Please write your proofs carefully and in complete English sentences. If you wish to use theorems from the text, make it clear which theorem you are using, by stating or describing it. Be careful to avoid using mathematical notation incorrectly. When in doubt, use English. Anything that the grader cannot understand may receive no credit.

Name: ________________________________

1. (10 points) Let a be a group element of order n and suppose that m is a positive integer that is relatively prime to n. Show that there is an element y such that $y^m = a$.

   Hint: The phrase "relatively prime" should make you think immediately of a certain equation involving m and n.

2. (a) (5 points) Give the definition of the *center*, $Z(G)$ of a group G.

(b) (5 points) Prove that $Z(G)$ is a subgroup of G.

3. True or false? If you think the statement is true, give a proof. If false, provide a concrete counterexample.

   (a) (3 points) The groups $U(n)$ are all cyclic.

   (b) (4 points) If a, b are elements of a group such that $ab = ba$, then for all positive integers n, we have
   $$(ab)^n = a^n b^n.$$

   (c) (3 points) If a group G has at least one element of infinite order, then all of its nonidentity elements have infinite order.