> If you wish to use theorems from the text, make it clear which theorem you are using, by stating or describing it.

Standard group notation.
D_n, the group of symmetries of the regular n-gon under function composition.
S_n, the group of permutations of the set $\{1, 2, \ldots, n\}$ under function composition.
Z_n, the set of nonnegative integers less than n under addition mod n.
$U(n)$, the set of nonnegative integers less than n and coprime to n under multiplication mod n.
$\mathrm{G}L(n, F)$, the group of invertible $n \times n$ matrices with entries from a field F.

1. (a) (4 points) State Lagrange's Theorem.

   (b) (4 points) Prove that for every element x of a finite group G, we have $x^{|}G| = e$.

2. (a) (4 points) Give the definition of a normal subgroup of a group.
   (b) (4 points) Prove that the intersection of two normal subgroups is a normal subgroup.

3. Let $\phi : G \to H$ be a homomorphism of groups.
   (a) (4 points) Define the **kernel** of ϕ.
   (b) (4 points) Prove that ϕ is one-one if and only if its kernel is the trivial subgroup.

4. (8 points) Determine up to isomorphism all abelian groups of order 2000. You should list all such groups and your list should not contain two isomorphic groups.

5. (a) (4 points) Let R be a ring. What additional properties must R satisfy in order to be an integral domain.

   (b) (4 points) Give an example of a commutative ring with a multiplicative identity which is not an integral domain. (Justify your example.)