < 0 \Rightarrow G_1 - c_1 - (G_2 - c_2) > r_H - r_L > 0$. Therefore, Lemma 2 implies:

$$\pi_1^{D1}(0,0) = G_1 - c_1 - (G_2 - c_2) = -3A + \Delta. \tag{83}$$

(83) and Proposition 1 imply:

$$\pi_1^S \gtreqless \pi_1^{D1}(0,0) \Leftrightarrow \frac{1}{2t}[t + |A|]^2 \gtreqless 3|A| + \Delta$$

$$\Leftrightarrow t^2 - 2[\Delta + 2|A|]t + |A|^2 \gtreqless 0. \tag{84}$$

(5) implies that the ("t") roots of the quadratic equation in (84) are given by:

$$t = \Delta + 2|A| \pm \sqrt{3|A|^2 + 4|A|\Delta + \Delta^2} \in \{t_{1L}, t_{1H}\}. \tag{85}$$

t_{1L}, the smaller root in (85), satisfies the maintained assumption that $t > |A|$ if and only if:

$$t_{1L} > |A| \Leftrightarrow \Delta + 2|A| - \sqrt{3|A|^2 + 4|A|\Delta + \Delta^2} > |A|$$

$$\Leftrightarrow \Delta^2 + 2|A|\Delta + |A|^2 > 3|A|^2 + 4|A|\Delta + \Delta^2 \Leftrightarrow 2|A|^2 + 2|A|\Delta < 0. \tag{86}$$

This inequality does not hold.

(5) implies that t_{1H} satisfies the maintained assumption that $t > |\,A\,|$ because:

$$t_{1H} \; > \; |\,A\,| \quad \Leftrightarrow \quad \Delta + |\,A\,| + \sqrt{3\,|\,A\,|^2 + 4\,|\,A\,|\,\Delta + \Delta^2} \; > \; 0\,. \tag{87}$$

(84) – (87) imply that because $t > |\,A\,|$ by assumption, $\pi_1^S - \pi_1^{D1}(0,0)$ is: (i) negative for $t < t_{1H}$; and (ii) positive for $t > t_{1H}$. Formally:

$$\pi_1^S \; > \; \pi_1^{D1}(0,0) \;\text{ if }\; t \; > \; t_{1H} \;\text{ and }\; \pi_1^S \; < \; \pi_1^{D1}(0,0) \;\text{ if }\; t \; \in \; (K_1, t_{1H}\,)\;. \tag{88}$$

The lower bound on t in the second conclusion in (88) (i.e., K_1) reflects the maintained assumption that $K_1 < t$.

(83) and Proposition 1 imply:

$$\pi_1^S - \pi_1^{D1}(0,0) \; = \; \frac{1}{2\,t}\,[\,t + |\,A\,|\,]^2 - 3\,|\,A\,| - \Delta\,. \tag{89}$$

(89) implies:

$$\frac{\partial\left(\pi_1^S - \pi_1^{D1}(0,0)\right)}{\partial\,|\,A\,|} \; = \; \frac{1}{t}\,[\,t + |\,A\,|\,] - 3 \; = \; \frac{|\,A\,|}{t} - 2 \; \lesseqgtr \; 0 \quad \Leftrightarrow \quad |\,A\,| \; < \; 2\,t$$

$$\Rightarrow \quad \frac{\partial\left(\pi_1^S - \pi_1^{D1}(0,0)\right)}{\partial\,|\,A\,|} \; < \; 0 \;\text{ for all }\; |\,A\,| \; < \; t\,. \tag{90}$$

The expression in (84) can be written as:

$$|\,A\,|^2 - 4\,t\,|\,A\,| + t^2 - 2\,\Delta\,t \; \gtreqless \; 0\,.$$

(5) implies that the ("$|\,A\,|$") roots of the associated quadratic equation are:

$$|\,A\,| = \frac{1}{2}\left[4\,t \pm \sqrt{16\,t^2 - 4\,t\,[\,t - 2\,\Delta\,]}\;\right] \; = \; 2\,t \pm \sqrt{3\,t^2 + 2\,t\,\Delta} \; \in \{\,|\,A\,|_{1L}\,,\,|\,A\,|_{1H}\,\}\,. \tag{91}$$

(5) implies:

$$|\,A\,|_{1L} \; > \; 0 \quad \Leftrightarrow \quad 2\,t \; > \; \sqrt{3\,t^2 + 2\,t\,\Delta} \quad \Leftrightarrow \quad t^2 \; > \; 2\,t\,\Delta \quad \Leftrightarrow \quad t \; > \; 2\,\Delta\,. \tag{92}$$

(5) further implies that $|\,A\,|_{1L} < \frac{t}{2}$ because:

$$|\,A\,|_{1L} \; < \; \frac{t}{2} \quad \Leftrightarrow \quad 2\,t \; - \; \sqrt{3\,t^2 + 2\,t\,\Delta} \; < \; \frac{t}{2} \quad \Leftrightarrow \quad \frac{3}{4}\,t^2 + 2\,t\,\Delta \; > \; 0\,. \tag{93}$$

(5) also implies that $|\,A\,|_{1H} > t$ because:

$$|\,A\,|_{1H} \; > \; t \quad \Leftrightarrow \quad 2\,t \; + \; \sqrt{3\,t^2 + 2\,t\,\Delta} \; > \; t \quad \Leftrightarrow \quad t + \sqrt{3\,t^2 + 2\,t\,\Delta} \; > \; 0. \tag{94}$$

(84) and (90) – (94) imply that $\pi_1^S - \pi_1^{D1}(0,0)$ is a decreasing function of $|A\,|$ for $|A\,| < t$.

Furthermore, this function is: (i) positive when $|A| < |A|_{1L}$ (which can occur if and only if $t > 2\Delta$); and (ii) negative for $|A| \in (|A|_{1L}, t)$ (because $t > |A|_{1H}$, by assumption). Formally:

$$\text{If } t > 2\Delta, \text{ then } \pi_1^S \begin{cases} > \pi_1^{D1}(0,0) & \text{if } |A| < |A|_{1L} \\ < \pi_1^{D1}(0,0) & \text{if } |A| \in (|A|_{1L}, t). \end{cases}$$

$$\text{If } t \leq 2\Delta, \text{ then } \pi_1^S < \pi_1^{D1}(0,0) \text{ for all } |A| \in (0, t). \quad \blacksquare \tag{95}$$

The proof of Lemma 4 parallels the proof of Lemma 3, and so is omitted.

Proof of Proposition 5. Condition (i) in the proposition implies:

$$A \equiv \frac{1}{3}[G_2 + r_H - c_2 - (G_1 + r_L - c_1)] = \frac{1}{3}[G_2 - c_2 - (G_1 - c_1)] + \frac{1}{3}[r_H - r_L] > 0.$$

Therefore, $K_1 > -A$ when $K_1 = 0$. Furthermore, condition (ii) ensures that $t > |A| = A$ and $r_H - c_2 - t < A < t + c_1 - r_L$. Therefore, all the maintained assumptions in Proposition 1 hold if $K_2 \in (A, t)$.

Lemma A2.3 implies that when $K_1 = K_2 = 0$, Firm 2's profit is nearly:

$$\pi_{2A} = G_2 - c_2 - (G_1 - c_1) = 3A - (r_H - r_L). \tag{96}$$

Proposition 1 implies that when $K_2 \in (A, t)$, there exists an equilibrium in which Firm 2's profit is:

$$\pi_{2B} = \frac{[t + A]^2}{2t}. \tag{97}$$

(96) and (97) imply:

$$\pi_{2B} > \pi_{2A} \Leftrightarrow t^2 + 2t[r_H - r_L - 2A] + A^2 > 0. \tag{98}$$

The roots of the quadratic equation associated with (98) are:

$$t^* = 2A - \Delta \pm \sqrt{[3A - \Delta][A - \Delta]}. \tag{99}$$

Condition (ii) ensures that $A > \Delta$, which implies $3A > \Delta$, so the roots in (99) are real. (5), (98), and (99) imply that $\pi_{2B} > \pi_{2A}$ if $t > 2A - \Delta + \sqrt{[3A - \Delta][A - \Delta]} = t_{2H}$.

Finally, Lemma A2.3 implies that Firm 1's equilibrium profit is 0 when $K_1 = K_2 = 0$. Firm 1's profit in the equilibrium identified in Proposition 1 is $\frac{1}{18t}[3t - A]^2 > 0$ when $K_1 = 0$ and $K_2 \in (A, t)$. $\blacksquare$

Proof of Proposition 6. Condition (i) implies that $K_2 > A$ when $K_2 = 0$. Therefore, conditions (i) – (iii) ensure that all the conditions in Proposition 1 hold if $K_1 \in (|A|, t)$.

Lemma 2 implies that when $K_1 = K_2 = 0$, Firm 1's profit is nearly:

$$\pi_{1A} \;=\; G_1 - c_1 - (\,G_2 - c_2\,) \;=\; r_H - r_L + 3\,|\,A\,|\;. \tag{100}$$

Proposition 1 implies that when $K_1 \in (\,|\,A\,|\,,\,t\,)$, there exists an equilibrium in which Firm 1's profit is:

$$\pi_{1B} \;=\; \frac{[\,t + |\,A\,|\,]^2}{2\,t}\,. \tag{101}$$

(100) and (101) imply:

$$\pi_{1B} \;>\; \pi_{1A} \;\;\Leftrightarrow\;\; t^2 - 2\,t\,[\,r_H - r_L + 2\,|\,A\,|\,] + A^2 \;>\; 0\,. \tag{102}$$

The roots of the quadratic equation associated with (102) are:

$$t_1^* \;=\; \Delta + 2\,|\,A\,| \pm \sqrt{3\,A^2 + 4\,|\,A\,|\,\Delta + \Delta^2}\,. \tag{103}$$

(5), (102), and (103) imply that if $t > \Delta + 2\,|\,A\,| \pm \sqrt{3\,A^2 + 4\,|\,A\,|\,\Delta + \Delta^2} = t_{1H}$, then $\pi_{1B} > \pi_{1A}$.

Finally, Lemma 2 implies that Firm 2's equilibrium profit is 0 when $K_1 = K_2 = 0$. Firm 2's profit in the equilibrium identified in Proposition 1 is $\frac{1}{18\,t}\,[\,3\,t - |\,A\,|\,]^2 > 0$ when $K_2 = 0$ and $K_1 \in (\,|\,A\,|\,,\,t\,)$. ■

Proof of Proposition 7. The proof proceeds by showing that neither firm can increase its profit by unilaterally changing its default-switching cost, regardless of the nature of the ensuing equilibrium.

To begin, observe that the proof of Proposition 1 establishes that the prices identified in the present Proposition are the unique prices that arise in a MS equilibrium. Furthermore, the equilibrium profits identified in Proposition 1 ($\pi_1^S > 0$ and $\pi_2^S > 0$) do not vary with K_1 and K_2. Therefore, Firm $i \in \{\,1,2\,\}$ cannot increase its profit by choosing $K_i \neq K_i^*$ if the resulting $\left(K_i, K_j^*\right)$ default-switching costs induce a MS equilibrium.

Next suppose that Firm $i \in \{\,1,2\,\}$ chooses a $K_i \neq K_i^*$ such that the resulting $\left(K_i, K_j^*\right)$ default-switching costs induce a MDj equilibrium (where $j \neq i,\; i,j \in \{1,2\,\}$). Then Firm i's profit will decline to 0. Consequently, Firm i cannot increase its profit by setting $K_i \neq K_i^*$ if the resulting $\left(K_i, K_j^*\right)$ default-switching costs induce a MDj equilibrium.

Now suppose that Firm 1 sets $K_1 \neq K_1^*$ such that the resulting (K_1, K_2^*) default-switching costs induce a MD1 equilibrium. The proof of Proposition 1 establishes that the maximum profit Firm 1 can secure in a MD1 equilibrium is

$$-\,2\,A + \frac{t - K_1}{2\,t}\,[\,2\,t + \Delta\,] \;\leq\; -\,2\,A + \frac{1}{2}\,[\,2\,t + \Delta\,]\,.$$

Condition (ii) in the present Proposition ensures that Firm 1's profit in the MS equilibrium identified in Proposition 1 exceeds $-\,2\,A + \frac{1}{2}\,[\,2\,t + \Delta\,]$. Therefore, Firm 1 cannot increase its profit by setting $K_1 \neq K_1^*$ if the resulting (K_1, K_2^*) default-switching costs induce a MD1 equilibrium.

Finally, suppose that Firm 2 sets a $K_2 \neq K_2^*$ such that the resulting (K_1^*, K_2) default-switching costs induce a MD2 equilibrium. The proof of Proposition 1 establishes that the maximum profit Firm 2 can secure in a MD2 equilibrium is

$$2\,A + \frac{t - K_2}{2\,t}\,[\,2\,t - \Delta\,] \;\leq\; 2\,A + \frac{1}{2}\,[\,2\,t - \Delta\,]\,.$$

The inequality here reflects condition (iii) in the present Proposition. Condition (ii) in this Proposition ensures that Firm 2's profit in the MS equilibrium identified in Proposition 1 exceeds $2\,A + \frac{1}{2}\,[\,2\,t - \Delta\,]$. Therefore, Firm 2 cannot increase its profit by setting $K_2 \neq K_2^*$ if the resulting (K_1^*, K_2) default-switching costs induce a MD2 equilibrium. ■

Proof of Proposition 8. The proof proceeds by showing that neither firm can increase its profit by unilaterally increasing its default-switching cost above 0, regardless of the nature of the ensuing equilibrium.

We first show that Firm 1 cannot strictly increase its profit by setting $K_1 > 0$ if the resulting $(K_1, 0)$ default-switching costs induce a MD1 equilibrium. The logic employed in the proof of Proposition 2 implies that Firm 1's profit in a MD1 equilibrium, given $K_1 > 0$ and $K_2 \geq 0$ is:

$$\pi_1^{D1}(K_1, K_2) \;=\; G_1 - G_2 + c_2 - c_1 - \frac{\Delta}{2} - K_1 \left[\frac{2\,t + \Delta}{2\,t}\right] \quad \Rightarrow \quad \frac{\partial \pi_1^{D1}(\cdot)}{\partial K_1} \;<\; 0\,. \tag{104}$$

(104) implies that Firm 1 cannot strictly increase its profit setting $K_1 > 0$ if a MD1 equilibrium ensues.

(104) further implies that when $K_1 = K_2 = 0$, Firm 1's profit is:

$$\pi_1^{D1}(0,0) \;=\; G_1 - c_1 - (G_2 - c_2) - \frac{\Delta}{2} \;>\; 0\,. \tag{105}$$

Firm 1's profit is 0 in any MD2 equilibrium. Therefore, (105) implies that Firm 1 cannot increase its profit by setting $K_1 > 0$ if the resulting $(K_1, 0)$ default-switching costs induce a MD2 equilibrium.

Proposition 1 establishes that Firm 1's profit in a MS equilibrium when $A < 0$ is $\pi_1^S = \frac{1}{2\,t}\,[\,t + |\,A\,|\,]^2$. Therefore, (105) and the maintained assumptions ensure that $\pi_1^{D1}(0,0) > \pi_1^S$. Consequently, Firm 1 cannot increase its profit by setting $K_1 > 0$ if the resulting $(K_1, 0)$ default-switching costs induce a MS equilibrium.

To initiate the demonstration that Firm 2 cannot increase its profit by unilaterally increasing K_2, observe that Firm 2's profit is 0 in all MD1 equilibria. Consequently, Firm 2 cannot increase its profit by implementing a $K_2 > 0$ that induces a MD1 equilibrium.

Next we establish that a MD2 equilibrium does not exist when $K_1 = 0$ and $A < 0$. To do so, suppose such an equilibrium exists. Then the consumer located at 0 prefers to buy a phone from Firm 2 than from Firm 1. Consequently:

$$G_2 - p_2 - \min\{K_2, t\} \;\geq\; G_1 - p_1 \;\;\Rightarrow\;\; p_2 \;\leq\; p_1 + G_2 - G_1 - \min\{K_2, t\}\,. \tag{106}$$

(106) reflects the fact that the consumer located at 0 who purchases a phone from Firm 2 will change the default PD setting on the phone if and only if $K_2 < t$.

Rather than serve no customers, Firm 1 will reduce its price at least to $c_1 - r_L$. Therefore, (106) implies that, to attract all consumers, Firm 2's price must satisfy:

$$p_2 \;\leq\; c_1 - r_L + G_2 - G_1 - \min\{K_2, t\}. \tag{107}$$

In any MD2 equilibrium in which (107) holds, Firm 2's profit is:

$$\begin{aligned} \pi_2 \;&\leq\; c_1 - r_L + G_2 - G_1 - \min\{K_2, t\} - c_2 + r_H \\ &<\; c_1 - r_L + G_2 - G_1 - c_2 + r_H \;=\; 3\,A \;<\; 0\,. \end{aligned} \tag{108}$$

The first inequality in (108) reflects the fact that Firm 2's revenue from advertisers cannot exceed r_H. The second inequality in (108) holds because $K_2 > 0$ (and $t > 0$), by assumption. The last inequality in (108) holds because $A < 0$, by assumption. (108) implies that a MD2 equilibrium does not exist under the maintained assumptions because Firm 2 must secure nonnegative profit in a MD2 equilibrium.

Finally, we establish that a MS equilibrium does not exist when $K_1 = 0$ and $A < 0$. To do so, suppose a MS equilibrium exists. Then there exists a consumer located at $x_0 \in (0, 1)$ who is indifferent between purchasing a phone from Firm 1 and purchasing a phone from Firm 2. Furthermore, the proof of Proposition 1 implies:

$$p_1 \;=\; c_1 - r_L + t - A, \quad p_2 \;=\; c_2 - r_H + t + A, \;\text{ and} \tag{109}$$

$$x_0 \;=\; \frac{1}{2} - \frac{A}{2\,t} \;>\; \frac{1}{2}\,. \tag{110}$$

The inequality in (110) holds because $A < 0$, by assumption.

Because $x_0 > \frac{1}{2}$ and $K_1 = 0$, the consumer located at x_0 will change the default PD setting on the phone he purchases if and only if he buys the phone from Firm 1. Therefore, because the consumer located at x_0 is indifferent between purchasing a phone from Firm 1 and purchasing a phone from Firm 2:

$$G_1 - p_1 - t\,[\,1 - x_0\,] \;=\; G_2 - p_2 - t\,[\,1 - x_0\,] \;\;\Rightarrow\;\; p_2 - p_1 \;=\; G_2 - G_1\,. \tag{111}$$

(109) implies:

$$p_2 - p_1 \;=\; c_2 - c_1 - r_H + r_L + 2\,A\,. \tag{112}$$

(111) and (112) imply:

$$G_2 - G_1 \;=\; c_2 - c_1 - r_H + r_L + 2\,A \;\;\Rightarrow\;\; 3\,A \;=\; 2\,A \;\;\Rightarrow\;\; A \;=\; 0\,. \tag{113}$$

(113) cannot hold because $A < 0$, by assumption. Therefore, by contradiction, a MS equi-

librium does not exist when $K_1 = 0$ and $A < 0$. ■

The proofs of Propositions 9 and 10 parallel the proof of Proposition 8, and so are omitted.

Proof of Proposition 11. First consider a putative MD1 equilibrium in which $K_1 \in (0, t)$ and $K_2 \geq 0$. Arguments analogous to those employed in the proof of Proposition 2 reveal that Firm 1 can increase its profit by reducing K_1 marginally. Therefore, the putative equilibrium cannot constitute an equilibrium.

Next consider a putative MD1 equilibrium in which $K_1 \geq t$ and $K_2 \geq 0$. It is readily verified that Firm 1's profit in this equilibrium is $\pi_1^{D1} = -3A - t$. Proposition 2 implies that if Firm 1 reduces K_1 below t, it can secure a profit of nearly $\pi_1^{D1'} = \frac{t - K_1}{2t} \Delta - 3A - K_1$. Observe that when $K_1 \in (0, t)$:

$$\pi_1^{D1'} > \pi_1^{D1} \Leftrightarrow \frac{t - K_1}{2t} \Delta - 3A - K_1 > -3A - t \Leftrightarrow \frac{\Delta}{2t} + 1 > 0. \tag{114}$$

Because the last inequality in (114) always holds, (114) implies that the putative equilibrium cannot constitute an equilibrium. ■

Proof of Proposition 12. First consider a putative MD2 equilibrium in which $K_1 \in [0, t)$ and $K_2 \in (0, t)$, $p_1 = c_1 - r_1$, and p_2 is marginally below $c_1 - r_1 + G_2 - G_1 - K_2$. The expression for π_2 in Proposition 3 implies that Firm 2 can increase its MD2 equilibrium profit by reducing K_2 marginally. Consequently, the identified putative equilibrium cannot constitute an equilibrium under the maintained conditions.

Now consider a putative MD2 equilibrium in which $K_1 \in [0, t)$, $K_2 \geq t$, $p_1 = c_1 - r_1$, and p_2 is marginally below $c_1 - r_1 + G_2 - G_1 - K_2$. Arguments analogous to those employed in the proof of Proposition 3 reveal that Firm 2's profit in this equilibrium is $\pi_2^{D2} = c_1 - r_1 + r_H - c_2 + G_2 - G_1 - t$. Proposition 3 implies that if Firm 2 reduces K_2 below t, its profit is nearly $\pi_2^{D2'} = c_1 - r_1 - c_2 + G_2 - G_1 - K_2 + \frac{1}{2}[r_H + r_L] + \frac{K_2}{2t} \Delta$. Observe that that when $K_2 \in (0, t)$:

$$\pi_2^{D2} < \pi_2^{D2'} \Leftrightarrow r_H - t < -K_2 + \frac{1}{2}[r_H + r_L] + \frac{K_2}{2t} \Delta \Leftrightarrow 2t > \Delta. \tag{115}$$

The last inequality in (115) holds, by assumption. Therefore, (115) implies that the identified putative equilibrium cannot constitute an equilibrium under the maintained conditions. ■

Proof of Proposition 13. First consider a putative equilibrium in which: (i) $K_1 \in [0, t)$ and $K_2 \in [0, t)$, where $(K_1, K_2) \neq (0, 0)$; (ii) $p_1 = c_1 - r_1$; and (iii) p_2 is marginally below $c_1 - r_1 + G_2 - G_1 - K_2$. The expression for π_2 in Proposition 3 implies that Firm 2 can increase its MD2 equilibrium profit by reducing K_2 marginally. Therefore, the identified putative equilibrium cannot constitute an equilibrium.

Now consider a putative equilibrium in which $K_1 = K_2 = 0$, $p_1 = c_1 - r_1$, and $p_2 = c_1 - \frac{1}{2}[r_L + r_H] + G_2 - G_1$. Arguments analogous to those employed in the proof of Proposition 3 reveal that Firm 2's profit in this equilibrium is nearly $\pi_2^{D2} = G_2 - c_2 - (G_1 - c_1)$. Proposition 3 implies that if Firm 2 increases K_2 marginally and reduces its price to $p_2 = c_1 - \frac{1}{2}[r_L + r_H] + G_2 - G_1 - \varepsilon_4$ (where $\varepsilon_4 > 0$ is arbitrarily small), then its profit would be nearly $\pi_2^{D2'} = G_2 - G_1 + c_1 - c_2 - \varepsilon_4 + \frac{\varepsilon_4}{2t}\Delta$. Observe that:

$$\pi_2^{D2} < \pi_2^{D2'} \Leftrightarrow -\varepsilon_4 + \frac{\varepsilon_4}{2t}\Delta > 0 \Leftrightarrow \frac{\Delta}{2t} > 1 \Leftrightarrow \Delta > 2t. \tag{116}$$

The last inequality in (116) holds, by assumption. Therefore, (116) implies that the identified putative equilibrium cannot constitute an equilibrium under the maintained conditions. ■

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