DETAILED PROOF OF PROPOSITION 4 IN "PRIVATELY-NEGOTIATED INPUT PRICES"

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The setting with multiple entrants is one in which there is no retail price regulation. In addition, the incumbent faces two entrants, entrant A and entrant B. The firms produce a homogeneous retail product. Each unit of the retail product is produced by combining one unit of the downstream input and one unit of the upstream input. Each firm has a constant unit cost of producing the inputs. Furthermore, for simplicity, the two entrants have the same production costs. The entrants' upstream and downstream unit costs are denoted by c_u^E and c_d^E, respectively. The incumbent's upstream and downstream unit costs are denoted by c_u^I and c_d^I, respectively. The incumbent is the least-cost supplier of the upstream input (so $c_u^I < c_u^E$), and the entrants are the least-cost suppliers of the downstream input (so $c_d^E < c_d^I$). When each firm uses its own inputs exclusively, the incumbent is the least cost supplier (i.e., $c_u^I + c_d^I < c_u^E + c_d^E$), as in the benchmark setting.

An entrant can produce the retail product by combining its downstream input either with its own input or with the incumbent's input. If entrant i buys the input from the incumbent, it pays the incumbent unit price w^i, for i = A, B. The incumbent negotiates these prices sequentially, first with entrant A and then with entrant B. The input price paid by entrant i cannot be linked explicitly to the input price paid by entrant j, where $j \neq i$ and i, j = A, B. The input prices are determined by Nash bargaining.

The timing in this setting with multiple entrants is as follows. First, the incumbent makes a binding commitment as to whether it will continue to serve retail customers (as it has historically) or it will provide wholesale services exclusively. Second, the incumbent engages in

Nash bargaining with entrant A. If the bargaining is successful, the two parties agree on input price w^A. Third, the incumbent bargains with entrant B. If this Nash bargaining is successful, the two parties agree on input price w^B. If an entrant fails to negotiate an input price with the incumbent, that entrant will compete for retail customers using exclusively its own inputs. Fourth, after all negotiations have been completed, the firms choose their retail prices simultaneously and independently. Because the firms' products are homogeneous, consumers purchase (at most one unit of) the product from the firm that offers the lowest price. For simplicity, all consumers are assumed to purchase the product from entrant A as long as it is one of the firms that sets the lowest retail price (and this price does not exceed v). Finally, if an entrant has chosen to buy the input from the incumbent, the entrant procures the quantity of the upstream input required to meet the equilibrium demand for its retail product. The incumbent's market participation decision, the negotiated input prices, and the entrants' decisions to make or buy the upstream input are all known publicly.

The solution to this problem is determined via backward induction, determining the outcome of the bargaining between the incumbent and entrant B first, taking as given the outcome of the bargaining between the incumbent and entrant A. Consider first the case where the incumbent decides to provide wholesale services exclusively. In this case, if both entrants successfully negotiate an input price with the incumbent, the equilibrium retail price will be the higher of the unit costs of the two entrants, $\max\{w^A + c_d^E, w^B + c_d^E\}$. Entrant B's profit function when it buys the input from the incumbent is $\Pi^B = [\max\{w^A + c_d^E, w^B + c_d^E\} - (w^B + c_d^E)]Q^B$, where Q^B is the equilibrium demand for entrant B's retail product when both entrants successfully negotiate an input price with the incumbent. In equilibrium, entrant B will either serve all retail customers or it will not produce at all.

If the incumbent and entrant B cannot agree on an input price, entrant B produces the upstream input itself. In this case, entrant B's (stand-alone) unit cost of producing the retail product is $c_u^E + c_d^E$ and the equilibrium retail price will be $\max\{w^A + c_d^E, c_u^E + c_d^E\}$ when entrant A has successfully negotiated input price w^A with the incumbent. Entrant B's profit will be $\overline{\Pi}^B = [\max\{w^A + c_d^E, c_u^E + c_d^E\} - (c_u^E + c_d^E)]\overline{Q}^B$, where $\overline{Q}^B$ is entrant B's retail sales when it does not buy the upstream input from the incumbent. This profit will be entrant B's threat point in its negotiations with the incumbent, following a successful negotiation of input price w^A for entrant A.

The incumbent's profit following successful negotiation with both entrants is $\Pi^I = w^A Q^A + w^B Q^B - c_u^I[Q^A + Q^B]$, where Q^A is the entrant A's retail sales when it buys the upstream input from the incumbent and entrant B buys the input from the incumbent at input price w^B.[1] If the negotiation between the incumbent and entrant A is successful but the incumbent and entrant B do not agree on an input price, the incumbent's profit will be $\overline{\Pi}^I = [w^A - c_u^I]\tilde{Q}^A$, where $\tilde{Q}^A$ is the level of entrant A's retail sales when the incumbent and entrant A successfully negotiate an input price, but the incumbent and entrant B fail to do so. This profit is the incumbent's threat level when it negotiates with entrant B, following successful negotiation with entrant A.

The input price, w^B, that arises from Nash bargaining in the second stage is the input price that maximizes the following function:

$$Z_B = [\Pi^B - \overline{\Pi}^B][\Pi^I - \overline{\Pi}^I]\ . \qquad \text{(E1)}$$

[1] This is the maximum profit the incumbent can secure by negotiating with the entrants. If this profit is less than the profit the incumbent can secure by refusing to negotiate with the entrants, the incumbent will not agree to negotiate. The incumbent always prefers to negotiate with both entrants than with just one entrant because the ability to negotiate with the second entrant should negotiations with the first entrant fail allows the incumbent to secure a larger portion of the available surplus in its negotiations with the first entrant.

Z_B is not differentiable, but can be analyzed graphically. Figure 1 below plots the difference between the profit entrant B secures when an input price is negotiated successfully and its corresponding profit absent such successful negotiation as a function of w^B. To derive Figure 1, observe that:

$$\Pi^B - \overline{\Pi}^B = [\max\{w^A + c_d^E,\ w^B + c_d^E\} - (w^B + c_d^E)]\, Q^B$$
$$- [\max\{w^A + c_d^E,\ c_u^E + c_d^E\} - (c_u^E + c_d^E)]\ \overline{Q}^B \ . \qquad \text{(E2)}$$

To explore how $\overline{\Pi}^B$ (entrant B's profit if its negotiations with the incumbent fail) varies with w^A (the input price negotiated by entrant A), first suppose w^A is less than the entrants' upstream unit cost, c_u^E. In this case, entrant B cannot compete profitably against entrant A in the retail market when entrant B produces the input itself because entrant A's cost of producing the retail product is less than entrant B's stand-alone cost. Consequently, if the input price negotiations between entrant B and the incumbent fail, entrant B will not serve any retail customers, i.e., $\overline{Q}^B = 0$. In this case, entrant A will serve all retail customers, so $\tilde{Q}^A = N$. Substituting these values into expression (E2) provides:

$$\Pi^B - \overline{\Pi}^B = [\max\{w^A + c_d^E,\ w^B + c_d^E\} - (w^B + c_d^E)]\ Q^B \quad \text{when } w^A \le c_u^E . \qquad \text{(E3)}$$

Entrant B can profitably serve all retail customers at price $w^A + c_d^E$ if it can secure a lower input price than entrant A has secured. Therefore, $Q^B = N$ if $w^B < w^A$, and so it follows from expression (E3) that:

$$\Pi^B - \overline{\Pi}^B = [w^A + c_d^E - (w^B + c_d^E)]\, N = [w^A - w^B]\, N \quad \text{when } w^B < w^A \le c_u^E . \qquad \text{(E4)}$$

If entrant B cannot secure a lower input price than entrant A has secured, entrant B cannot compete profitably against entrant A in the retail market, i.e., $Q^B = 0$ for $w^B \ge w^A$. The equilibrium retail price in this case will be $w^B + c_d^E$, and entrant A will serve all retail customers. Consequently, from expression (E3):

$$\Pi^B - \overline{\Pi}^B = 0 \quad \text{when} \quad w^B \geq w^A. \tag{E5}$$

Expressions (E4) and (E5) are reflected in Figure 1.

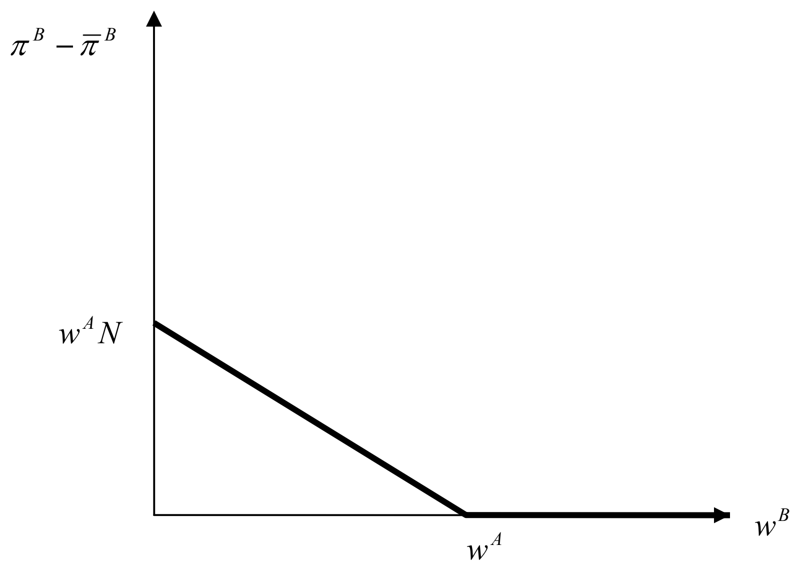

Figure 1

The corresponding difference between the incumbent's profit when it successfully negotiates an input price with entrant B and its threat level of profit is:

$$\Pi^I - \overline{\Pi}^I = w^A Q^A + w^B Q^B - c_u^I [Q^A + Q^B] - [w^A \tilde{Q}^A - c_u^I \tilde{Q}^A]. \tag{E6}$$

If entrant A is able to secure an input price below its upstream unit cost of production (i.e., if $w^A \leq c_u^E$), entrant A will serve all retail customers if the negotiations between the incumbent and entrant B fail. Consequently, $\tilde{Q}^A = N$ in this case.

Expression (E6) simplifies according to which entrant secures the lower input price. If entrant B secures an input price below entrant A's input price (i.e., if $w^B < w^A$), entrant B will serve all retail customers. Consequently, $Q^A = 0$ and $Q^B = N$ in this case. Substituting these values into expression (E6) provides:

$$\Pi^I - \overline{\Pi}^I = [w^B - w^A]\,N < 0 \qquad \text{when } w^B < w^A . \tag{E7}$$

If entrant B cannot secure a lower input price than entrant A has secured, then entrant A will serve all retail customers, and so $Q^B = 0$ and $Q^A = N$. Consequently, from expression (E6):

$$\Pi^I - \overline{\Pi}^I = 0 \qquad \text{when } w^B \geq w^A . \tag{E8}$$

Expressions (E7) and (E8) are reflected in Figure 2.

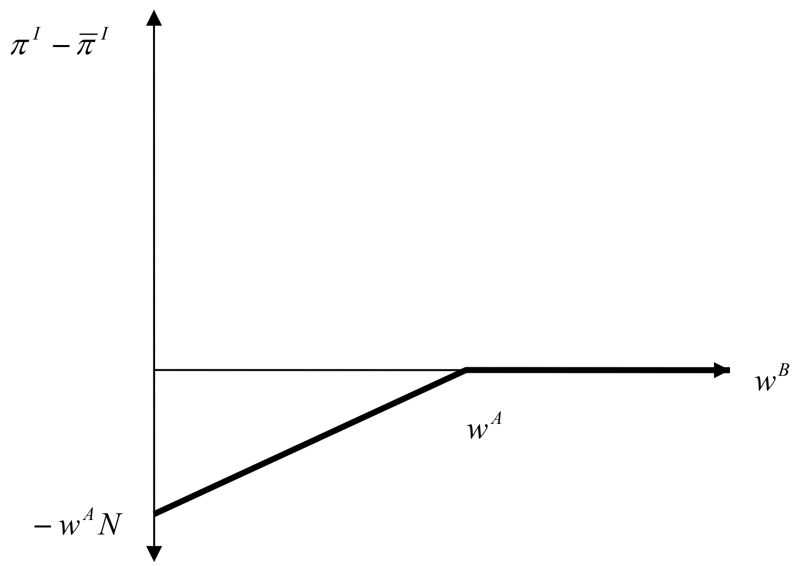

Figure 2

Equilibrium outcomes when $w^A \le c_u^E$ can be determined by comparing Figures 1 and 2. If entrant B secures a lower input price than entrant A has secured (so $w^B < w^A$), entrant B will serve all retail customers in equilibrium, and the incumbent's profit will be $\Pi^I = [w^B - c_u^I] N$. When $w^B < w^A$, this profit is lower than the profit the incumbent would secure if it did not negotiate an input price with entrant B, and so entrant A served all retail customers. Therefore, the incumbent will not agree on an input price for entrant B that is below w^A. Consequently, entrant B's unit production cost will exceed entrant A's unit production cost, whether entrant B buys the upstream input from the incumbent or employs its own upstream input. Therefore, entrant B will not be able to operate profitably in the retail market when $w^A \le c_u^E$, and so will be indifferent among all input prices that are at least as high as w^A.

In summary, if entrant A secures an input price below c_u^E, the incumbent and entrant B will subsequently negotiate an input price that is at least as high as w^A. Formally:

$$w^B \ge w^A \quad \text{when} \quad w^A \le c_u^E \;. \tag{E9}$$

As noted, when expression (E9) holds, entrant A will serve all retail customers (so $Q^A = N$ and $Q^B = 0$) by setting a price equal to entrant B's unit cost of production. Thus, if entrant B and the incumbent negotiate input price w^B, the equilibrium retail price in this case will be:

$$p = w^B + c_d^E \;. \tag{E10}$$

Because entrant B is indifferent among all input prices that are at least as high as w^A when $w^A \le c_u^E$, Nash bargaining could produce multiple values for w^B. To account for these multiple values, let $x \ge 0$ denote the difference between w^B and w^A. Formally:

$$w^B = w^A + x \,. \tag{E11}$$

Expressions (E10) and (E11) imply that when $w^A \le c_u^E$, the profits of the three firms will be:

$$\Pi^A = [w^B + c_d^E - (w^A + c_d^E)] \, N = [w^B - w^A] \, N = xN \,; \tag{E12}$$

$$\Pi^I = [w^A - c_u^I] N \,; \text{ and} \tag{E13}$$

$$\Pi^B = 0 \,. \tag{E14}$$

If the negotiated input price for entrant A exceeds c_u^E and the negotiations between entrant B and the incumbent fail, entrant B can profitably drive entrant A from the retail market when entrant A has committed to purchase the input from the incumbent. This is because entrant B's stand-alone cost will be lower than entrant A's unit cost under these circumstances. Consequently, from expression (E2):

$$\begin{aligned} \Pi^B - \overline{\Pi}^B &= [\max\{w^A + c_d^E,\, w^B + c_d^E\} - (w^B + c_d^E)] \, Q^B \\ &\qquad - [w^A + c_d^E - (c_u^E + c_d^E)] \, N \\ &= [\max\{w^A + c_d^E, w^B + c_d^E\} - (w^B + c_d^E)] \, Q^B \\ &\qquad - [w^A - c_u^E] \, N \quad \text{when } w^A \ge c_u^E \,. \end{aligned} \tag{E15}$$

If entrant B secures an input price below w^A, it will be able to serve all retail customers profitably (so $Q^B = N$ and $Q^A = 0$). Consequently, from expression (E15):

$$\Pi^B - \overline{\Pi}^B = [w^A + c_d^E - (w^B + c_d^E)]N - [w^A - c_u^E]N$$

$$= [w^A - w^B]N - [w^A - c_u^E]N = [c_u^E - w^B]N \quad \text{when } w^B < w^A. \qquad \text{(E16)}$$

In contrast, if entrant B negotiates an input price above w^A, entrant A will serve all retail customers (and so $Q^A = N$ and $Q^B = 0$). Consequently, from expression (E15):

$$\Pi^B - \overline{\Pi}^B = -[w^A - c_u^E]N \qquad \text{when } w^B \geq w^A. \qquad \text{(E17)}$$

Expressions (E16) and (E17) are reflected in Figure 3.

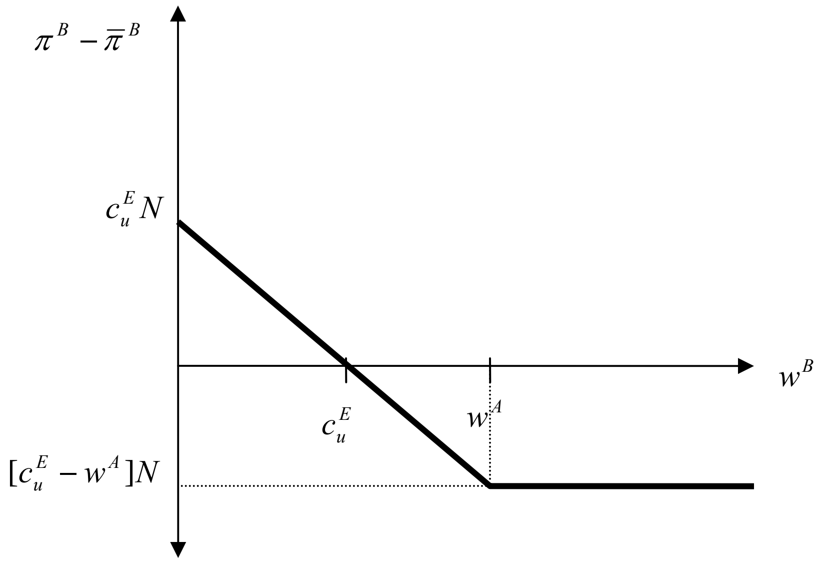

Figure 3

Now consider the corresponding payoffs for the incumbent. It follows from expression (E6) that:

$$\Pi^I - \overline{\Pi}^I = w^A Q^A + w^B Q^B - c_u^I [Q^A + Q^B] \quad \text{when} \quad w^A > c_u^E . \tag{E18}$$

If $w^A > w^B$, entrant B will serve all retail customers in equilibrium. Therefore, in this case, expression (E18) becomes:

$$\Pi^I - \overline{\Pi}^I = [w^B - c_u^I]\, N \quad \text{when} \quad w^A > \max\{c_u^E, w^B\} . \tag{E19}$$

In contrast, if $w^A \le w^B$, entrant A will serve all retail customers. Consequently, from expression (E18):

$$\Pi^I - \overline{\Pi}^I = [w^A - c_u^I]\, N \quad \text{when} \quad c_u^E < w^A \le w^B . \tag{E20}$$

Expressions (E19) and (E20) are reflected in Figure 4.

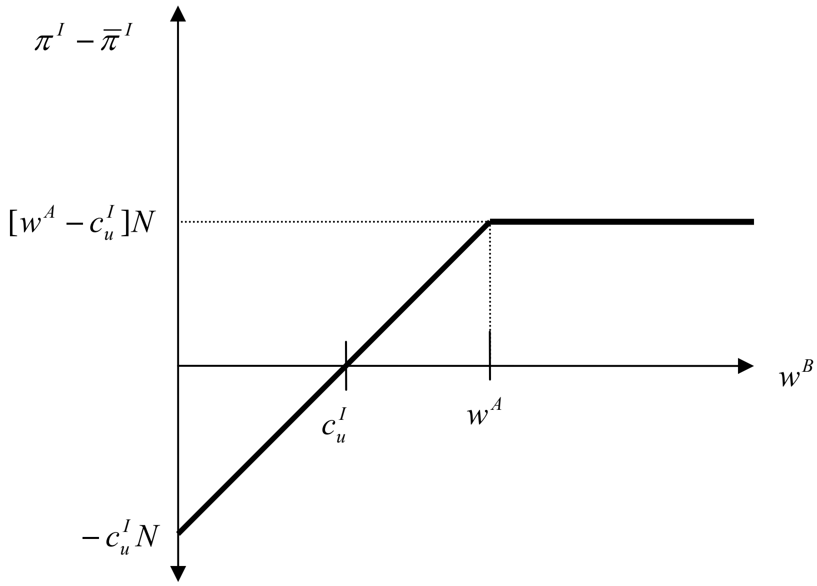

Figure 4

A comparison of Figures 3 and 4 reveals that Z_B is negative except on the interval $[c_u^I, c_u^E]$. Consequently, the value of w^B that maximizes expression (E1) in this interval will be the outcome of the Nash bargaining between the incumbent and entrant B when $w^A > c_u^E$. Substituting expressions (E16) and (E19) into expression (E1) provides:

$$\mathrm{Z_B} = [c_u^E - w^B][w^B - c_u^I]\, N^2 \quad \text{when} \quad w^A > c_u^E \,. \tag{E21}$$

Setting the derivative of expression (E21) equal to zero and solving for w^B provides:

$$\frac{\partial Z_B}{\partial w^B} = [-w^B - c_u^I + c_u^E - w^B]\, N^2 = 0$$

$$=> \qquad w^B = [c_u^I + c_u^E]/2 \,. \tag{E22}$$

Expression (E22) reveals that the negotiated input price for entrant B will be the average of its upstream unit cost and the incumbent's corresponding cost when $w^A > c_u^E$. Furthermore, because entrant B will serve all retail customers in equilibrium:

$$\Pi^B = [w^A + c_d^E - (w^B + c_d^E)]\ N = \left[w^A - (c_u^I + c_u^E)/2\right] N \ ; \tag{E23}$$

$$\Pi^I = [w^B - c_u^I]\ N = N\,[c_u^E - c_u^I]/2 \quad ; \text{ and} \tag{E24}$$

$$\Pi^A = 0. \tag{E25}$$

These second stage outcomes inform the determination of the first stage outcomes. The Nash bargaining between the incumbent and entrant A will produce an input price, w^A, that maximizes:

$$\mathrm{Z_A} = [\Pi^A - \overline{\Pi}^A][\Pi^I - \overline{\Pi}^I]\ . \tag{E26}$$

Z_A is not differentiable, but can be analyzed graphically. To do so, notice first that when the incumbent and entrant A agree on input price w^A, entrant A's profit, Π^A, will be:

$$\Pi^A = [\max\{w^A + c_d^E, w^B + c_d^E\} - (w^A + c_d^E)]\, Q^A. \tag{E27}$$

The threat level for entrant A, $\overline{\Pi}^A$, in its Nash bargaining with the incumbent is the level of profit entrant A can secure if it competes using its own upstream and downstream inputs:

$$\overline{\Pi}^A = [\max\{c_u^E + c_d^E,\ w^B + c_d^E\} - (c_u^E + c_d^E)]\ \overline{Q}^A. \tag{E28}$$

Recall from the discussion preceding expression (E9) that if the incumbent and entrant A agree on an input price that is less than entrant A's upstream cost (i.e., if $w^A \le c_u^E$), entrant A will serve all retail customers and entrant B will not operate in equilibrium. Substituting $Q^B = 0$, $Q^A = N$ and $w^B = w^A + x$ into expressions (E27) and (E28) provides:

$$\Pi^A - \overline{\Pi}^A = [w^B - w^A] N - [\max\{ c_u^E + c_d^B, \ w^B + c_d^E \} - (c_u^E + c_d^E)] \overline{Q}^A$$

$$= x N - [\max\{c_u^E + c_d^E, \ w^A + x + c_d^E\} - (c_u^E + c_d^E)] \overline{Q}^A \quad \text{for } w^A \leq c_u^E . \quad \text{(E29)}$$

If x, the difference between w^B and w^A, is sufficiently small compared to entrant A's upstream cost, entrant A will not be able to serve retail customers profitably if its negotiations with the incumbent fail. In particular, if $c_u^E > w^A + x$, the equilibrium retail price will be the stand alone cost of entrant A, $\overline{Q}^A$ will be zero, and $\tilde{Q}^B$, entrant B's retail output when it successfully negotiates an input price but entrant A fails to do so, will be N. In this case, expression (E29) becomes:

$$\Pi^A - \overline{\Pi}^A = x N \quad \text{when } c_u^E > w^A + x . \quad \text{(E30)}$$

In contrast, if x is sufficiently large, then even if the negotiations between the incumbent and entrant A fail, entrant B will not be able to operate profitably. In particular, if $c_u^E \leq w^A + x$, then $\overline{Q}^A = N$ and $\tilde{Q}^B = 0$. Consequently, from expression (E29):

$$\Pi^A - \overline{\Pi}^A = [w^B - w^A] N - [w^B - c_u^E] N$$

$$= [c_u^E - w^A] N \quad \text{when } c_u^E \leq w^A + x . \quad \text{(E31)}$$

Recall from the discussion leading up to expression (E25) that if entrant A and the incumbent agree on an input price that is higher than the entrants' upstream cost (i.e. if $w^A > c_u^E$), entrant A will not be able to serve retail customers profitably in equilibrium. Consequently, Q^A and $\overline{Q}^A$ both will be zero. Substituting $Q^A = 0$ and $\overline{Q}^A = 0$ into expressions (E27) and (E28) provides:

$$\Pi^A - \overline{\Pi}^A = 0 \quad \text{when} \quad w^A > c_u^E . \quad \text{(E32)}$$

Expressions (E30), (E31), and (E32) provide the relationship depicted in Figure 5.

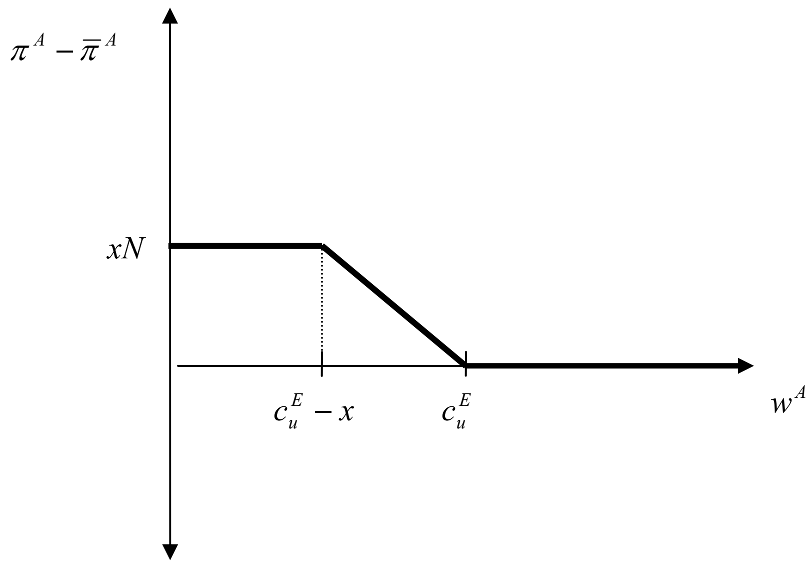

Figure 5

If the negotiations between the incumbent and entrant A fail, the incumbent is still able to negotiate with entrant B. The incumbent's threat level in its Nash bargaining with entrant A, $\overline{\Pi}^I$, is the profit the incumbent can secure from bargaining with entrant B:

$$\overline{\Pi}^I = [w^B - c_u^I]\,\tilde{Q}^B \,. \tag{E33}$$

Recall from the discussion leading up to expression (E14) that if the incumbent and entrant A agree on an input price below c_u^E, entrant B will not be able to produce profitably in equilibrium, and so entrant A will serve all retail customers. Consequently, the difference between the incumbent's profit and its threat level will be:

$$\Pi^I - \overline{\Pi}^I = [w^A - c_u^I]\,N - [w^B - c_u^I]\,\tilde{Q}^B \quad \text{when } w^A \leq c_u^E \,. \tag{E34}$$

$\tilde{Q}^B$ varies with x, the difference between w^B and w^A. Recall from the discussion preceding expression (E30) that entrant A will not be able to serve retail customers profitably if

x is sufficiently small (i.e., if $c_u^E > w^A + x$), and so $\overline{Q}^A = 0$ and $\tilde{Q}^B = N$. Consequently, from expression (E34):

$$\Pi^I - \overline{\Pi}^I = [w^A - c_u^I]\, N - [w^B - c_u^I]\, N$$

$$= [w^A - w^B]\, N = -x\, N \qquad \text{when } c_u^E > w^A + x. \tag{E35}$$

In contrast, recall from the discussion preceding expression (E31) that entrant B will not be able to serve retail customers profitably if x is sufficiently large (i.e., if $c_u^E \leq w^A + x$). Consequently, $\tilde{Q}^B = 0$, and expression (E34) reveals:

$$\Pi^I - \overline{\Pi}^I = [w^A - c_u^I]\, N \qquad \text{when } c_u^E \leq w^A + x. \tag{E36}$$

Recall from the discussion leading up to expression (E22) that if the incumbent and entrant A initially agree on an input price above c_u^E, entrant A will not be able to compete profitably against entrant B in equilibrium. Therefore, if $w^A > c_u^E$, it follows from expression (E22) that the incumbent and entrant B will negotiate input price $w^B = [c_u^I + c_u^E]/2$. Substituting this value for w^B into expression (E34) reveals:

$$\Pi^I - \overline{\Pi}^I = [w^A - c_u^I]N - [\frac{c_u^I + c_u^E}{2} - c_u^I]N$$

$$= [w^A - (\frac{c_u^I + c_u^E}{2})]N \qquad \text{when } w^A > c_u^E. \tag{E37}$$

Expressions (E35) through (E37) provide Figure 6:

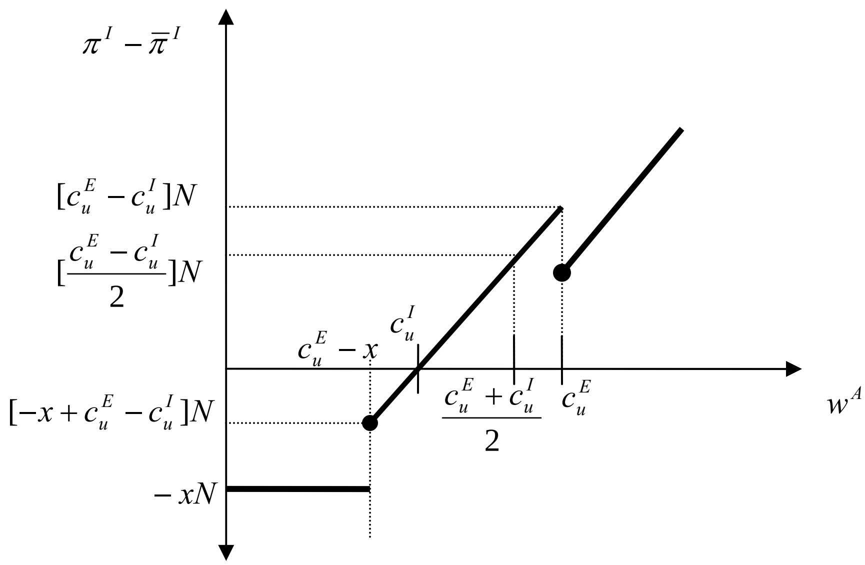

Figure 6

Figures 5 and 6 reveal that Z_A is strictly negative when $w^A < c_u^E - x$, and zero when $w^A \geq c_u^E$. Therefore, Nash bargaining will produce an input price in the interval $[c_u^E - x,\ c_u^E)$. The value of w^A that maximizes Z_A in this interval will vary with the magnitude of x. If x is sufficiently large compared to the incumbent's upstream cost advantage (i.e., if $x \geq [c_u^E - c_u^I]/2$), expression (E26) implies:

$$\text{Z}_\text{A} = [c_u^E - w^A][w^A - c_u^I]N^2 \quad \text{when} \quad x \geq [c_u^E - c_u^I]/2 \,. \tag{E38}$$

In this case, the negotiated input price is determined by:

$$\frac{\partial Z_A}{\partial w^A} = [-w^A - c_u^I + c_u^E - w^A]\ N^2 = 0$$

$$\Rightarrow \qquad w^A = \frac{c_u^I + c_u^E}{2} \,. \tag{E39}$$

If $x \geq [c_u^E - c_u^I]/2$ and so the input price for entrant A is as specified in expression (E39), $w^B = x + [c_u^I + c_u^E]/2$, entrant A will serve all retail customers at price $p = c_d^E + x + [c_u^I + c_u^E]/2$, and the profits of entrant A and the incumbent will be:

$$\Pi^A = xN \text{ ; and} \tag{E40}$$

$$\Pi^I = [c_u^E - c_u^I]N/2 \quad \text{when } x \geq [c_u^E - c_u^I]/2 . \tag{E41}$$

Entrant B will not agree to an input price below its stand-alone upstream unit cost of production. Therefore, the largest feasible value for x is $[c_u^E - c_u^I]/2$. At this value of x, the profits of the incumbent and entrant A are given by expressions (E40) and (E41), respectively.

If $x < [c_u^E - c_u^I]/2$, the value of w^A identified in expression (E39) will lie outside the interval $[c_u^E - x, c_u^E)$. Consequently, Z_A is negative at $w^A = [c_u^I + c_u^E]/2$. Z_A is decreasing in w^A at $w^A = c_u^E - x$, and so Nash bargaining will not produce an input price above $c_u^E - x$. Z_A is negative when $w^A < c_u^E - x$. Therefore, the negotiated input price for entrant A will be $w^A = c_u^E - x$ if $x < [c_u^E - c_u^I]/2$. From expression (E11), this input price for entrant A will lead to a negotiated input price of $w^B = c_u^E$ for entrant B. The corresponding equilibrium retail price will be $p = c_u^E + c_d^E$. Entrant B will earn zero profit under these circumstances, and the equilibrium profits of entrant A and the incumbent will be:

$$\Pi^A = xN \text{ ; and} \tag{E42}$$

$$\Pi^I = [c_u^E - c_u^I - x]N \quad \text{when } x < [c_u^E - c_u^I]/2 . \tag{E43}$$

The discussions leading to expressions (E40) and (E42) reveal that in equilibrium, entrant B is never able to secure an input price below w^A. Because entrant B does not serve any retail customers in equilibrium in this case, it will be indifferent among feasible values of x. Furthermore, because x cannot exceed $[c_u^E - c_u^I]/2$, the value of Π^I in expression (E43) will exceed the corresponding value in expression (E41).

Recall from the discussion following expression (E41) that the largest possible value of x is $[c_u^E - c_u^I]/2$. At this value of x, $w^B = c_u^E$ (since $w^B = x + [c_u^I + c_u^E]/2$) and $w^A = c_u^E - x$ (since $w^A = [c_u^I + c_u^E]/2$). Also recall from the discussion leading to expression (E42) that when $x < [c_u^E - c_u^I]/2$, $w^B = c_u^E$ and $w^A = c_u^E - x$. Therefore, in equilibrium, for every feasible value of x (i.e., for all $x \in [0, (c_u^E - c_u^I)/2]$), the incumbent will negotiate input price $w^B = c_u^E$ with entrant B and input price $w^A = c_u^E - x$, with entrant A. Furthermore, the equilibrium retail price will be $p = c_u^E + c_d^E$ for every x in the interval $[0, (c_u^E - c_u^I)/2]$. Expression (E43) reveals that the incumbent will prefer the smallest value of x that induces entrant A to negotiate with the incumbent.

It remains to consider the possibility that the incumbent might choose not to negotiate with the entrants. When the incumbent competes against the entrants in the retail market and all firms employ their own inputs, the equilibrium retail price will be $c_u^E + c_d^E$, the entrants' stand-alone unit cost. Due to its overall unit cost advantage, the incumbent will serve all retail customers and earn profit $\Pi^I = [c_u^E + c_d^E - c_u^I - c_d^I]N$. If $x > c_d^I - c_d^E$, this profit will exceed the level of profit identified in expression (E43), and so the incumbent will choose not to negotiate with the entrants. In this case, industry costs will not be minimized because the incumbent will serve retail customers even though its downstream unit cost exceeds the downstream unit cost of the entrants. However, if entrant A will negotiate an input price with the incumbent whenever doing so ensures entrant A at least the profit it can secure by refusing to negotiate with the incumbent,

x will be zero in equilibrium, the incumbent will negotiate with the entrants, and industry costs will be minimized.