Please write your proofs carefully and in complete English sentences. If you wish to use theorems from the text, make it clear which theorem you are using, by stating or describing it. Be careful to avoid using mathematical notation incorrectly. When in doubt, use English. Anything that the grader cannot understand may receive no credit.

Name: ______________________________

1. Let σ be the permutation
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 3 & 4 & 5 & 2 & 1 & 6 \end{pmatrix}.$$

   (a) (5 points) Write σ as a product of disjoint cycles.

   (b) (5 points) Write σ as a product of 2-cycles.

2. (10 points) Prove that $\mathrm{A}ut(Z_8) \cong U(8)$.

3. True or false? If you think the statement is true, give a proof. If false, provide a concrete counterexample.

   (a) (3 points) A finite group G has a subgroup of order d if and only if d divides the order of G.

   (b) (3 points) If a group G has an element of order p and one of order q, where p and q are distinct primes, then G has an element of order pq.

   (c) (4 points) Let $\mathrm{I}nn(G)$ be the group of inner automorphisms of the group G. Then $|\mathrm{I}nn(G)| = 1$ if and only if G is abelian.

4. (5 points) (Extra credit) Prove that if a subgroup H of S_n has odd order, then in fact $H \leq A_n$.