Please write your proofs carefully and in complete English sentences. If you wish to use theorems from the text, make it clear which theorem you are using, by stating or describing it. Be careful to avoid using mathematical notation incorrectly. When in doubt, use English. Anything that the grader cannot understand may receive no credit.

Name: ______________________________

1. Let $\phi : G \to H$ be a homomorphism of groups.
   (a) (5 points) Show that if K is a subgroup of H, then the set
   $$X = \{g \in G \mid \phi(g) \in K\}$$
   is a subgroup of G.

   (b) (5 points) Show that if K is a normal subgroup of H then X is a normal subgroup of G.

2. (10 points) Find, up to isomorphism, all Abelian groups of order 600.

3. True or false? If you think the statement is true, give a proof, stating any theorems you need. If false, provide a concrete counterexample.

   (a) (3 points) If a factor group G/N has an element of order n then G has an element of order n.

   (b) (3 points) If G is a group of permutations of a set S and $s \in S$, then the stabilizer $\mathrm{S}tab_G(s)$ is a normal subgroup of G.

   (c) (4 points) The number of elements of order 4 in $\mathbf{Z}_4 \oplus \mathbf{Z}_8$ is 12.

4. (5 points) (Extra credit) Determine the positive integers n for which every Abelian group of order n is cyclic.