

MAP 2302  
FINAL EXAM

Please write your answers in full detail.

1. (6 points) Find the general solution of the equation

$$y'' + y = \tan x$$

2. (5 points) Find the general solution of

$$\frac{1}{x} \frac{dy}{dx} - \frac{2y}{x^2} = x \cos x, \quad x > 0.$$

3. (6 points)

The motion of a mass-spring system with damping is governed by the equations

$$y''(t) + by'(t) + 16y(t) = 0, \quad y(0) = 1, y'(0) = 0.$$

Find the equation of motion and sketch its graph for  $b = 0, 8,$  and  $10$ .

4. (5 points) If  $\mathcal{L}\{f(t)\}(s) = F(s)$  and  $\mathcal{L}\{g(t)\}(s) = G(s)$  express

$$\mathcal{L}\{e^t [f'(t) * (g(t-5)u(t-5))]\}(s)$$

in terms of  $F(s)$  and  $G(s)$ , explaining clearly which properties of the Laplace transform you use in each step. (Pay attention to the brackets!)

5. (6 points) Solve the initial value problem

$$y'' + 3y' + 2y = e^{-3t}u(t-2), \quad y(0) = 2, \quad y'(0) = 0.$$

6. (6 points) Find the first 4 nonzero terms of the Taylor series of the solution of the initial value problem

$$y'' + \sin y = 0, \quad y(0) = 1, y'(0) = -1.$$

You may leave your answer in terms of values of the sine function.

7. (6 points) Find a the power series expansion about  $x = 0$  for the general solution of the differential equation.

$$(x^2 + 1)y'' - xy' + y = 0.$$

Your answer should include a general formula for the coefficients.

#### FORMULAE

$$(1) \quad \mathcal{L}\{e^{at}f(t)\}(s) = F(s - a)$$

$$(2) \quad \mathcal{L}\{f'(t)\}(s) = sF(s) - f(0)$$

$$(3) \quad \mathcal{L}\{tf(t)\}(s) = -\frac{d}{ds}F(s)$$

$$(4) \quad \mathcal{L}\{f(t - a)u(t - a)\}(s) = e^{-as}F(s)$$

$$(5) \quad \mathcal{L}\{(f * g)(t)\}(s) = F(s)G(s)$$