Assignment 1 Solutions

1.3.22 (First recall from Example 3 of section 1.2 that $\mathcal{F}(F_1, F_2)$ is a vector space over the field F_2.) Let E denote the subset of even functions in $\mathcal{F}(F_1, F_2)$, and by O we denote the subset of odd functions. By f_0 we denote the zero vector of $\mathcal{F}(F_1, F_2)$. For any $t \in F_1$, we have $f_0(-t) = 0 = f_0(t) = 0 = -f_0(-t)$. Thus f_0 is an element of both E and O.

Consider two even functions $f, g \in E$ and a scalar $c \in F_2$. Now for any $t \in F_1$, as f and g are even, we have: a) $(f+g)(-t) = f(-t) + g(-t) = f(t) + g(t) = (f+g)(t)$; and b) $(cf)(-t) = c(f(-t)) = cf(t) = (cf)(t)$. We proved that for any $f, g \in E$ and $c \in F_2$, the functions $f+g$ and cf are elements of E. We already saw that $f_0 \in E$. Now by Theorem 1.3, the subset E is a subspace.

Consider two odd functions $f, g \in O$ and a scalar $c \in F_2$. Now for any $t \in F_1$, as f and g are odd, we have: a) $(f+g)(-t) = f(-t) + g(-t) = -f(t) + (-g(t)) = -((f+g)(t))$; and b) $(cf)(-t) = c(f(-t)) = -cf(t) = -((cf)(t))$. We proved that for any $f, g \in O$ and $c \in F_2$, the functions $f+g$ and cf are elements of O. We already saw that $f_0 \in O$. Now by Theorem 1.3, the subset O is a subspace.

1.5.5 Assume that

$$a_0 + a_1 x + \ldots + a_n x^n = 0,$$

for some scalars $a_0, a_1, \ldots a_n$ in F.

Equating the coefficients of x^k on both sides of this equation for $k = 1, 2, \ldots, n$, we obtain $0 = a_0 = a_1 = \ldots = a_n$. Thus the zero vector of $\mathrm{P}_n(F)$ (i.e 0) has no non-trivial representation as a linear combination of elements of the set $\{1, x, \ldots x^n\}$. Therefore $\{1, x, \ldots x^n\}$ is linearly independent in $\mathrm{P}_n(F)$.