Technical Appendix to Accompany

"Motivating Regulated Suppliers to Assess Alternative Technologies, Protocols, and Capital Structures"

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The purpose of the technical appendix is to provide additional characterization of the solution to [RP-I]$'$ when the density functions for S under the alternative capital structure are triangular.

The $F_L(S)$ distribution.

Consider the triangular density $f_L(S)$ with support on $\left[\,\underline{S}, \overline{S}\,\right]$. The peak of the density must be $\frac{2}{\overline{S}-\underline{S}}$ to ensure an area of 1 under the density ($\frac{h}{2}\left[\,\overline{S} - \underline{S}\,\right] = 1 \;\Leftrightarrow\; h = \frac{2}{\overline{S}-\underline{S}}$). We assume the $f_L(S)$ density is symmetric, so the value of S at which the density attains its peak is $S_L = \frac{1}{2}\left[\,\underline{S} + \overline{S}\,\right]$. The slope of the density on $[\,\underline{S}, S_L\,]$ is $\frac{\frac{2}{\overline{S}-\underline{S}}}{S_L - \underline{S}} = \frac{2}{[\overline{S}-\underline{S}][\frac{1}{2}(\underline{S}+\overline{S})-\underline{S}]} = \frac{4}{[\overline{S}-\underline{S}]^2}$. The slope of the density on $\left[\,S_L\,, \overline{S}\,\right]$ is $-\frac{4}{[\overline{S}-\underline{S}]^2}$. Consequently, since $\frac{2}{\overline{S}-\underline{S}} - \frac{4\,[\,S-S_L\,]}{[\overline{S}-\underline{S}]^2} = \frac{2\left[\overline{S}-\underline{S}\right]-4\,S+2\left[\underline{S}+\overline{S}\right]}{[\overline{S}-\underline{S}]^2} = \frac{4\left[\overline{S}-S\right]}{[\overline{S}-\underline{S}]^2}$:

$$f_L(S) \;=\; \begin{cases} \frac{4\,[\,S-\underline{S}\,]}{[\overline{S}-\underline{S}]^2} & \text{for } \; S \in [\,\underline{S}, S_L\,] \\ \frac{4\left[\,\overline{S}-S\,\right]}{[\overline{S}-\underline{S}]^2} & \text{for } \; S \in \left[\,S_L\,, \overline{S}\,\right]. \end{cases} \tag{1}$$

(1) implies:

$$F_L(S) \;=\; \frac{4}{\left[\,\overline{S}\,-\,\underline{S}\,\right]^2}\int_{\underline{S}}^{S}\left[\,\widetilde{S}\,-\,\underline{S}\,\right]d\,\widetilde{S} \;=\; \frac{4}{\left[\,\overline{S}\,-\,\underline{S}\,\right]^2}\left\{\frac{1}{2}\left[\,S^2-(\underline{S})^2\,\right]-\underline{S}\,[\,S-\underline{S}\,]\right\}$$

$$=\; \frac{4}{\left[\,\overline{S}\,-\,\underline{S}\,\right]^2}\left[\frac{1}{2}\right][\,S-\underline{S}\,]\,[\,S+\underline{S}-2\,\underline{S}\,] \;=\; \frac{2\,[\,S-\underline{S}\,]^2}{\left[\,\overline{S}\,-\,\underline{S}\,\right]^2} \quad \text{for } \; S \in [\,\underline{S}, S_L\,]. \tag{2}$$

An interior minimum requires $\frac{\partial^2 M}{\partial S^2} > 0 \;\Leftrightarrow\; p_L''(\cdot) = f_L'(\cdot) > 0$. Therefore, an interior minimum must occur on $[\,\underline{S}, S_L\,]$.

Case 1. $S_L - S_H \;\geq\; \frac{k}{\phi_L\,\phi_H}$ (as in Proposition 5 in the text).

Recall the regulator optimally sets $R = R_D$ in this case. Furthermore, since $\pi_i = R_D - K_0 + S_i$ when $R = R_D \in [\,K_0 - S_L + \frac{k}{\phi_L},\, K_0 - \underline{S}\,]$:

$$\pi_H + \frac{k}{\phi_H} \gtreqless 0 \;\Leftrightarrow\; R_D - K_0 + S_H \gtreqless -\frac{k}{\phi_H} \;\Leftrightarrow\; R_D \gtreqless K_0 - S_H - \frac{k}{\phi_H}\,.$$

Recall from the text that $\pi_0 = \max\left\{ 0,\, \pi_H + \frac{k}{\phi_H} \right\}$. Therefore:

$$\begin{aligned} M &= \phi_L \,[\, R_D - K_0 + S_L \,] + \phi_L \, p_L(R_D)\, D \\ &\quad + \begin{cases} 0 & \text{for } R_D \in [\, K_0 - S_L + \frac{k}{\phi_L},\, K_0 - S_H - \frac{k}{\phi_H} \,] \\ \phi_H \left[R_D - K_0 + S_H + \frac{k}{\phi_H} \right] & \text{for } R_D \in (\, K_0 - S_H - \frac{k}{\phi_H},\, K_0 - \underline{S} \,]\,. \end{cases} \end{aligned} \tag{3}$$

It is convenient to view the regulator as choosing $S = K_0 - R_D$. (3) can be written as:

$$\begin{aligned} M &= \phi_L \,[\, S_L - S \,] + \phi_L \, F_L(S)\, D \\ &\quad + \begin{cases} 0 & \text{for } -S \in [\, -S_L + \frac{k}{\phi_L},\, -S_H - \frac{k}{\phi_H} \,] \\ \phi_H \left[S_H - S + \frac{k}{\phi_H} \right] & \text{for } -S \in (\, -S_H - \frac{k}{\phi_H},\, -\underline{S} \,] \end{cases} \end{aligned}$$

or

$$\begin{aligned} M &= \phi_L \,[\, S_L - S \,] + \phi_L \, F_L(S)\, D \\ &\quad + \begin{cases} \phi_H \left[S_H - S + \frac{k}{\phi_H} \right] & \text{for } S \in [\, \underline{S},\, S_H + \frac{k}{\phi_H} \,) \\ 0 & \text{for } S \in [\, S_H + \frac{k}{\phi_H},\, S_L - \frac{k}{\phi_L} \,]\,. \end{cases} \end{aligned} \tag{4}$$

(4) implies:

$$\frac{\partial M}{\partial S} = \begin{cases} -1 + \phi_L \, f_L(S)\, D & \text{for } S \in (\, \underline{S},\, S_H + \frac{k}{\phi_H} \,) \\ -\phi_L + \phi_L \, f_L(S)\, D & \text{for } S \in (\, S_H + \frac{k}{\phi_H},\, S_L - \frac{k}{\phi_L} \,). \end{cases} \tag{5}$$

There are five candidates for the value of S that minimizes M.

<u>Case (1a)</u>. $S^* = \underline{S}$ because $\left.\frac{\partial M^+}{\partial S}\right|_{S = \underline{S}} \geq 0 \;\Leftrightarrow\; -1 + \phi_L \, f_L(\underline{S})\, D \,\geq\, 0.$

This case cannot arise because the triangular density ensures $f_L(\underline{S}) = 0$.

<u>Case (1b)</u>. $S^* \in \left(\underline{S},\, S_H + \frac{k}{\phi_H} \right)$ when $\left.\frac{\partial M^+}{\partial S}\right|_{S = \underline{S}} < 0$ and $\left.\frac{\partial M^-}{\partial S}\right|_{S = S_H + \frac{k}{\phi_H}} > 0.$

From (5), this case arises when:

$$-1 + \phi_L \, f_L(S_H + \frac{k}{\phi_H})\, D \;>\; 0 \;\Leftrightarrow\; \frac{1}{\phi_L \, D} \;<\; \frac{4 \left[S_H + \frac{k}{\phi_H} \,-\, \underline{S} \right]}{\left[\overline{S} \,-\, \underline{S} \right]^2}$$

$$\Leftrightarrow \quad \frac{\left[\,\overline{S} \,-\, \underline{S}\,\right]^2}{4\,\phi_L\,D} + \underline{S} \;<\; S_H + \frac{k}{\phi_H}\,. \tag{6}$$

In this case:

$$-\,1 + \phi_L\, f_L(S^*)\, D \;=\; 0 \quad\Leftrightarrow\quad \frac{4\,[\,S^* - \underline{S}\,]}{\left[\,\overline{S} \,-\, \underline{S}\,\right]^2} \;=\; \frac{1}{\phi_L\,D} \quad\Leftrightarrow\quad S^* \;=\; \underline{S} + \frac{\left[\,\overline{S} \,-\, \underline{S}\,\right]^2}{4\,\phi_L\,D}\,. \tag{7}$$

<u>Case (1c)</u>. $S^* \;=\; S_H + \frac{k}{\phi_H}$ when $\left.\frac{\partial M^-}{\partial S}\right|_{S\,=\,S_H + \frac{k}{\phi_H}} \le 0$ and $\left.\frac{\partial M^+}{\partial S}\right|_{S\,=\,S_H + \frac{k}{\phi_H}} \ge 0$.

From (5), this case arises when:

$$\frac{1}{D} \;\le\; f_L(S_H + \frac{k}{\phi_H}) \;\le\; \frac{1}{\phi_L\,D} \quad\Leftrightarrow\quad \frac{1}{D} \;\le\; \frac{4\left[\,S_H + \frac{k}{\phi_H} \,-\, \underline{S}\,\right]}{\left[\,\overline{S} \,-\, \underline{S}\,\right]^2} \;\le\; \frac{1}{\phi_L\,D}$$

$$\Leftrightarrow \quad \frac{\left[\,\overline{S} \,-\, \underline{S}\,\right]^2}{4\,D} + \underline{S} \;\le\; S_H + \frac{k}{\phi_H} \;\le\; \frac{\left[\,\overline{S} \,-\, \underline{S}\,\right]^2}{4\,\phi_L\,D} + \underline{S}\,. \tag{8}$$

<u>Case (1d)</u>. $S^* \in \left(S_H + \frac{k}{\phi_H}\,,\, S_L - \frac{k}{\phi_L}\right)$ when $\left.\frac{\partial M^+}{\partial S}\right|_{S\,=\,S_H + \frac{k}{\phi_H}} < 0$ and $\left.\frac{\partial M^-}{\partial S}\right|_{S\,=\,S_L - \frac{k}{\phi_L}} > 0$.

From (5), this case arises when:

$$f_L(S_H{+}\frac{k}{\phi_H}) \;\le\; \frac{1}{D} \;<\; f_L(S_L{-}\frac{k}{\phi_L}) \quad\Leftrightarrow\quad \frac{4\left[\,S_H + \frac{k}{\phi_H} \,-\, \underline{S}\,\right]}{\left[\,\overline{S} \,-\, \underline{S}\,\right]^2} \;\le\; \frac{1}{D} \;\le\; \frac{4\left[\,S_L - \frac{k}{\phi_L} \,-\, \underline{S}\,\right]}{\left[\,\overline{S} \,-\, \underline{S}\,\right]^2}$$

$$\Leftrightarrow \quad S_H + \frac{k}{\phi_H} - \underline{S} \;\le\; \frac{\left[\,\overline{S} - \underline{S}\,\right]^2}{4\,D} \;\le\; S_L - \frac{k}{\phi_L} - \underline{S}\,. \tag{9}$$

In this case:

$$-\,\phi_L + \phi_L\, f_L(S^*)\, D \;=\; 0 \quad\Leftrightarrow\quad \frac{4\,[\,S^* - \underline{S}\,]}{\left[\,\overline{S} \,-\, \underline{S}\,\right]^2} \;=\; \frac{1}{D} \quad\Leftrightarrow\quad S^* \;=\; \underline{S} + \frac{\left[\,\overline{S} \,-\, \underline{S}\,\right]^2}{4\,D}\,. \tag{10}$$

<u>Case (1e)</u>. $S^* \,=\, S_L - \frac{k}{\phi_L}$ when $\left.\frac{\partial M^-}{\partial S}\right|_{S\,=\,S_L - \frac{k}{\phi_L}} \le 0$.

From (5), this case arises when:

$$f_L(S_L - \frac{k}{\phi_L}) \le \frac{1}{D} \quad\Leftrightarrow\quad \frac{4\left[\,S_L - \frac{k}{\phi_L} - \underline{S}\,\right]}{\left[\,\overline{S} \,-\, \underline{S}\,\right]^2} \;\le\; \frac{1}{D} \quad\Leftrightarrow\quad S_L - \frac{k}{\phi_L} \;\le\; \underline{S} + \frac{\left[\,\overline{S} - \underline{S}\,\right]^2}{4\,D}\,. \tag{11}$$

We now calculate the minimized value of M in the four relevant cases.

Case (1b).

$$\begin{aligned}
M &= \phi_L \left[S_L - S + F_L(S)\, D \right] + \phi_H \left[S_H - S + \frac{k}{\phi_H} \right] \\
&= \phi_L\, S_L + \phi_H\, S_H + k - S + \phi_L\, F_L(S)\, D \\
&= \phi_L\, S_L + \phi_H\, S_H + k - \underline{S} - \frac{\left[\overline{S} - \underline{S} \right]^2}{4\, \phi_L\, D} + \phi_L\, D\, \frac{2 \left[\underline{S} + \frac{\left[\overline{S} - \underline{S} \right]^2}{4\, \phi_L\, D} - \underline{S} \right]^2}{\left[\overline{S} - \underline{S} \right]^2} \\
&= \phi_L\, S_L + \phi_H\, S_H + k - \underline{S} - \frac{\left[\overline{S} - \underline{S} \right]^2}{4\, \phi_L\, D} + \frac{2\, \phi_L\, D}{\left[\overline{S} - \underline{S} \right]^2}\, \frac{\left[\overline{S} - \underline{S} \right]^4}{16\, (\, \phi_L\, D\,)^2} \\
&= \phi_L\, S_L + \phi_H\, S_H + k - \underline{S} - \frac{\left[\overline{S} - \underline{S} \right]^2}{4\, \phi_L\, D} + \frac{\left[\overline{S} - \underline{S} \right]^2}{8\, \phi_L\, D} \\
&= \phi_L\, S_L + \phi_H\, S_H + k - \underline{S} - \frac{\left[\overline{S} - \underline{S} \right]^2}{8\, \phi_L\, D}\, . \qquad (12)
\end{aligned}$$

Case (1c).

$$\begin{aligned}
M &= \phi_L \left[S_L - S + F_L(S)\, D \right] \\
&= \phi_L \left[S_L - S_H - \frac{k}{\phi_H} + D\, \frac{2 \left[S_H + \frac{k}{\phi_H} - \underline{S} \right]^2}{\left[\overline{S} - \underline{S} \right]^2} \right] \\
&= \phi_L \left[S_L - S_H - \frac{k}{\phi_H} \right] + \frac{2\, \phi_L\, D \left[S_H + \frac{k}{\phi_H} - \underline{S} \right]^2}{\left[\overline{S} - \underline{S} \right]^2}\, . \qquad (13)
\end{aligned}$$

Case (1d).

$$\begin{aligned}
M &= \phi_L \left[S_L - S + F_L(S)\, D \right] \\
&= \phi_L\, S_L - \phi_L \left[\underline{S} + \frac{\left[\overline{S} - \underline{S} \right]^2}{4\, D} \right] + \phi_L\, D\, \frac{2 \left[\underline{S} + \frac{\left[\overline{S} - \underline{S} \right]^2}{4\, D} - \underline{S} \right]^2}{\left[\overline{S} - \underline{S} \right]^2} \\
&= \phi_L\, S_L - \phi_L\, \underline{S} - \frac{\phi_L \left[\overline{S} - \underline{S} \right]^2}{4\, D} + \frac{2\, \phi_L\, D}{\left[\overline{S} - \underline{S} \right]^2}\, \frac{\left[\overline{S} - \underline{S} \right]^4}{16\, D^2}
\end{aligned}$$

$$= \phi_L S_L - \phi_L \underline{S} - \frac{\phi_L \left[\overline{S} - \underline{S}\right]^2}{4D} + \frac{\phi_L \left[\overline{S} - \underline{S}\right]^2}{8D}$$

$$= \phi_L S_L - \phi_L \underline{S} - \frac{\phi_L \left[\overline{S} - \underline{S}\right]^2}{8D} = \phi_L \left[S_L - \underline{S} - \frac{\left(\overline{S} - \underline{S}\right)^2}{8D} \right]. \tag{14}$$

Case (1e).

$$M = \phi_L \left[S_L - S + F_L(S)\, D \right]$$

$$= \phi_L S_L - \phi_L \left[S_L - \frac{k}{\phi_L} \right] + \phi_L D \, \frac{2 \left[S_L - \frac{k}{\phi_L} - \underline{S} \right]^2}{\left[\overline{S} - \underline{S}\right]^2}$$

$$= k + \frac{2\, \phi_L D \left[S_L - \frac{k}{\phi_L} - \underline{S} \right]^2}{\left[\overline{S} - \underline{S}\right]^2}. \tag{15}$$

The $F_H(S)$ distribution.

We again consider a triangular density with support on $\left[\underline{S}, \overline{S}\right]$ and height $\frac{2}{\overline{S} - \underline{S}}$. S_C denotes the value of S at which the density reaches its peak. The slope of the density on $[\underline{S}, S_C]$ is $\frac{\frac{2}{\overline{S} - \underline{S}}}{S_C - \underline{S}} = \frac{2}{\left[\overline{S} - \underline{S}\right][S_C - \underline{S}]}$. The slope of the density on $\left[S_C, \overline{S}\right]$ is $-\frac{\frac{2}{\overline{S} - \underline{S}}}{\overline{S} - S_C} = -\frac{2}{\left[\overline{S} - \underline{S}\right]\left[\overline{S} - S_C\right]}$. Consequently, since $\frac{2}{\overline{S} - \underline{S}} - \frac{2\,[S - S_C]}{\left[\overline{S} - \underline{S}\right]\left[\overline{S} - S_C\right]} = \frac{2\left[\overline{S} - S\right]}{\left[\overline{S} - \underline{S}\right]\left[\overline{S} - S_C\right]}$:

$$f_H(S) = \begin{cases} \frac{2\,[S - \underline{S}]}{\left[\overline{S} - \underline{S}\right][S_C - \underline{S}]} & \text{for } S \in [\underline{S}, S_C] \\ \frac{2\left[\overline{S} - S\right]}{\left[\overline{S} - \underline{S}\right]\left[\overline{S} - S_C\right]} & \text{for } S \in \left[S_C, \overline{S}\right]. \end{cases} \tag{16}$$

(16) implies:

$$S_H = \int_{\underline{S}}^{\overline{S}} S\, f_H(S) = \frac{2}{\overline{S} - \underline{S}} \left\{ \frac{1}{S_C - \underline{S}} \int_{\underline{S}}^{S_C} S\, [S - \underline{S}]\, dS + \frac{1}{\overline{S} - S_C} \int_{S_C}^{\overline{S}} S \left[\overline{S} - S\right] dS \right\}$$

$$= \frac{2}{\left[\overline{S} - \underline{S}\right][S_C - \underline{S}]\left[\overline{S} - S_C\right]} \left\{ \left[\overline{S} - S_C\right] \int_{\underline{S}}^{S_C} \left[S^2 - S\,\underline{S}\right] dS + [S_C - \underline{S}] \int_{S_C}^{\overline{S}} \left[S\,\overline{S} - S^2\right] dS \right\}$$

$$
\begin{aligned}
&= \frac{2}{\left[\,\overline{S}-\underline{S}\,\right]\left[\,S_C-\underline{S}\,\right]\left[\,\overline{S}-S_C\,\right]}\Bigg\{\left[\,\overline{S}-S_C\,\right]\left[\frac{S^3}{3}-\frac{\underline{S}}{2}\,S^2\right]_{\underline{S}}^{S_C} \\
&\qquad\qquad + \left[\,S_C-\underline{S}\,\right]\left[\frac{\overline{S}}{2}\,S^2-\frac{S^3}{3}\right]_{S_C}^{\overline{S}}\Bigg\} \\
&= \frac{1}{3\left[\,\overline{S}-\underline{S}\,\right]\left[\,S_C-\underline{S}\,\right]\left[\,\overline{S}-S_C\,\right]}\Big\{\left[\,\overline{S}-S_C\,\right]\left[\,2\,S^3-3\,\underline{S}\;S^2\,\right]_{\underline{S}}^{S_C} \\
&\qquad\qquad + \left[\,S_C-\underline{S}\,\right]\left[\,3\,\overline{S}\,S^2-2\,S^3\,\right]_{S_C}^{\overline{S}}\Big\} \\
&= \frac{1}{3\left[\,\overline{S}-\underline{S}\,\right]\left[\,S_C-\underline{S}\,\right]\left[\,\overline{S}-S_C\,\right]}\Big\{\left[\,\overline{S}-S_C\,\right]\left[\,2\,(S_C)^3-3\,\underline{S}\;(S_C)^2-2\,(\underline{S})^3+3\,(\underline{S})^3\,\right] \\
&\qquad\qquad + \left[\,S_C-\underline{S}\,\right]\left[\,3\left(\overline{S}\right)^3-2\left(\overline{S}\right)^3-3\,\overline{S}\,(S_C)^2+2\,(S_C)^3\,\right]\Big\} \\
&= \frac{1}{3\left[\,\overline{S}-\underline{S}\,\right]\left[\,S_C-\underline{S}\,\right]\left[\,\overline{S}-S_C\,\right]}\Big\{\left[\,\overline{S}-S_C\,\right]\left[\,2\,(S_C)^3-3\,\underline{S}\;(S_C)^2+(\underline{S})^3\,\right] \\
&\qquad\qquad + \left[\,S_C-\underline{S}\,\right]\left[\left(\overline{S}\right)^3-3\,\overline{S}\,(S_C)^2+2\,(S_C)^3\right]\Big\} \\
&= \frac{1}{3\left[\,\overline{S}-\underline{S}\,\right]\left[\,S_C-\underline{S}\,\right]\left[\,\overline{S}-S_C\,\right]}\Big\{\,2\,(S_C)^3\left[\,\overline{S}-\underline{S}\,\right]+\left[\,\overline{S}-S_C\,\right]\left[\,(\underline{S})^3-3\,\underline{S}\;(S_C)^2\,\right] \\
&\qquad\qquad + \left[\,S_C-\underline{S}\,\right]\left[\left(\overline{S}\right)^3-3\,\overline{S}\,(S_C)^2\right]\Big\} \\
&= \frac{1}{3\left[\,\overline{S}-\underline{S}\,\right]\left[\,S_C-\underline{S}\,\right]\left[\,\overline{S}-S_C\,\right]}\Big\{\,2\,(S_C)^3\left[\,\overline{S}-\underline{S}\,\right] \\
&\qquad - 3\,(S_C)^2\left[\,\underline{S}\left(\,\overline{S}-S_C\,\right)+\overline{S}\,(\,S_C-\underline{S}\,)\right]+\;(\underline{S})^3\left[\,\overline{S}-S_C\,\right]+\left(\overline{S}\right)^3\left[\,S_C-\underline{S}\,\right]\Big\} \\
&= \frac{1}{3\left[\,\overline{S}-\underline{S}\,\right]\left[\,S_C-\underline{S}\,\right]\left[\,\overline{S}-S_C\,\right]}\Big\{\,2\,(S_C)^3\left[\,\overline{S}-\underline{S}\,\right]-3\,(S_C)^3\left[\,\overline{S}-\underline{S}\,\right] \\
&\qquad\qquad + \;(\underline{S})^3\left[\,\overline{S}-S_C\,\right]+\left(\overline{S}\right)^3\left[\,S_C-\underline{S}\,\right]\Big\} \\
&= \frac{-\;(S_C)^3\left[\,\overline{S}-\underline{S}\,\right]+(\underline{S})^3\left[\,\overline{S}-S_C\,\right]+\left(\overline{S}\right)^3\left[\,S_C-\underline{S}\,\right]}{3\left[\,\overline{S}-\underline{S}\,\right]\left[\,S_C-\underline{S}\,\right]\left[\,\overline{S}-S_C\,\right]} \\
&= \frac{\left(\overline{S}\right)^3\left[\,S_C-\underline{S}\,\right]+(\underline{S})^3\left[\,\overline{S}-\underline{S}-(S_C-\underline{S})\,\right]-\;(S_C)^3\left[\,\overline{S}-\underline{S}\,\right]}{3\left[\,\overline{S}-\underline{S}\,\right]\left[\,S_C-\underline{S}\,\right]\left[\,\overline{S}-S_C\,\right]} \\
&= \frac{\left[\left(\overline{S}\right)^3-(\underline{S})^3\right]\left[\,S_C-\underline{S}\,\right]-\left[\,(S_C)^3-(\underline{S})^3\,\right]\left[\,\overline{S}-\underline{S}\,\right]}{3\left[\,\overline{S}-\underline{S}\,\right]\left[\,S_C-\underline{S}\,\right]\left[\,\overline{S}-S_C\,\right]}\,. \qquad (17)
\end{aligned}
$$

Recall: $(x)^3 - (y)^3 = [x - y][x^2 + xy + y^2]$. Therefore, (17) can be written as:

$$S_H = \frac{[\overline{S} - \underline{S}]\left[(\overline{S})^2 + \underline{S}\,\overline{S} + (\underline{S})^2\right][S_C - \underline{S}] - [S_C - \underline{S}]\left[(S_C)^2 + \underline{S}\,S_C + (\underline{S})^2\right][\overline{S} - \underline{S}]}{3[\overline{S} - \underline{S}][S_C - \underline{S}][\overline{S} - S_C]}$$

$$= \frac{(\overline{S})^2 + \underline{S}\,\overline{S} + (\underline{S})^2 - \left[(S_C)^2 + \underline{S}\,S_C + (\underline{S})^2\right]}{3[\overline{S} - S_C]} = \frac{(\overline{S})^2 + \underline{S}\,\overline{S} - (S_C)^2 - \underline{S}\,S_C}{3[\overline{S} - S_C]}$$

$$= \frac{(\overline{S})^2 - (S_C)^2 + \underline{S}\,[\overline{S} - S_C]}{3[\overline{S} - S_C]} = \frac{1}{3}[\overline{S} + S_C + \underline{S}]. \tag{18}$$

(18) implies:

$$S_H \le 0 \quad \Leftrightarrow \quad S_C \le -[\underline{S} + \overline{S}] \quad \text{where } \underline{S} < 0 < \overline{S}.$$

If $S_C = -[\underline{S} + \overline{S}]$ so $S_H = 0$, then from (16):

$$f_H(S) = \begin{cases} -\frac{2[S - \underline{S}]}{[\overline{S} - \underline{S}][2\underline{S} + \overline{S}]} & \text{for } S \in [\underline{S}, -(\underline{S} + \overline{S})] \\ \frac{2[\overline{S} - S]}{[\overline{S} - \underline{S}][\underline{S} + 2\overline{S}]} & \text{for } S \in [-(\underline{S} + \overline{S}), \overline{S}]. \end{cases} \tag{19}$$

Note. We must have $\underline{S} < S_C = -[\underline{S} + \overline{S}] \Leftrightarrow 2\underline{S} + \overline{S} < 0$.

To verify that $G(S) \equiv F_H(S) - F_L(S) > 0$ for all $S \in (\underline{S}, \overline{S})$, define S_I to be the unique value of $S \in (\underline{S}, \overline{S})$ at which $f_H(S) = f_L(S)$. It is apparent that $G(\underline{S}) = G(\overline{S}) = 0$. Furthermore, $G'(S) = f_H(S) - f_L(S) \gtreqless 0$ as $S \lesseqgtr S_I$. Consequently, $G(\cdot)$ is 0 at $\underline{S}$, strictly increasing on $(\underline{S}, S_I)$, strictly decreasing on $(S_I, \overline{S})$, and 0 at $\overline{S}$. Therefore, $G(S) > 0$ for all $S \in (\underline{S}, \overline{S})$.

In summary, when $S_H = 0$ and $S_L - \frac{k}{\phi_L} \ge \frac{k}{\phi_H}$, (6), (8), (9), and (11) – (15) provide:

Case (1b). $\underline{S} + \frac{[\overline{S} - \underline{S}]^2}{4\phi_L D} < \frac{k}{\phi_H}$.

$$S^* = \underline{S} + \frac{[\overline{S} - \underline{S}]^2}{4\phi_L D}, \quad R^* = R_D^* = K_0 - \left(\underline{S} + \frac{[\overline{S} - \underline{S}]^2}{4\phi_L D}\right), \text{ and}$$

$$M = \phi_L S_L + k - \underline{S} - \frac{[\overline{S} - \underline{S}]^2}{8\phi_L D}. \tag{20}$$

Case (1c). $\underline{\underline{S} + \frac{\left[\overline{S} - \underline{S}\right]^2}{4\,D} \;\le\; \frac{k}{\phi_H} \;\le\; \underline{S} + \frac{\left[\overline{S} - \underline{S}\right]^2}{4\,\phi_L\,D}}$.

$$S^* \;=\; \frac{k}{\phi_H}, \quad R^* \;=\; R_D^* \;=\; K_0 - \frac{k}{\phi_H}, \text{ and}$$

$$M \;=\; \phi_L \left[S_L - \frac{k}{\phi_H} \right] + \frac{2\,\phi_L\,D \left[\frac{k}{\phi_H} - \underline{S} \right]^2}{\left[\overline{S} - \underline{S}\right]^2}. \tag{21}$$

Case (1d). $\underline{\frac{k}{\phi_H} \;\le\; \underline{S} + \frac{\left[\overline{S} - \underline{S}\right]^2}{4\,D} \;\le\; S_L - \frac{k}{\phi_L}}$.

$$S^* \;=\; \underline{S} + \frac{\left[\overline{S} - \underline{S}\right]^2}{4\,D}, \quad R^* \;=\; R_D^* \;=\; K_0 - \left(\underline{S} + \frac{\left[\overline{S} - \underline{S}\right]^2}{4\,D} \right), \text{ and}$$

$$M \;=\; \phi_L\,S_L - \phi_L\,\underline{S} - \frac{\phi_L\left[\overline{S} - \underline{S}\right]^2}{8\,D} \;=\; \phi_L \left[S_L - \underline{S} - \frac{\left(\overline{S} - \underline{S}\right)^2}{8\,D} \right]. \tag{22}$$

Case (1e). $\underline{S_L - \frac{k}{\phi_L} \;\le\; \underline{S} + \frac{\left[\overline{S} - \underline{S}\right]^2}{4\,D}}$.

$$S^* \;=\; S_L - \frac{k}{\phi_L}, \quad R^* \;=\; R_D^* \;=\; K_0 - \left(S_L - \frac{k}{\phi_L} \right), \text{ and}$$

$$M \;=\; k + \frac{2\,\phi_L\,D \left[S_L - \frac{k}{\phi_L} - \underline{S} \right]^2}{\left[\overline{S} - \underline{S}\right]^2}. \tag{23}$$

Also, $\pi_H \;=\; R - K_0$ and $\pi_L \;=\; S_L + \pi_H$ in each case.

Case 2. $S_L - S_H \;<\; \frac{k}{\phi_L\,\phi_H}$ (as in Proposition 6 in the text)

Denote by $\underline{R}$ the right-most intersection of the boundaries defined by the equalities in expressions (35) and (36) in the text. From Proposition 6 in the text, $\pi_0 \;=\; \pi_H + \frac{k}{\phi_H} \;=\; \pi_L - \frac{k}{\phi_L}$. Therefore, the regulator chooses $R_D \in [\,\underline{R},\, K_0 - \underline{S}\,)$ to minimize:

$$M \;=\; \phi_L\,\pi_L + \phi_H \left[\pi_L - \frac{k}{\phi_L} \right] + \phi_L\,p_L(R_D)\,D \;=\; \pi_L - \frac{\phi_H}{\phi_L}\,k + \phi_L\,p_L(R_D)\,D. \tag{24}$$

From equation (39) in the text:

$$\frac{\partial R}{\partial R_D} = 1 - \frac{\left[S_H + \frac{k}{\phi_H} - \left(S_L - \frac{k}{\phi_L} \right) \right] [\, p'_H(R_D) - p'_L(R_D)\,]}{[\, p_H(R_D) - p_L(R_D)\,]^2}$$

$$= 1 - \frac{[\, R - R_D\,]\,[\, p'_H(R_D) - p'_L(R_D)\,]}{p_H(R_D) - p_L(R_D)}\,. \tag{25}$$

From (24):

$$\frac{\partial M}{\partial R_D} = p_L(R_D) + p'_L(R_D)\,R_D - p'_L(R_D)\,R + [\,1 - p_L(R_D)\,]\,\frac{\partial R}{\partial R_D} + \phi_L\,p'_L(R_D)\,D$$

$$= p_L(R_D) - p'_L(R_D)\,[\,R - R_D\,] + [\,1 - p_L(R_D)\,]\,\frac{\partial R}{\partial R_D} + \phi_L\,p'_L(R_D)\,D\,. \tag{26}$$

(25), (26), and equation (39) in the text provide:

$$\frac{\partial M}{\partial R_D} = \phi_L\,p'_L(\cdot)\,D + p_L(\cdot) - p'_L(\cdot)\,[\,R - R_D\,] + [\,1 - p_L(\cdot)\,]\left[1 - \frac{[\,R - R_D\,]\,[\,p'_H(\cdot) - p'_L(\cdot)\,]}{p_H(\cdot) - p_L(\cdot)}\right]$$

$$= 1 + \phi_L\,p'_L(\cdot)\,D - p'_L(\cdot)\,[\,R - R_D\,] - [\,1 - p_L(\cdot)\,]\,\frac{[\,R - R_D\,]\,[\,p'_H(\cdot) - p'_L(\cdot)\,]}{p_H(\cdot) - p_L(\cdot)}$$

$$= 1 + \phi_L\,p'_L(\cdot)\,D - \left[p'_L(\cdot) + (\,1 - p_L(\cdot)\,)\,\frac{p'_H(\cdot) - p'_L(\cdot)}{p_H(\cdot) - p_L(\cdot)} \right] \frac{S_H - S_L + \frac{k}{\phi_L\,\phi_H}}{p_H(\cdot)\,-\,p_L(\cdot)}\,. \tag{27}$$

The value of R_D at which M is minimized will either be at the corner where $R_D = \underline{R}$ or at an interior value of R_D at which $\frac{\partial M}{\partial R_D} = 0$.