

Technical Appendix to Accompany
“Revealing Transactions Data to Third Parties:
Implications of Privacy Regimes for Welfare in Online Markets”

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This Technical Appendix provides proofs of the key conclusions in the text. Section I characterizes the pure-strategy perfect Bayesian equilibria (PPBE) that arise when a sophisticated consumer interacts sequentially with two merchants. Section II employs these results to prove Propositions 1 – 9 in the paper. Section III considers the extensions of the model discussed in Section 6 of the paper.

I. Characterizing Pure-Strategy Perfect Bayesian Equilibria

The analysis in this section pertains to the setting in the paper where a generic sophisticated consumer (or buyer, B) interacts first with merchant (or seller) $S1$, and then with merchant $S2$. B 's reservation value for each unit of each seller's product is $r \in \{\underline{r}, \bar{r}\}$.

We first characterize transactions outcomes under privacy, where it is common knowledge that no data from B 's interaction with $S1$ will be revealed to $S2$. Then we characterize outcomes under transparency, where it is common knowledge that all transactions data from B 's interaction with $S1$ will be revealed to $S2$.

Lemmas 1 – 4 in the text follow directly from Claims 1 – 3 below and the explanations in the text. Lemmas 5 and 6 in the text follow from the remaining theorems and corollaries in this Appendix. Specifically, Lemma 5 follows from Theorems 1 – 4, 9, 10, 14, and 17 and their corollaries. Lemma 6 follows from Theorems 5 – 8, 11 – 13, 15, 16, and 18 and their corollaries.

Claim 1. *Under privacy, S_i maximizes her expected profit by setting price $p_i = \bar{r}$ if $c_i > \hat{c} \equiv \bar{r} - \frac{\bar{r} - \underline{r}}{1 - \phi}$ and price $p_i = \underline{r}$ if $c_i \leq \hat{c}$.*

Proof. S_i earns 0 with probability one if she sets $p_i > \bar{r}$. Her (expected) payoff is $[\bar{r} - c_i] \phi n_i$ if she sets $p_i = \bar{r}$. S_i earns certain payoff $[\underline{r} - c_i] n_i$ if she sets $p_i = \underline{r}$. Because S_i 's payoff is strictly less than $[\underline{r} - c_i] n_i$ if she sets $p_i < \underline{r}$ and strictly less than $[\bar{r} - c_i] \phi n_i$ if she sets any $p_i \in (\underline{r}, \bar{r})$, S_i 's optimal price is either $p_i = \underline{r}$ or $p_i = \bar{r}$.

Comparing the payoffs from these two prices reveals that S_i secures a strictly higher expected payoff by setting $p_i = \bar{r}$ if and only if:

$$[\bar{r} - c_i] \phi n_i > [\underline{r} - c_i] n_i$$

$$\Leftrightarrow c_i > \frac{\underline{r} - \phi \bar{r}}{1 - \phi} = \frac{[1 - \phi] \bar{r} - [\bar{r} - \underline{r}]}{1 - \phi} = \bar{r} - \frac{\bar{r} - \underline{r}}{1 - \phi} \equiv \hat{c}.$$

Similarly, S_i optimally sets $p_i = \underline{r}$ if $c_i \leq \hat{c}$. ■

Claim 2. Under privacy, S_i 's equilibrium payoff is: (i) $\phi n_i [\bar{r} - c_i]$ if $c_i > \hat{c}$; and (ii) $n_i [\underline{r} - c_i]$ if $c_i \leq \hat{c}$.

Proof. The conclusion follows directly from Claim 1. ■

Claim 3. Under privacy, B 's equilibrium welfare is:

$$\begin{aligned}
0 & \quad \text{if } r = \underline{r} \\
0 & \quad \text{if } r = \bar{r} \text{ and } c_1 > \hat{c} \text{ and } c_2 > \hat{c} \\
[\bar{r} - \underline{r}] n_1 & \quad \text{if } r = \bar{r} \text{ and } c_1 \leq \hat{c} \text{ and } c_2 > \hat{c} \\
[\bar{r} - \underline{r}] n_2 & \quad \text{if } r = \bar{r} \text{ and } c_1 > \hat{c} \text{ and } c_2 \leq \hat{c} \\
[\bar{r} - \underline{r}] [n_1 + n_2] & \quad \text{if } r = \bar{r} \text{ and } c_1 \leq \hat{c} \text{ and } c_2 \leq \hat{c}.
\end{aligned} \tag{1}$$

Proof. The conclusion follows directly from Claim 1. ■

To prove Lemmas 5 and 6 in the paper, we now characterize the PPBE that can arise under transparency for each of the possible configurations of the sellers' production costs. The following definitions are employed in the ensuing analysis.

Definitions

1. $\phi_{n_1}(p_1)$ is S_2 's *ex post* assessment of the probability that $r = \bar{r}$ after observing B purchase $n_1 > 0$ units from S_1 at price p_1 .
2. $\phi_0(p_1)$ is S_2 's *ex post* assessment of the probability that $r = \bar{r}$ after after observing B purchase 0 units from S_1 at price p_1 .
3. $c^* \equiv \hat{c} + \phi \frac{n_2}{n_1} \left[\frac{\bar{r} - r}{1 - \phi} \right]$.
4. A separating PPBE is a PPBE in which B 's action in his interaction with S_1 varies with his reservation value, r .
5. A pooling PPBE is a PPBE in which B 's action in his interaction with S_1 does not vary with his reservation value, r .

We begin by presenting three conclusions that hold for all possible configurations of the sellers' costs.

Theorem 1. *Suppose $n_1 \leq n_2$. Then a separating PPBE does not exist under transparency.*

Proof. Initially suppose a separating PPBE exists in which B buys from $S1$ at some \tilde{p}_1 if and only if $r = \bar{r}$. Because beliefs must satisfy Bayes Rule along the equilibrium path, $\phi_{n_1}(\tilde{p}_1) = 1$ and $\phi_0(\tilde{p}_1) = 0$. Consequently, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$ at \tilde{p}_1 , whereas $S2$ will set $p_2 = \underline{r}$ if B does not buy from $S1$. B will buy from $S2$ if and only if the price she sets does not exceed B 's valuation, r .

First suppose $r = \bar{r}$. B 's welfare if he buys from $S1$ at price \tilde{p}_1 is:

$$\pi_B^{n_1}(\tilde{p}_1, \bar{r}) = n_1[\bar{r} - \tilde{p}_1] + n_2[\bar{r} - \bar{r}] = n_1[\bar{r} - \tilde{p}_1]. \quad (2)$$

B 's welfare if he does not buy from $S1$ at price $p_1 = \tilde{p}_1$ is:

$$\pi_B^0(\tilde{p}_1, \bar{r}) = n_2[\bar{r} - \underline{r}]. \quad (3)$$

(2) and (3) imply that B will buy from $S1$ when $p_1 = \tilde{p}_1$ if and only if:

$$\begin{aligned} n_1[\bar{r} - \tilde{p}_1] \geq n_2[\bar{r} - \underline{r}] &\Leftrightarrow n_1\bar{r} - n_1\tilde{p}_1 \geq n_2[\bar{r} - \underline{r}] \\ \Leftrightarrow n_1\bar{r} - n_2[\bar{r} - \underline{r}] \geq n_1\tilde{p}_1 &\Leftrightarrow \bar{r} - \frac{n_2}{n_1}[\bar{r} - \underline{r}] \geq \tilde{p}_1. \end{aligned}$$

Because $n_2 \geq n_1$:

$$\bar{r} - \frac{n_2}{n_1}[\bar{r} - \underline{r}] \leq \bar{r} - \frac{n_2}{n_2}[\bar{r} - \underline{r}] = \underline{r}.$$

Therefore, if such a separating PPBE exists, B buys from $S1$ at price $p_1 = \tilde{p}_1$ if and only if:

$$\tilde{p}_1 \leq \bar{r} - \frac{n_2}{n_1}[\bar{r} - \underline{r}] \leq \underline{r}. \quad (4)$$

Now suppose $r = \underline{r}$. B 's welfare if he buys from $S1$ at price \tilde{p}_1 (and subsequently does not buy from $S2$ at price $p_2 = \bar{r}$) is:

$$\pi_B^{n_1}(\tilde{p}_1, \underline{r}) = n_1[\underline{r} - \tilde{p}_1]. \quad (5)$$

B 's welfare if he does not buy from $S1$ at price \tilde{p}_1 is:

$$\pi_B^0(\tilde{p}_1, \underline{r}) = n_2[\underline{r} - \underline{r}] = 0. \quad (6)$$

(5) and (6) imply that B will not buy from $S1$ at price, \tilde{p}_1 if and only if:

$$n_1[\underline{r} - \tilde{p}_1] < 0 \Leftrightarrow \tilde{p}_1 > \underline{r}. \quad (7)$$

The last inequality in (7) contradicts (4). Therefore, when $n_2 \geq n_1$, there does not exist a separating PPBE in which B buys from $S1$ at some price \tilde{p}_1 if and only if $r = \bar{r}$.

To conclude the proof, suppose a separating PPBE exists in which B buys from $S1$ at price $p_1 = \tilde{p}_1$ if and only if $r = \underline{r}$. Because beliefs must satisfy Bayes Rule along the equilibrium path, $\phi_{n_1}(\tilde{p}_1) = 0$ and $\phi_0(\tilde{p}_1) = 1$. Consequently, $S2$ will set price $p_2 = \underline{r}$ if B buys from $S1$ at price \tilde{p}_1 , whereas $S2$ will set $p_2 = \bar{r}$ if B does not buy from $S1$.

First suppose $r = \bar{r}$. B 's welfare if she buys from $S1$ at price \tilde{p}_1 is:

$$\pi_B^{n_1}(\tilde{p}_1, \bar{r}) = n_1[\bar{r} - \tilde{p}_1] + n_2[\bar{r} - \underline{r}].$$

B 's welfare if he does not buy from $S1$ at price \tilde{p}_1 is:

$$\pi_B^0(\tilde{p}_1, \bar{r}) = n_2 [\bar{r} - \bar{r}] = 0.$$

Therefore, B will not buy from $S1$ at price \tilde{p}_1 if and only if:

$$\begin{aligned} n_1 [\bar{r} - \tilde{p}_1] + n_2 [\bar{r} - \underline{r}] < 0 &\Leftrightarrow [\bar{r} - \tilde{p}_1] + \frac{n_2}{n_1} [\bar{r} - \underline{r}] < 0 \\ \Leftrightarrow \bar{r} + \frac{n_2}{n_1} [\bar{r} - \underline{r}] < \tilde{p}_1. \end{aligned}$$

Because $\bar{r} < \bar{r} + \frac{n_2}{n_1} [\bar{r} - \underline{r}]$, B will not buy from from $S1$ at price \tilde{p}_1 if and only if:

$$\bar{r} < \bar{r} + \frac{n_2}{n_1} [\bar{r} - \underline{r}] < \tilde{p}_1. \quad (8)$$

Now suppose $r = \underline{r}$. B 's welfare if he buys from $S1$ at price \tilde{p}_1 (and subsequently buys from $S2$ at price $p_2 = \underline{r}$) is:

$$\pi_B^{n_1}(\tilde{p}_1, \underline{r}) = n_1 [\underline{r} - \tilde{p}_1] + n_2 [\underline{r} - \underline{r}] = n_1 [\underline{r} - \tilde{p}_1].$$

B 's welfare if he does not buy from $S1$ at price \tilde{p}_1 (and subsequently does not buy from $S2$ at price $p_2 = \bar{r}$) is 0. Therefore, B will buy from $S1$ at \tilde{p}_1 if and only if:

$$n_1 [\underline{r} - \tilde{p}_1] \geq 0 \Leftrightarrow \tilde{p}_1 \leq \underline{r}. \quad (9)$$

(9) contradicts (8). Therefore, there does not exist a separating PPBE when $n_2 \geq n_1$. ■

Theorem 2. *There does not exist a PPBE under transparency in which: (i) $S1$ sets $p_1 = \bar{r}$; (ii) B buys from $S1$ if and only if $r = \bar{r}$; (iii) $S2$ always sets $p_2 = \bar{r}$.*

Proof. Suppose there exists a PPBE in which $S1$ sets $p_1 = \bar{r}$, B buys at this price if and only if $r = \bar{r}$, and $S2$ always sets $p_2 = \bar{r}$. Suppose $S2$ observes that B did not buy from $S1$ at price $p_1 = \bar{r}$. Because $S2$'s beliefs must satisfy Bayes' Rule along the equilibrium path, $\phi_0(\bar{r}) = 0$. Consequently, $S2$'s payoff is $n_2 [\underline{r} - c_2] > 0$ if she sets $p_2 = \underline{r}$ and 0 if she sets $p_2 = \bar{r}$. The fact that $S2$'s payoff is strictly higher when she sets $p_2 = \underline{r}$ after seeing that B did not buy from $S1$ at $p_1 = \bar{r}$ contradicts the premise that $S2$ always sets \bar{r} in the putative equilibrium. ■

Theorem 3. *Suppose $n_1 > n_2$. Then under transparency, $S1$ will set $p_1 = \hat{p}_1 \equiv \bar{r} - \frac{n_2}{n_1} [\bar{r} - \underline{r}]$ in any separating PPBE.*

Proof. First consider a separating PPBE in which B buys from $S1$ when she sets price \tilde{p}_1 if and only if $r = \bar{r}$. Because beliefs must satisfy Bayes Rule along the equilibrium path, $\phi_{n_1}(\tilde{p}_1) = 1$ and $\phi_0(\tilde{p}_1) = 0$. Consequently, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$ at price \tilde{p}_1 and $S2$ will set $p_2 = \underline{r}$ otherwise.

B will buy from $S2$ if and only if $p_2 \leq r$. Therefore, B 's welfare if he buys from $S1$ at price \tilde{p}_1 when $r = \bar{r}$ is:

$$\pi_B^{n_1}(\tilde{p}_1, \bar{r}) = n_1 [\bar{r} - \tilde{p}_1] + n_2 [\bar{r} - \bar{r}] = n_1 [\bar{r} - \tilde{p}_1].$$

B 's welfare if he does not buy from $S1$ at price $p_1 = \tilde{p}_1$ is:

$$\pi_B^0(\tilde{p}_1, \bar{r}) = n_2 [\bar{r} - \underline{r}].$$

Therefore, B will buy from $S1$ at price $p_1 = \tilde{p}_1$ if and only if:

$$n_1 [\bar{r} - \tilde{p}_1] \geq n_2 [\bar{r} - \underline{r}] \Leftrightarrow \tilde{p}_1 \leq \bar{r} - \frac{n_2}{n_1} [\bar{r} - \underline{r}] \equiv \hat{p}_1. \quad (10)$$

B 's welfare if he buys from $S1$ at price $p_1 = \tilde{p}_1$ when $r = \underline{r}$ (and subsequently does not buy from $S2$ at price $p_2 = \bar{r}$) is:

$$\pi_B^{n_1}(\tilde{p}_1, \underline{r}) = n_1 [\underline{r} - \tilde{p}_1].$$

B 's welfare if he does not buy from $S1$ at price $p_1 = \tilde{p}_1$ is:

$$\pi_B^0(\tilde{p}_1, \underline{r}) = n_2 [\underline{r} - \underline{r}] = 0.$$

Therefore, B will not buy from $S1$ at price $p_1 = \tilde{p}_1$ if and only if:

$$n_1 [\underline{r} - \tilde{p}_1] < 0 \Leftrightarrow \tilde{p}_1 > \underline{r}. \quad (11)$$

(10) and (11) imply that $S1$'s price must lie in the interval $(\underline{r}, \hat{p}_1]$ in any separating PPBE in which B buys from $S1$ at price $\tilde{p}_1 \in (\underline{r}, \hat{p}_1]$ if and only if $r = \bar{r}$. (10) and (11) also imply that when $S1$ sets $\tilde{p}_1 \in (\underline{r}, \hat{p}_1)$, her payoff is $\phi n_1 [\tilde{p}_1 - c_1] < \phi n_1 [\hat{p}_1 - c_1]$. Because $S1$ can secure a higher payoff by charging \hat{p}_1 than by charging $\tilde{p}_1 \in (\underline{r}, \hat{p}_1)$, any such \tilde{p}_1 cannot arise in a separating PPBE. Consequently, $\tilde{p}_1 = \hat{p}_1$ in any separating PPBE in which B buys from $S1$ at price $p_1 = \tilde{p}_1$ if and only if $r = \bar{r}$.

Now suppose a separating PPBE exists in which B buys from $S1$ at price $p_1 = \tilde{p}_1$ if and only if $r = \underline{r}$. Because beliefs must satisfy Bayes Rule along the equilibrium path, $\phi_{n_1}(\tilde{p}_1) = 0$ and $\phi_0(\tilde{p}_1) = 1$. Consequently, $S2$ will set $p_2 = \underline{r}$ if B buys from $S1$ at price \tilde{p}_1 , whereas $S2$ will set $p_2 = \bar{r}$ if B does not buy from $S1$ at price \tilde{p}_1 .

B will buy from $S2$ if and only if $p_2 \leq r$. Therefore, B 's welfare if he buys from $S1$ at price \tilde{p}_1 when $r = \bar{r}$ is:

$$\pi_B^{n_1}(\tilde{p}_1, \bar{r}) = n_1 [\bar{r} - \tilde{p}_1] + n_2 [\bar{r} - \underline{r}].$$

B 's welfare if he does not buy from $S1$ at price \tilde{p}_1 is:

$$\pi_B^{n_1}(\tilde{p}_1, \bar{r}) = n_2 [\bar{r} - \bar{r}] = 0.$$

Therefore, B will not buy from $S1$ at price \tilde{p}_1 if and only if:

$$n_1 [\bar{r} - \tilde{p}_1] + n_2 [\bar{r} - \underline{r}] < 0 \Leftrightarrow \tilde{p}_1 > \bar{r} + \frac{n_2}{n_1} [\bar{r} - \underline{r}] > \bar{r}. \quad (12)$$

B 's welfare if he buys from $S1$ at price p_1 when $r = \underline{r}$ is:

$$\pi_B^{n_1}(\tilde{p}_1, \underline{r}) = n_1 [\underline{r} - \tilde{p}_1] + n_2 [\underline{r} - \underline{r}] = n_1 [\underline{r} - \tilde{p}_1].$$

B 's welfare if he does not buy from $S1$ at price \tilde{p}_1 (and subsequently does not buy from $S2$ at price $p_2 = \bar{r}$) is 0. Therefore, B will buy from $S1$ at price \tilde{p}_1 if and only if:

$$n_1 [\underline{r} - \tilde{p}_1] \geq 0 \Leftrightarrow \tilde{p}_1 \leq \underline{r}. \quad (13)$$

(13) provides a contradiction of (12). Therefore, no separating PPBE exists in which B buys from $S1$ at price \tilde{p}_1 if and only if $r = \underline{r}$. ■

We now characterize the equilibria that can arise for each of the possible configurations of the sellers' costs (and relative values of n_1 and n_2).

Setting 1A. $c_1 > \hat{c}$, $c_2 \leq \hat{c}$, and $n_1 > n_2$.

Theorem 4. *Suppose $n_1 > n_2$, $c_1 > c^* > \hat{c}$, and $c_2 \leq \hat{c}$. Then under transparency, a separating PPBE exists in which: (i) $S1$ sets $\hat{p}_1 \equiv \bar{r} - \frac{n_2}{n_1} [\bar{r} - \underline{r}]$; (ii) B buys from $S1$ if and only if $r = \bar{r}$; (iii) $S2$ sets $p_2 = \bar{r}$ if B buys from $S1$, and sets $p_2 = \underline{r}$ if B does not buy from $S1$; and (iv) B buys from $S2$ if and only if $p_2 \leq r$.*

Proof. The beliefs that support the identified equilibrium actions are: (i) $\phi_{n_1}(\underline{r}) = \phi$; (ii) $\phi_0(\underline{r}) \leq \phi$; (iii) $\phi_{n_1}(p_1) = 1$ and $\phi_0(p_1) \leq \phi$ for all $p_1 > \bar{r}$; (iv) $\phi_{n_1}(p_1) = 1$ and $\phi_0(p_1) = 0$ for all $p_1 \in (\underline{r}, \bar{r}]$; and (v) $\phi_{n_1}(p_1) = \phi$ and $\phi_0(p_1) \leq \phi$ for all $p_1 < \underline{r}$.

The proof proceeds by backward induction. We first prove that $S2$'s equilibrium actions are optimal, given her beliefs. Then we prove that B 's equilibrium actions are optimal, given $S2$'s beliefs. Next we prove that $S1$'s equilibrium actions are optimal. Finally, we verify that $S2$'s beliefs satisfy Bayes Rule along the equilibrium path.

A. Prove that $S2$'s equilibrium actions are optimal.

$\phi_{n_1}(\hat{p}_1) = 1$. Therefore, $S2$ maximizes her payoff by setting $p_2 = \bar{r}$ if B buys from $S1$ at price $p_1 = \hat{p}_1$.

$\phi_0(\hat{p}_1) = 0$. Therefore, $S2$ optimally sets $p_2 = \underline{r}$ if B does not buy from $S1$ at price $p_1 = \hat{p}_1$.

B. Prove that B 's equilibrium strategy is optimal.

Because the game ends following B 's interaction with $S2$, B maximizes his welfare by buying from $S2$ if and only if $p_2 \leq r$.

We now prove that B maximizes his welfare by buying from $S1$ at price $p_1 = \hat{p}_1$ if and only if $r = \bar{r}$.

First suppose $r = \bar{r}$. If B buys from $S1$ at price $p_1 = \hat{p}_1$, $S2$ will set $p_2 = \bar{r}$ because $\phi_{n_1}(\hat{p}_1) = 1$. Therefore, B 's welfare is:

$$\begin{aligned} \pi_B^{n_1}(\hat{p}_1, \bar{r}) &= n_1 [\bar{r} - \hat{p}_1] + n_2 [\bar{r} - \bar{r}] = n_1 [\bar{r} - \hat{p}_1] \\ &= n_1 \left[\bar{r} - \left(\bar{r} - \frac{n_2}{n_1} [\bar{r} - \underline{r}] \right) \right] = n_2 [\bar{r} - \underline{r}]. \end{aligned}$$

If B does not buy from $S1$ when $p_1 = \hat{p}_1$, $S2$ will set $p_2 = \underline{r}$ because $\phi_0(\hat{p}_1) = 0$. In this case, B 's welfare is:

$$\pi_B^0(\widehat{p}_1, \bar{r}) = n_2 [\bar{r} - \underline{r}].$$

Because $\pi_B^{n_1}(\widehat{p}_1, \bar{r}) = \pi_B^0(\widehat{p}_1, \bar{r})$, B optimally buys from $S1$ when she sets $p_1 = \widehat{p}_1$ when $r = \bar{r}$.

Now suppose $r = \underline{r}$. If B buys from $S1$ at price $p_1 = \widehat{p}_1$, $S2$ will set $p_2 = \bar{r}$ because $\phi_{n_1}(\widehat{p}_1) = 1$. B will not buy from $S2$ because $\underline{r} < p_2$. Therefore, B 's welfare is:

$$\pi_B^{n_1}(\widehat{p}_1, \underline{r}) = n_1 [\underline{r} - \widehat{p}_1] < 0.$$

If B instead does not buy from $S1$ at price $p_1 = \widehat{p}_1$, $S2$ will set $p_2 = \underline{r}$ because $\phi_0(\widehat{p}_1) = 0$. Therefore, B 's welfare is:

$$\pi_B^0(\widehat{p}_1, \underline{r}) = n_2 [\underline{r} - \underline{r}] = 0.$$

Because $\pi_B^0(\widehat{p}_1, \underline{r}) > \pi_B^{n_1}(\widehat{p}_1, \underline{r})$, B optimally does not buy from $S1$ when $p_1 = \widehat{p}_1$ and $r = \underline{r}$.

C. Prove that $S1$'s equilibrium actions are optimal.

1. We begin by characterizing B 's optimal response to out-of-equilibrium prices by $S1$.

Result C1. When $r = \bar{r}$, B optimally does not buy from $S1$ at any price $p_1 > \widehat{p}_1$, and buys from $S1$ at any price $p_1 < \widehat{p}_1$.

Proof. Initially suppose that $S1$ sets $p_1 > \bar{r}$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 > \bar{r}$, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$. Consequently, B 's welfare is:

$$\pi_B^{n_1}(p_1, \bar{r}) = n_1 [\bar{r} - p_1] + n_2 [\bar{r} - \bar{r}] = n_1 [\bar{r} - p_1] < 0.$$

Because $\phi_0(p_1) \leq \phi$ for all $p_1 > \bar{r}$, if B does not buy from $S1$ at price $p_1 > \bar{r}$, $S2$ will set $p_2 = \underline{r}$ (because $c_2 \leq \widehat{c}$, by assumption). Therefore, B 's welfare is:

$$\pi_B^0(p_1, \bar{r}) = n_2 [\bar{r} - \underline{r}] > 0.$$

Consequently, when $r = \bar{r}$, B optimally does not buy from $S1$ if $p_1 > \bar{r}$.

Next suppose that $S1$ sets $p_1 \in (\widehat{p}_1, \bar{r}]$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 \in (\widehat{p}_1, \bar{r}]$, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$. Consequently, B 's welfare is:

$$\pi_B^{n_1}(p_1, \bar{r}) = n_1 [\bar{r} - p_1] + n_2 [\bar{r} - \bar{r}] < n_1 [\bar{r} - \widehat{p}_1] = n_2 [\bar{r} - \underline{r}].$$

Because $\phi_0(p_1) = 0$ for all $p_1 \in (\widehat{p}_1, \bar{r}]$, $S2$ will set $p_2 = \underline{r}$ if B does not buy from $S1$. Consequently, B 's welfare is:

$$\pi_B^0(p_1, \bar{r}) = n_2 [\bar{r} - \underline{r}].$$

Because $\pi_B^{n_1}(p_1, \bar{r}) < \pi_B^0(p_1, \bar{r})$, B will not buy from $S1$ at any price $p_1 \in (\widehat{p}_1, \bar{r}]$ when $r = \bar{r}$.

Next suppose that $S1$ sets $p_1 \in (\underline{r}, \widehat{p}_1)$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 \in (\underline{r}, \widehat{p}_1)$, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$. Consequently, B 's welfare is:

$$\pi_B^{n_1}(p_1, \bar{r}) = n_1 [\bar{r} - p_1] + n_2 [\bar{r} - \bar{r}] > n_1 [\bar{r} - \widehat{p}_1] = n_2 [\bar{r} - \underline{r}].$$

Because $\phi_0(p_1) = 0$ for all $p_1 \in (\underline{r}, \widehat{p}_1)$, $S2$ will set $p_2 = \underline{r}$ if B does not buy from $S1$. Consequently, B 's welfare is:

$$\pi_B^0(p_1, \bar{r}) = n_2 [\bar{r} - \underline{r}].$$

Because $\pi_B^{n_1}(p_1, \bar{r}) > \pi_B^0(p_1, \bar{r})$, B will buy from $S1$ at any price $p_1 \in (\underline{r}, \hat{p}_1)$ when $r = \bar{r}$.

Finally, suppose that $S1$ sets $p_1 \leq \underline{r}$. Because $\phi_{n_1}(p_1) = \phi$ for all $p_1 \leq \underline{r}$, $S2$ will set $p_2 = \underline{r}$ if B buys from $S1$ (since $c_2 \leq \hat{c}$). Consequently, B 's welfare is:

$$\pi_B^{n_1}(p_1, \bar{r}) = n_1 [\bar{r} - p_1] + n_2 [\bar{r} - \underline{r}] \geq [n_1 + n_2] [\bar{r} - \underline{r}].$$

Because $\phi_0(p_1) \leq \phi$ for all $p_1 \leq \underline{r}$, $S2$ will set $p_2 = \underline{r}$ if B does not buy from $S1$ (since $c_2 \leq \hat{c}$). Consequently, B 's welfare is:

$$\pi_B^0(p_1, \bar{r}) = n_2 [\bar{r} - \underline{r}].$$

Because $\pi_B^{n_1}(p_1, \bar{r}) > \pi_B^0(p_1, \bar{r})$, B will buy from $S1$ at any price $p_1 \leq \underline{r}$ when $r = \bar{r}$.

Result C2. When $r = \underline{r}$ and $p_1 \neq \hat{p}_1$, B optimally does not buy from $S1$ at any price $p_1 > \underline{r}$ and buys from $S1$ at any price $p_1 \leq \underline{r}$.

Proof. Initially suppose that $S1$ sets $p_1 > \underline{r}$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 > \underline{r}$, if B buys from $S1$, $S2$ will set $p_2 = \bar{r}$. Consequently, B 's welfare is:

$$\pi_B^{n_1}(p_1, \underline{r}) = n_1 [\underline{r} - p_1] + 0 [\bar{r} - \underline{r}] = n_1 [\underline{r} - p_1] < 0.$$

Because $\phi_0(p_1) \leq \phi$ for all $p_1 > \underline{r}$, $S2$ will set $p_2 = \underline{r}$ if B does not buy from $S1$ (since $c_2 < \hat{c}$). Consequently, B 's welfare is:

$$\pi_B^0(p_1, \underline{r}) = n_2 [\bar{r} - \underline{r}] = 0.$$

Because $\pi_B^{n_1}(p_1, \underline{r}) < \pi_B^0(p_1, \underline{r})$, B will not buy from $S1$ at any price $p_1 > \underline{r}$ when $r = \underline{r}$.

Next suppose that $S1$ sets $p_1 \leq \underline{r}$. Because $\phi_{n_1}(p_1) = \phi$ for all $p_1 \leq \underline{r}$, if B buys from $S1$, $S2$ will set $p_2 = \underline{r}$ (since $c_2 < \hat{c}$). Consequently, B 's welfare is:

$$\pi_B^{n_1}(p_1, \underline{r}) = n_1 [\underline{r} - p_1] + n_2 [\underline{r} - \underline{r}] = n_1 [\underline{r} - p_1] \geq 0.$$

Because $\phi_0(p_1) \leq \phi$ for all $p_1 \leq \underline{r}$, $S2$ will set $p_2 = \underline{r}$ (since $c_2 < \hat{c}$) if B does not buy from $S1$. Consequently, B 's welfare is:

$$\pi_B^0(p_1, \underline{r}) = n_2 [\bar{r} - \underline{r}] = 0.$$

Because $\pi_B^{n_1}(p_1, \underline{r}) \geq \pi_B^0(p_1, \underline{r})$, B will buy from $S1$ at any price $p_1 \leq \underline{r}$ when $r = \underline{r}$.

2. We now prove that $S1$'s equilibrium actions are optimal.

When $S1$ sets $p_1 = \hat{p}_1$, B will buy from $S1$ if and only if $r = \bar{r}$. Consequently, $S1$'s payoff is:

$$\pi_1(\hat{p}_1) = \phi n_1 [\hat{p}_1 - c_1] > 0.$$

Results C1 and C2 imply that B will not buy from $S1$ at any price $p_1 > \hat{p}_1$. Consequently, $S1$'s payoff is $\pi_1(p_1) = 0$ for all $p_1 > \hat{p}_1$.

Results C1 and C2 imply that if $S1$ sets $p_1 \in (\underline{r}, \hat{p}_1)$, B will buy from $S1$ if $r = \bar{r}$ and not buy from $S1$ if $r = \underline{r}$. Consequently, $S1$'s payoff is:

$$\pi_1(p_1) = \phi n_1 [p_1 - c_1] < \phi n_1 [\hat{p}_1 - c_1].$$

Results C1 and C2 imply that B will buy from $S1$ if she sets $p_1 \leq \underline{r}$. Consequently, $S1$'s

payoff from a price $p_1 \leq \underline{r}$ is:

$$\pi_1(p_1) = n_1 [p_1 - c_1] \leq n_1 [\underline{r} - c_1] < \phi n_1 [\widehat{p}_1 - c_1]. \quad (14)$$

The inequality in (14) holds because:

$$\begin{aligned} \underline{r} - c_1 < \phi [\widehat{p}_1 - c_1] &\Leftrightarrow [1 - \phi] c_1 > \underline{r} - \phi \widehat{p}_1 = \underline{r} - \phi \left[\bar{r} - \frac{n_2}{n_1} (\bar{r} - \underline{r}) \right] \\ &\Leftrightarrow [1 - \phi] c_1 > \underline{r} - \phi \bar{r} + \phi \frac{n_2}{n_1} [\bar{r} - \underline{r}] \\ &\Leftrightarrow [1 - \phi] c_1 > [1 - \phi] \bar{r} - (\bar{r} - \underline{r}) + \phi \frac{n_2}{n_1} [\bar{r} - \underline{r}] \\ &\Leftrightarrow c_1 > \bar{r} - \frac{\bar{r} - \underline{r}}{1 - \phi} + \phi \frac{n_2}{n_1} [\bar{r} - \underline{r}] = \widehat{c} + \phi \frac{n_2}{n_1} \left[\frac{\bar{r} - \underline{r}}{1 - \phi} \right]. \end{aligned} \quad (15)$$

The inequality in (15) holds because $c_1 > c^*$ by hypothesis. Therefore, $S1$ maximizes her payoff by setting $p_1 = \widehat{p}_1$.

D. Prove that $S2$'s beliefs satisfy Bayes Rule along the equilibrium path.

$$\begin{aligned} \Pr(r = \bar{r} | B \text{ buys at } \widehat{p}_1) &= \frac{\Pr(r = \bar{r} \text{ and } B \text{ buys at } \widehat{p}_1)}{\Pr(B \text{ buys at } \widehat{p}_1)} \\ &= \frac{\Pr(B \text{ buys at } \widehat{p}_1 | r = \bar{r}) \Pr(r = \bar{r})}{\Pr(B \text{ buys at } \widehat{p}_1 | r = \bar{r}) \Pr(r = \bar{r}) + \Pr(B \text{ buys at } \widehat{p}_1 | r = \underline{r}) \Pr(r = \underline{r})} \\ &= \frac{1[\phi]}{1[\phi] + 0[1 - \phi]} = 1 = \phi_{n_1}(\widehat{p}_1). \end{aligned}$$

$$\begin{aligned} \Pr(r = \bar{r} | B \text{ does not buy at } \widehat{p}_1) &= \frac{\Pr(r = \bar{r} \text{ and } B \text{ does not buy at } \widehat{p}_1)}{\Pr(B \text{ does not buy at } \widehat{p}_1)} \\ &= \frac{\Pr(B \text{ does not buy at } \widehat{p}_1 | r = \bar{r}) \Pr(r = \bar{r})}{\Pr(B \text{ does not buy at } \widehat{p}_1 | r = \bar{r}) \Pr(r = \bar{r}) + \Pr(B \text{ does not buy at } \widehat{p}_1 | r = \underline{r}) \Pr(r = \underline{r})} \\ &= \frac{0[\phi]}{0[\phi] + 1[1 - \phi]} = 0 = \phi_0(\widehat{p}_1). \quad \blacksquare \end{aligned}$$

Observation. Theorem 3 implies that the equilibrium identified in Theorem 4 is the unique separating PPBE under the specified conditions.

Corollary 1. *Under the conditions specified in Theorem 4, B 's equilibrium welfare is the same under transparency and privacy for each realization of r .*

Proof. Claim 3 implies that under privacy, B 's equilibrium welfare is: (i) 0 when $r = \underline{r}$; and (ii) $n_2 [\bar{r} - \underline{r}]$ when $r = \bar{r}$.

Theorem 4 implies that under transparency, B 's equilibrium welfare is: (i) $n_1 [\bar{r} - \hat{p}_1] = n_2 [\bar{r} - \underline{r}]$ when $r = \bar{r}$; and (ii) 0 when $r = \underline{r}$. ■

Corollary 2. *Under the conditions specified in Theorem 4, transparency reduces the equilibrium payoff of $S1$ and increases the equilibrium payoff of $S2$ (by the same amount).*

Proof. Claim 2 implies that under privacy, the payoffs of $S1$ and $S2$ are:

$$\pi_1^V = \phi n_1 [\bar{r} - c_1] \quad \text{and} \quad \pi_2^V = n_2 [\underline{r} - c_2]. \quad (16)$$

Theorem 4 implies that under transparency, the payoffs of $S1$ and $S2$, are:

$$\begin{aligned} \pi_1^T &= \phi n_1 [\hat{p}_1 - c_1] < \phi n_1 [\bar{r} - c_1], \quad \text{and} \\ \pi_2^T &= \phi n_2 [\bar{r} - c_2] + [1 - \phi] n_2 [\underline{r} - c_2] > n_2 [\underline{r} - c_2]. \end{aligned} \quad (17)$$

(16) and (17) imply:

$$\pi_1^T - \pi_1^V = \phi n_1 [\hat{p}_1 - \bar{r}] = -\phi n_2 [\bar{r} - \underline{r}] = -[\pi_2^T - \pi_2^V]. \quad \blacksquare$$

Corollary 3. *Under the conditions specified in Theorem 4, transparency does not affect expected industry welfare.*

Proof. Because industry welfare is the sum of B 's welfare and the payoffs of $S1$ and $S2$, the conclusion follows directly from Corollaries 1 and 2. ■

Theorem 5. *Suppose $c_1 > c^* > \hat{c}$ and $c_2 \leq \hat{c}$. Then a pooling PPBE does not exist under transparency.*

Proof. Because $c_2 \leq \hat{c}$ and $S2$'s beliefs must satisfy Bayes Rule along the equilibrium path, $S2$ will always set $p_2 = \underline{r}$ in any pooling PPBE. B will always buy from $S2$ at this price. Consequently, B 's welfare if he always buys from $S1$ at price p_1 is:

$$\begin{aligned} &n_1 [\underline{r} - p_1] \quad \text{when } r = \underline{r}, \quad \text{and} \\ &n_1 [\bar{r} - p_1] + n_2 [\bar{r} - \underline{r}] \quad \text{when } r = \bar{r}. \end{aligned} \quad (18)$$

First consider a pooling PPBE in which B always buys from $S1$ when $p_1 > \underline{r}$. (18) implies that B 's welfare will be negative when $r = \underline{r}$. B can secure nonnegative welfare by not buying from $S1$ at price $p_1 > \underline{r}$ when $r = \underline{r}$. Therefore, a pooling PPBE in which B always buys from $S1$ at price $p_1 > \underline{r}$ does not exist.

Now consider a pooling PPBE in which B always buys from $S1$ when $p_1 \leq \underline{r}$. $S1$'s payoff is $n_1 [p_1 - c_1]$, which is maximized at price $p_1 = \underline{r}$. Consequently, in any such pooling PPBE, $S1$ will set $p_1 = \underline{r}$ and secure payoff $n_1 [\underline{r} - c_1] > 0$ (which exceeds $S1$'s payoff in any pooling PPBE equilibrium in which B does not buy from $S1$).

(18) implies that when $S1$ sets $p_1 = \underline{r}$, B 's welfare is 0 when $r = \underline{r}$ whether he buys or does not buy from $S1$. When $r = \bar{r}$, B 's welfare is $[n_1 + n_2][\bar{r} - \underline{r}]$ if he buys from $S1$ at price $p_1 = \underline{r}$. His corresponding welfare is $n_2 [\bar{r} - \underline{r}]$ if he does not buy from $S1$ at price $p_1 = \underline{r}$. Therefore, B will always buy from $S1$ at price $p_1 = \underline{r}$ and $S1$ secures payoff $n_1 [\underline{r} - c_1]$.

Theorem 4 implies that if $S1$ deviates by setting price $p_1 = \hat{p}_1$, B will buy at this price when $r = \underline{r}$. Therefore, $S1$'s payoff from this deviation is

$$\pi_1(\hat{p}_1) = \phi n_1 [\hat{p}_1 - c_1] > n_1 [\underline{r} - c_1]$$

where the strict inequality holds because:

$$\begin{aligned} \underline{r} - c_1 < \phi [\hat{p}_1 - c_1] &\Leftrightarrow [1 - \phi] c_1 > \underline{r} - \phi \hat{p}_1 = \underline{r} - \phi \left[\bar{r} - \frac{n_2}{n_1} (\bar{r} - \underline{r}) \right] \\ &\Leftrightarrow [1 - \phi] c_1 > \underline{r} - \phi \bar{r} + \phi \frac{n_2}{n_1} [\bar{r} - \underline{r}] \\ &\Leftrightarrow [1 - \phi] c_1 > [1 - \phi] \bar{r} - (\bar{r} - \underline{r}) + \phi \frac{n_2}{n_1} [\bar{r} - \underline{r}] \\ &\Leftrightarrow c_1 > \bar{r} - \frac{\bar{r} - \underline{r}}{1 - \phi} + \phi \frac{n_2}{n_1} [\bar{r} - \underline{r}] = \hat{c} + \phi \frac{n_2}{n_1} \left[\frac{\bar{r} - \underline{r}}{1 - \phi} \right] \equiv c^*. \end{aligned}$$

Because $S1$ earns a higher payoff by deviating to charge \hat{p}_1 , there does not exist a pooling PPBE under the conditions specified in Theorem 5. ■

Theorem 6. *Suppose $c_1 \in (\hat{c}, c^*)$ and $c_2 \leq \hat{c}$. Then the unique PPBE under transparency is the pooling PPBE in which $S1$ and $S2$ both charge \underline{r} and B always purchases from both sellers at this price.*

Proof. Claims 1 and 2 imply that if a pooling PPBE exists, $S1$ will set $p_1 = \underline{r}$ and secure payoff $n_1 [\underline{r} - c_1]$. Theorems 3 and 4 imply that if a separating PPBE exists, $S1$ will charge price \hat{p}_1 and secure payoff $\phi n_1 [\hat{p}_1 - c_1]$, which is strictly less than $n_1 [\underline{r} - c_1]$ when $c_1 \in (\hat{c}, c^*)$ as demonstrated above. Therefore, $S1$ will set $p_1 = \underline{r}$, B will always buy from $S1$ at this price, and consequently, $S2$ will set $p_2 = \underline{r}$ and B will buy from $S2$ at this price. ■

Observation. Theorems 3, 4, 5, and 6 imply that when $c_2 \leq \hat{c} < c_1$, the unique PPBE under transparency is: (i) the separating PPBE in which $S1$ sets $p_1 = \hat{p}_1$ if $c_1 > c^*$ and (ii) the pooling PPBE in which $p_1 = p_2 = \underline{r}$ if $c_1 \in (\hat{c}, c^*)$.

Corollary 4. *Under the conditions specified in Theorem 6, transparency increases B 's welfare when $r = \bar{r}$ and does not change B 's welfare when $r = \underline{r}$.*

Proof. Claim 3 implies that under privacy, B 's equilibrium welfare is: (i) $n_2 [\bar{r} - \underline{r}]$ when $r = \bar{r}$; and (ii) 0 when $r = \underline{r}$.

Theorem 6 implies that under transparency, B 's equilibrium welfare is: (i) $[n_1 + n_2] [\bar{r} - \underline{r}]$ when $r = \bar{r}$; and (ii) 0 when $r = \underline{r}$. ■

Corollary 5. *Under the conditions specified in Theorem 6, transparency: (i) reduces $S1$'s equilibrium payoff; and (ii) does not change $S2$'s equilibrium payoff.*

Proof. Claim 3 implies that under privacy, the payoffs of $S1$ and $S2$ are:

$$\pi_1^V = \phi n_1 [\bar{r} - c_1] \quad \text{and} \quad \pi_2^V = n_2 [\underline{r} - c_2]. \quad (19)$$

Theorem 6 implies that under transparency, the payoffs of $S1$ and $S2$ are:

$$\pi_i^T = n_i [\underline{r} - c_i] \quad \text{for } i = 1, 2. \quad (20)$$

The conclusion follows because $\pi_2^V = \pi_2^T$ and $n_1 [\underline{r} - c_1] < \phi n_1 [\bar{r} - c_1]$ since $c_1 > \hat{c}$. ■

Corollary 6. *Under the conditions specified in Theorem 6, transparency increases equilibrium industry welfare.*

Proof. Corollaries 4 and 5 and their proofs imply that under privacy, equilibrium industry welfare is:

$$W^V = \phi n_2 [\bar{r} - \underline{r}] + \phi n_1 [\bar{r} - c_1] + n_2 [\underline{r} - c_2]. \quad (21)$$

(20) and the proof of Corollary 4 imply that under transparency, equilibrium industry welfare is:

$$W^T = \phi [n_1 + n_2] [\bar{r} - \underline{r}] + n_1 [\underline{r} - c_1] + n_2 [\underline{r} - c_2]. \quad (22)$$

(21) and (22) imply:

$$\begin{aligned} W^T - W^V &= \phi [n_1 + n_2] [\bar{r} - \underline{r}] + n_1 [\underline{r} - c_1] - \phi n_2 [\bar{r} - \underline{r}] - \phi n_1 [\bar{r} - c_1] \\ &= \phi [n_1 + n_2] [\bar{r} - \underline{r}] - \phi n_2 [\bar{r} - \underline{r}] + n_1 [\underline{r} - c_1] - \phi n_1 [\bar{r} - c_1] \\ &= \phi n_1 [\bar{r} - \underline{r}] - \phi n_1 [\bar{r} - c_1] + n_1 [\underline{r} - c_1] \\ &= \phi n_1 [\bar{r} - \underline{r} - \bar{r} + c_1] + n_1 [\underline{r} - c_1] \\ &= -\phi n_1 [\underline{r} - c_1] + n_1 [\underline{r} - c_1] \\ &= [1 - \phi] n_1 [\underline{r} - c_1] > 0. \quad \blacksquare \end{aligned}$$

Setting 1B. $c_1 > \hat{c}$, $c_2 \leq \hat{c}$, and $n_2 \geq n_1$.

Theorem 7. *Suppose $n_2 \geq n_1$, $c_1 > \hat{c}$, and $c_2 \leq \hat{c}$. Then under transparency, a pooling PPBE exists in which: (i) $S1$ sets $p_1 = \underline{r}$; (ii) $S2$ sets $p_1 = \underline{r}$; and (iii) B always buys from $S1$ and from $S2$.*

Proof. The beliefs that support the identified equilibrium actions are: (i) $\phi_{n_1}(\underline{r}) = \phi$; (ii) $\phi_0(\underline{r}) \leq \phi$; (iii) $\phi_{n_1}(p_1) = 1$ and $\phi_0(p_1) \leq \phi$ for all $p_1 > \bar{r}$; (iv) $\phi_{n_1}(p_1) = 1$ and $\phi_0(p_1) = 0$ for all $p_1 \in (\underline{r}, \bar{r}]$; and (v) $\phi_{n_1}(p_1) = \phi$ and $\phi_0(p_1) \leq \phi$ for all $p_1 < \underline{r}$.

The proof proceeds by backward induction. We first prove that $S2$'s equilibrium action is optimal, given her beliefs. Then we prove that B 's equilibrium actions are optimal, given $S2$'s beliefs. Next we prove that $S1$'s equilibrium action is optimal. Finally, we verify that $S2$'s beliefs satisfy Bayes Rule along the equilibrium path.

A. Prove that $S2$'s equilibrium action is optimal.

$\phi_{n_1}(\underline{r}) = \phi$. Therefore, after B buys from $S1$ at price $p_1 = \underline{r}$, $S2$ maximizes her payoff by acting as she does under privacy. Claim 1 implies that because $c_2 \leq \hat{c}$, $S2$ will set $p_2 = \underline{r}$.

$\phi_0(\underline{r}) \leq \phi$. Therefore, because $c_2 \leq \hat{c}$, $S2$ will set $p_2 = \underline{r}$ if B does not buy from $S1$.

B. Prove that B 's equilibrium actions are optimal.

Because the game ends following B 's interaction with $S2$, B maximizes his welfare by buying from $S2$ at price $p_2 = \underline{r}$.

We now prove that B maximizes his welfare by buying from $S1$ when she sets price $p_1 = \underline{r}$. First suppose $r = \bar{r}$. If B buys from $S1$ at price $p_1 = \underline{r}$, $S2$ will set $p_2 = \underline{r}$ because $\phi_{n_1}(\underline{r}) = \phi$ and $c_2 \leq \hat{c}$. Therefore, B 's welfare is:

$$\pi_B^{n_1}(p_1 = \underline{r}, \bar{r}) = n_1 [\bar{r} - \underline{r}] + n_2 [\bar{r} - \underline{r}] = [n_1 + n_2] [\bar{r} - \underline{r}].$$

If B instead does not buy from $S1$ at price $p_1 = \underline{r}$, $S2$ will set $p_2 = \underline{r}$ because $\phi_0(\underline{r}) \leq \phi$ and $c_2 \leq \hat{c}$. Therefore, B 's welfare is:

$$\pi_B^0(p_1 = \underline{r}, \bar{r}) = n_2 [\bar{r} - \underline{r}] < [n_1 + n_2] [\bar{r} - \underline{r}].$$

Because $\pi_B^{n_1}(p_1 = \underline{r}, \bar{r}) > \pi_B^0(p_1 = \underline{r}, \bar{r})$, B optimally buys from $S1$ at price $p_1 = \underline{r}$ when $r = \bar{r}$.

Now suppose $r = \underline{r}$. If B buys from $S1$ at price $p_1 = \underline{r}$, $S2$ will set $p_2 = \underline{r}$ because $\phi_{n_1}(\underline{r}) = \phi$ and $c_2 \leq \hat{c}$. Therefore, B 's welfare is:

$$\pi_B^{n_1}(p_1 = \underline{r}, \underline{r}) = n_1 [\underline{r} - \underline{r}] + n_2 [\underline{r} - \underline{r}] = 0.$$

If B does not buy from $S1$ at price $p_1 = \underline{r}$, $S2$ will set $p_2 = \underline{r}$ because $\phi_{n_1}(\underline{r}) \leq \phi$ and $c_2 \leq \hat{c}$. Therefore, B 's welfare is:

$$\pi_B^0(p_1 = \underline{r}, \underline{r}) = n_2 [\underline{r} - \underline{r}] = 0.$$

Because $\pi_B^{n_1}(p_1 = \underline{r}, \underline{r}) = \pi_B^0(p_1 = \underline{r}, \underline{r})$, B optimally buys from $S1$ at price $p_1 = \underline{r}$ when $r = \underline{r}$.

C. Prove that $S1$'s equilibrium action is optimal.

1. We begin by characterizing B 's optimal response to out-of-equilibrium prices by $S1$.

Result Ci. When $r = \bar{r}$, B optimally does not buy from $S1$ at any price $p_1 > \underline{r}$ and buys from $S1$ at any price $p_1 < \underline{r}$.

Proof. Initially suppose that $S1$ sets $p_1 > \bar{r}$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 > \bar{r}$, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$. Consequently, B 's welfare is:

$$\pi_B^{n_1}(p_1, \bar{r}) = n_1 [\bar{r} - p_1] + n_2 [\bar{r} - \bar{r}] = n_1 [\bar{r} - p_1] < 0.$$

Because $\phi_0(p_1) \leq \phi$ for all $p_1 > \bar{r}$, if B does not buy from $S1$ at price $p_1 > \bar{r}$, $S2$ will set $p_2 = \underline{r}$ (because $c_2 \leq \hat{c}$, by assumption). Consequently, B 's welfare is:

$$\pi_B^0(p_1, \bar{r}) = n_2 [\bar{r} - \underline{r}] > 0.$$

Because $\pi_B^0(p_1, \bar{r}) > \pi_B^{n_1}(p_1, \bar{r})$, B optimally does not buy from $S1$ when she sets $p_1 > \bar{r}$ and $r = \bar{r}$.

Next suppose that $S1$ sets $p_1 \in (\underline{r}, \bar{r}]$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 \in (\underline{r}, \bar{r}]$, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$. Consequently, B 's welfare is:

$$\pi_B^{n_1}(p_1, \bar{r}) = n_1 [\bar{r} - p_1] + n_2 [\bar{r} - \bar{r}] = n_1 [\bar{r} - p_1] < n_1 [\bar{r} - \underline{r}].$$

Because $\phi_0(p_1) = 0$ for all $p_1 \in (\underline{r}, \bar{r}]$, if B does not buy from $S1$, $S2$ will set $p_2 = \underline{r}$. Because $n_2 \geq n_1$, B 's welfare is:

$$\pi_B^0(p_1, \bar{r}) = n_2 [\bar{r} - \underline{r}] \geq n_1 [\bar{r} - \underline{r}]. \quad (23)$$

Because $\pi_B^0(p_1, \bar{r}) \geq \pi_B^{n_1}(p_1, \bar{r})$, B optimally does not buy from $S1$ when she sets any price $p_1 \in (\underline{r}, \bar{r}]$ and $r = \bar{r}$.

Now suppose that $S1$ sets $p_1 < \underline{r}$. Because $\phi_{n_1}(p_1) = \phi$ for all $p_1 < \underline{r}$, $S2$ will set $p_2 = \underline{r}$ if B buys from $S1$ at this price (since $c_2 \leq \hat{c}$). In this case B 's welfare is:

$$\pi_B^{n_1}(p_1, \bar{r}) = n_1 [\bar{r} - p_1] + n_2 [\bar{r} - \underline{r}] > [n_1 + n_2] [\bar{r} - \underline{r}].$$

Because $c_2 \leq \hat{c}$ and $\phi_0(p_1) \leq \phi$ for all $p_1 < \underline{r}$, if B does not buy from $S1$, $S2$ will set $p_2 = \underline{r}$. Consequently, B 's welfare is:

$$\pi_B^0(p_1, \bar{r}) = n_2 [\bar{r} - \underline{r}] < [n_1 + n_2] [\bar{r} - \underline{r}].$$

Because $\pi_B^{n_1}(p_1, \bar{r}) > \pi_B^0(p_1, \bar{r})$, B optimally buys from $S1$ for any $p_1 < \underline{r}$ when $r = \bar{r}$.

Result Cii. When $r = \underline{r}$, B optimally does not buy from $S1$ if $p_1 > \underline{r}$ and buys from $S1$ if $p_1 < \underline{r}$.

Proof. Initially suppose that $S1$ sets $p_1 > \underline{r}$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 > \underline{r}$, if B buys from $S1$, he will subsequently not buy from $S2$ at price $p_2 = \bar{r}$. Consequently, his welfare

is $\pi_B^{n_1}(p_1, \underline{r}) = n_1 [\underline{r} - p_1] < 0$. Because $\phi_0(p_1) \leq \phi$ for all $p_1 > \underline{r}$, if B does not buy from $S1$ at price $p_1 > \underline{r}$, $S2$ will set $p_2 = \underline{r}$ (because $c_2 \leq \hat{c}$). Consequently, B 's welfare is $\pi_B^0(p_1, \underline{r}) = n_2 [\underline{r} - \underline{r}] = 0$. Because $\pi_B^0(p_1, \underline{r}) > \pi_B^{n_1}(p_1, \underline{r})$, B optimally does not buy from $S1$ when she sets any price $p_1 > \underline{r}$ and $r = \underline{r}$.

Next suppose that $S1$ sets $p_1 \in (\underline{r}, \bar{r}]$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 \in (\underline{r}, \bar{r}]$, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$ and he will subsequently not buy from $S2$. Consequently, his welfare is $\pi_B^{n_1}(p_1, \underline{r}) = n_1 [\underline{r} - p_1] < 0$. Because $\phi_0(p_1) = 0$ for all $p_1 \in (\underline{r}, \bar{r}]$, if B does not buy from $S1$ at price $p_1 \in (\underline{r}, \bar{r}]$, $S2$ will set $p_2 = \underline{r}$ (because $c_2 \leq \hat{c}$). Consequently, B 's welfare is $\pi_B^0(p_1, \underline{r}) = n_2 [r - \underline{r}] = 0$. Because $\pi_B^0(p_1, \underline{r}) > \pi_B^{n_1}(p_1, \underline{r})$, B optimally does not buy from $S1$ at any $p_1 \in (\underline{r}, \bar{r}]$ when $r = \underline{r}$.

Now suppose that $S1$ sets $p_1 < \underline{r}$. Because $\phi_{n_1}(p_1) = \phi$ for all $p_1 < \underline{r}$, if B buys from $S1$ at this price, $S2$ will set $p_2 = \underline{r}$ (because $c_2 \leq \hat{c}$). Consequently, B 's welfare is $\pi_B^{n_1}(p_1, \underline{r}) = n_1 [\underline{r} - p_1] + n_2 [\underline{r} - \underline{r}] > 0$. Because $\phi_0(p_1) \leq \phi$ for all $p_1 < \underline{r}$, if B does not buy from $S1$, $S2$ will set $p_2 = \underline{r}$ (since $c_2 \leq \hat{c}$). Consequently, B 's welfare is $\pi_B^0(p_1, \underline{r}) = n_2 [\underline{r} - \underline{r}] = 0$. Because $\pi_B^0(p_1, \underline{r}) < \pi_B^{n_1}(p_1, \underline{r})$, B optimally buys from $S1$ at any $p_1 < \underline{r}$ when $r = \underline{r}$.

2. We now prove that $S1$'s equilibrium strategy is optimal.

Because B always buys from $S1$ at price $p_1 = \underline{r}$, $S1$'s payoff in the putative equilibrium is:

$$\pi_1(\underline{r}) = n_1 [\underline{r} - c_1] > 0.$$

Results Ci and Cii imply that this payoff exceeds the payoff that $S1$ earns by setting any price $p_1 > \underline{r}$. In particular, B does not buy from $S1$ if $p_1 > \underline{r}$, so $\pi_1(p_1) = 0$ for all $p_1 > \underline{r}$.

Results Ci and Cii imply that B will buy from $S1$ if $p_1 < \underline{r}$. Consequently, $S1$'s payoff from setting $p_1 < \underline{r}$ is:

$$\pi_1(p_1) = n_1 [p_1 - c_1] < n_1 [\underline{r} - c_1] = \pi_1(\underline{r}).$$

Therefore, $S1$ maximizes her payoff by setting $p_1 = \underline{r}$.

D. Prove that $B2$'s beliefs satisfy Bayes Rule along the equilibrium path.

$$\begin{aligned} \Pr(r = \bar{r} | B \text{ buys at } p_1 = \underline{r}) &= \frac{\Pr(r = \bar{r} \text{ and } B \text{ buys at } p_1 = \underline{r})}{\Pr(B \text{ buys at } p_1 = \underline{r})} \\ &= \frac{\Pr(B \text{ buys at } p_1 = \underline{r} | r = \bar{r}) \Pr(r = \bar{r})}{\Pr(B \text{ buys at } p_1 = \underline{r} | r = \bar{r}) \Pr(r = \bar{r}) + \Pr(B \text{ buys at } p_1 = \underline{r} | r = \underline{r}) \Pr(r = \underline{r})} \\ &= \frac{1[\phi]}{1[\phi] + 1[1 - \phi]} = \phi = \phi_{n_1}(\underline{r}). \quad \blacksquare \end{aligned}$$

Corollary 7. *Under the conditions specified in Theorem 7, B 's equilibrium welfare is: (i) strictly higher under transparency than under privacy when $r = \bar{r}$; and (ii) the same under transparency and privacy when $r = \underline{r}$.*

Proof. Claim 3 implies that under privacy, B 's equilibrium welfare is: (i) $n_2 [\underline{r} - \bar{r}]$ when $r = \bar{r}$; and (ii) 0 when $r = \underline{r}$.

Theorem 7 implies that under transparency, B 's equilibrium welfare is: (i) 0 when $r = \underline{r}$; and (ii) $[n_1 + n_2] [\underline{r} - \bar{r}]$ when $r = \bar{r}$. ■

Corollary 8. *Under the conditions specified in Theorem 7, transparency reduces the equilibrium payoff of $S1$ and leaves the equilibrium payoff of $S2$ unchanged.*

Proof. Claim 2 implies that under privacy, the payoffs of $S1$ and $S2$ are:

$$\pi_1^V = \phi n_1 [\bar{r} - c_1] \quad \text{and} \quad \pi_2^V = n_2 [\underline{r} - c_2]. \quad (24)$$

Theorem 7 shows that, under transparency, B will always buy from $S1$ and from $S2$ at price \underline{r} , so the equilibrium payoffs of $S1$ and $S2$ are:

$$\pi_1^T = n_1 [\underline{r} - c_1] < \phi n_1 [\bar{r} - c_1] = \pi_1^V \quad \text{and} \quad \pi_2^T = n_2 [\underline{r} - c_2] = \pi_2^V. \quad (25)$$

The strict inequality in (25) holds because $c_1 > \hat{c}$, by assumption. ■

Corollary 9. *Under the conditions specified in Theorem 7, transparency increases equilibrium industry welfare.*

Proof. Corollaries 4 and 5 and their proofs imply that under privacy, equilibrium expected industry welfare is:

$$W^V = \phi n_2 [\bar{r} - \underline{r}] + \phi n_1 [\bar{r} - c_1] + n_2 [\underline{r} - c_2]. \quad (26)$$

(25) and the proof of Corollary 7 imply that under transparency, equilibrium industry welfare is:

$$W^T = \phi [n_1 + n_2] [\bar{r} - \underline{r}] + n_1 [\underline{r} - c_1] + n_2 [\underline{r} - c_2]. \quad (27)$$

(26) and (27) imply:

$$\begin{aligned} W^T - W^V &= \phi n_1 [\bar{r} - \underline{r}] + n_1 [\underline{r} - c_1] - \phi n_1 [\bar{r} - c_1] \\ &= n_1 [\phi \bar{r} - \phi \underline{r} + \underline{r} - c_1 - \phi \bar{r} + \phi c_1] \\ &= n_1 [-\phi \underline{r} + \underline{r} - c_1 + \phi c_1] \\ &= n_1 [(1 - \phi) \underline{r} - (1 - \phi) c_1] \\ &= [1 - \phi] n_1 [\underline{r} - c_1] > 0. \quad \blacksquare \end{aligned}$$

Theorem 8. *Under the conditions specified in Theorem 7, the equilibrium identified in the theorem is the unique pooling PPBE.*

Proof. $S1$'s payoff is $n_1 [\underline{r} - c_1] > 0$ in the identified pooling PPBE. $S1$'s payoff is 0 in any pooling PPBE in which B never buys from $S1$. $S1$'s payoff is also less than $n_1 [\underline{r} - c_1]$ in any pooling PPBE in which B always buys from $S1$ at $p_1 < \underline{r}$. Therefore, in any alternative candidate pooling PPBE, B must always buy from $S1$ at $p_1 > \underline{r}$.

When $r = \underline{r}$, B 's welfare if he buys from $S1$ at price $p_1 > \underline{r}$ is $n_1 [\underline{r} - p_1] < 0$. B 's welfare is non-negative if he does not buy from $S1$ at this price. Because B will not always buy from $S1$ at $p_1 > \underline{r}$, the equilibrium in which $S1$ sets $p_1 = \underline{r}$ is the unique pooling PPBE under the specified conditions. ■

Setting 2A. $c_1 > \hat{c}$, $c_2 > \hat{c}$, and $n_1 > n_2$.

Theorem 9. *Suppose $n_1 > n_2$, $c_1 > c^* > \hat{c}$, and $c_2 \geq \hat{c}$. Then under transparency, a PPBE exists in which: (i) $S1$ sets $\hat{p}_1 \equiv \bar{r} - \frac{n_2}{n_1} [\bar{r} - \underline{r}]$; (ii) $S2$ sets $p_2 = \bar{r}$ if B buys from $S1$, whereas $S2$ sets $p_2 = \underline{r}$ if B does not buy from $S1$; (iii) B buys from $S1$ if $r = \bar{r}$, but does not buy from $S1$ if $r = \underline{r}$; and (iv) B buys from $S2$ if and only $p_2 \leq r$.*

Proof. The beliefs that support the identified equilibrium actions are: (i) $\phi_{n_1}(p_1) = 1$ for all $p_1 > \underline{r}$; (ii) $\phi_0(p_1) = 0$ for all $p_1 \leq \bar{r}$; (iii) $\phi_0(p_1) = \phi$ for all $p_1 > \bar{r}$; and (iv) $\phi_{n_1}(p_1) = \phi$ for all $p_1 \leq \underline{r}$.

The proof proceeds by backward induction. We first prove that $S2$'s equilibrium actions are optimal, given her beliefs. Then we prove that B 's equilibrium actions are optimal, given $S2$'s beliefs. Next we prove that $S1$'s equilibrium actions are optimal. Finally, we verify that $S2$'s beliefs satisfy Bayes Rule along the equilibrium path.

A. Prove that $S2$'s equilibrium actions are optimal.

First suppose B purchases n_1 units from $S1$ at price \hat{p}_1 . Because $\phi_{n_1}(\hat{p}_1) = 1$, $S2$'s payoff is $\bar{r} - c_2 > 0$ if she sets $p_2 = \bar{r}$. Because $S2$'s secures payoff zero if she sets $p_2 > \bar{r}$ and secures payoff $p_2 - c_2 < \bar{r} - c_2$ if she sets $p_2 < \bar{r}$, $S2$ maximizes her payoff by setting $p_2 = \bar{r}$.

Now suppose B does not buy from $S1$ at price \hat{p} . Because $\phi_0(\hat{p}_1) = 0$, $S2$'s payoff is $\underline{r} - c_2 > 0$ if she sets $p_2 = \underline{r}$. Because $S2$ secures payoff zero if she sets $p_2 > \underline{r}$ and secures payoff $p_2 - c_2 < \underline{r} - c_2$ if she sets $p_2 < \underline{r}$, $S2$ maximizes her payoff by setting $p_2 = \underline{r}$.

B. Prove that B 's equilibrium actions are optimal.

Because the game ends following B 's interaction with $S2$, B maximizes his welfare by buying from $S2$ if and only if $p_2 \leq r$.

We now prove that B maximizes his welfare by buying from $S1$ at price $p_1 = \hat{p}_1$ if $r = \bar{r}$ and not buying from $S1$ if $r = \underline{r}$.

First suppose $r = \bar{r}$. If B buys from $S1$ at price \hat{p}_1 , $S2$ will set $p_2 = \bar{r}$ because $\phi_{n_1}(\hat{p}_1) = 1$. Therefore, B 's welfare is:

$$\begin{aligned}
\pi_B^{n_1}(\widehat{p}_1, \bar{r}) &= n_1 [\bar{r} - \widehat{p}_1] + n_2 [\bar{r} - \bar{r}] = n_1 [\bar{r} - \widehat{p}_1] \\
&= n_1 \left[\bar{r} - \left(\bar{r} - \frac{n_2}{n_1} [\bar{r} - \underline{r}] \right) \right] = n_2 [\bar{r} - \underline{r}]. \tag{28}
\end{aligned}$$

If B does not buy from $S1$ at price \widehat{p}_1 , $S2$ will set $p_2 = \underline{r}$ because $\phi_0(\widehat{p}_1) = 0$. Therefore, B 's welfare is:

$$\pi_B^0(\widehat{p}_1, \bar{r}) = n_2 [\bar{r} - \underline{r}].$$

Because $\pi_B^{n_1}(\widehat{p}_1, \bar{r}) = \pi_B^0(\widehat{p}_1, \bar{r})$, B will buy from $S1$ at price $p_1 = \widehat{p}_1$ when $r = \bar{r}$.

Now suppose $r = \underline{r}$. If B buys from $S1$ at price \widehat{p}_1 , $S2$ will set $p_2 = \bar{r}$ because $\phi_{n_1}(\widehat{p}_1) = 1$. B will not buy from $S2$ at this price. Consequently, B 's welfare is:

$$\begin{aligned}
\pi_B^{n_1}(\widehat{p}_1, \underline{r}) &= n_1 [\underline{r} - \widehat{p}_1] = n_1 \left[\underline{r} - \left(\bar{r} - \frac{n_2}{n_1} [\bar{r} - \underline{r}] \right) \right] \\
&= n_1 \underline{r} - n_1 \bar{r} + n_2 \bar{r} - n_2 \underline{r} = [n_2 - n_1] [\bar{r} - \underline{r}] < 0.
\end{aligned}$$

If B does not buy from $S1$ at price \widehat{p}_1 , $S2$ will set $p_2 = \underline{r}$ because $\phi_0(\widehat{p}_1) = 0$. Therefore, B 's welfare is:

$$\pi_B^0(\widehat{p}_1, \underline{r}) = n_2 [\underline{r} - \underline{r}] = 0.$$

Because $\pi_B^0(\widehat{p}_1, \underline{r}) > \pi_B^{n_1}(\widehat{p}_1, \underline{r})$, B optimally does not buy from $S1$ at price \widehat{p}_1 when $r = \underline{r}$.

C. Prove that $S1$'s equilibrium action is optimal.

1. We begin by characterizing B 's optimal response to out-of-equilibrium prices by $S1$.

Result C1. When $r = \bar{r}$, B optimally buys from $S1$ if $p_1 < \widehat{p}_1$ and does not buy from $S1$ if $p_1 > \widehat{p}_1$.

Proof. Initially suppose that $S1$ sets $p_1 > \bar{r}$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 > \underline{r}$, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$. Consequently, B 's welfare is:

$$\pi_B^{n_1}(p_1, \bar{r}) = n_1 [\bar{r} - p_1] + n_2 [\bar{r} - \bar{r}] = n_1 [\bar{r} - p_1] < 0.$$

Because $\phi_0(p_1) = \phi$ for all $p_1 > \bar{r}$, if B does not buy from $S1$ at price $p_1 > \bar{r}$, $S2$ will either set $p_2 = \bar{r}$ (if $c_2 > \widehat{c}$) or $p_2 = \underline{r}$ (if $c_2 = \widehat{c}$). Consequently, B 's welfare is:

$$\pi_B^0(p_1, \bar{r}) \geq \min \{ n_2 [\bar{r} - \bar{r}], n_2 [\bar{r} - \underline{r}] \} = n_2 [\bar{r} - \bar{r}] = 0.$$

Because $\pi_B^0(p_1, \bar{r}) > \pi_B^{n_1}(p_1, \bar{r})$, B will not buy from $S1$ at any price $p_1 > \bar{r}$ when $r = \bar{r}$.

Next suppose that $S1$ sets $p_1 \in (\widehat{p}_1, \bar{r}]$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 > \underline{r}$, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$. Consequently, B 's welfare is:

$$\pi_B^{n_1}(p_1, \bar{r}) = n_1 [\bar{r} - p_1] + n_2 [\bar{r} - \bar{r}] = n_1 [\bar{r} - p_1] < n_1 [\bar{r} - \widehat{p}_1] = n_2 [\bar{r} - \underline{r}],$$

where the last equality follows from the analysis that underlies (28). Because $\phi_0(p_1) = 0$ for all $p_1 \leq \bar{r}$, if B does not buy from $S1$ at price $p_1 \in (\widehat{p}_1, \bar{r}]$, $S2$ will set $p_2 = \underline{r}$. Consequently, B 's welfare is:

$$\pi_B^0(p_1, \bar{r}) = n_2 [\bar{r} - \underline{r}].$$

Because $\pi_B^0(p_1, \bar{r}) > \pi_B^{n_1}(p_1, \bar{r})$, B optimally does not buy from $S1$ at price $p_1 \in (\widehat{p}_1, \bar{r}]$

when $r = \bar{r}$.

Next suppose that $S1$ sets $p_1 \in (\underline{r}, \hat{p}_1)$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 > \underline{r}$, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$. Consequently, B 's welfare is (again using the analysis that underlies (28)):

$$\pi_B^{n_1}(p_1, \bar{r}) = n_1[\bar{r} - p_1] + n_2[\bar{r} - \bar{r}] > n_1[\bar{r} - \hat{p}_1] = n_2[\bar{r} - \underline{r}].$$

Because $\phi_0(p_1) = 0$ for all $p_1 \leq \bar{r}$, if B does not buy from $S1$ at price $p_1 \in (\underline{r}, \hat{p}_1)$, $S2$ will set $p_2 = \underline{r}$. Consequently, B 's welfare is:

$$\pi_B^0(p_1, \bar{r}) = n_2[\bar{r} - \underline{r}].$$

Because $\pi_B^{n_1}(p_1, \bar{r}) > \pi_B^0(p_1, \bar{r})$, B will buy from $S1$ for any $p_1 \in (\hat{p}_1, \underline{r})$ when $r = \bar{r}$.

Now suppose that $S1$ sets $p_1 \leq \underline{r}$. Because $\phi_{n_1}(p_1) = \phi$ for all $p_1 \leq \underline{r}$, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$ (because $c_2 \geq \hat{c}$). Therefore, if B buys from $S1$, his welfare is:

$$\pi_B^{n_1}(p_1, \bar{r}) = n_1[\bar{r} - p_1] + n_2[\bar{r} - \bar{r}] = n_1[\bar{r} - p_1] \geq n_1[\bar{r} - \underline{r}].$$

Because $\phi_0(p_1) = 0$ for all $p_1 \leq \bar{r}$, if B does not buy from $S1$ at price $p_1 \leq \underline{r}$, $S2$ will set $p_2 = \underline{r}$. Consequently, B 's welfare is:

$$\pi_B^0(p_1, \bar{r}) = n_2[\bar{r} - \underline{r}].$$

$\pi_B^{n_1}(p_1, \bar{r}) > \pi_B^0(p_1, \bar{r})$ because $n_1 > n_2$. Consequently, B optimally buys from $S1$ at any price $p_1 \leq \underline{r}$ when $r = \bar{r}$.

Result C2. When $r = \underline{r}$, B optimally: (i) does not buy from $S1$ if $p_1 > \underline{r}$ ($p_1 \neq \hat{p}_1$); and (ii) buys from $S1$ if $p_1 \leq \underline{r}$.

Proof. Initially suppose that $S1$ sets $p_1 > \bar{r}$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 > \underline{r}$, if B buys from $S1$ at this price, he will subsequently not buy from $S2$ when she sets $p_2 = \bar{r}$. Consequently, B 's welfare is $\pi_B^{n_1}(p_1, \underline{r}) = n_1[\underline{r} - p_1] < 0$. Because $\phi_0(p_1) = \phi$ for all $p_1 > \bar{r}$, if B does not buy from $S1$ when she sets $p_1 > \bar{r}$, $S2$ will set $p_2 = \bar{r}$ (because $c_2 \geq \hat{c}$). B will not buy from $S2$ at this price, so B 's welfare is $\pi_B^0(p_1, \underline{r}) = 0$. Because $\pi_B^0(p_1, \underline{r}) > \pi_B^{n_1}(p_1, \underline{r})$, B optimally does not buy from $S1$ at any price $p_1 > \bar{r}$ when $r = \underline{r}$.

Next suppose that $S1$ sets $p_1 \in (\underline{r}, \bar{r}]$ ($p_1 \neq \hat{p}_1$). Because $\phi_{n_1}(p_1) = 1$ for all $p_1 > \underline{r}$, if B buys from $S1$ at this price, he will subsequently not buy from $S2$ when she sets price $p_2 = \bar{r}$. Consequently, B 's welfare is $\pi_B^{n_1}(p_1, \underline{r}) = n_1[\underline{r} - p_1] < 0$. Because $\phi_0(p_1) = 0$ for all $p_1 \leq \bar{r}$, if B does not buy from $S1$ when she sets $p_1 \in (\underline{r}, \bar{r}]$, $S2$ will set $p_2 = \underline{r}$. Consequently, B 's welfare is $\pi_B^0(p_1, \underline{r}) = n_2[\underline{r} - \underline{r}] = 0$. Because $\pi_B^0(p_1, \underline{r}) > \pi_B^{n_1}(p_1, \underline{r})$, B will not buy from $S1$ if she sets $p_1 \in (\underline{r}, \bar{r}]$ ($p_1 \neq \hat{p}_1$) when $r = \underline{r}$.

Now suppose that $S1$ sets $p_1 \leq \underline{r}$. Because $\phi_{n_1}(p_1) = \phi$ for all $p_1 \leq \underline{r}$, if B buys from $S1$ at this price, he will subsequently not buy from $S2$ when she sets $p_2 = \bar{r}$ (because $c_2 \geq \hat{c}$). Consequently, B 's welfare is $\pi_B^{n_1}(p_1, \underline{r}) = n_1[\underline{r} - p_1] \geq 0$. Because $\phi_0(p_1) = 0$ for all $p_1 \leq \bar{r}$, if B does not buy from $S1$ at price $p_1 \leq \underline{r}$, $S2$ will set $p_2 = \underline{r}$. Consequently, B 's welfare is $\pi_B^0(p_1, \underline{r}) = n_2[\underline{r} - \underline{r}] = 0$. Because $\pi_B^{n_1}(p_1, \underline{r}) \geq \pi_B^0(p_1, \underline{r})$, B optimally buys from $S1$ at price $p_1 \leq \underline{r}$ when $r = \underline{r}$.

2. We now prove that $S1$'s equilibrium action is optimal.

When $S1$ sets $p_1 = \hat{p}_1$, B buys from $S1$ if and only if $r = \bar{r}$. Consequently, $S1$'s payoff in the putative equilibrium is:

$$\pi_1(\hat{p}_1) = \phi n_1 [\hat{p}_1 - c_1].$$

Results C1 and C2 imply that B will not buy from $S1$ at price $p_1 > \hat{p}_1$. Consequently, $S1$'s payoff is $\pi_1(p_1) = 0$ for all $p_1 > \hat{p}_1$.

Results C1 and C2 imply that if $S1$ sets $p_1 \in (\underline{r}, \hat{p}_1)$, B will buy from $S1$ if $r = \bar{r}$ and not buy from $S1$ if $r = \underline{r}$. Consequently, $S1$'s payoff is:

$$\pi_1(p_1) = \phi n_1 [p_1 - c_1] < \phi n_1 [\hat{p}_1 - c_1].$$

Results C1 and C2 imply that B will buy from $S1$ if $p_1 \leq \underline{r}$. Consequently, $S1$'s payoff from setting $p_1 \leq \underline{r}$ is:

$$\pi_1(p_1) = n_1 [p_1 - c_1] \leq n_1 [\underline{r} - c_1] < \phi n_1 [\hat{p}_1 - c_1].$$

The strict inequality here holds because:

$$\begin{aligned} \underline{r} - c_1 < \phi [\hat{p}_1 - c_1] &\Leftrightarrow [1 - \phi] c_1 > \underline{r} - \phi \hat{p}_1 = \underline{r} - \phi \left[\bar{r} - \frac{n_2}{n_1} (\bar{r} - \underline{r}) \right] \\ \Leftrightarrow [1 - \phi] c_1 > \underline{r} - \phi \bar{r} + \phi \frac{n_2}{n_1} [\bar{r} - \underline{r}] &= [1 - \phi] \bar{r} - (\bar{r} - \underline{r}) + \phi \frac{n_2}{n_1} [\bar{r} - \underline{r}] \\ \Leftrightarrow c_1 > \bar{r} - \frac{\bar{r} - \underline{r}}{1 - \phi} + \left[\frac{\phi}{1 - \phi} \right] \frac{n_2}{n_1} [\bar{r} - \underline{r}] &= \hat{c} + \phi \frac{n_2}{n_1} \left[\frac{\bar{r} - \underline{r}}{1 - \phi} \right] \equiv c^*. \end{aligned}$$

Therefore, $S1$ maximizes her expected payoff by setting $p_1 = \hat{p}_1$.

D. Prove that $S2$'s beliefs satisfy Bayes Rule along the equilibrium path.

$$\begin{aligned} \Pr(r = \bar{r} | B \text{ buys at } \hat{p}_1) &= \frac{\Pr(r = \bar{r} \text{ and } B \text{ buys at } \hat{p}_1)}{\Pr(B \text{ buys at } \hat{p}_1)} \\ &= \frac{\Pr(B \text{ buys at } \hat{p}_1 | r = \bar{r}) \Pr(r = \bar{r})}{\Pr(B \text{ buys at } \hat{p}_1 | r = \bar{r}) \Pr(r = \bar{r}) + \Pr(B \text{ buys at } \hat{p}_1 | r = \underline{r}) \Pr(r = \underline{r})} \\ &= \frac{1[\phi]}{1[\phi] + 0[1 - \phi]} = 1 = \phi_{n_1}(\hat{p}_1). \end{aligned}$$

$$\begin{aligned} \Pr(r = \bar{r} | B \text{ does not buy at } \hat{p}_1) &= \frac{\Pr(r = \bar{r} \text{ and } B \text{ does not buy at } \hat{p}_1)}{\Pr(B \text{ does not buy at } \hat{p}_1)} \\ &= \frac{\Pr(B \text{ does not buy at } \hat{p}_1 | r = \bar{r}) \Pr(r = \bar{r})}{\Pr(B \text{ does not buy at } \hat{p}_1 | r = \bar{r}) \Pr(r = \bar{r}) + \Pr(B \text{ does not buy at } \hat{p}_1 | r = \underline{r}) \Pr(r = \underline{r})} \end{aligned}$$

$$= \frac{0[\phi]}{0[\phi] + 1[1 - \phi]} = 0 = \phi_0(\hat{p}_1). \quad \blacksquare$$

Corollary 10. *Under the conditions specified in Theorem 9, B's equilibrium welfare is: (i) strictly higher under transparency than under privacy when $r = \bar{r}$; and (ii) the same under transparency and privacy when $r = \underline{r}$.*

Proof. Claim 3 demonstrates that under privacy, B's equilibrium welfare is 0 in the present setting both when $r = \bar{r}$ and when $r = \underline{r}$.

Under transparency, B's equilibrium welfare when $r = \bar{r}$ is:

$$n_1[\bar{r} - \hat{p}_1] = n_1 \frac{n_2}{n_1} [\bar{r} - \underline{r}] = n_2[\bar{r} - \underline{r}] > 0. \quad (29)$$

Under transparency when $r = \underline{r}$, B's equilibrium welfare is $n_2[\underline{r} - \underline{r}] = 0$. \blacksquare

Corollary 11. *Under the conditions specified in Theorem 9, transparency: (i) reduces the equilibrium payoff of S1; (ii) increases the equilibrium payoff of S2; and (iii) reduces the aggregate equilibrium payoff of the sellers if $c_2 > \hat{c}$, and (iv) does not change the aggregate equilibrium payoff of the sellers if $c_2 = \hat{c}$.*

Proof. Claim 2 implies that under privacy, the payoffs of S1 and S2 are:

$$\pi_1^V = \phi n_1[\bar{r} - c_1] \quad \text{and} \quad \pi_2^V = \phi n_2[\bar{r} - c_2]. \quad (30)$$

Under transparency, B buys from S1 at price \hat{p}_1 (and so S2 will set $p_2 = \bar{r}$) if and only if $r = \bar{r}$. If $r = \underline{r}$, B does not buy from S1 at price \hat{p}_1 and so S2 sets $p_2 = \underline{r}$. Therefore, S1's equilibrium payoff is lower under transparency:

$$\pi_1^T = \phi n_1[\hat{p}_1 - c_1] < \phi n_1[\bar{r} - c_1] = \pi_1^V. \quad (31)$$

S2's equilibrium payoff is higher under transparency:

$$\pi_2^T = \phi n_2[\bar{r} - c_2] + [1 - \phi] n_2[\underline{r} - c_2] > \phi n_2[\bar{r} - c_2] = \pi_2^V. \quad (32)$$

(30) implies that the aggregate equilibrium payoff of the sellers under privacy is:

$$\pi_S^V = \phi n_1[\bar{r} - c_1] + \phi n_2[\bar{r} - c_2]. \quad (33)$$

(31) and (32) imply that under transparency, the aggregate equilibrium payoff of the sellers is:

$$\pi_S^T = \phi n_1[\hat{p}_1 - c_1] + \phi n_2[\bar{r} - c_2] + [1 - \phi] n_2[\underline{r} - c_2]. \quad (34)$$

(33) and (34) imply:

$$\begin{aligned} \pi_S^T - \pi_S^V &= \phi n_1[\hat{p}_1 - c_1] + \phi n_2[\bar{r} - c_2] + [1 - \phi] n_2[\underline{r} - c_2] \\ &\quad - (\phi n_1[\bar{r} - c_1] + \phi n_2[\bar{r} - c_2]) \\ &= \phi n_1[\hat{p}_1 - c_1] - \phi n_1[\bar{r} - c_1] + [1 - \phi] n_2[\underline{r} - c_2] \end{aligned}$$

$$\begin{aligned}
&= \phi n_1 \widehat{p}_1 - \phi n_1 c_1 - \phi n_1 \bar{r} + \phi n_1 c_1 + [1 - \phi] n_2 [\underline{r} - c_2] \\
&= \phi n_1 \widehat{p}_1 - \phi n_1 \bar{r} + [1 - \phi] n_2 [\underline{r} - c_2] \\
&= \phi n_1 [\widehat{p}_1 - \bar{r}] + [1 - \phi] n_2 [\underline{r} - c_2] \\
&\leq \phi n_1 [\widehat{p}_1 - \bar{r}] + [1 - \phi] n_2 [\underline{r} - \widehat{c}] \\
&= \phi n_1 \left[\bar{r} - \frac{n_2}{n_1} (\bar{r} - \underline{r}) - \bar{r} \right] + [1 - \phi] n_2 \left[\underline{r} - \left(\bar{r} - \frac{\bar{r} - \underline{r}}{1 - \phi} \right) \right] \\
&= -\phi n_2 [\bar{r} - \underline{r}] - [1 - \phi] n_2 [\bar{r} - \underline{r}] + n_2 [\bar{r} - \underline{r}] = 0.
\end{aligned}$$

The weak inequality here holds strictly when $c_2 > \widehat{c}$, and holds as an equality when $c_2 = \widehat{c}$. ■

Corollary 12. *Under the conditions specified in Theorem 9, transparency increases equilibrium industry welfare.*

Proof. Under privacy, $S1$ and $S2$ both charge \bar{r} and B 's welfare is 0. Therefore, industry welfare is aggregate supplier welfare:

$$W^V = \phi n_1 [\bar{r} - c_1] + \phi n_2 [\bar{r} - c_2]. \quad (35)$$

Under transparency, B 's welfare is as specified in (29) with probability ϕ , and the sellers receive the payoffs in (31) and (32). Therefore, under transparency, equilibrium industry welfare is:

$$W^T = \phi n_2 [\bar{r} - \underline{r}] + \phi n_1 [\widehat{p}_1 - c_1] + \phi n_2 [\bar{r} - c_2] + [1 - \phi] n_2 [\underline{r} - c_2]. \quad (36)$$

(35) and (36) imply:

$$\begin{aligned}
W^T - W^V &= \phi n_2 [\bar{r} - \underline{r}] + \phi n_1 [\widehat{p}_1 - c_1] + \phi n_2 [\bar{r} - c_2] + [1 - \phi] n_2 [\underline{r} - c_2] \\
&\quad - (\phi n_1 [\bar{r} - c_1] + \phi n_2 [\bar{r} - c_2]) \\
&= \phi n_2 [\bar{r} - \underline{r}] + \phi n_1 [\widehat{p}_1 - c_1] + [1 - \phi] n_2 [\underline{r} - c_2] - \phi n_1 [\bar{r} - c_1] \\
&= \phi n_2 [\bar{r} - \underline{r}] + \phi n_1 \left[\bar{r} - \frac{n_2}{n_1} (\bar{r} - \underline{r}) - c_1 \right] + [1 - \phi] n_2 [\underline{r} - c_2] - \phi n_1 [\bar{r} - c_1] \\
&= [1 - \phi] n_2 [\underline{r} - c_2] > 0. \quad \blacksquare
\end{aligned}$$

Theorem 10. *Suppose $n_1 > n_2$, $c^* > c_1 > \widehat{c}$, and $c_2 \geq \widehat{c}$. Then a separating PPBE does not exist under transparency.*

Proof. Initially suppose a separating PPBE exists in which B buys from $S1$ at price \widetilde{p}_1 if and only if $r = \bar{r}$. Because beliefs must satisfy Bayes Rule along the equilibrium path, $\phi_{n_1}(\widetilde{p}_1) = 1$ and $\phi_0(\widetilde{p}_1) = 0$. Consequently, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$ at price \widetilde{p}_1 , whereas $S2$ will set $p_2 = \underline{r}$ if B does not buy from $S1$ at price \widetilde{p}_1 .

B will buy from $S2$ if and only if $p_2 \leq r$. Therefore, B 's welfare if he buys from $S1$ at price \tilde{p}_1 when $r = \bar{r}$ is:

$$\pi_B^{n_1}(\tilde{p}_1, \bar{r}) = n_1[\bar{r} - \tilde{p}_1] + n_2[\bar{r} - \bar{r}] = n_1[\bar{r} - \tilde{p}_1].$$

B 's welfare if he does not buy from $S1$ at price \tilde{p}_1 is:

$$\pi_B^0(\tilde{p}_1, \bar{r}) = n_2[\bar{r} - \underline{r}].$$

Therefore, B will buy from $S1$ at price \tilde{p}_1 if and only if:

$$n_1[\bar{r} - \tilde{p}_1] \geq n_2[\bar{r} - \underline{r}] \Leftrightarrow \tilde{p}_1 \leq \bar{r} - \frac{n_2}{n_1}[\bar{r} - \underline{r}]. \quad (37)$$

B 's welfare if he buys from $S1$ at price \tilde{p}_1 when $r = \underline{r}$ (and subsequently does not buy from $S2$ when she sets $p_2 = \bar{r}$) is:

$$\pi_B^{n_1}(\tilde{p}_1, \underline{r}) = n_1[\underline{r} - \tilde{p}_1].$$

B 's welfare if he does not buy from $S1$ at price \tilde{p}_1 is:

$$\pi_B^0(\tilde{p}_1, \underline{r}) = n_2[\underline{r} - \underline{r}] = 0.$$

Therefore, B will not buy from $S1$ at price \tilde{p}_1 if and only if:

$$n_1[\underline{r} - \tilde{p}_1] < 0 \Leftrightarrow \tilde{p}_1 > \underline{r}. \quad (38)$$

These conditions together imply that $\tilde{p}_1 \in (\underline{r}, \bar{r} - \frac{n_2}{n_1}[\bar{r} - \underline{r}])$ in any candidate separating PPBE.

We now demonstrate that $\pi_1(\underline{r}) > \pi_1(\tilde{p}_1)$, i.e., that $S1$ secures a higher payoff if she sets $p_1 = \underline{r}$ than if she sets $p_1 = \tilde{p}_1 \in (\underline{r}, \bar{r} - \frac{n_2}{n_1}[\bar{r} - \underline{r}])$. To do so, we first show that B will always buy from $S1$ if she sets price $p_1 = \underline{r}$. Because $\phi_{n_1}(\underline{r}) = \phi$ and $c_2 \geq \hat{c}$, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$ when she sets $p_1 = \underline{r}$. Because $\phi_0(\underline{r}) = 0$, $S2$ will set $p_2 = \underline{r}$ if B does not buy from $S1$ when she sets $p_1 = \underline{r}$. Therefore, when $r = \bar{r}$, B 's welfare if he buys from $S1$ at this price is:

$$\pi_B^{n_1}(\underline{r}, \bar{r}) = n_1[\bar{r} - \underline{r}] + n_2[\bar{r} - \bar{r}] = n_1[\bar{r} - \underline{r}].$$

B 's corresponding welfare if he does not buy from $S1$ at this price is:

$$\pi_B^0(\underline{r}, \bar{r}) = n_2[\bar{r} - \underline{r}] < n_1[\bar{r} - \underline{r}].$$

Because $\pi_B^{n_1}(\underline{r}, \bar{r}) > \pi_B^0(\underline{r}, \bar{r})$, B will buy from $S1$ at price $p_1 = \underline{r}$ when $r = \bar{r}$.

When $r = \underline{r}$, B 's welfare if he buys from $S1$ at price $p_1 = \underline{r}$ is:

$$\pi_B^{n_1}(\underline{r}, \underline{r}) = n_1[\underline{r} - \underline{r}] = 0.$$

B 's corresponding welfare if he does not buy from $S1$ at this price is:

$$\pi_B^0(\underline{r}, \underline{r}) = n_2[\underline{r} - \underline{r}] = 0.$$

Because $\pi_B^{n_1}(\underline{r}, \underline{r}) = \pi_B^0(\underline{r}, \underline{r})$, B will buy from $S1$ at price $p_1 = \underline{r}$ when $c = \underline{r}$.

It remains to demonstrate that $\pi_1(\underline{r}) > \pi_1(\tilde{p}_1)$. To do so, it is sufficient to prove that:

$$\pi_1(\underline{r}) = n_1[\underline{r} - c_1] > \phi n_1[\widehat{p}_1 - c_1] = \pi_1(\widetilde{p}_1) \quad (39)$$

or equivalently:

$$[1 - \phi]c_1 < \underline{r} - \phi \widehat{p}_1. \quad (40)$$

From (37):

$$\underline{r} - \phi \widehat{p}_1 \geq \underline{r} - \phi \left[\bar{r} - \frac{n_2}{n_1} (\bar{r} - \underline{r}) \right].$$

Therefore, (40) holds if:

$$\begin{aligned} [1 - \phi]c_1 &< \underline{r} - \phi \left[\bar{r} - \frac{n_2}{n_1} (\bar{r} - \underline{r}) \right] \\ &= \underline{r} - \phi \bar{r} + \phi \frac{n_2}{n_1} [\bar{r} - \underline{r}] = [1 - \phi]\bar{r} - (\bar{r} - \underline{r}) + \phi \frac{n_2}{n_1} [\bar{r} - \underline{r}] \\ \Leftrightarrow c_1 &< \bar{r} - \frac{\bar{r} - \underline{r}}{1 - \phi} + \phi \frac{n_2}{n_1} [\bar{r} - \underline{r}] = \widehat{c} + \phi \frac{n_2}{n_1} \left[\frac{\bar{r} - \underline{r}}{1 - \phi} \right] \equiv c^*. \end{aligned} \quad (41)$$

(41) holds because $c_1 < c^*$ by hypothesis.

Now suppose a separating PPBE exists in which B buys from $S1$ at price \widetilde{p}_1 if and only if $r = \underline{r}$. Because beliefs must satisfy Bayes Rule along the equilibrium path, $\phi_{n_1}(\widetilde{p}_1) = 0$ and $\phi_0(\widetilde{p}_1) = 1$. Consequently, $S2$ will set $p_2 = \underline{r}$ if B buys from $S1$ at price \widetilde{p}_1 , whereas $S2$ will set $p_2 = \bar{r}$ if B does not buy from $S1$ at price \widetilde{p}_1 .

B will buy from $S2$ if and only if $p_2 \leq r$. Therefore, B 's welfare if he buys from $S1$ at price \widetilde{p}_1 when $r = \bar{r}$ is:

$$\pi_B^{n_1}(\widetilde{p}_1, \bar{r}) = n_1[\bar{r} - \widetilde{p}_1] + n_2[\bar{r} - \underline{r}].$$

B 's welfare if he does not buy from $S1$ at price \widetilde{p}_1 is:

$$\pi_B^0(\widetilde{p}_1, \bar{r}) = n_2[\bar{r} - \bar{r}] = 0.$$

Therefore, B will not buy from $S1$ at price \widetilde{p}_1 if and only if:

$$n_1[\bar{r} - \widetilde{p}_1] + n_2[\bar{r} - \underline{r}] < 0 \Leftrightarrow \widetilde{p}_1 > \bar{r} + \frac{n_2}{n_1}[\bar{r} - \underline{r}] > \bar{r}. \quad (42)$$

B 's welfare if he buys from $S1$ at price \widetilde{p}_1 when $r = \underline{r}$ is:

$$\pi_B^{n_1}(\widetilde{p}_1, \underline{r}) = n_1[\underline{r} - \widetilde{p}_1] + n_2[\underline{r} - \underline{r}] = n_1[\underline{r} - \widetilde{p}_1].$$

B 's welfare if he does not buy from $S1$ at price \widetilde{p}_1 (and subsequently does not buy from $S2$ when she sets $p_2 = \bar{r}$) is 0. Therefore, B will buy from $S1$ at price \widetilde{p}_1 if and only if:

$$n_1[\underline{r} - \widetilde{p}_1] > 0 \Leftrightarrow \widetilde{p}_1 < \underline{r}. \quad (43)$$

(43) provides a contradiction of (42). ■

Theorem 11. *Suppose $c_1 > \widehat{c}$ and $c_2 \geq \widehat{c}$. Then a pooling PPBE does not exist under transparency.*

Proof. Because $c_2 \geq \widehat{c}$ and $S2$'s beliefs must satisfy Bayes Rule along the equilibrium path,

$S2$ will always set $p_2 = \bar{r}$ in any pooling PPBE. B will buy from $S2$ at this price if and only if $r = \bar{r}$, and so secures zero welfare from his interaction with $S2$.

First suppose B always buys from $S1$. Then the putative equilibrium price cannot exceed \underline{r} . Otherwise, B could increase his welfare by not buying from $S1$ when $r = \underline{r}$. Therefore, $S1$'s payoff in the putative equilibrium cannot exceed $\pi_1(\underline{r}) = n_1[\underline{r} - c_1]$. However, because $c_1 > \hat{c}$, we have a contradiction: By setting $p_1 = \bar{r}$, $S1$ can increase her payoff to $\phi n_1[\bar{r} - c_1] > n_1[\underline{r} - c_1]$.¹

Now suppose B never buys from $S1$. Then the price $S1$ sets must exceed \bar{r} . Otherwise, B would buy from $S1$ when $r = \bar{r}$. Therefore, $S1$'s payoff in the putative equilibrium is zero. But because $c_1 > \hat{c}$, we have a contradiction: By setting $p_1 = \bar{r}$, $S1$ can increase her payoff to $\phi n_1[\bar{r} - c_1] > 0$. ■

Setting 2B. $c_1 > \hat{c}$, $c_2 > \hat{c}$, and $n_1 \leq n_2$.

Theorem 1 indicates that a separating PPBE does not exist under transparency in this setting when $n_2 \geq n_1$.

Theorem 11 indicates that a pooling PPBE does not exist under transparency in this setting.

Observation. Theorems 1, 9, 10, and 11 imply that a PPBE does not exist in this setting if $n_2 \geq n_1$ or if $n_1 > n_2$ and $c_1 \in (\hat{c}, c^*)$. The unique PPBE when $c_1 > c^* > \hat{c}$ and $n_1 > n_2$ is a separating PPBE in which $S1$ sets $\hat{p}_1 \equiv \bar{r} - \frac{n_2}{n_1}[\bar{r} - \underline{r}]$.

Setting 3. $c_1 \leq \hat{c}$ and $c_2 \leq \hat{c}$.

Theorem 12. *Suppose $c_1 \leq \hat{c}$ and $c_2 \leq \hat{c}$. Then under transparency, a pooling PPBE exists in which: (i) $S1$ sets \underline{r} ; (ii) $S2$ sets \underline{r} ; and (iii) B always buys from both $S1$ and $S2$ at these prices.*

Proof. The following beliefs support the identified equilibrium actions: (i) $\phi_{n_1}(p_1) = 1$ for all $p_1 > \underline{r}$; (ii) $\phi_{n_1}(p_1) = \phi$ for all $p_1 \leq \underline{r}$; (iii) $\phi_0(p_1) = \phi$ for all $p_1 > \bar{r}$; and (iv) $\phi_0(p_1) = 0$ for all $p_1 < \bar{r}$.

The proof proceeds by backward induction. We first prove that $S2$'s equilibrium action is optimal, given his beliefs. Then we prove that B 's equilibrium actions are optimal, given $S2$'s beliefs. Next we prove that $S1$'s equilibrium action is optimal. Finally, we verify that $S2$'s beliefs satisfy Bayes Rule along the equilibrium path.

¹This inequality holds because $c_1 > \hat{c}$, by assumption.

A. Prove that $S2$'s equilibrium action is optimal.

$\phi_{n_1}(\underline{r}) = \phi$. Therefore, after B buys from $S1$ at price $p_1 = \underline{r}$, $S2$ will maximize her expected payoff by acting as she does under privacy. Claim 1 implies that because $c_2 \leq \hat{c}$, $S2$ will set $p_2 = \underline{r}$.

$\phi_0(\underline{r}) = 0$. Therefore, after B does not buy from $S1$ at price $p_1 = \underline{r}$, $S2$ secures payoff: (i) 0 if she sets $p_2 > \underline{r}$; and (ii) $p_2 - c_2$ if she sets $p_2 \leq \underline{r}$. Consequently, $S2$ will set $p_2 = \underline{r}$.

B. Prove that B 's equilibrium actions are optimal.

Because the game ends following B 's interaction with $S2$, B maximizes his welfare by buying from $S2$ at price $p_2 = \underline{r}$.

We now prove that B maximizes his payoff by buying from $S1$ at price $p_1 = \underline{r}$.

First suppose $r = \bar{r}$. If B buys from $S1$ at price $p_1 = \underline{r}$, $S2$ will set $p_2 = \underline{r}$ because $\phi_{n_1}(\underline{r}) = \phi$ and $c_2 < \hat{c}$. Therefore, B 's welfare is:

$$\pi_B^{n_1}(p_1 = \underline{r}, \bar{r}) = n_1[\bar{r} - \underline{r}] + n_2[\bar{r} - \underline{r}] = [n_1 + n_2][\bar{r} - \underline{r}].$$

If B does not buy from $S1$ at price $p_1 = \underline{r}$, $S2$ will set $p_2 = \underline{r}$ because $\phi_0(\underline{r}) = 0$. Therefore, B 's welfare is:

$$\pi_B^0(p_1 = \underline{r}, \bar{r}) = n_2[\bar{r} - \underline{r}] < [n_1 + n_2][\bar{r} - \underline{r}].$$

Because $\pi_B^{n_1}(p_1 = \underline{r}, \bar{r}) > \pi_B^0(p_1 = \underline{r}, \bar{r})$, B will buy from $S1$ at price $p_1 = \underline{r}$ when $r = \bar{r}$.

Now suppose $r = \underline{r}$. If B buys from $S1$ at price $p_1 = \underline{r}$, $S2$ will set $p_2 = \underline{r}$ because $\phi_{n_1}(\underline{r}) = \phi$ and $c_2 < \hat{c}$. Therefore, B 's welfare is:

$$\pi_B^{n_1}(p_1 = \underline{r}, \underline{r}) = n_1[\underline{r} - \underline{r}] + n_2[\underline{r} - \underline{r}] = 0.$$

If B does not buy from $S1$ at price $p_1 = \underline{r}$, $S2$ will set $p_2 = \underline{r}$ because $\phi_0(\underline{r}) = 0$. Therefore, B 's welfare is:

$$\pi_B^0(p_1 = \underline{r}, \underline{r}) = n_2[\underline{r} - \underline{r}] = 0.$$

Because $\pi_B^0(p_1 = \underline{r}, \underline{r}) = \pi_B^{n_1}(p_1 = \underline{r}, \underline{r})$, B will buy from $S1$ at price $p_1 = \underline{r}$ when $r = \underline{r}$.

C. Prove that $S1$'s equilibrium action is optimal.

1. We begin by characterizing B 's optimal response to out-of-equilibrium prices by $S1$.

Result C1. When $r = \bar{r}$, B optimally does not buy from $S1$ at any price $p_1 > \underline{r}$ and buys from $S1$ at any price $p_1 < \underline{r}$.

Proof. Initially suppose that $S1$ sets $p_1 > \bar{r}$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 > \underline{r}$, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$. Consequently, B 's welfare is:

$$\pi_B^{n_1}(p_1, \bar{r}) = n_1[\bar{r} - p_1] + n_2[\bar{r} - \bar{r}] = n_1[\bar{r} - p_1] < 0.$$

Because $\phi_0(p_1) = \phi$ for all $p_1 > \bar{r}$, if B does not buy from $S1$ at price $p_1 > \bar{r}$, $S2$ will set $p_2 = \underline{r}$ (because $c_2 \leq \hat{c}$, by assumption). Consequently, B 's welfare is:

$$\pi_B^0(p_1, \bar{r}) = n_2[\bar{r} - \underline{r}] > 0.$$

Because $\pi_B^0(p_1, \bar{r}) > \pi_B^{n_1}(p_1, \bar{r})$, B will not buy from $S1$ at price $p_1 > \bar{r}$ when $r = \bar{r}$.

Next suppose that $S1$ sets $p_1 \in (\underline{r}, \bar{r}]$ and $n_2 \geq n_1$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 > \underline{r}$, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$. Consequently, B 's welfare is:

$$\pi_B^{n_1}(p_1, \bar{r}) = n_1[\bar{r} - p_1] + n_2[\bar{r} - \bar{r}] = n_1[\bar{r} - p_1] < n_1[\bar{r} - \underline{r}].$$

Because $\phi_0(p_1) = 0$ for all $p_1 \in (\underline{r}, \bar{r}]$, $S2$ will set $p_2 = \underline{r}$ if B does not buy from $S1$ at price $p_1 \in (\underline{r}, \bar{r}]$. Consequently, B 's expected payoff is:

$$\pi_B^0(p_1, \bar{r}) = n_2[\bar{r} - \underline{r}] \geq n_1[\bar{r} - \underline{r}].$$

Because $\pi_B^0(p_1, \bar{r}) > \pi_B^{n_1}(p_1, \bar{r})$, B will not buy from $S1$ at price $p_1 \in (\underline{r}, \bar{r}]$ when $r = \bar{r}$ and $n_2 \geq n_1$.

Next suppose that $S1$ sets $p_1 \in (\underline{r}, \bar{r}]$ and $n_1 > n_2$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 > \underline{r}$, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$. Consequently, B 's welfare is:

$$\pi_B^{n_1}(p_1, \bar{r}) = n_1[\bar{r} - p_1] + n_2[\bar{r} - \bar{r}] = n_1[\bar{r} - p_1].$$

Because $\phi_0(p_1) = 0$ for all $p_1 \in (\underline{r}, \bar{r}]$, $S2$ will set $p_2 = \underline{r}$ if B does not buy from $S1$ at price $p_1 \in (\underline{r}, \bar{r}]$. Consequently, B 's welfare is:

$$\pi_B^0(p_1, \bar{r}) = n_2[\bar{r} - \underline{r}] > n_1[\bar{r} - p_1] \Leftrightarrow p_1 > \bar{r} - \frac{n_2}{n_1}[\bar{r} - \bar{r}] > \underline{r}. \quad (44)$$

Inequality (44) holds because $p_1 > \underline{r}$ by assumption in this case. Therefore, $\pi_B^0(p_1, \bar{r}) > \pi_B^{n_1}(p_1, \bar{r})$, so B will not buy from $S1$ at price $p_1 \in (\underline{r}, \bar{r}]$ when $r = \bar{r}$ and $n_1 > n_2$.

Now suppose that $S1$ sets $p_1 < \underline{r}$. Because $\phi_{n_1}(p_1) = \phi$ for all $p_1 < \underline{r}$, $S2$ will set $p_2 = \underline{r}$ if B buys from $S1$ (because $c_2 < \hat{c}$). Therefore, if B buys from $S1$ at price $p_1 < \underline{r}$, his welfare is:

$$\pi_B^{n_1}(p_1, \bar{r}) = n_1[\bar{r} - p_1] + n_2[\bar{r} - \underline{r}] > [n_1 + n_2][\bar{r} - \underline{r}].$$

Because $\phi_0(p_1) = 0$ for all $p_1 \leq \underline{r}$, if B does not buy from $S1$ at price $p_1 < \underline{r}$, $S2$ will set $p_2 = \underline{r}$. Consequently, B 's welfare is:

$$\pi_B^0(p_1, \bar{r}) = n_2[\bar{r} - \underline{r}] < [n_1 + n_2][\bar{r} - \underline{r}].$$

Because $\pi_B^{n_1}(p_1, \bar{r}) > \pi_B^0(p_1, \bar{r})$, B will buy from $S1$ at price $p_1 > \underline{r}$ when $r = \bar{r}$.

Result C2. When $r = \underline{r}$, B optimally does not buy from $S1$ at any $p_1 > \underline{r}$ and buys from $S1$ at any $p_1 < \underline{r}$.

Proof. Initially suppose that $S1$ sets $p_1 > \bar{r}$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 > \underline{r}$, if B buys from $S1$ at this price, he will subsequently not buy from $S2$ at price $p_2 = \bar{r}$. Consequently, his welfare is $\pi_B^{n_1}(p_1, \underline{r}) = n_1[\underline{r} - p_1] < 0$. Because $\phi_0(p_1) = \phi$ for all $p_1 > \bar{r}$, if B does not buy from $S1$ at price $p_1 > \bar{r}$, $S2$ will set $p_2 = \underline{r}$ (because $c_2 < \hat{c}$). Consequently, B 's welfare is $\pi_B^0(p_1, \underline{r}) = n_2[\underline{r} - \underline{r}] = 0$. Because $\pi_B^0(p_1, \underline{r}) > \pi_B^{n_1}(p_1, \underline{r})$, B will not buy from $S1$ at any $p_1 > \bar{r}$ when $r = \underline{r}$.

Next suppose that $S1$ sets $p_1 \in (\underline{r}, \bar{r}]$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 > \underline{r}$, if B buys from $S1$ at this price, he will subsequently not buy from $S2$ when she sets $p_2 = \bar{r}$. Consequently,

B 's welfare is $\pi_B^{n_1}(p_1, \underline{r}) = n_1[\underline{r} - p_1] < 0$. Because $\phi_0(p_1) = 0$ for all $p_1 \in (\underline{r}, \bar{r}]$, if B does not buy from $S1$ at price $p_1 \in (\underline{r}, \bar{r}]$, $S2$ will set $p_2 = \underline{r}$. Consequently, B 's welfare is $\pi_B^0(p_1, \underline{r}) = n_2[\underline{r} - \underline{r}] = 0$. Because $\pi_B^{n_1}(p_1, \underline{r}) > \pi_B^0(p_1, \underline{r})$, B will not buy from $S1$ at any $p_1 \in (\underline{r}, \bar{r}]$ when $r = \underline{r}$.

Now suppose that $S1$ sets $p_1 < \underline{r}$. Because $\phi_{n_1}(p_1) = \phi$ for all $p_1 < \underline{r}$, $S2$ will set $p_2 = \underline{r}$ (because $c_2 < \hat{c}$) if B buys from $S1$ at price $p_1 < \underline{r}$. Consequently, B 's welfare is $\pi_B^{n_1}(p_1, \underline{r}) = n_1[\underline{r} - p_1] + n_2[\underline{r} - \underline{r}] > 0$. Because $\phi_0(p_1) = 0$ for all $p_1 < \underline{r}$, $S2$ will set $p_2 = \underline{r}$ if B does not buy from $S1$ at price $p_1 < \underline{r}$. Consequently, B 's welfare is $\pi_B^0(p_1, \underline{r}) = n_2[\underline{r} - \underline{r}] = 0$. Because $\pi_B^{n_1}(p_1, \underline{r}) > \pi_B^0(p_1, \underline{r})$, B will buy from $S1$ at any price $p_1 < \underline{r}$ when $r = \underline{r}$.

2. We now prove that $S1$'s equilibrium action is optimal.

Because B always buys from $S1$ at $p_1 = \underline{r}$, $S1$'s payoff is:

$$\pi_1(\underline{r}) = n_1[\underline{r} - c_1] > 0.$$

Results C1 and C2 imply that B will not buy from $S1$ at any price $p_1 > \underline{r}$. Consequently, $S1$'s payoff is $\pi_1(p_1) = 0$ for all $p_1 > \underline{r}$.

Results C1 and C2 imply that B will buy from $S1$ at any price $p_1 \leq \underline{r}$. Consequently, $S1$'s payoff when she sets $p_1 < \underline{r}$ is:

$$\pi_1(p_1) = n_1[p_1 - c_1] < n_1[\underline{r} - c_1].$$

Therefore, $S1$ maximizes her payoff by setting $p_1 = \underline{r}$.

D. Prove that $S2$'s beliefs satisfy Bayes Rule along the equilibrium path.

$$\begin{aligned} \Pr(r = \bar{r} | B \text{ buys at } p_1 = \underline{r}) &= \frac{\Pr(r = \bar{r} \text{ and } B \text{ buys at } p_1 = \underline{r})}{\Pr(B \text{ buys at } p_1 = \underline{r})} \\ &= \frac{\Pr(B \text{ buys at } p_1 = \underline{r} | r = \bar{r}) \Pr(r = \bar{r})}{\Pr(B \text{ buys at } p_1 = \underline{r} | r = \bar{r}) \Pr(r = \bar{r}) + \Pr(B \text{ buys at } p_1 = \underline{r} | r = \underline{r}) \Pr(r = \underline{r})} \\ &= \frac{1[\phi]}{1[\phi] + 1[1 - \phi]} = \phi = \phi_{n_1}(\underline{r}). \quad \blacksquare \end{aligned}$$

Corollary 13. *Under the conditions specified in Theorem 12, B 's equilibrium welfare is the same under transparency and under privacy.*

Proof. Claim 1 implies that under privacy, $S1$ and $S2$ both charge \underline{r} . Both sellers also charge \underline{r} under transparency. Furthermore, B always buys from $S1$ and from $S2$ in both settings. Therefore, B 's equilibrium welfare in both settings is: (i) $[n_1 + n_2][\bar{r} - \underline{r}] > 0$ when $r = \bar{r}$; and (ii) 0 when $r = \underline{r}$. \blacksquare

Corollary 14. *Under the conditions specified in Theorem 12, $S1$ and $S2$ each secure the same equilibrium payoff under transparency that it secures under privacy.*

Proof. Claim 1 implies that under privacy, $S1$ and $S2$ both charge \underline{r} . Both sellers also charge \underline{r} under transparency. Furthermore, B always buys from $S1$ and from $S2$ in both settings. Therefore, S_i 's equilibrium payoff in both regimes is $n_i[\underline{r} - c_i] > 0$ for $i = 1, 2$. ■

Theorem 13. *Under the conditions specified in Theorem 12, the equilibrium identified in the theorem is the unique pooling PPBE.*

Proof. $S1$'s payoff is $n_1[\underline{r} - c_1] > 0$ in the identified PPBE. $S1$'s payoff is 0 in any pooling PPBE in which B never buys from $S1$ at any given price p_1 . $S1$'s payoff is less than $n_1[\underline{r} - c_1]$ in any pooling PPBE in which B always buys from $S1$ at a price $p_1 < \underline{r}$. Therefore, in any alternative candidate pooling PPBE, B must always buy from $S1$ at price $p_1 > \underline{r}$.

When $r = \underline{r}$, B 's welfare if he buys from $S1$ at price $p_1 > \underline{r}$ is $n_1[\underline{r} - p_1] < 0$. B 's welfare is non-negative if he does not buy from $S1$ at this price. Because B will not always buy from $S1$ at a price $p_1 > \underline{r}$, the equilibrium in which $S1$ sets $p_1 = \underline{r}$ is the unique PPBE under the specified conditions. ■

Theorem 14. *Suppose $c_1 < \hat{c}$ and $c_2 < \hat{c}$. Then a separating PPBE does not exist under transparency.*

Proof. Initially suppose a separating PPBE exists in which B buys from $S1$ at price \tilde{p}_1 if and only if $r = \bar{r}$. Because beliefs must satisfy Bayes Rule along the equilibrium path, $\phi_{n_1}(\tilde{p}_1) = 1$ and $\phi_0(\tilde{p}_1) = 0$. Consequently, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$ at price \tilde{p}_1 . $S2$ will set $p_2 = \underline{r}$ if B does not buy from $S1$ at price \tilde{p}_1 .

B will buy from $S2$ if and only if $p_2 \leq r$. Therefore, B 's welfare if he buys from $S1$ at price \tilde{p}_1 when $r = \bar{r}$ is:

$$\pi_B^{n_1}(\tilde{p}_1, \bar{r}) = n_1[\bar{r} - \tilde{p}_1] + n_2[\bar{r} - \bar{r}] = n_1[\bar{r} - \tilde{p}_1].$$

B 's welfare if he does not buy from $S1$ at price \tilde{p}_1 is:

$$\pi_B^0(\tilde{p}_1, \bar{r}) = n_2[\bar{r} - \underline{r}].$$

Therefore, B will buy from $S1$ at price \tilde{p}_1 if and only if:

$$n_1[\bar{r} - \tilde{p}_1] \geq n_2[\bar{r} - \underline{r}] \Leftrightarrow \tilde{p}_1 \leq \bar{r} - \frac{n_2}{n_1}[\bar{r} - \underline{r}]. \quad (45)$$

B 's welfare if he buys from $S1$ at price \tilde{p}_1 (and subsequently does not buy from $S2$ at price $p_2 = \bar{r}$) when $r = \underline{r}$ is $\pi_B^{n_1}(\tilde{p}_1, \underline{r}) = n_1[\underline{r} - \tilde{p}_1]$. B 's welfare if he does not buy from $S1$ at price \tilde{p}_1 is $\pi_B^0(\tilde{p}_1, \underline{r}) = n_2[\underline{r} - \underline{r}] = 0$. Therefore, B will not buy from $S1$ at price \tilde{p}_1 if and only if:

$$n_1[\underline{r} - \tilde{p}_1] < 0 \Leftrightarrow \tilde{p}_1 > \underline{r}. \quad (46)$$

This completes this part of the proof if $n_2 \geq n_1$ because (46) contradicts (45) in this case.

Now suppose $n_1 > n_2$. We will demonstrate that $S1$ secures a higher payoff by setting $p_1 = \underline{r}$ than by setting $\tilde{p}_1 \in (\underline{r}, \bar{r} - \frac{n_2}{n_1} [\bar{r} - \underline{r}])$ when $n_1 > n_2$. We first show that B will always buy from $S1$ when she sets $p_1 = \underline{r}$. Because $\phi_{n_1}(\underline{r}) = \phi$ and $c_2 < \hat{c}$, $S2$ will set $p_2 = \underline{r}$ if B buys from $S1$ at price $p_1 = \underline{r}$. Because $\phi_0(\underline{r}) = 0$, $S2$ will set $p_2 = \underline{r}$ if B does not buy from $S1$ at price $p_1 = \underline{r}$. Therefore, when $r = \bar{r}$, B 's welfare if he buys from $S1$ is:

$$\pi_B^{n_1}(\underline{r}, \bar{r}) = n_1 [\bar{r} - \underline{r}] + n_2 [\bar{r} - \underline{r}] = [n_1 + n_2] [\bar{r} - \underline{r}].$$

B 's corresponding payoff if he does not buy from $S1$ is:

$$\pi_B^0(\underline{r}, \bar{r}) = n_2 [\bar{r} - \underline{r}].$$

Because $\pi_B^{n_1}(\underline{r}, \bar{r}) > \pi_B^0(\underline{r}, \bar{r})$, B will buy from $S1$ at price $p_1 = \underline{r}$ when $r = \bar{r}$.

When $r = \underline{r}$, B 's welfare if he buys from $S1$ at price $p_1 = \underline{r}$ is:

$$\pi_B^{n_1}(\underline{r}, \underline{r}) = n_1 [\underline{r} - \underline{r}] + n_2 [\underline{r} - \underline{r}] = 0.$$

B 's corresponding payoff if he does not buy from $S1$ is:

$$\pi_B^0(\underline{r}, \underline{r}) = n_2 [\underline{r} - \underline{r}] = 0.$$

Because $\pi_B^{n_1}(\underline{r}, \underline{r}) \geq \pi_B^0(\underline{r}, \underline{r})$, B buys from $S1$ at $p_1 = \underline{r}$ when $r = \underline{r}$.

We now establish that $S1$ will not set $p_1 = \tilde{p}_1$ under the specified conditions because $S1$ can earn a strictly higher payoff by setting $p_1 = \underline{r}$. Because B always buys from $S1$ when she sets $p_1 = \underline{r}$ under the specified conditions, $S1$'s payoff when she sets $p_1 = \underline{r}$ is:

$$\pi_1(\underline{r}) = n_1 [\underline{r} - c_1] > \phi n_1 [\bar{r} - c_1] > \phi n_1 [\tilde{p}_1 - c_1] = \pi_1(\tilde{p}_1).$$

The first inequality here reflects Claim 1 and the hypothesis that $c_1 < \hat{c}$. The second inequality reflects (45), which implies:

$$\tilde{p}_1 \leq \bar{r} - \frac{n_2}{n_1} [\bar{r} - \underline{r}] < \bar{r}.$$

Now suppose a separating PPBE exists in which B buys from $S1$ at price \tilde{p}_1 if and only if $r = \underline{r}$. Because beliefs must satisfy Bayes Rule along the equilibrium path, $\phi_{n_1}(\tilde{p}_1) = 0$ and $\phi_0(\tilde{p}_1) = 1$. Consequently, $S2$ will set $p_2 = \underline{r}$ if B buys from $S1$ at price \tilde{p}_1 , whereas $S2$ will set $p_2 = \bar{r}$ if B does not buy from $S1$ at price \tilde{p}_1 .

First suppose $r = \bar{r}$. B 's welfare if he buys from $S1$ at price \tilde{p}_1 is:

$$\pi_B^{n_1}(\tilde{p}_1, \bar{r}) = n_1 [\bar{r} - \tilde{p}_1] + n_2 [\bar{r} - \underline{r}].$$

B 's welfare if he does not buy from $S1$ at price \tilde{p}_1 is:

$$\pi_B^0(\tilde{p}_1, \bar{r}) = n_2 [\bar{r} - \bar{r}] = 0.$$

Therefore, B will buy from $S1$ at price \tilde{p}_1 if and only if:

$$n_1 [\bar{r} - \tilde{p}_1] + n_2 [\bar{r} - \underline{r}] \geq 0 \Leftrightarrow \tilde{p}_1 \leq \bar{r} + \frac{n_2}{n_1} [\bar{r} - \underline{r}].$$

Now suppose $r = \underline{r}$. B 's welfare if he buys from $S1$ at price \tilde{p}_1 is:

$$\pi_B^{n_1}(\tilde{p}_1, \underline{r}) = n_1 [\underline{r} - \tilde{p}_1] + n_2 [\underline{r} - \underline{r}] = n_1 [\underline{r} - \tilde{p}_1].$$

B 's welfare if he does not buy from $S1$ at price \tilde{p}_1 (and subsequently does not buy from $S2$ when she sets $p_2 = \bar{r}$) is 0. Therefore, B will buy from $S1$ at price \tilde{p}_1 if and only if:

$$n_1 [\underline{r} - \tilde{p}_1] \geq 0 \Leftrightarrow \tilde{p}_1 \leq \underline{r}. \quad (47)$$

(47) implies that in the postulated separating PPBE, $S1$'s price cannot exceed \underline{r} if B is to always buy from $S1$. However, B will always buy from $S1$ when $p_1 \leq \underline{r}$ under the specified conditions. Therefore, the postulated separating PPBE cannot exist. ■

Setting 4A. $c_1 \leq \hat{c}$, $c_2 > \hat{c}$, and $n_1 \geq n_2$.

Theorem 15. *Suppose $n_1 \geq n_2$, $c_1 \leq \hat{c}$, and $c_2 > \hat{c}$. Then under transparency, a pooling PPBE exists in which: (i) $S1$ sets $p_1 = \underline{r}$; (ii) $S2$ sets $p_2 = \bar{r}$; (iii) B always buys from $S1$; and (iv) B buys from $S2$ if and only if $r = \bar{r}$.*

Proof. The beliefs that support the identified equilibrium actions are: (i) $\phi_{n_1}(p_1) = 1$ for all $p_1 > \underline{r}$; (ii) $\phi_0(p_1) = 0$ for all $p_1 \leq \bar{r}$; (iii) $\phi_0(p_1) = \phi$ for all $p_1 > \bar{r}$; and (iv) $\phi_{n_1}(p_1) = \phi$ for all $p_1 \leq \underline{r}$.

The proof proceeds by backward induction. We first prove that $S2$'s equilibrium action is optimal, given her beliefs. Then we prove that B 's equilibrium actions are optimal, given $S2$'s beliefs. Next we prove that $S1$'s equilibrium action is optimal. Finally, we verify that $S2$'s beliefs satisfy Bayes Rule along the equilibrium path.

A. Prove that $S2$'s equilibrium action is optimal.

$\phi_{n_1}(\underline{r}) = \phi$. Therefore, because $c_2 > \hat{c}$, $S2$ will set $p_2 = \bar{r}$ after B buys from $S1$ at price $p_1 = \underline{r}$.

B. Prove that B 's equilibrium actions are optimal.

Because the game ends following B 's interaction with $S2$, B maximizes his welfare by buying from $S2$ at price $p_2 = \bar{r}$ if and only if $r = \bar{r}$.

We now prove that B maximizes his welfare by always buying from $S1$ at price $p_1 = \underline{r}$.

First suppose $r = \bar{r}$. If B buys from $S1$ at price $p_1 = \underline{r}$, $S2$ will set $p_2 = \bar{r}$ because $\phi_{n_1}(\underline{r}) = \phi$ and $c_2 > \hat{c}$. Therefore, B 's welfare is:

$$\pi_B^{n_1}(p_1, \bar{r}) = n_1 [\bar{r} - \underline{r}] + n_2 [\bar{r} - \bar{r}] = n_1 [\bar{r} - \underline{r}].$$

If B does not buy from $S1$ at price $p_1 = \underline{r}$, $S2$ will set $p_2 = \underline{r}$ because $\phi_0(\underline{r}) = 0$. Therefore, B 's welfare is:

$$\pi_B^0(p_1, \bar{r}) = n_2 [\bar{r} - \underline{r}] \leq n_1 [\bar{r} - \underline{r}].$$

Because $\pi_B^{n_1}(p_1, \bar{r}) \geq \pi_B^0(p_1, \bar{r})$, B will buy from $S1$ at price $p_1 = \underline{r}$ when $r = \bar{r}$.

Now suppose $r = \underline{r}$. If B buys from $S1$ at price $p_1 = \underline{r}$, $S2$ will set $p_2 = \bar{r}$ because $\phi_{n_1}(\underline{r}) = \phi$ and $c_2 > \hat{c}$. B will not buy from $S2$ at this price. Consequently, B 's welfare is:

$$\pi_B^{n_1}(p_1, \underline{r}) = [\underline{r} - \underline{r}] = 0.$$

If B does not buy from $S1$ when she sets $p_1 = \underline{r}$, $S2$ will set $p_2 = \underline{r}$ because $\phi_0(\underline{r}) = 0$. Therefore, B 's welfare is:

$$\pi_B^0(p_1, \underline{r}) = n_2 [\underline{r} - \underline{r}] = 0.$$

Because $\pi_B^0(p_1, \underline{r}) = \pi_B^{n_1}(p_1, \underline{r})$, B will buy from $S1$ at price $p_1 = \underline{r}$ when $r = \underline{r}$.

C. Prove that $S1$'s equilibrium action is optimal.

1. We begin by characterizing B 's optimal response to out-of-equilibrium prices by $S1$.

Result C1. When $r = \bar{r}$, B optimally does not buy from $S1$ if $p_1 > \hat{p}_1 \equiv \bar{r} - \frac{n_2}{n_1} [\bar{r} - \underline{r}]$ and buys from $S1$ if $p_1 \leq \hat{p}_1$.

Proof. Initially suppose that $S1$ sets $p_1 > \bar{r}$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 > \underline{r}$, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$. Consequently, B 's welfare is:

$$\pi_B^{n_1}(p_1, \bar{r}) = n_1 [\bar{r} - p_1] + n_2 [\bar{r} - \bar{r}] = n_1 [\bar{r} - p_1] < 0.$$

Because $\phi_0(p_1) = \phi$ for all $p_1 > \bar{r}$, if B does not buy from $S1$ at price $p_1 > \bar{r}$, $S2$ will set $p_2 = \bar{r}$ (because $c_2 > \hat{c}$). Consequently, B 's welfare is:

$$\pi_B^0(p_1, \bar{r}) = n_2 [\bar{r} - \bar{r}] = 0.$$

Because $\pi_B^0(p_1, \bar{r}) > \pi_B^{n_1}(p_1, \bar{r})$, B will not buy from $S1$ at price $p_1 > \bar{r}$ when $r = \bar{r}$.

Next suppose that $S1$ sets $p_1 \in (\hat{p}_1, \bar{r}]$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 > \underline{r}$, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$. Consequently, B 's welfare is:

$$\pi_B^{n_1}(p_1, \bar{r}) = n_1 [\bar{r} - p_1] + n_2 [\bar{r} - \bar{r}] = n_1 [\bar{r} - p_1] < n_1 [\bar{r} - \hat{p}_1] = n_2 [\bar{r} - \underline{r}].$$

Because $\phi_0(p_1) = 0$ for all $p_1 \leq \bar{r}$, if B does not buy from $S1$ at price $p_1 \in (\hat{p}_1, \bar{r}]$, $S2$ will set $p_2 = \underline{r}$. Consequently, B 's welfare is:

$$\pi_B^0(p_1, \bar{r}) = n_2 [\bar{r} - \underline{r}].$$

Because $\pi_B^0(p_1, \bar{r}) > \pi_B^{n_1}(p_1, \bar{r})$, B will not buy from $S1$ at any $p_1 \in (\hat{p}_1, \bar{r}]$ when $r = \bar{r}$.

Next suppose that $S1$ sets $p_1 \in (\underline{r}, \hat{p}_1]$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 > \underline{r}$, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$. Consequently, B 's welfare is:

$$\pi_B^{n_1}(p_1, \bar{r}) = n_1 [\bar{r} - p_1] + n_2 [\bar{r} - \bar{r}] \geq n_1 [\bar{r} - \hat{p}_1] = n_2 [\bar{r} - \underline{r}].$$

Because $\phi_0(p_1) = 0$ for all $p_1 \leq \bar{r}$, if B does not buy from $S1$ at price $p_1 \in (\underline{r}, \hat{p}_1]$, $S2$ will set $p_2 = \underline{r}$. Consequently, B 's welfare is:

$$\pi_B^0(p_1, \bar{r}) = n_2 [\bar{r} - \underline{r}].$$

Because $\pi_B^{n_1}(p_1, \bar{r}) \geq \pi_B^0(p_1, \bar{r})$, B will buy from $S1$ at any $p_1 \in (\underline{r}, \hat{p}_1]$ when $r = \bar{r}$.

Now suppose that $S1$ sets $p_1 < \underline{r}$. Because $\phi_{n_1}(p_1) = \phi$ for all $p_1 \leq \underline{r}$, if B buys from $S1$, $S2$ will set $p_2 = \bar{r}$ (because $c_2 > \hat{c}$). Therefore, if B buys from $S1$, his payoff is:

$$\pi_B^{n_1}(p_1, \bar{r}) = n_1 [\bar{r} - p_1] + n_2 [\bar{r} - \bar{r}] > n_1 [\bar{r} - \underline{r}].$$

Because $\phi_0(p_1) = 0$ for all $p_1 \leq \bar{r}$, if B does not buy from $S1$ at price $p_1 < \underline{r}$, $S2$ will set

$p_2 = \underline{r}$. Consequently, B 's welfare is:

$$\pi_B^0(p_1, \bar{r}) = n_2 [\bar{r} - \underline{r}] < n_1 [\underline{r} - \bar{r}].$$

Because $\pi_B^{n_1}(p_1, \bar{r}) > \pi_B^0(p_1, \bar{r})$, B will buy from $S1$ at price $p_1 < \underline{r}$ when $r = \bar{r}$.

Result C2. When $r = \underline{r}$, B optimally: (i) buys from $S1$ whenever $p_1 < \underline{r}$; and (ii) does not buy from $S1$ whenever $p_1 > \underline{r}$.

Proof. Initially suppose that $S1$ sets $p_1 > \bar{r}$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 > \underline{r}$, if B buys from $S1$ at this price, he will subsequently not buy from $S2$ at price $p_2 = \bar{r}$. Consequently, B 's welfare is $\pi_B^{n_1}(p_1, \underline{r}) = n_1 [\underline{r} - p_1] < 0$. Because $\phi_0(p_1) = \phi$ for all $p_1 > \bar{r}$, if B does not buy from $S1$ at price $p_1 > \bar{r}$, $S2$ will set $p_2 = \bar{r}$ (because $c_2 > \hat{c}$). B will not buy from $S2$ at this price, so B 's welfare is $\pi_B^0(p_1, \underline{r}) = 0$. Because $\pi_B^0(p_1, \underline{r}) > \pi_B^{n_1}(p_1, \underline{r})$, B will not buy from $S1$ at price $p_1 > \bar{r}$ when $r = \underline{r}$.

Next suppose that $S1$ sets $p_1 \in (\underline{r}, \bar{r})$. Because $\phi_{n_1}(p_1) = 1$ for all $p_1 > \underline{r}$, if B buys from $S1$ at this price, he will subsequently not buy from $S2$ at price $p_2 = \bar{r}$. Consequently, B 's welfare is $\pi_B^{n_1}(p_1, \underline{r}) = n_1 [\underline{r} - p_1] < 0$. Because $\phi_0(p_1) = 0$ for all $p_1 \in (\underline{r}, \bar{r})$, if B does not buy from $S1$, $S2$ will set $p_2 = \underline{r}$. Consequently, B 's welfare is $\pi_B^0(p_1, \underline{r}) = n_2 [\underline{r} - \underline{r}] = 0$. Because $\pi_B^0(p_1, \underline{r}) > \pi_B^{n_1}(p_1, \underline{r})$, B will not buy from $S1$ when she sets any price $p_1 \in (\underline{r}, \bar{r})$ when $r = \underline{r}$.

Now suppose that $S1$ sets $p_1 < \underline{r}$. Because $\phi_{n_1}(p_1) = \phi$ for all $p_1 \leq \underline{r}$, if B buys from $S1$ at this price, he will subsequently not buy from $S2$ at price $p_2 = \bar{r}$. (Recall $c_2 > \hat{c}$). Consequently, B 's welfare is $\pi_B^{n_1}(p_1, \underline{r}) = n_1 [\underline{r} - p_1] > 0$. Because $\phi_0(p_1) = 0$ for all $p_1 \leq \bar{r}$, if B does not buy from $S1$, $S2$ will set $p_2 = \underline{r}$. Consequently, B 's welfare is $\pi_B^0(p_1, \underline{r}) = n_2 [\underline{r} - \underline{r}] = 0$. Because $\pi_B^{n_1}(p_1, \underline{r}) > \pi_B^0(p_1, \underline{r})$, B will buy from $S1$ at price $p_1 > \underline{r}$ when $r = \underline{r}$. \square

2. We now prove that $S1$'s equilibrium action is optimal.

When $S1$ sets $p_1 = \underline{r}$, B always buys from $S1$. Consequently, $S1$'s payoff is:

$$\pi_1(\underline{r}) = n_1 [\underline{r} - c_1] > 0.$$

Results C1 and C2 imply that B will not buy from $S1$ at price $p_1 > \hat{p}_1$. Consequently, $S1$'s payoff is $\pi_1(p_1) = 0$ for all $p_1 > \hat{p}_1$.

Results C1 and C2 imply that if $S1$ sets $p_1 \in (\underline{r}, \hat{p}_1]$, B will buy from $S1$ if $r = \bar{r}$ and not buy from $S1$ if $r = \underline{r}$. Consequently, $S1$'s payoff from setting $p_1 \in (\underline{r}, \hat{p}_1]$ is:

$$\pi_1(p_1) = \phi n_1 [p_1 - c_1] < \phi n_1 [\bar{r} - c_1] \leq n_1 [\underline{r} - c_1].$$

The strict inequality here holds because $p_1 < \bar{r}$. The weak inequality follows from Claim 1 and the maintained assumption that $c_1 \leq \hat{c}$.

Results C1 and C2 imply that $S1$'s payoff from setting $p_1 < \underline{r}$ is:

$$\pi_1(p_1) = n_1 [p_1 - c_1] < n_1 [\underline{r} - c_1].$$

Therefore, $S1$ maximizes her payoff by setting $p_1 = \underline{r}$.

D. Prove that $S2$'s beliefs satisfy Bayes Rule along the equilibrium path.

$$\begin{aligned}
\Pr(r = \bar{r} | B \text{ buys at } \underline{r}) &= \frac{\Pr(r = \bar{r} \text{ and } B \text{ buys at } \underline{r})}{\Pr(B \text{ buys at } \underline{r})} \\
&= \frac{\Pr(B \text{ buys at } \underline{r} | r = \bar{r}) \Pr(r = \bar{r})}{\Pr(B \text{ buys at } \underline{r} | r = \bar{r}) \Pr(r = \bar{r}) + \Pr(B \text{ buys at } \underline{r} | r = \underline{r}) \Pr(r = \underline{r})} \\
&= \frac{1[\phi]}{1[\phi] + 1[1 - \phi]} = \phi = \phi_{n_1}(\underline{r}). \quad \blacksquare
\end{aligned}$$

Corollary 15. *Under the conditions specified in Theorem 15, transparency does not alter B 's equilibrium welfare or the equilibrium payoffs of $S1$ or $S2$.*

Proof. From Claim 1 and Theorem 15, equilibrium behavior is identical in the two regimes: $S1$ sets $p_1 = \underline{r}$; $S2$ sets $p_2 = \bar{r}$; B always buys from $S1$ at price $p_1 = \underline{r}$; and B buys from $S2$ at price $p_1 = \bar{r}$ if and only if $r = \bar{r}$. \blacksquare

Theorem 16. *Under the conditions specified in Theorem 15, the equilibrium identified in the theorem is the unique pooling PPBE.*

Proof. $S1$'s payoff is $n_1[\underline{r} - c_1] > 0$ in the identified pooling PPBE. $S1$'s payoff is 0 in any pooling PPBE in which B never buys from $S1$. $S1$'s payoff is also less than $n_1[\underline{r} - c_1]$ in any pooling PPBE in which B always buys from $S1$ at price $p_1 < \underline{r}$. Therefore, in any alternative candidate pooling PPBE, B must always buy from $S1$ at a price $p_1 > \underline{r}$.

When $r = \underline{r}$, B 's welfare if he buys from $S1$ at price $p_1 > \underline{r}$ is $n_1[\underline{r} - p_1] < 0$. B 's welfare is non-negative if he does not buy from $S1$ at this price. Because B will not always buy from $S1$ when she sets $p_1 > \underline{r}$, the equilibrium in which $S1$ sets $p_1 = \underline{r}$ is the unique pooling PPBE under the specified conditions. \blacksquare

Theorem 17. *Suppose $c_1 \leq \hat{c}$ and $c_2 > \hat{c}$. Then a separating PPBE does not exist under transparency.*

Proof. Initially suppose a separating PPBE exists in which B buys from $S1$ at price \tilde{p}_1 if and only if $r = \bar{r}$. Because beliefs must satisfy Bayes Rule along the equilibrium path, $\phi_{n_1}(\tilde{p}_1) = 1$ and $\phi_0(\tilde{p}_1) = 0$. Consequently, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$ at price \tilde{p}_1 , whereas $S2$ will set $p_2 = \underline{r}$ if B does not buy from $S1$ when she sets price \tilde{p}_1 .

B will buy from $S2$ if and only if $p_2 \leq r$. Therefore, B 's welfare if he buys from $S1$ at price \tilde{p}_1 when $r = \bar{r}$ is:

$$\pi_B^{n_1}(\tilde{p}_1, \bar{r}) = n_1[\bar{r} - \tilde{p}_1] + n_2[\bar{r} - \bar{r}] = n_1[\bar{r} - \tilde{p}_1]. \quad (48)$$

B 's welfare if he does not buy from $S1$ at price \tilde{p}_1 is:

$$\pi_B^0(\tilde{p}_1, \bar{r}) = n_2[\bar{r} - \underline{r}]. \quad (49)$$

(48) and (49) imply that B will buy from $S1$ at \tilde{p}_1 if and only if:

$$n_1[\bar{r} - \tilde{p}_1] \geq n_2[\bar{r} - \underline{r}] \Leftrightarrow \tilde{p}_1 \leq \bar{r} - \frac{n_2}{n_1}[\bar{r} - \underline{r}]. \quad (50)$$

B 's welfare if he buys from $S1$ at price \tilde{p}_1 (and subsequently does not buy from $S2$ at price $p_2 = \bar{r}$) when $r = \underline{r}$ is:

$$\pi_B^{n_1}(\tilde{p}_1, \underline{r}) = n_1[\underline{r} - \tilde{p}_1]. \quad (51)$$

B 's welfare if he does not buy from $S1$ at price \tilde{p}_1 is:

$$\pi_B^0(\tilde{p}_1, \underline{r}) = n_2[\underline{r} - \underline{r}] = 0. \quad (52)$$

(51) and (52) imply that B will not buy from $S1$ at \tilde{p}_1 if and only if:

$$n_1[\underline{r} - \tilde{p}_1] < 0 \Leftrightarrow \tilde{p}_1 > \underline{r}. \quad (53)$$

(53) provides a contradiction of (50) when $n_2 \geq n_1$.

We now demonstrate that $S1$ earns a higher payoff by setting $p_1 = \underline{r}$ than by setting $\tilde{p}_1 \in (\underline{r}, \bar{r} - \frac{n_2}{n_1}[\bar{r} - \underline{r}])$ when $n_1 > n_2$ (and hence, $\tilde{p}_1 < \bar{r}$). We first show that B will always buy from $S1$ at price $p_1 = \underline{r}$. Because $\phi_{n_1}(\underline{r}) = \phi$ and $c_2 \geq \hat{c}$, $S2$ will set $p_2 = \bar{r}$ if B buys from $S1$ at price $p_1 = \underline{r}$. Because $\phi_0(\underline{r}) = 0$, $S2$ will set $p_2 = \underline{r}$ if B does not buy from $S1$ at price $p_1 = \underline{r}$. Therefore, when $r = \bar{r}$, B 's welfare if he buys from $S1$ at this price is:

$$\pi_B^{n_1}(\underline{r}, \bar{r}) = n_1[\bar{r} - \underline{r}] + n_2[\bar{r} - \underline{r}] = [n_1 + n_2][\bar{r} - \underline{r}].$$

B 's corresponding welfare if he does not buy from $S1$ at this price is:

$$\pi_B^0(\underline{r}, \bar{r}) = n_2[\bar{r} - \underline{r}].$$

Because $\pi_B^{n_1}(\underline{r}, \bar{r}) > \pi_B^0(\underline{r}, \bar{r})$, B will buy from $S1$ at price $p_1 = \underline{r}$ when $r = \bar{r}$.

When $r = \underline{r}$, B 's welfare if he buys from $S1$ at price $p_1 = \underline{r}$ (and subsequently does not buy from $S2$ at price $p_2 = \bar{r}$) is:

$$\pi_B^{n_1}(\underline{r}, \underline{r}) = n_1[\underline{r} - \underline{r}] = 0.$$

B 's corresponding welfare if he does not buy from $S1$ at this price is:

$$\pi_B^0(\underline{r}, \underline{r}) = n_2[\underline{r} - \underline{r}] = 0.$$

Because $\pi_B^{n_1}(\underline{r}, \underline{r}) \geq \pi_B^0(\underline{r}, \underline{r})$, B will buy from $S1$ at price $p_1 = \underline{r}$ when $r = \underline{r}$.

We now demonstrate that $S1$ secures a higher payoff by setting $p_1 = \underline{r}$ than by setting $p_1 = \tilde{p}_1$. Because B always buys from $S1$ when she sets $p_1 = \underline{r}$, $S1$ secures payoff $\pi_1(\underline{r}) = n_1[\underline{r} - c_1]$ if she sets $p_1 = \underline{r}$. If $S1$ sets $p_1 = \tilde{p}_1$, she secures payoff:

$$\pi_1(\tilde{p}_1) = \phi n_1[\tilde{p}_1 - c_1] < \phi n_1[\bar{r} - c_1] \leq n_1[\underline{r} - c_1] = \pi_1(\underline{r}).$$

The strict inequality here holds because $\tilde{p}_1 < \bar{r}$. The weak inequality reflects Claim 1 and the maintained assumption that $c_1 \leq \hat{c}$.

Now suppose a separating PPBE exists in which B buys from $S1$ at price \tilde{p}_1 if and only if $r = \underline{r}$. Because beliefs must satisfy Bayes Rule along the equilibrium path, $\phi_{n_1}(\tilde{p}_1) = 0$ and $\phi_0(\tilde{p}_1) = 1$. Consequently, $S2$ will set $p_2 = \underline{r}$ if B buys from $S1$ at price \tilde{p}_1 , whereas $S2$ will set $p_2 = \bar{r}$ if B does not buy from $S1$ at price \tilde{p}_1 .

B will buy from $S2$ if and only if $p_2 \leq r$. Therefore, B 's welfare if he buys from $S1$ at price \tilde{p}_1 when $r = \bar{r}$ is:

$$\pi_B^{n_1}(\tilde{p}_1, \bar{r}) = n_1 [\bar{r} - \tilde{p}_1] + n_2 [\bar{r} - \underline{r}].$$

B 's welfare if he does not buy from $S1$ at price \tilde{p}_1 when $r = \bar{r}$ is:

$$\pi_B^0(\tilde{p}_1, \bar{r}) = n_2 [\bar{r} - \bar{r}] = 0.$$

Therefore, B will buy from $S1$ at price \tilde{p}_1 when $r = \bar{r}$ if and only if:

$$n_1 [\bar{r} - \tilde{p}_1] + n_2 [\bar{r} - \underline{r}] \geq 0 \quad \Leftrightarrow \quad \tilde{p}_1 \leq \bar{r} - \frac{n_2}{n_1} [\bar{r} - \underline{r}]. \quad (54)$$

B 's welfare if he buys from $S1$ at price \tilde{p}_1 when $r = \underline{r}$ is:

$$\pi_B^{n_1}(\tilde{p}_1, \underline{r}) = n_1 [\underline{r} - \tilde{p}_1] + n_2 [\underline{r} - \underline{r}] = n_1 [\underline{r} - \tilde{p}_1].$$

B 's welfare if he does not buy from $S1$ at price \tilde{p}_1 (and subsequently buys from $S2$ at price $p_2 = \underline{r}$) when $r = \underline{r}$ is 0. Therefore, B will not buy from $S1$ at price \tilde{p}_1 when $r = \underline{r}$ if and only if:

$$n_1 [\underline{r} - \tilde{p}_1] < 0 \quad \Leftrightarrow \quad \tilde{p}_1 > \underline{r}. \quad (55)$$

(55) implies that in the postulated separating PPBE, B will not buy from $S1$ at any price above \underline{r} when $r = \underline{r}$. (54) implies that when $r = \bar{r}$, B will buy from $S1$ when she sets a price above \underline{r} . Therefore, the postulated separating PPBE cannot exist. ■

Setting 4B. $c_1 \leq \hat{c}$, $c_2 > \hat{c}$, and $n_1 < n_2$.

Observe that Theorem 17 holds both in Setting 4A and in Setting 4B.

Theorem 18. *Suppose $c_1 \leq \hat{c}$, $c_2 > \hat{c}$, and $n_1 < n_2$. Further suppose a pooling PPBE exists under transparency.² Then this equilibrium is the one specified in Theorem 15.*

Proof. The proof is identical to the proof of Theorem 16. ■

²It can be shown that, under the conditions specified in Theorem 18, a pooling PPBE exists (does not exist) if out-of-equilibrium beliefs are passive (if $\phi_0(\underline{r}) = 0$).

II. Proofs of Propositions in the Text

Propositions 1 – 7 follow directly from Theorems 1 – 18 above. Propositions 8 and 9 consider settings where price discrimination is not permitted and $r \in \{\underline{r}, \bar{r}\}$ for each of the N consumers. Proposition 8 follows from Conclusions 1 and 2 below.

Conclusion 1. *Suppose a separating PPBE arises for a sophisticated consumer in the absence of privacy when price discrimination is permitted. Then in the absence of privacy in the large numbers setting, a ban on price discrimination: (i) increases the welfare of unsophisticated consumers with $r = \bar{r}$; and (ii) does not otherwise affect consumer welfare.*

Proof. Lemma 5 in the paper implies that $n_1 > n_2$, $c_1 > c^*$ ($> \hat{c}$), and $c_2 \leq \hat{c}$ in the present setting.

First consider merchant interactions with sophisticated consumers. Lemma 5 in the paper implies that when price discrimination is permitted in the present setting, Merchant 1 (S1) sets $p_1 = \hat{p}_1 \in (\underline{r}, \bar{r})$ for a sophisticated consumer, and Merchant 2 (S2) sets $p_2 = \bar{r}$ ($p_2 = \underline{r}$) for a sophisticated consumer who buys (does not buy) S1's product. Recall from equation (1) in the paper that \hat{p}_1 is determined by:

$$[\bar{r} - \hat{p}_1] n_1 = [\bar{r} - \underline{r}] n_2. \quad (56)$$

A sophisticated consumer with $r = \bar{r}$ buys the products of both merchants and secures payoff:

$$[\bar{r} - \hat{p}_1] n_1 + [\bar{r} - \bar{r}] n_2 = [\bar{r} - \hat{p}_1] n_1. \quad (57)$$

A sophisticated consumer with $r = \underline{r}$ buys only S2's product and secures payoff:

$$[\underline{r} - \hat{p}_1] 0 + [\underline{r} - \underline{r}] n_2 = 0. \quad (58)$$

Now suppose price discrimination is prohibited. In this case, S1 sets $p_1 = \bar{r}$ for all consumers (because $c_1 > \hat{c}$) and S2 sets $p_2 = \underline{r}$ for all consumers (because $c_2 \leq \hat{c}$).

A sophisticated consumer with $r = \bar{r}$ buys the products of both merchants and secures payoff:

$$[\bar{r} - \bar{r}] n_1 + [\bar{r} - \underline{r}] n_2 = [\bar{r} - \underline{r}] n_2. \quad (59)$$

A sophisticated consumer with $r = \underline{r}$ buys only S2's product and secures payoff:

$$[\underline{r} - \bar{r}] 0 + [\underline{r} - \underline{r}] n_2 = 0. \quad (60)$$

(56), (57), and (59) imply that a ban on price discrimination does not affect the welfare of sophisticated consumers with $r = \bar{r}$. (58) and (60) imply that a ban on price discrimination does not affect the welfare of sophisticated consumers with $r = \underline{r}$.

Now consider merchant interactions with unsophisticated consumers. Because $c_1 > \hat{c}$, S1 sets $p_1 = \bar{r}$ for an unsophisticated consumer and the consumer buys her product if and only if his reservation value is \bar{r} . Consequently, S2 learns the true reservation value of each unsophisticated consumer.

Case 1. $c_1 > \hat{c}$ and $c_2 > \hat{c}$.

When price discrimination is permitted in this case, S2 sets p_2 to eliminate the surplus of each unsophisticated consumer. When price discrimination is not permitted in this case, Merchant 2 sets $p_2 = \bar{r}$ for all consumers (because $c_2 > \hat{c}$). Consequently, a consumer buys S2's product if and only if $p_1 = \bar{r}$, so each consumer derives payoff 0 from his interaction with S2. Therefore, each consumer secures the same welfare whether price discrimination is permitted or banned.

Case 2. $c_1 > \hat{c}$ and $c_2 \leq \hat{c}$.

When price discrimination is permitted in this case, S2 sets p_2 to eliminate the surplus of each unsophisticated consumer. When price discrimination is not permitted in this case, Merchant 2 sets $p_2 = \underline{r}$ for all consumers (because $c_2 \leq \hat{c}$). Consequently, all consumers buy S2's product. Each consumer with $r = \bar{r}$ secures strictly positive welfare $([\bar{r} - \underline{r}] n_2)$ from his interaction with S2. Therefore, a ban on price discrimination increases the welfare of unsophisticated consumers with $r = \bar{r}$ and does not change the welfare of unsophisticated consumers with $r = \underline{r}$. ■

Conclusion 2. *Suppose a pooling PPBE arises for a sophisticated consumer in the absence of privacy when price discrimination is permitted. Then in the absence of privacy in the large numbers setting, a ban on price discrimination: (i) reduces the welfare of sophisticated consumers with $r = \bar{r}$ when $c_1 > \hat{c}$; (ii) increases the welfare of unsophisticated consumers with $r = \bar{r}$ when $c_1 > \hat{c}$ and $c_2 \leq \hat{c}$; and (iii) otherwise does not affect consumer welfare.*

Proof. The conclusion is proved by considering Case A and Case B.

Case A. Merchants interact with sophisticated consumers.

There are three sub-cases to consider.

Case A1. $n_1 \leq n_2$.

The discussion in the text explains why the only PPBE that arises when $n_1 \leq n_2$ is the pooling equilibrium in which S1 sets $p_1 = \underline{r}$.

First suppose $c_2 \leq \hat{c}$. Then when price discrimination is permitted, S1 sets $p_1 = \underline{r}$ for sophisticated consumers in the postulated pooling equilibrium. Because all such consumers buy S1's product at this price, S2 sets $p_2 = \underline{r}$ for sophisticated consumers (since $c_2 \leq \hat{c}$). Therefore, each sophisticated consumer buys both merchants' products and secures payoff:

$$(i) [\bar{r} - \underline{r}] [n_1 + n_2] \text{ when } r = \bar{r}; \text{ and } (ii) 0 \text{ when } r = \underline{r}. \quad (61)$$

Now suppose $c_2 > \hat{c}$. Then when price discrimination is permitted, S1 sets $p_1 = \underline{r}$ for sophisticated consumers in the postulated pooling equilibrium. Because all such consumers buy S1's product at this price, S2 sets $p_2 = \bar{r}$ for sophisticated consumers (since $c_2 > \hat{c}$). Therefore, a sophisticated consumer with $r = \bar{r}$ buys both merchants' products whereas a

sophisticated consumer with $r = \underline{r}$ only purchases S1's product. A sophisticated consumer thereby secures payoff:

$$(i) [\bar{r} - \underline{r}] n_1 \quad \text{when } r = \bar{r}; \quad \text{and} \quad (ii) 0 \quad \text{when } r = \underline{r}. \quad (62)$$

When price discrimination is not permitted, Merchant $i \in \{1, 2\}$ sets: (i) $p_i = \underline{r}$ when $c_i \leq \hat{c}$; and (ii) $p_i = \bar{r}$ when $c_i > \hat{c}$. Therefore, a consumer with $r = \underline{r}$ secures payoff 0 whereas a consumer with $r = \bar{r}$ secures payoff:

$$\begin{aligned} (i) & [\bar{r} - \underline{r}] [n_1 + n_2] \quad \text{when } c_1 \leq \hat{c} \text{ and } c_2 \leq \hat{c}; \\ (ii) & [\bar{r} - \underline{r}] n_1 \quad \text{when } c_1 \leq \hat{c} \text{ and } c_2 > \hat{c}; \\ (iii) & [\bar{r} - \underline{r}] n_2 \quad \text{when } c_1 > \hat{c} \text{ and } c_2 \leq \hat{c}; \quad \text{and} \\ (iv) & 0 \quad \text{when } c_1 > \hat{c} \text{ and } c_2 > \hat{c}. \end{aligned} \quad (63)$$

(61) – (63) imply that a ban on price discrimination does not affect the welfare of sophisticated consumers with $r = \underline{r}$. (61) and (63) also imply that for sophisticated consumers with $r = \bar{r}$, a ban on price discrimination: (i) does not affect their welfare when $c_1 \leq \hat{c}$; and (ii) reduces their welfare when $c_1 > \hat{c}$.

Case A2. $n_1 > n_2$ and $c_2 > \hat{c}$.

When price discrimination is permitted, S1 sets $p_1 = \underline{r}$ for sophisticated consumers in the postulated pooling equilibrium. Because all sophisticated consumers buy S1's product at this price, S2 sets $p_2 = \bar{r}$ for sophisticated consumers (since $c_2 > \hat{c}$). Therefore, a sophisticated consumer's welfare is 0 when $r = \underline{r}$, whereas the consumer's welfare is $[\bar{r} - \underline{r}] n_1$ when $r = \bar{r}$.

When price discrimination is not permitted, S1 sets price: (i) $p_1 = \underline{r}$ if $c_1 \leq \hat{c}$; and (ii) $p_1 = \bar{r}$ if $c_1 > \hat{c}$. S2 sets $p_2 = \bar{r}$ because $c_2 > \hat{c}$. Therefore, a consumer's payoff is 0 if $r = \underline{r}$, whereas the consumer's payoff when $r = \bar{r}$ is:

$$(i) [\bar{r} - \underline{r}] n_1 \quad \text{when } c_1 \leq \hat{c}; \quad \text{and} \quad (ii) 0 \quad \text{when } c_1 > \hat{c}.$$

Consequently, a ban on price discrimination does not affect the welfare of sophisticated consumers with $r = \underline{r}$. For sophisticated consumers with $r = \bar{r}$, the ban: (i) does not affect their welfare when $c_1 \leq \hat{c}$; and (ii) reduces their welfare when $c_1 > \hat{c}$.

Case A3. $n_1 > n_2$ and $c_2 \leq \hat{c}$.

When price discrimination is permitted, S1 sets $p_1 = \underline{r}$ for sophisticated consumers in the postulated pooling equilibrium. Because all sophisticated consumers buy S1's product at this price, S2 sets $p_2 = \underline{r}$ for sophisticated consumers (since $c_2 \leq \hat{c}$). Therefore, the payoff of a sophisticated consumer is: (i) 0 if $r = \underline{r}$; and (ii) $[\bar{r} - \underline{r}] [n_1 + n_2]$ if $r = \bar{r}$.

When price discrimination is not permitted, S1 sets price: (i) $p_1 = \underline{r}$ if $c_1 \leq \hat{c}$; and (ii) $p_1 = \bar{r}$ if $c_1 > \hat{c}$. S2 sets $p_2 = \underline{r}$ because $c_2 \leq \hat{c}$. Therefore, a consumer's payoff is 0 if $r = \underline{r}$, whereas the consumer's payoff if $r = \bar{r}$ is:

(i) $[\bar{r} - \underline{r}][n_1 + n_2]$ when $c_1 \leq \hat{c}$; and (ii) $[\bar{r} - \underline{r}]n_2$ when $c_1 > \hat{c}$.

Consequently, a ban on price discrimination does not affect the welfare of a sophisticated consumer with $r = \underline{r}$. For sophisticated consumers with $r = \bar{r}$, the ban: (i) does not affect their welfare when $c_1 \leq \hat{c}$; and (ii) reduces their welfare when $c_1 > \hat{c}$.

Case B. Merchants interact with unsophisticated consumers.

First suppose $c_1 \leq \hat{c}$. Then S1 sets $p_1 = \underline{r}$ for unsophisticated consumers both when price discrimination is permitted and when it is not permitted. Because all unsophisticated consumers buy S1's product at this price, S2 learns nothing about the reservation value of any unsophisticated consumer. Therefore, S2 sets the same price for unsophisticated consumers whether price discrimination is permitted or banned, so such a ban does not affect their welfare.

Now suppose $c_1 > \hat{c}$. In this case, S1 sets $p_1 = \bar{r}$ for unsophisticated consumers, and such a consumer buys her product if and only if his reservation value is \bar{r} . Consequently, S2 learns the true reservation value of each unsophisticated consumer.

Case B1. $c_1 > \hat{c}$ and $c_2 > \hat{c}$.

When price discrimination is permitted in this case, S2 sets p_2 to eliminate the surplus of each unsophisticated consumer. When price discrimination is not permitted in this case, Merchant 2 sets $p_2 = \bar{r}$ for all consumers (because $c_2 > \hat{c}$). Consequently, a consumer buys S2's product if and only if $p_1 = \bar{r}$, so each consumer derives payoff 0 from his interaction with S2. Therefore, each unsophisticated consumer secures the same payoff whether price discrimination is permitted or banned.

Case B2. $c_1 > \hat{c}$ and $c_2 \leq \hat{c}$.

When price discrimination is permitted in this case, S2 sets p_2 to eliminate the surplus of each unsophisticated consumer. When price discrimination is not permitted in this case, Merchant 2 sets $p_2 = \underline{r}$ for all consumers (because $c_2 \leq \hat{c}$). Consequently, all consumers buy S2's product. Each consumer with $r = \bar{r}$ secures a strictly positive payoff ($[\bar{r} - \underline{r}]n_2$) from his interaction with S2. Therefore, a ban on price discrimination increases the welfare of an unsophisticated consumer with $r = \bar{r}$ and does not change the welfare of an unsophisticated consumer with $r = \underline{r}$. ■

Proposition 9 follows from Conclusions 3 and 4 below.

Conclusion 3. *Under the conditions specified in Conclusion 1, a ban on price discrimination: (i) reduces the total welfare from transactions with unsophisticated consumers when $r = \underline{r}$ and $c_2 > \hat{c}$; and (ii) otherwise does not affect total welfare.*

Proof. First consider the total welfare from transactions with sophisticated consumers in the setting of Conclusion 1. The proof of the Conclusion establishes that a sophisticated

consumer with $r = \bar{r}$ purchases the products of S1 and S2 both when price discrimination is permitted and when it is prohibited. Consequently, the total welfare from transactions with a sophisticated consumer with $r = \bar{r}$ in both cases in this setting is $[\bar{r} - c_1]n_1 + [\bar{r} - c_2]n_2$.

The proof of the Conclusion also establishes that a sophisticated consumer with $r = \underline{r}$ only purchases S2's product, both when price discrimination is permitted and when it is prohibited. Consequently, the total welfare from transactions with a sophisticated consumer with $r = \underline{r}$ in both cases in this setting is $[\underline{r} - c_2]n_2$. Therefore, a ban on price discrimination does not affect the total welfare from transactions with sophisticated consumers in the setting of Conclusion 1.

Now consider total welfare from transactions with unsophisticated consumers in the setting of Conclusion 1. Recall from the proof of the Conclusion that when price discrimination is permitted in this setting, S1 sets $p_1 = \bar{r}$ for unsophisticated consumers and S2 sets p_2 to eliminate the surplus of each unsophisticated consumer. Total welfare from the transactions with an unsophisticated consumer in this case is:

$$\begin{aligned} \text{(i)} \quad & [\bar{r} - c_1]n_1 + [\bar{r} - c_2]n_2 \quad \text{when } r = \bar{r}; \\ \text{(ii)} \quad & [\underline{r} - c_2]n_2 \quad \text{when } r = \underline{r}. \end{aligned} \tag{64}$$

When price discrimination is not permitted in the setting of Conclusion 1, S1 sets $p_1 = \bar{r}$ (because $c_1 > \hat{c}$). S2 sets: (i) $p_2 = \bar{r}$ if $c_2 > \hat{c}$; and (ii) $p_2 = \underline{r}$ if $c_2 \leq \hat{c}$. Therefore, the total welfare from the transactions of an unsophisticated consumer in this case is:

$$\begin{aligned} \text{(i)} \quad & [\bar{r} - c_1]n_1 + [\bar{r} - c_2]n_2 \quad \text{when } r = \bar{r}; \\ \text{(ii)} \quad & [\underline{r} - c_2]n_2 \quad \text{when } r = \underline{r} \text{ and } c_2 \leq \hat{c}; \text{ and} \\ \text{(iii)} \quad & 0 \quad \text{when } r = \underline{r} \text{ and } c_2 > \hat{c}. \end{aligned} \tag{65}$$

(64) and (65) imply that a ban on price discrimination in this setting: (i) reduces the total welfare from transactions with unsophisticated consumers if $r = \underline{r}$ and $c_2 > \hat{c}$; and (ii) otherwise does not affect this total welfare. ■

Conclusion 4. *Under the conditions specified in Conclusion 2, a ban on price discrimination: (i) reduces the total welfare from transactions with sophisticated consumers when $r = \underline{r}$ and $c_1 > \hat{c}$; (ii) reduces the total welfare from transactions with unsophisticated consumers when $r = \underline{r}$, $c_1 > \hat{c}$, and $c_2 > \hat{c}$; and (iii) otherwise does not affect total welfare.*

Proof. First consider the total welfare from transactions with sophisticated consumers in the setting of Conclusion 2. A sophisticated consumer always purchases both merchants' products when $r = \bar{r}$ in this setting. Therefore, the total welfare from the transactions of a sophisticated consumer with $r = \bar{r}$ is $[\bar{r} - c_1]n_1 + [\bar{r} - c_2]n_2$ in this setting, both when price discrimination is permitted and when it is prohibited.

Now consider each of Cases A1 – A3 in the proof of Conclusion 2. In Case A1, when price discrimination is permitted: (i) S1 sets price $p_1 = \underline{r}$ for sophisticated consumers; and

(ii) S2 sets $p_2 = \underline{r}$ ($p_2 = \bar{r}$) when $c_2 \leq \hat{c}$ ($c_2 > \hat{c}$) for these consumers. Therefore, when price discrimination is permitted in Case 1, the total welfare from the transactions of a sophisticated consumer with $r = \underline{r}$ is:

$$\begin{aligned} \text{(i)} \quad & [\underline{r} - c_1]n_1 + [\underline{r} - c_2]n_2 \quad \text{when } c_2 \leq \hat{c}; \text{ and} \\ \text{(ii)} \quad & [\underline{r} - c_1]n_1 \quad \text{when } c_2 > \hat{c}. \end{aligned} \tag{66}$$

When price discrimination is not permitted, Mi sets $p_i = \underline{r}$ ($p_i = \bar{r}$) when $c_i \leq \hat{c}$ ($c_i > \hat{c}$) for $i \in \{1, 2\}$. Therefore, the total welfare from the transactions of a consumer with $r = \underline{r}$ is:

$$\begin{aligned} \text{(i)} \quad & [\underline{r} - c_1]n_1 + [\underline{r} - c_2]n_2 \quad \text{when } c_1 \leq \hat{c} \text{ and } c_2 \leq \hat{c}; \\ \text{(ii)} \quad & [\underline{r} - c_1]n_1 \quad \text{when } c_1 \leq \hat{c} \text{ and } c_2 > \hat{c}; \\ \text{(iii)} \quad & [\underline{r} - c_2]n_2 \quad \text{when } c_1 > \hat{c} \text{ and } c_2 \leq \hat{c}; \text{ and} \\ \text{(iv)} \quad & 0 \quad \text{when } c_1 > \hat{c} \text{ and } c_2 > \hat{c}. \end{aligned} \tag{67}$$

(66) and (67) imply that for a sophisticated consumer with $r = \underline{r}$, a ban on price discrimination in Case A1: (i) does not affect the total welfare from his transactions when $c_1 \leq \hat{c}$; and (ii) reduces this total welfare when $c_1 > \hat{c}$.

In Case A2, when price discrimination is permitted: (i) S1 sets price $p_1 = \underline{r}$ for sophisticated consumers; and (ii) S2 sets $p_2 = \bar{r}$ for these consumers (because $c_2 > \hat{c}$). Therefore, the total welfare from the transactions of a sophisticated consumer with $r = \underline{r}$ is $[\underline{r} - c_1]n_1$. (67) implies that the corresponding total welfare when $r = \underline{r}$ and price discrimination is not permitted in this case is: (i) $[\underline{r} - c_1]n_1$ when $c_1 \leq \hat{c}$; and (ii) 0 when $c_1 > \hat{c}$. Therefore, when $r = \underline{r}$, a ban on price discrimination: (i) does not affect the total welfare from the transactions of a sophisticated consumer when $c_1 \leq \hat{c}$; and (ii) reduces this total welfare when $c_1 > \hat{c}$.

In Case A3, when price discrimination is permitted, S1 and S2 both set price \underline{r} for sophisticated consumers. Therefore, the total welfare from the transactions of a sophisticated consumer with $r = \underline{r}$ is $[\underline{r} - c_1]n_1 + [\underline{r} - c_2]n_2$. (67) implies that the corresponding total welfare when $r = \underline{r}$ and price discrimination is not permitted in this case is: (i) $[\underline{r} - c_1]n_1 + [\underline{r} - c_2]n_2$ when $c_1 \leq \hat{c}$; and (ii) $[\underline{r} - c_2]n_2$ when $c_1 > \hat{c}$. Therefore, when $r = \underline{r}$, a ban on price discrimination: (i) does not affect the total welfare from the transactions of a sophisticated consumer when $c_1 \leq \hat{c}$; and (ii) reduces this total welfare when $c_1 > \hat{c}$.

Finally, consider total welfare from transactions with unsophisticated consumers in the setting of Conclusion 2. The proof of the Conclusion establishes that when $c_1 \leq \hat{c}$, the merchants set the same prices for unsophisticated consumers whether price discrimination is permitted or prohibited. Therefore, the total welfare from transactions with unsophisticated consumers is the same in both cases.

In Cases B1 and B2 (where $c_1 > \hat{c}$), S1 sets $p_1 = \bar{r}$ for unsophisticated consumers and S2 sets her prices to fully extract the rent of these consumers when price discrimination is

permitted. Therefore, the total welfare from the transactions of an unsophisticated consumer when price discrimination is permitted in this case is:

$$\begin{aligned}
& \text{(i)} \quad [\bar{r} - c_1] n_1 + [\bar{r} - c_2] n_2 \quad \text{when } r = \bar{r}; \text{ and} \\
& \text{(ii)} \quad [\underline{r} - c_2] n_2 \quad \text{when } r = \underline{r}.
\end{aligned} \tag{68}$$

If price discrimination is not permitted in Cases B1 and B2, S1 sets $p_1 = \bar{r}$ (because $c_1 > \hat{c}$). S2 sets price: (i) $p_2 = \bar{r}$ when $c_2 > \hat{c}$; and (ii) $p_2 = \underline{r}$ when $c_2 \leq \hat{c}$. Therefore, the total welfare from the transactions of each consumer in this case is:

$$\begin{aligned}
& \text{(i)} \quad [\bar{r} - c_1] n_1 + [\bar{r} - c_2] n_2 \quad \text{when } r = \bar{r}; \\
& \text{(ii)} \quad [\underline{r} - c_2] n_2 \quad \text{when } r = \underline{r} \text{ and } c_2 \leq \hat{c}; \text{ and} \\
& \text{(iii)} \quad 0 \quad \text{when } r = \underline{r} \text{ and } c_2 > \hat{c}.
\end{aligned} \tag{69}$$

(68) and (69) imply that a ban on price discrimination in Cases B1 and B2 (where $c_1 > \hat{c}$): (i) reduces the total welfare from transactions with unsophisticated consumers when $r = \underline{r}$ and $c_2 > \hat{c}$; and (ii) does not affect this total welfare otherwise. ■

III. Model Extensions

This section considers the extensions discussed in Section 6 of the paper. Section A considers a setting where sophisticated and unsophisticated consumers are both present simultaneously, and merchants cannot distinguish between sophisticated and unsophisticated consumers. Section B considers a setting where consumer demands for the merchants' products are non-binary.

A. Incomplete Information about Consumer Sophistication.

Suppose a fraction $\alpha \in [0, 1]$ of the N consumers are sophisticated, whereas the remaining fraction are unsophisticated. Merchants cannot observe directly whether a consumer is sophisticated or unsophisticated. As in the paper: (i) a generic consumer purchases either 0 or n_i units from Merchant (Seller) i (S_i), $i \in \{1, 2\}$; and (ii) a generic consumer interacts with S1 before interacting with S2. The present analysis considers the setting where $n_1 > n_2$, each consumer's reservation value (r) is either \underline{r} or \bar{r} , and ϕ is the probability that $r = \bar{r}$ for each consumer.

The ensuing analysis employs the following definitions.

1. $\hat{c} \equiv \bar{r} - \frac{\bar{r} - \underline{r}}{1 - \phi}$.
2. $c^* \equiv \hat{c} + \phi \frac{n_2}{n_1} \left[\frac{\bar{r} - \underline{r}}{1 - \phi} \right]$.
3. $\hat{p}_1 \equiv \frac{n_2}{n_1} \underline{r} + \left[1 - \frac{n_2}{n_1} \right] \bar{r} \in (\underline{r}, \bar{r})$.

4. $c_{1H} \equiv \frac{1}{\alpha} \left[\frac{n_2}{n_1} \underline{r} + \left(\alpha - \frac{n_2}{n_1} \right) \bar{r} \right] < \underline{r}$ when $\alpha < \frac{n_2}{n_1}$.
5. $\tilde{c}_2 \equiv \frac{\underline{r} - \phi \bar{r}}{1 - \phi} < \underline{r}$.
6. $\tilde{c}_1 \equiv \frac{\underline{r} - \phi [1 - \alpha] \bar{r}}{1 - \phi [1 - \alpha]} < \underline{r}$.
7. $\alpha^* \equiv \frac{[1 - \phi] \frac{n_2}{n_1}}{1 - \phi \frac{n_2}{n_1}} < \frac{n_2}{n_1}$.

The inequalities in definitions 4, 5, 6, and 7 hold because:

$$\begin{aligned} c_{1H} < \underline{r} &\Leftrightarrow \frac{1}{\alpha} \left[\frac{n_2}{n_1} \underline{r} + \left(\alpha - \frac{n_2}{n_1} \right) \bar{r} \right] < \underline{r} \\ &\Leftrightarrow \frac{n_2}{n_1} \underline{r} + \left[\alpha - \frac{n_2}{n_1} \right] \bar{r} < \alpha \underline{r} \Leftrightarrow \left[\frac{n_2}{n_1} - \alpha \right] \bar{r} > \left[\frac{n_2}{n_1} - \alpha \right] \underline{r}; \end{aligned}$$

$$\tilde{c}_2 < \underline{r} \Leftrightarrow \underline{r} - \phi \bar{r} < \underline{r} [1 - \phi] \Leftrightarrow \phi \underline{r} < \phi \bar{r} \Leftrightarrow \underline{r} < \bar{r};$$

$$\begin{aligned} \tilde{c}_1 < \underline{r} &\Leftrightarrow \underline{r} - \phi [1 - \alpha] \bar{r} < \underline{r} [1 - \phi (1 - \alpha)] \\ &\Leftrightarrow \phi [1 - \alpha] \underline{r} < \phi [1 - \alpha] \bar{r} \Leftrightarrow \underline{r} < \bar{r}; \text{ and} \end{aligned}$$

$$\alpha^* < \frac{n_2}{n_1} \Leftrightarrow \frac{[1 - \phi] \frac{n_2}{n_1}}{1 - \phi \frac{n_2}{n_1}} < \frac{n_2}{n_1} \Leftrightarrow \frac{n_2}{n_1} - \frac{n_2}{n_1} \phi < \frac{n_2}{n_1} - \left(\frac{n_2}{n_1} \right)^2 \phi \Leftrightarrow \frac{n_2}{n_1} < 1.$$

Finding 1. Suppose $\alpha \geq \alpha^*$, $c_1 \in (c^*, c_{1H})$, and $c_2 \leq \tilde{c}_2$. Then in the absence of privacy, there exists a separating equilibrium in which S1 sets price $p_1 = \hat{p}_1$ for the generic consumer. The consumer purchases n_1 units of S1's product at this price if and only if his reservation value is $r = \bar{r}$. S2 sets price $p_2 = \bar{r}$ if the consumer purchases S1's product and otherwise charges price $p_2 = \underline{r}$. The consumer always buys n_2 units from S2.

Proof. The optimality of the consumer's behavior and S2's behavior follows from the definition of \hat{p}_1 . Therefore, it suffices to demonstrate that S1 will set price \hat{p}_1 . To induce a sophisticated consumer with reservation value $r = \bar{r}$ to purchase S1's product when $c_2 \leq \tilde{c}_2$, S1 must reduce p_1 to \hat{p}_1 , where:

$$n_1 [\bar{r} - \hat{p}_1] = n_2 [\bar{r} - \underline{r}] \Leftrightarrow \hat{p}_1 = \frac{n_2}{n_1} \underline{r} + \left[1 - \frac{n_2}{n_1} \right] \bar{r} \in (\underline{r}, \bar{r}). \quad (70)$$

If S1 sets price $p_1 = \hat{p}_1$ ($< \bar{r}$), she sells her product to all consumers with reservation value $r = \bar{r}$, and secures payoff:

$$\pi_1(\hat{p}_1) = N \phi [\hat{p}_1 - c_1] = N \phi \left[\frac{n_2}{n_1} \underline{r} + \left(1 - \frac{n_2}{n_1} \right) \bar{r} - c_1 \right]. \quad (71)$$

If S1 sets price \underline{r} , she sells her product to all consumers and secures payoff:

$$\pi_1(\underline{r}) = N[\underline{r} - c_1]. \quad (72)$$

If S1 sets price \bar{r} , she sells her product to unsophisticated consumers with reservation value $r = \bar{r}$ and secures payoff:

$$\pi_1(\bar{r}) = N[1 - \alpha]\phi[\bar{r} - c_1]. \quad (73)$$

(71) and (73) imply:

$$\begin{aligned} \pi_1(\hat{p}_1) > \pi_1(\bar{r}) &\Leftrightarrow N\phi\left[\frac{n_2}{n_1}\underline{r} + \left(1 - \frac{n_2}{n_1}\right)\bar{r} - c_1\right] > N[1 - \alpha]\phi[\bar{r} - c_1] \\ &\Leftrightarrow \frac{n_2}{n_1}\underline{r} + \left[1 - \frac{n_2}{n_1}\right]\bar{r} - c_1 > [1 - \alpha][\bar{r} - c_1] \\ &\Leftrightarrow \alpha c_1 < \frac{n_2}{n_1}\underline{r} + \left[1 - \frac{n_2}{n_1} - (1 - \alpha)\right]\bar{r} \\ &\Leftrightarrow c_1 < \frac{1}{\alpha}\left[\frac{n_2}{n_1}\underline{r} + \left(\alpha - \frac{n_2}{n_1}\right)\bar{r}\right] \equiv c_{1H}. \end{aligned} \quad (74)$$

(71) and (72) imply:

$$\begin{aligned} \pi_1(\hat{p}_1) > \pi_1(\underline{r}) &\Leftrightarrow N\phi\left[\frac{n_2}{n_1}\underline{r} + \left(1 - \frac{n_2}{n_1}\right)\bar{r} - c_1\right] > N[\underline{r} - c_1] \\ &\Leftrightarrow \phi\left[\frac{n_2}{n_1}\underline{r} + \left(1 - \frac{n_2}{n_1}\right)\bar{r} - c_1\right] > \underline{r} - c_1 \\ &\Leftrightarrow [1 - \phi]c_1 > \left[1 - \phi\frac{n_2}{n_1}\right]\underline{r} - \phi\left[1 - \frac{n_2}{n_1}\right]\bar{r} \\ &\Leftrightarrow c_1 > \frac{1}{1 - \phi}\left\{\left[1 - \phi\frac{n_2}{n_1}\right]\underline{r} - \phi\left[1 - \frac{n_2}{n_1}\right]\bar{r}\right\} = c^*. \end{aligned} \quad (75)$$

The equality in (75) holds because:

$$\begin{aligned} c^* &\equiv \hat{c} + \phi\frac{n_2}{n_1}\left[\frac{\bar{r} - \underline{r}}{1 - \phi}\right] = \bar{r} - \frac{\bar{r} - \underline{r}}{1 - \phi} + \phi\frac{n_2}{n_1}\left[\frac{\bar{r} - \underline{r}}{1 - \phi}\right] \\ &= \frac{1}{1 - \phi}\left\{\bar{r}[1 - \phi] - \bar{r} + \underline{r} + \phi\frac{n_2}{n_1}[\bar{r} - \underline{r}]\right\} = \frac{1}{1 - \phi}\left\{-\phi\bar{r} + \underline{r} + \phi\frac{n_2}{n_1}[\bar{r} - \underline{r}]\right\} \\ &= \frac{1}{1 - \phi}\left\{\left[1 - \phi\frac{n_2}{n_1}\right]\underline{r} - \phi\left[1 - \frac{n_2}{n_1}\right]\bar{r}\right\}. \end{aligned}$$

The condition in (75) is feasible because:

$$\begin{aligned}
c^* < \underline{r} &\Leftrightarrow \left[1 - \phi \frac{n_2}{n_1}\right] \underline{r} - \phi \left[1 - \frac{n_2}{n_1}\right] \bar{r} < [1 - \phi] \underline{r} \\
&\Leftrightarrow \phi \left[1 - \frac{n_2}{n_1}\right] \underline{r} < \phi \left[1 - \frac{n_2}{n_1}\right] \bar{r} \Leftrightarrow \underline{r} < \bar{r}.
\end{aligned}$$

(74) and (75) imply that S1 strictly prefers to set $p_1 = \hat{p}_1$ than to set $p_1 = \underline{r}$ or $p_1 = \bar{r}$ if and only if $c_1 \in (c^*, c_{1H})$. Furthermore, it is readily verified that $\pi_1(p_1) < \max\{\pi_1(\underline{r}), \pi_1(\hat{p}_1), \pi_1(\bar{r})\}$ for all $p_1 \in (\underline{r}, \bar{r})$, $p_1 \neq \hat{p}_1$.³

To ensure $c^* \leq c_{1H}$, it must be the case that:

$$\begin{aligned}
&\frac{1}{1 - \phi} \left\{ \left[1 - \phi \frac{n_2}{n_1}\right] \underline{r} - \phi \left[1 - \frac{n_2}{n_1}\right] \bar{r} \right\} \leq \frac{1}{\alpha} \left[\frac{n_2}{n_1} \underline{r} + \left(\alpha - \frac{n_2}{n_1}\right) \bar{r} \right] \\
\Leftrightarrow & [1 - \phi] \frac{n_2}{n_1} \underline{r} + [1 - \phi] \left[\alpha - \frac{n_2}{n_1} \right] \bar{r} \geq \alpha \left[1 - \phi \frac{n_2}{n_1}\right] \underline{r} - \alpha \phi \left[1 - \frac{n_2}{n_1}\right] \bar{r} \\
\Leftrightarrow & \left\{ [1 - \phi] \alpha - [1 - \phi] \frac{n_2}{n_1} + \alpha \phi - \alpha \phi \frac{n_2}{n_1} \right\} \bar{r} \geq \left\{ \alpha \left[1 - \phi \frac{n_2}{n_1}\right] - [1 - \phi] \frac{n_2}{n_1} \right\} \underline{r} \\
\Leftrightarrow & \left\{ \alpha - [1 - \phi] \frac{n_2}{n_1} - \alpha \phi \frac{n_2}{n_1} \right\} \bar{r} \geq \left\{ \alpha \left[1 - \phi \frac{n_2}{n_1}\right] - [1 - \phi] \frac{n_2}{n_1} \right\} \underline{r} \\
\Leftrightarrow & \left\{ \alpha \left[1 - \phi \frac{n_2}{n_1}\right] - [1 - \phi] \frac{n_2}{n_1} \right\} \bar{r} \geq \left\{ \alpha \left[1 - \phi \frac{n_2}{n_1}\right] - [1 - \phi] \frac{n_2}{n_1} \right\} \underline{r}.
\end{aligned}$$

This inequality holds if and only if:

$$\alpha \left[1 - \phi \frac{n_2}{n_1}\right] - [1 - \phi] \frac{n_2}{n_1} \geq 0 \Leftrightarrow \alpha \geq \frac{[1 - \phi] \frac{n_2}{n_1}}{1 - \phi \frac{n_2}{n_1}} = \alpha^*. \quad (76)$$

The optimality of \hat{p}_1 follows from (74), (75), and (76). ■

B. The Setting with Non-Binary Demand.

We now establish that the existence of a profit-maximizing price for S1, $p_1 \in (\underline{r}, \bar{r})$, that induces a sophisticated consumer to buy S1's product if and only if $r = \bar{r}$ is not an artifact of binary consumer demand. To do so, let n_i be the maximum number of units a consumer will purchase from S_i ($i \in \{1, 2\}$). $F_j(p)$ is the fraction of n_i that, in the absence of strategic considerations, a j -consumer ($j \in \{L, H\}$) purchases when S_i sets price $p_i = p$. We assume that for $j \in \{L, H\}$:

$$\begin{aligned}
F_j(p) &= f_j - b_j p \text{ for } p \in [0, \frac{f_j}{b_j}] \\
\Rightarrow P_j(F) &= \frac{1}{b_j} [f_j - F] \text{ for } F \in [0, f_j],
\end{aligned} \quad (77)$$

³Specifically: (i) $\pi_1(p_1) < \pi_1(\hat{p}_1)$ if $p_1 \in (\underline{r}, \hat{p}_1)$; and (ii) $\pi_1(p_1) < \pi_1(\bar{r})$ if $p_1 \in (\hat{p}_1, \bar{r})$.

where $f_H > f_L$, $b_H \geq b_L$, and $\frac{f_H}{b_H} > \frac{f_L}{b_L}$. Thus, the H -consumer's demand for each merchant's product is higher and less price-sensitive than the corresponding demand of the L -consumer. The inverse demand curve of the H -consumer ($P_H(F)$) lies to the right of and is less steeply-sloped than the inverse demand curve of the L -consumer ($P_L(F)$). We also assume $f_j - b_j c_i > 0$ for all $j \in \{L, H\}$ and $i \in \{1, 2\}$.

Let $\delta \in [0, 1]$ denote a merchant's perceived probability that a consumer is an H -consumer. Then in the absence of strategic considerations, the profit-maximizing price that S_i will set for a generic consumer is determined by:

$$\text{Maximize}_p \quad n_i [p - c_i] \{ \delta [f_H - b_H p] + [1 - \delta] [f_L - b_L p] \}. \quad (78)$$

Differentiating (78) provides:

$$\begin{aligned} & - [p - c_i] [\delta b_H + (1 - \delta) b_L] + \delta [f_H - b_H p] + [1 - \delta] [f_L - b_L p] = 0 \\ \Rightarrow & 2p [\delta b_H + (1 - \delta) b_L] = [\delta b_H + (1 - \delta) b_L] c_i + \delta f_H + [1 - \delta] f_L \\ \Rightarrow & p_i^*(\delta) = \frac{\delta [f_H + b_H c_i] + [1 - \delta] [f_L + b_L c_i]}{2 [\delta b_H + (1 - \delta) b_L]}. \end{aligned} \quad (79)$$

(79) implies:

$$\begin{aligned} \frac{\partial p_i^*(\delta)}{\partial \delta} &= [\delta b_H + (1 - \delta) b_L] [f_H + b_H c_i - f_L - b_L c_i] \\ &\quad - \{ \delta [f_H + b_H c_i] + [1 - \delta] [f_L + b_L c_i] \} [b_H - b_L] \\ &= [f_H + b_H c_i] [\delta b_H + (1 - \delta) b_L - \delta b_H + \delta b_L] \\ &\quad - [f_L + b_L c_i] [\delta b_H + (1 - \delta) b_L + (1 - \delta) b_H - (1 - \delta) b_L] \\ &= b_L [f_H + b_H c_i] - b_H [f_L + b_L c_i] = b_L f_H - b_H f_L > 0. \end{aligned} \quad (80)$$

(77) implies that the welfare a j -consumer secures when he purchases $F_j(p_i) n_i$ units of S_i 's product at unit price p_i is:

$$\begin{aligned} W_j(F_j(p_i) n_i) &= \frac{1}{2} F_j(p_i) \left[\frac{f_j}{b_j} - p_i \right] n_i \\ &= \frac{n_i}{2} [f_j - b_j p_i] \left[\frac{f_j}{b_j} - p_i \right] = \frac{n_i}{2 b_j} [f_j - b_j p_i]^2. \end{aligned} \quad (81)$$

Similarly, (77) implies that the welfare an H -consumer secures when he purchases $F_L(p_i) n_i$ units of S_i 's product at unit price p_i is:

$$\begin{aligned}
W_H(F_L(p_i) n_i) &= n_i \int_0^{F_L(p_i)} \left(\frac{1}{b_H} [f_H - F] - p_i \right) dF \\
&= n_i \left\{ \left[\frac{f_H}{b_H} - p_i \right] F_L(p_i) - \frac{1}{2b_H} [F_L(p_i)]^2 \right\} = n_i F_L(p_i) \left\{ \frac{f_H}{b_H} - p_i - \frac{1}{2b_H} F_L(p_i) \right\} \\
&= \frac{n_i}{2b_H} F_L(p_i) [2f_H - 2b_H p_i - F_L(p_i)] \\
&= \frac{n_i}{2b_H} [f_L - b_L p_i] [2f_H - f_L - (2b_H - b_L) p_i]. \tag{82}
\end{aligned}$$

We seek to determine if S1 will ever reduce p_1 below $p_1^*(\delta)$ to induce the H -consumer to purchase $n_1 F_H(p_1)$ units of her product, even though doing so leads S2 to set price $p_2^*(\delta = 1)$. We do so under the assumption that when S1 sets price p_1 for a consumer: (i) if the consumer purchases $n_1 F_L(p_1)$ units from S1, S2 infers the consumer is an L -consumer, and so sets $p_2 = p_2^*(\delta = 0)$ for the consumer; and (ii) otherwise, S2 infers the consumer is an H -consumer, and so sets $p_2 = p_2^*(\delta = 1)$.

(79) and (82) imply that when an H -consumer purchases $n_1 F_H(p_1)$ units of S1's product at unit price p_1 and $n_2 F_H(p_2^*(\delta = 1))$ units of S2's product at unit price $p_2^*(\delta = 1) = \frac{f_H + b_H c_2}{2b_H}$, his welfare is:

$$\begin{aligned}
W_H^R &\equiv W_H(F_H(p_1) n_1) + W_H(F_H(p_2^*(\delta = 1)) n_2) \\
&= \frac{n_1}{2b_H} [f_H - b_H p_1]^2 + \frac{n_2}{2b_H} \left[f_H - b_H \left(\frac{f_H + b_H c_2}{2b_H} \right) \right]^2 \\
&= \frac{1}{2b_H} \left\{ [f_H - b_H p_1]^2 n_1 + \left[f_H - \frac{b_H}{2} (f_H + b_H c_2) \right]^2 n_2 \right\} \\
&= \frac{1}{2b_H} \left\{ [f_H - b_H p_1]^2 n_1 + \frac{1}{4} [f_H - b_H c_2]^2 n_2 \right\}. \tag{83}
\end{aligned}$$

(79) and (82) imply that when an H -consumer purchases $n_1 F_L(p_1)$ units of S1's product at unit price p_1 and $n_2 F_H(p_2^*(\delta = 0))$ units of S2's product at unit price $p_2^*(\delta = 0) = \frac{f_L + b_L c_2}{2b_L}$, his welfare is:⁴

$$\begin{aligned}
W_H^C &\equiv W_H(F_L(p_1) n_1) + W_H(F_H(p_2^*(\delta = 0)) n_2) \\
&= \frac{n_1}{2b_H} [f_L - b_L p_1] [2f_H - f_L - (2b_H - b_L) p_1] + \frac{n_2}{2b_H} \left[f_H - b_H \left(\frac{f_L + b_L c_2}{2b_L} \right) \right]^2. \tag{84}
\end{aligned}$$

⁴The “ R ” in W_H^R denotes “reveal.” The “ C ” in W_H^C denotes “conceal.”

Finding 2. Suppose $b_L = b_H = b$. Then for any relevant p_1 :

$$W_H^C \gtrless W_H^R \Leftrightarrow n_1 < n_2 \left[\frac{2(f_H - b c_2) + f_H - f_L}{4(f_H - f_L)} \right].$$

Proof. (83) and (84) imply that when $b_L = b_H = b$:

$$\begin{aligned} W_H^C \gtrless W_H^R &\Leftrightarrow n_1 [f_L - b p_1] [2 f_H - f_L - b p_1] + n_2 \left[f_H - \frac{1}{2} (f_L + b c_2) \right]^2 \\ &> n_1 [f_H - b p_1]^2 + \frac{n_2}{4} [f_H - b c_2]^2 \\ \Leftrightarrow n_1 [f_L - b p_1] [f_H - b p_1 + f_H - f_L] + \frac{n_2}{4} [2 f_H - f_L - b c_2]^2 \\ &> n_1 [f_H - b p_1]^2 + \frac{1}{4} [f_H - b c_2]^2 \\ \Leftrightarrow n_1 [f_L - b p_1] [f_H - b p_1] + n_1 [f_H - f_L] [f_L - b p_1] + \frac{n_2}{4} [f_H - b c_2 + f_H - f_L]^2 \\ &> n_1 [f_H - b p_1]^2 + \frac{1}{4} [f_H - b c_2]^2 \\ \Leftrightarrow n_1 [f_H - b p_1 - (f_H - f_L)] [f_H - b p_1] + n_1 [f_H - f_L] [f_L - b p_1] + \frac{n_2}{4} [f_H - b c_2]^2 \\ &\quad + \frac{n_2}{4} [2(f_H - f_L)(f_H - b c_2) + (f_H - f_L)^2] > n_1 [f_H - b p_1]^2 + \frac{n_2}{4} [f_H - b c_2]^2 \\ \Leftrightarrow n_1 [f_H - f_L] [f_L - b p_1] - n_1 [f_H - f_L] [f_H - b p_1] \\ &\quad + \frac{n_2}{4} [2(f_H - f_L)(f_H - b c_2) + (f_H - f_L)^2] > 0 \\ \Leftrightarrow n_1 [f_L - b p_1] - n_1 [f_H - b p_1] + \frac{n_2}{4} [2(f_H - b c_2) + f_H - f_L] > 0 \\ \Leftrightarrow \frac{n_2}{4} [2(f_H - b c_2) + f_H - f_L] > n_1 [f_H - f_L] \\ \Leftrightarrow n_1 < n_2 \left[\frac{2(f_H - b c_2) + f_H - f_L}{4(f_H - f_L)} \right]. \blacksquare \end{aligned}$$

Finding 2 implies that when the inverse demand curves of an H -consumer and an L -consumer are parallel, the trade-off the H -consumer faces in deciding whether to conceal his true product valuation (his “type”) does not vary with the price S_1 sets. If $\frac{n_1}{n_2}$ is sufficiently small, the H -consumer conceals his type by reducing his purchase of S_1 's product because the corresponding (relatively small) welfare reduction he incurs in his interaction with S_1 enables him to secure a (relatively large) increase in welfare in his interaction with S_2 .

Finding 3 considers settings where the demand curves of the H -consumer and the L -consumer are not parallel. The Finding characterizes the p_1 at which an H -consumer is indifferent between revealing and concealing his type through his interaction with S1 (so $W_H^C = W_H^R$).

Finding 3. Suppose $b_H > b_L$ and there exists a p_1 at which $W_H^C = W_H^R$. Then this p_1 is determined by:

$$\tilde{p}_1^* = \frac{1}{b_H - b_L} \left[f_H - f_L - \sqrt{\frac{n_2}{n_1} A} \right] \quad (85)$$

$$\text{where } A \equiv \left[f_H - b_H \left(\frac{f_L + b_L c_2}{2 b_L} \right) \right]^2 - \frac{1}{4} [f_H - b_H c_2]^2. \quad (86)$$

Proof. (83) and (84) imply that if there exists a p_1 at which $W_H^C = W_H^R$, it is determined by:

$$\begin{aligned} & n_1 [f_L - b_L p_1] [2 f_H - f_L - (2 b_H - b_L) p_1] + n_2 \left[f_H - b_H \left(\frac{f_L + b_L c_2}{2 b_H} \right) \right]^2 \\ &= [f_H - b_H p_1]^2 n_1 + \frac{n_2}{4} [f_H - b_H c_2]^2 \\ \Leftrightarrow & [f_H - b_H p_1]^2 - [f_L - b_L p_1] [2 f_H - f_L - (2 b_H - b_L) p_1] = \frac{n_2}{n_1} A. \end{aligned} \quad (87)$$

Observe that:

$$\begin{aligned} & [f_H - b_H p_1]^2 - [f_L - b_L p_1] [2 f_H - f_L - (2 b_H - b_L) p_1] \\ &= f_H^2 - 2 b_H f_H p_1 + b_H^2 p_1^2 - f_L [2 f_H - f_L - 2 b_H p_1 + b_L p_1] \\ &\quad + b_L p_1 [2 f_H - f_L - 2 b_H p_1 + b_L p_1] \\ &= f_H^2 - 2 b_H f_H p_1 + b_H^2 p_1^2 - 2 f_L f_H + f_L^2 + 2 b_H f_L p_1 - f_L b_L p_1 \\ &\quad + 2 b_L f_H p_1 - b_L f_L p_1 - 2 b_L b_H p_1^2 + b_L^2 p_1^2 \\ &= f_H^2 - 2 f_L f_H + f_L^2 + p_1^2 [b_H^2 - 2 b_L b_H + b_L^2] \\ &\quad + p_1 [-2 b_H f_H + 2 b_H f_L - 2 f_L b_L + 2 b_L f_H] \\ &= [f_H - f_L]^2 - 2 p_1 [b_H (f_H - f_L) - b_L (f_H - f_L)] + p_1^2 [b_H - b_L]^2 \\ &= [f_H - f_L]^2 - 2 p_1 [b_H - b_L] [f_H - f_L] + p_1^2 [b_H - b_L]^2 \\ &= [f_H - f_L - (b_H - b_L) p_1]^2. \end{aligned} \quad (88)$$

(87) and (88) imply that if $b_H > b_L$ and if the p_1 in question exists, then it is determined by:

$$f_H - f_L - [b_H - b_L]p_1 = \sqrt{\frac{n_2}{n_1} A} \Rightarrow \tilde{p}_1^* = \frac{1}{b_H - b_L} \left[f_H - f_L - \sqrt{\frac{n_2}{n_1} A} \right]. \quad \blacksquare$$

Finding 4 specifies how \tilde{p}_1^* varies with the maximum demands for the merchants' products.

Finding 4. *When the conditions specified in Finding 3 hold:*

$$\frac{\partial \tilde{p}_1^*}{\partial n_1} > 0 \quad \text{and} \quad \frac{\partial \tilde{p}_1^*}{\partial n_2} < 0 \quad \Rightarrow \quad \frac{\partial \tilde{p}_1^*}{\partial (n_1/n_2)} > 0. \quad (89)$$

Proof. From (86):

$$\begin{aligned} A &= \left[f_H - b_H \left(\frac{f_L + b_L c_2}{2 b_L} \right) \right]^2 - \frac{1}{4} [f_H - b_H c_2]^2 \\ &= \left[\frac{2 b_L f_H - b_H (f_L + b_L c_2)}{2 b_L} \right]^2 - \frac{1}{4} [f_H^2 - 2 b_H f_H c_2 + b_H^2 c_2^2] \\ &= \frac{1}{4 b_L^2} \{ [2 b_L f_H - b_H (f_L + b_L c_2)]^2 - b_L^2 [f_H^2 - 2 b_H f_H c_2 + b_H^2 c_2^2] \} \\ &= \frac{1}{4 b_L^2} \{ 4 b_L^2 f_H^2 - 4 b_L b_H f_H [f_L + b_L c_2] + b_H^2 [f_L + b_L c_2]^2 \\ &\quad - b_L^2 f_H^2 + 2 b_L^2 b_H f_H c_2 - b_L^2 b_H^2 c_2^2 \} \\ &= \frac{1}{4 b_L^2} \{ 3 b_L^2 f_H^2 - 4 b_L b_H f_L f_H - 4 b_L^2 b_H f_H c_2 + b_H^2 [f_L^2 + 2 b_L f_L c_2 + b_L^2 c_2^2] \\ &\quad + 2 b_L^2 b_H f_H c_2 - b_L^2 b_H^2 c_2^2 \} \\ &= \frac{1}{4 b_L^2} \{ 3 b_L^2 f_H^2 - 4 b_L b_H f_L f_H - 2 b_L^2 b_H f_H c_2 + b_H^2 f_L^2 + 2 b_L b_H^2 f_L c_2 \} \\ &= \frac{1}{4 b_L^2} \{ 3 b_L^2 f_H^2 - 4 b_L b_H f_L f_H + b_H^2 f_L^2 + 2 b_L b_H [b_H f_L - b_L f_H] c_2 \} \\ &= \frac{1}{4 b_L^2} \{ [3 b_L f_H - b_H f_L] [b_L f_H - b_H f_L] - 2 b_L b_H [b_L f_H - b_H f_L] c_2 \} \\ &= \frac{b_L f_H - b_H f_L}{4 b_L^2} [3 b_L f_H - b_H f_L - 2 b_L b_H c_2] \end{aligned} \quad (90)$$

$$= \frac{b_L f_H - b_H f_L}{4 b_L^2} [b_L f_H - b_H f_L + 2 b_L (f_H - b_H c_2)] > 0. \quad (91)$$

(85) and (91) imply that (89) holds if $b_H > b_L$. \blacksquare

Finding 4 reflects the fact that as the H -consumer's interaction with S2 becomes relatively less important (in the sense that n_2/n_1 declines), S1 does not have to reduce p_1 as much to induce the H -consumer to reveal his type (by purchasing $n_1 F_H(p_1)$, rather than $n_1 F_L(p_1)$, units of S1's product).

It remains to identify conditions under which S1 finds it profitable to induce H -consumers to reveal their relatively high valuation of the merchants' products. If an H -consumer conceals his type by acting as if he is an L -consumer, the profit-maximizing price for S1 is $p_1^*(\delta = 0) = \frac{f_L + b_L c_1}{2b_L}$. When she sets this price, the profit S1 secures from her interaction with the consumer is:

$$\begin{aligned}
\pi_1(p_1^*(\delta = 0)) &= n_1 [p_1^*(\delta = 0) - c_1] F_L(p_1^*(\delta = 0)) \\
&= n_1 \left[\frac{f_L + b_L c_1}{2b_L} - c_1 \right] \left[f_L - b_L \left(\frac{f_L + b_L c_1}{2b_L} \right) \right] \\
&= n_1 \left[\frac{f_L + b_L c_1 - 2b_L c_1}{2b_L} \right] \left[\left(\frac{2b_L f_L - b_L f_L - b_L^2 c_1}{2b_L} \right) \right] = \frac{n_1}{4b_L} [f_L - b_L c_1]^2. \quad (92)
\end{aligned}$$

Define $f^e \equiv \delta f_H + [1 - \delta] f_L$ and $b^e \equiv \delta b_H + [1 - \delta] b_L$. If an H -consumer reveals his type when S1 sets price $p_1 = \tilde{p}_1^*$, then the profit S1 secures from her interaction with the consumer is:

$$\begin{aligned}
\pi_1(\tilde{p}_1^*) &= n_1 [\tilde{p}_1^* - c_1] [\delta F_H(\tilde{p}_1^*) + (1 - \delta) F_L(\tilde{p}_1^*)] \\
&= n_1 [\tilde{p}_1^* - c_1] [\delta (f_H - b_H \tilde{p}_1^*) + (1 - \delta) (f_L - b_L \tilde{p}_1^*)] \\
&= n_1 [\tilde{p}_1^* - c_1] [f^e - b^e \tilde{p}_1^*] = n_1 [(f^e + b^e c_1) \tilde{p}_1^* - b^e (\tilde{p}_1^*)^2 - c_1 f^e] \\
&= n_1 \left[(f^e + b^e c_1) \left(\frac{f_H - f_L - \sqrt{\frac{n_2}{n_1} A}}{b_H - b_L} \right) - b^e \left(\frac{f_H - f_L - \sqrt{\frac{n_2}{n_1} A}}{b_H - b_L} \right)^2 - c_1 f^e \right] \\
&= \frac{n_1}{[b_H - b_L]^2} \left\{ [f^e + b^e c_1] [b_H - b_L] \left[f_H - f_L - \sqrt{\frac{n_2}{n_1} A} \right] \right. \\
&\quad \left. - b^e \left[f_H - f_L - \sqrt{\frac{n_2}{n_1} A} \right]^2 - c_1 f^e [b_H - b_L]^2 \right\}. \quad (93)
\end{aligned}$$

To verify that there are conditions under which $\pi_1(\tilde{p}_1^*) > \pi_1(p_1^*(\delta = 0))$, consider the following parameter values:

$$c_1 = c_2 = 1; \delta = \frac{1}{2}; n_1 = 1; f_H = 16; f_L = 4; b_H = 3; b_L = 1. \quad (94)$$

(85), (90), and (92) imply that for the parameter values in (94):

$$\begin{aligned}
A &= \frac{16 - 12}{4} [3(16) - 12 - 6] = 48 - 18 = 30; \\
\widehat{p}_1^* &= \frac{1}{2} [12 - \sqrt{30 n_2}] = 6 - 2.739 \sqrt{n_2}; \text{ and} \\
\pi_1(p_1^*(\delta = 0)) &= \frac{1}{4} [4 - 1]^2 = \frac{9}{4}.
\end{aligned} \tag{95}$$

Because $f^e = 10$ and $b^e = 2$ for the parameter values in (94), (93) implies:

$$\begin{aligned}
\pi_1(\widehat{p}_1^*) &= \frac{1}{4} \left[12(2) \left(12 - \sqrt{30} \sqrt{n_2} \right) - 2 \left(12 - \sqrt{30} \sqrt{n_2} \right)^2 - 40 \right] \\
&= \frac{1}{2} \left[12 \left(12 - \sqrt{30} \sqrt{n_2} \right) - \left(12 - \sqrt{30} \sqrt{n_2} \right)^2 - 20 \right] \\
&= \frac{1}{2} \left[124 - 12 \sqrt{30} \sqrt{n_2} - \left(2 - \sqrt{30} \sqrt{n_2} \right)^2 \right] \\
&= \frac{1}{2} \left[124 - 65.727 \sqrt{n_2} - \left(2 - 5.477 \sqrt{n_2} \right)^2 \right].
\end{aligned} \tag{96}$$

Suppose $n_2 = 1$. Then (96) implies:

$$\pi_1(\widehat{p}_1^*) = \frac{1}{2} [124 - 65.727 - (2 - 5.477)^2] = \frac{1}{2} [58.273 - 12.090] = 23.092 > \frac{9}{4}.$$

Furthermore, from (95):

$$\begin{aligned}
\widehat{p}_1^* &= 6 - 2.739 = 3.261 \in [0, 4]; \quad f_H - b_H \widehat{p}_1^* = 16 - 3[3.261] = 6.217 > 0; \\
&\text{and} \quad f_L - b_L \widehat{p}_1^* = 4 - 3.261 = 0.739 > 0. \quad \blacksquare
\end{aligned}$$

This analysis demonstrates that a separating equilibrium can arise with non-binary demand even when $n_1 \leq n_2$. This is the case because a consumer can reveal his high product valuation without increasing his consumption of S1's product by n_1 units. Consequently, S1 can sometimes (profitably) reduce p_1 to a level that induces H -consumers to reveal their relatively high product valuation without reducing p_1 below \underline{r} .