Technical Appendix to Accompany

"Targeting Rivals: Moving from 'Whether' to 'Whom'"

by David M. Mandy, John W. Mayo, and David E. M. Sappington

This technical appendix extends the analysis in section 4 of the paper to consider the setting where the VIP and its rivals set their retail prices simultaneously and noncooperatively. The notation here is the same as in the paper, except the ρ_i's are no longer employed whereas the following new variables are employed, for $i,\, j,\, k \in \{1, 2, 3\}$ all distinct:[1]

$$\theta_i \;\equiv\; z_i + s_i + u_i\,; \quad \rho_{ii} \;\equiv\; 4\,\beta^2 - \gamma_{jk}^2\,; \quad \rho_{ij} \;\equiv\; 2\,\beta\,\gamma_{ij} + \gamma_{ik}\,\gamma_{jk}\,. \tag{1}$$

Each of these variables is positive, and $\rho_{ij} \;=\; \rho_{ji}$.

The necessary conditions for equilibrium prices are as specified in (9) in the paper:

$$\beta\,\theta_i + \sum_{j \neq i} \gamma_{ij}\,p_j - 2\,\beta\,p_i \;=\; 0\,. \tag{2}$$

(2) implies that the equilibrium prices solve the system:

$$\begin{bmatrix} 2\,\beta & -\,\gamma_{12} & -\,\gamma_{13} \\ -\,\gamma_{12} & 2\,\beta & -\,\gamma_{23} \\ -\,\gamma_{13} & -\,\gamma_{23} & 2\,\beta \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} \;=\; \beta \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}. \tag{3}$$

Denote the determinant of the Jacobian of (3) by

$$J \;\equiv\; 2\left[\,4\,\beta^3 - \gamma_{12}\,\gamma_{13}\,\gamma_{23} - \beta(\gamma_{12}^2 + \gamma_{13}^2 + \gamma_{23}^2)\,\right] \;>\; 0\,.$$

Applying Cramer's Rule to (3) and using (1) provides the equilibrium prices:

$$\begin{aligned} p_i^e \;&=\; \frac{\beta}{J}\left[\,\left(4\,\beta^2 - \gamma_{jk}^2\right)\theta_i + \left(2\,\beta\,\gamma_{ij} + \gamma_{ik}\,\gamma_{jk}\right)\theta_j + \left(2\,\beta\,\gamma_{ik} + \gamma_{ij}\,\gamma_{kj}\right)\theta_k\,\right] \\ &=\; \frac{\beta}{J}\left[\,\rho_{ii}\,\theta_i + \rho_{ij}\,\theta_j + \rho_{ik}\,\theta_k\,\right]. \end{aligned} \tag{4}$$

Differentiating (4), using (1), provides:

$$\frac{\partial p_i^e}{\partial s_j} \;=\; \frac{\beta\,\rho_{ij}}{J} \;>\; 0\,. \tag{5}$$

[1] Recall that $z_i \;\equiv\; \frac{\alpha_i}{\beta} + c_i + w_i$ and $u_i \;\equiv\; \frac{1}{\beta}\,\frac{\partial \pi_i^u}{\partial p_i}$ for $i = 1, 2, 3$.

To analyze the VIP's sabotage choice, label the VIP firm 1, without loss of generality. Then $s_1 = 0$, $w_1 = c^u$, $u_1 = \frac{1}{\beta}\left[\,(w_2 - c^u)\,\gamma_{12} + (w_3 - c^u)\,\gamma_{13}\,\right]$, and $u_2 = u_3 = 0$. The VIP's objective function is:

$$\pi_1(s_2, s_3) \;=\; [\,p_1^e - c_1 - c^u\,]\,Q_1^e + [\,w_2 - c^u\,]\,Q_2^e + [\,w_3 - c^u\,]\,Q_3^e - K(s_2, s_3)\,, \tag{6}$$

where $Q_i^e = \alpha_i - \beta\, p_i^e + \sum\limits_{j\neq i} \gamma_{ij}\, p_j^e$ is the demand for firm i's product evaluated at the equilibrium prices. Differentiating (6), using (5) and the envelope theorem, yields for $j = 2, 3$:

$$\begin{aligned}
\frac{\partial \pi_1}{\partial s_j} \;&=\; [\,p_1^e - c_1 - c^u\,]\left[\gamma_{12}\,\frac{\partial p_2^e}{\partial s_j} + \gamma_{13}\,\frac{\partial p_3^e}{\partial s_j}\right] + [\,w_2 - c^u\,]\left[-\beta\,\frac{\partial p_2^e}{\partial s_j} + \gamma_{23}\,\frac{\partial p_3^e}{\partial s_j}\right] \\
&\quad + \;[\,w_3 - c^u\,]\left[-\beta\,\frac{\partial p_3^e}{\partial s_j} + \gamma_{23}\,\frac{\partial p_2^e}{\partial s_j}\right] - \frac{\partial K(s_2, s_3)}{\partial s_j} \\
&=\; \frac{\beta}{J}\Big[\,(p_1^e - c_1 - c^u)\,\big(\gamma_{12}\,\rho_{2j} + \gamma_{13}\,\rho_{3j}\big) - (w_2 - c^u)\,\big(\beta\,\rho_{2j} - \gamma_{23}\,\rho_{3j}\big) \\
&\qquad - (w_3 - c^u)\,\big(\beta\,\rho_{3j} - \gamma_{23}\,\rho_{2j}\big)\,\Big] - \frac{\partial K(s_2, s_3)}{\partial s_j}\,.
\end{aligned} \tag{7}$$

Straightforward calculations, employing (1), provide:

$$\gamma_{12}\,\rho_{2j} + \gamma_{13}\,\rho_{3j} \;=\; \gamma_{1j}\left[\,4\,\beta^2 - \gamma_{1k}^2\,\right] + \gamma_{1k}\,[\,2\,\beta\,\gamma_{23} + \gamma_{12}\,\gamma_{13}\,] \;=\; 2\,\beta\,\rho_{1j}\,. \tag{8}$$

(7) and (8) imply:

$$\begin{aligned}
\frac{\partial \pi_1}{\partial s_j} \;&=\; \frac{\beta}{J}\,[\,(p_1^e - c_1 - c^u)\,2\,\beta\,\rho_{1j} - (w_2 - c^u)\,\big(\beta\,\rho_{2j} - \gamma_{23}\,\rho_{3j}\big) \\
&\qquad - (w_3 - c^u)\,\big(\beta\,\rho_{3j} - \gamma_{23}\,\rho_{2j}\big)\,] - \frac{\partial K(s_2, s_3)}{\partial s_j}\,.
\end{aligned} \tag{9}$$

Differentiating (9), using (5), provides, for $j,\, k = 2,\, 3$:

$$\frac{\partial^2 \pi_1}{\partial s_j\,\partial s_k} \;=\; \frac{\beta}{J}\left[\,2\,\beta\,\rho_{1j}\,\frac{\partial p_1^e}{\partial s_k}\right] - \frac{\partial^2 K(s_2, s_3)}{\partial s_2\,\partial s_3} \;=\; \frac{2\,\beta^3 \rho_{1j}\,\rho_{1k}}{J^2} - \frac{\partial^2 K(s_2, s_3)}{\partial s_2\,\partial s_3}\,. \tag{10}$$

(10) implies that the Hessian of $\pi_1(s_2, s_3)$ is:

$$\frac{2\,\beta^3}{J^2}\begin{bmatrix} \rho_{12}^2 & \rho_{12}\,\rho_{13} \\ \rho_{12}\,\rho_{13} & \rho_{13}^2 \end{bmatrix} - H_K\,, \tag{11}$$

where H_K is the Hessian of $K(s_2, s_3)$.

Proposition TA1. *Absent direct sabotage costs, $\pi_1(s_2, s_3)$ is a convex function.*

Proof. Ignoring the positive coefficient $\frac{2\beta^3}{J^2}$ and H_K, the diagonal elements of the Hessian in (11) are positive and the determinant is zero. Therefore, the Hessian is singular and positive semidefinite. ■

Proposition TA2. *Absent direct sabotage costs, if the VIP charges input prices equal to upstream marginal cost, then the VIP will foreclose both rivals.*

Proof. (9) implies that under the stated conditions, for $j = 1, 2$:

$$\frac{\partial \pi_1}{\partial s_j} = \frac{2\,\beta^2 \rho_{1j}}{J} \left[\, p_1^e - c_1 - c^u \,\right] > 0\,. \quad \blacksquare$$

Proposition TA3. *Suppose the VIP experiences no diseconomies of scope in sabotaging rivals (so $\frac{\partial^2 K(s_2,s_3)}{\partial s_2 \partial s_3} \leq 0$). Then the rate at which the VIP's profit increases with its sabotage of one rival increases with the level of sabotage directed at the other rival.*

Proof. (5) and (10) imply that under the stated conditions:

$$\frac{\partial^2 \pi_1}{\partial s_2\, \partial s_3} = \frac{2\,\beta^3 \rho_{12}\, \rho_{13}}{J} - \frac{\partial^2 K(s_2, s_3)}{\partial s_2\, \partial s_3} > 0\,. \quad \blacksquare$$

Assume now that $K(s_2, s_3)$ is sufficiently convex to ensure that $\pi_1(s_2, s_3)$ is a strictly concave function and the optimal sabotage choice is interior.

Proposition TA4. *Suppose firm 2 is the VIP's closest rival (so $\gamma_{12} > \gamma_{13}$) and the VIP's rivals pay the same input price. Then the VIP's sabotage of firm 2 exceeds its sabotage of firm 3, provided the marginal cost of sabotaging the closer rival never exceeds the marginal cost of sabotaging the other rival at equal levels of sabotage (so $\frac{\partial K(s_2,s_3)}{\partial s_2} \leq \frac{\partial K(s_2,s_3)}{\partial s_3}$ for all $s_2 = s_3$).*

Proof. Denote the common input price by $w \equiv w_2 = w_3$. Then (9) implies that under the stated conditions, for $j = 2, 3$:

$$\frac{\partial \pi_1}{\partial s_j} = \frac{\beta}{J}\left[(p_1^e - c_1 - c^u)\, 2\, \beta\, \rho_{1j} - (w - c^u)\, (\beta - \gamma_{23})\, \big(\rho_{jj} + \rho_{23}\big) - \frac{\partial K(\cdot)}{\partial s_j} \right]. \tag{12}$$

(12) implies that, for $s_2 = s_3$:

$$\frac{\partial \pi_1}{\partial s_2} - \frac{\partial \pi_1}{\partial s_3} \geq \frac{\beta}{J}\left[(p_1^e - c_1 - c^u)\, 2\, \beta\, (\rho_{12} - \rho_{13}) - (w - c^u)\, (\beta - \gamma_{23})\, (\rho_{22} - \rho_{33}) \right]. \tag{13}$$

From (1):

$$\rho_{12} - \rho_{13} = 2\,\beta\,\gamma_{12} + \gamma_{13}\,\gamma_{23} - (2\,\beta\,\gamma_{13} + \gamma_{12}\,\gamma_{23}) = [\,2\,\beta - \gamma_{23}\,]\,[\,\gamma_{12} - \gamma_{13}\,] > 0\,; \text{ and}$$

$$\rho_{22} - \rho_{33} = 4\,\beta^2 - \gamma_{13}^2 - \big(4\,\beta^2 - \gamma_{12}^2\big) = \gamma_{12}^2 - \gamma_{13}^2 > 0\,. \tag{14}$$

The remainder of the proof follows from applying the second-order Taylor series, as in the proof of Proposition 4 in the paper. ■

Proposition TA5. *The rate at which the VIP's profit increases with increased sabotage of either rival increases as: (i) the demand intercept of any firm increases; or (ii) the downstream production cost of either rival increases.*

Proof. For $j = 2, 3$ and $i = 1, 2, 3$:

$$\frac{\partial^2 \pi_1}{\partial s_j\, \partial \theta_i} = \frac{2\,\beta^2 \rho_{1j}}{J}\, \frac{\partial p_1^e}{\partial \theta_i} = \frac{2\,\beta^2 \rho_{1j}\, \rho_{1i}}{J^2} > 0\,. \tag{15}$$

The proposition follows from (15) because, from (1):

$$\frac{\partial \theta_i}{\partial \alpha_i} = \frac{\partial z_i}{\partial \alpha_i} = \frac{1}{\beta} > 0 \quad \text{and} \quad \frac{\partial \theta_i}{\partial c_i} = \frac{\partial z_i}{\partial c_i} = 1\,. \quad ■$$