Proof of Claim in

"On the Profitability of Self-Sabotage"

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Claim. Our focus on a single $(\underline{r}, \overline{r})$ compensation structure is without essential loss of generality.

More precisely, the buyer's maximum expected payoff with one $(\underline{r}, \overline{r})$ pair is the same as the maximum expected payoff she could achieve if she offered two $(\underline{r}, \overline{r})$ pairs, one for each possible supplier report of his cost of delivering cost-reducing effort ($k \in \{\underline{k}, \overline{k}\}$).

Proof. The Claim follows from the following two observations.

First, Lemmas 2 and 3 report that the supplier secures no rent both when he never delivers cost-reducing effort and when he delivers this effort selectively. Therefore, the buyer secures with a single $(\underline{r}, \overline{r})$ pair her maximum potential expected payoff when she induces these two patterns of effort delivery.

Second, the buyer also secures with a single $(\underline{r}, \overline{r})$ pair her maximum potential expected payoff when she induces the supplier to always deliver cost-reducing effort. To see why, note that Lemma 1 reports that when the buyer employs a single $(\underline{r}, \overline{r})$ pair, the supplier earns no rent when $k = \overline{k}$ and he earns rent $\overline{k} - \underline{k}$ when $k = \underline{k}$. It remains to verify that the supplier secures at least these rent levels when the buyer employs two $(\underline{r}, \overline{r})$ pairs.

To prove this conclusion, consider the following notation. Let $\underline{r}_L$ ($\overline{r}_L$) denote the payment to the supplier when he reports $k = \underline{k}$ and project cost $\underline{c}$ ($\overline{c}$) is subsequently realized. Similarly, let $\underline{r}_H$ ($\overline{r}_H$) denote the payment to the supplier when he reports $k = \overline{k}$ and project cost $\underline{c}$ ($\overline{c}$) is subsequently realized. To ensure the supplier secures non-negative expected profit when he reports $k = \overline{k}$ (truthfully), it must be the case that:

$$p_L[\, \underline{r}_H - \underline{c}\,] + [1 - p_L][\, \overline{r}_H - \overline{c}\,] - \overline{k} \;\geq\; 0. \tag{1}$$

To ensure the supplier truthfully reports $k = \underline{k}$ and delivers cost-reducing effort rather than reporting $k = \overline{k}$ and delivering this effort, it must be the case that:

$$p_L[\underline{r}_L - \underline{c}] + [1 - p_L][\overline{r}_L - \overline{c}] - \underline{k} \;\geq\; p_L[\underline{r}_H - \underline{c}] + [1 - p_L][\overline{r}_H - \overline{c}] - \underline{k} \;\geq\; \overline{k} - \underline{k}. \tag{2}$$

The last inequality in expression (2) reflects expression (1). Expressions (1) and (2) (and Lemma 1) imply that when the buyer always induces the supplier to deliver cost-reducing effort, her expected payoff when she employs a single $(\underline{r}, \overline{r})$ pair optimally is at least as large as her expected payoff when she employs two $(\underline{r}, \overline{r})$ pairs.

In settings where the buyer can employ multiple $(\underline{r}, \overline{r})$ pairs, the $(\underline{r}, \overline{r})$ pair identified in Lemma 2 is not the unique solution to the buyer's problem when she induces the supplier to deliver cost-reducing effort selectively. The buyer can secure the same expected payoff, for example, by setting $\underline{r}_H = \underline{c}$, $\overline{r}_H = \overline{c}$, and $\underline{r}_L = \overline{r}_L = p_L\, \underline{c} + [1 - p_L]\overline{c} + \underline{k}_L$. However, this pair of reward structures generates the same expected payoff for the buyer as the single reward structure specified in Lemma 2. ■