

EXTENSIONS OF SIMPLE MODULES FOR $\mathrm{Sp}_4(2^n)$ AND $\mathrm{Suz}(2^m)$

PETER SIN

ABSTRACT

The group of extensions between any two irreducible 2-modular representations of the groups $\mathrm{Sp}_4(2^n)$ and $\mathrm{Suz}(2^m)$ is determined.

1. Introduction

In this note we shall determine the group of extensions between any two irreducible 2-modular representations of the groups $\mathrm{Sp}_4(2^n)$ and $\mathrm{Suz}(2^m)$. Our proof is inspired by the inductive argument of Alperin [1], where the problem is solved for $\mathrm{SL}_2(2^n)$. Another important source is the work [4] of Chastkofsky and Feit, valuable not only for some results on projective characters but also for the idea that these two families should be treated simultaneously. It is interesting to observe that with the notation of [4], the statement and the inductive step of [1] carry over to the present case without change.

We shall adopt the notation of [4]; thus, for each natural number m , we define

$$G := G_m := \begin{cases} \mathrm{Sp}_4(2^{m/2}), & \text{if } m \text{ is even,} \\ \mathrm{Suz}(2^m), & \text{if } m \text{ is odd.} \end{cases}$$

Let K be an algebraic closure of \mathbb{F}_2 . We denote by G_∞ the algebraic group $\mathrm{Sp}_4(K)$ and by σ its standard Frobenius map (squaring matrix entries). There is an endomorphism τ of G_∞ , arising from a symmetry of the Dynkin diagram, such that $\tau^2 = \sigma$. Then G_m is the subgroup of G_∞ consisting of all elements fixed by τ^m . Thus τ and σ are automorphisms of G_m .

Let V be the natural 4-dimensional module for G_∞ .

If M is any KG -module and θ is an automorphism of G , let M^θ denote the module obtained by letting $g \in G$ act on M^θ as g^θ acts on M .

Let $N := N_m := \mathbb{Z}/m\mathbb{Z}$. For each subset I of N , let $|I|$ be the cardinality of I .

For $i \in N$ we define $V_i = V^{i^t}$, and for $I \subseteq N$ we define $V_I = \bigotimes_{i \in I} V_i$, with V_\emptyset taken to be the trivial module K . Then, as is well known, the 2^m modules V_I form a complete set of nonisomorphic simple KG -modules. The module V_N is the Steinberg module. All of these modules are self-dual since $V_0 = V$ is, by definition, and we shall never make any distinction between V_I and V_I^* .

We may now state the Theorem.

THEOREM. *Suppose $m \geq 3$. Then for $I, J \subseteq N$ we have $\mathrm{Ext}_{KG}^1(V_I, V_J) = 0$ unless $I \cup J = (I \cap J) \cup \{r\}$, where neither r nor $r-1$ belongs to $I \cap J$, in which case $\mathrm{Ext}_{KG}^1(V_I, V_J) \cong K$.*

Received 13 November 1990.

1980 *Mathematics Subject Classification* 20G05, 20C20.

Supported by NSF grant number DMS 900-1273.

Bull. London Math. Soc. 24 (1992) 159–164

(For $m = 2$, $G_m = \text{Sp}_4(2) \cong S_6$ and the answer is well known, but different from the case $m \geq 3$.)

For $G = \text{Suz}(2^m)$ and $|I| = |J| = 1$, the result is due to R. P. Martineau [6].

REMARK. The simple (rational) G_∞ -modules are indexed by finite subsets of natural numbers in the obvious way, so that if m is greater than the largest number s in such a set I , then the restriction of V_I to G_m is the simple KG_m -module V_I . The G_∞ -extensions are then given by the same statement as for the finite groups, except that I and J are finite subsets of natural numbers and we require that the index r be different from 0 (that is, for $r = 0$ the G_∞ -extensions are trivial.) This can be seen as follows. It is known [2, Proposition 2.7] that the restriction of a nonsplit extension of two simple (rational) G_∞ -modules to $\text{Sp}_4(2^n)$ does not split as long as n is so large that the highest weights of the two modules are 2^n -restricted. On the other hand, it will be clear (see the remark after Corollary 6) that each of the nonsplit extensions in the statement of the Theorem is the restriction of a G_∞ -extension between the simple G_∞ -modules with the ‘same’ indices, as long as $r \neq 0$. It remains to show the triviality of $\text{Ext}_{G_\infty}^1(V_{I \cup \{0\}}, V_I)$ for $0 \notin I$. For $I = \emptyset$, we note that V_0 is a Weyl module, so $\text{Ext}_{G_\infty}^1(V_0, K) = 0$. Then, in general, we can see that all composition factors of $V_I \otimes V_I$ have the form V_S with $0 \notin S$. The result now follows from the long exact sequence, using the case $I = \emptyset$, Lemma 3 below and the result quoted above about restricting extensions to the finite groups.

2. Preparatory results

We shall denote by P_I the projective cover of V_I . In order to apply Alperin’s inductive argument, we need to study the projective module $P_{N \setminus \{i\}}$. The first lemma is a statement of the relevant parts of [4, Lemma 5.1].

- LEMMA 1. (a) $V_i \otimes V_N \cong P_{N \setminus \{i\}}$
 (b) $V_{\{i, i+1\}} \otimes V_N \cong \begin{cases} P_{N \setminus \{i, i+1\}}, & \text{if } m > 3, \\ P_{i+2} \oplus V_N \oplus V_N, & \text{if } m = 3. \end{cases}$

Proof. See [4, Lemma 5.1]; clearly $\{i, i+1\}$ is not circular if $m > 3$.

To continue our study of $P_{N \setminus \{i\}} \cong V_i \otimes V_N = (V_i \otimes V_i) \otimes V_{N \setminus \{i\}}$, we next determine the structure of $V_i \otimes V_i$. Our aim is to exploit filtrations of $P_{N \setminus \{i\}}$ induced by filtrations of $V_i \otimes V_i$.

We shall often use two natural exact sequences for vector spaces over perfect fields of characteristic two (see [7] or [8]) which in our special situation are

$$0 \rightarrow \Lambda^2(V_i) \rightarrow V_i \otimes V_i \rightarrow S^2(V_i) \rightarrow 0 \tag{2.1 a}$$

and

$$0 \rightarrow V_{i+2} \rightarrow S^2(V_i) \rightarrow \Lambda^2(V_i) \rightarrow 0. \tag{2.1 b}$$

(The surjective map in 2.1a sends bilinear forms to quadratic forms by ‘restriction to the diagonal’, and the surjection in 2.1b is the map sending a quadratic form to its associated (symplectic) bilinear form.)

- LEMMA 2. (a) $V_i \otimes V_i$ is uniserial, with composition factors (in descending order)
 $K, V_{i+1}, K, V_{i+2}, K, V_{i+1}, K.$
 (b) $\Lambda^2(V_i)$ is uniserial with series $K, V_{i+1}, K.$
 (c) $S^2(V_i)$ is uniserial with series $K, V_{i+1}, K, V_{i+2}.$

Proof. The composition factors of $V_i \otimes V_i$ are given by a Brauer character calculation [4, Theorem 3.4]. The exact sequences (2.1) then yield immediately the composition factors of $\Lambda^2(V_i)$ and $S^2(V_i)$. It is easy to see that $V_i \otimes V_i$ has a unique maximal submodule with quotient isomorphic to K , so the same is true of $\Lambda^2(V_i)$. Since V_i is self-dual, so is $\Lambda^2(V_i)$, and part (b) follows. Moreover, G does not lie in an orthogonal group of degree 4, so $S^2(V_i)^G = 0$. This proves (c). Part (a) now follows from the self-duality of $V_i \otimes V_i$.

We shall now consider some special cases of the Theorem.

LEMMA 3. If $|I \Delta J| > 1$ then $\text{Ext}_{KG}^1(V_i, V_j) = 0$.

Proof. Since $\text{Ext}_{KG}^1(V_i, V_j) \cong \text{Ext}_{KG}^1(V_{I \cup J}, V_{I \cap J})$, we may assume $I \supseteq J$. Suppose E is a nonsplit extension of V_i by V_j . We shall show that for $r \in N \setminus I$, the tensor product $E \otimes V_r$ is a nonsplit extension (of $V_{I \cup \{r\}}$ by $V_{J \cup \{r\}}$). Then since $(I \cup \{r\}) \setminus (J \cup \{r\}) = I \setminus J$, we may repeat the argument, leading eventually to a contradiction because V_N is projective.

It is easy to see from the composition factors of $V_i \otimes V_i$ that the composition factors V_T of $(V_r \otimes V_r) \otimes V_j$ satisfy $|T| \leq |J| + 1$. In particular, V_i is not one of them. Thus,

$$\text{Hom}_{KG}(E \otimes V_r, V_j \otimes V_r) \cong \text{Hom}_{KG}(E, (V_r \otimes V_r) \otimes V_j) = 0,$$

so the lemma is proved.

The next result is well known (see, for example, [3] and [5]).

LEMMA 4. $H^1(G, V_i) \cong K$.

At this point it will be very convenient to introduce the notation $U(L_1, L_2, \dots, L_t)$ to mean a uniserial module with composition factors (in descending order) isomorphic to L_1, L_2, \dots, L_t . We do not know at this point that such a module is unique up to isomorphism, so we shall usually say only ' M is a $U(L_1, \dots, L_t)$ ', but we shall permit ' $M \cong U(L_1, \dots, L_t)$ ' in the event that M is the unique uniserial module with this series up to isomorphism. For example, Lemma 2 states that $V_i \otimes V_i$ is a $U(K, V_{i+1}, K, V_{i+2}, K, V_{i+1}, K)$. Furthermore, we have $\text{Ext}_{KG}^1(V_i, V_j) = 0$ if $i \neq j$, by Lemma 3, and it is clear that $H^1(G, K) = 0$. Thus, by Lemma 4 and the long exact sequence of group cohomology, we see that $\Lambda^2(V_i) \cong U(K, V_{i+1}, K)$, $S^2(V_i) \cong U(K, V_{i+1}, K, V_{i+2})$ and $J(V_i \otimes V_i) / J^5(V_i \otimes V_i) \cong U(V_{i+1}, K, V_{i+2}, K)$. (As usual, we let $J^t M$ denote the t th radical of the module M , and $\text{soc}^t(M)$ its t th socle.)

The next lemma describes some more tensor products. Part (a) is not needed for the proof of the Theorem, but is included for completeness.

- LEMMA 5. (a) $\Lambda^2(V_i) \otimes V_i \cong V_i \oplus V_i \oplus V_{\{i, i+1\}}$.
 (b) $\Lambda^2(V_i) \otimes V_{i+1}$ is a $U(V_{i+1}, K, V_{i+2}, K, V_{i+3}, K, V_{i+2}, K, V_{i+1})$.
 (c) $\Lambda^2(V_i) \otimes V_j$ is a $U(V_j, V_{J \cup \{i+1\}}, V_j)$ if neither i nor $i+1$ belongs to J .

Proof. (a) From the structure of $V_i \otimes V_i$ given in Lemma 2, we have

$$\dim_K \text{Hom}_{KG}(\Lambda^2(V_i) \otimes V_i, V_i) = \dim_K \text{Hom}_{KG}(\Lambda^2(V_i), V_i \otimes V_i) = 2.$$

Since $\Lambda^2(V_i) \otimes V_i$ is self-dual, with composition factors V_i , V_i and $V_{\{i, i+1\}}$, part (a) is proved.

(b) The sequences (2.1) show that $\Lambda^2(V_i) \otimes V_{i+1}$ is both a quotient and a submodule of $V_i \otimes V_i \otimes V_{i+1}$, which in turn is a tensor factor of $V_i \otimes V_N \cong P_{N \setminus \{i\}}$. Therefore, the head and socle of $\Lambda^2(V_i) \otimes V_{i+1}$ are simple, and isomorphic to V_{i+1} by Lemma 2(b). The uniseriality now follows from the uniseriality of $V_{i+1} \otimes V_{i+1}$.

(c) Again, the module $\Lambda^2(V_i) \otimes V_j$ is both a submodule and a quotient of $P_{N \setminus \{i\}}$, so has simple socle and head, which in this case are isomorphic to V_j , by Lemma 2(b), which also gives the middle composition factor.

We single out an important consequence of Lemma 4(c).

COROLLARY 6. *If $I = J \cup \{r + 1\}$ where neither r nor $r + 1$ belongs to J , then $\text{Ext}_{KG}^1(V_I, V_J) \neq 0$.*

REMARK. Since G_∞ acts on the modules V_j and $\Lambda^2(V_j)$, Corollary 6 is valid with G replaced by G_∞ .

We are now in a position to prove our main technical result.

LEMMA 7.

$$\text{Ext}_{KG}^1(V_{N \setminus \{i\}}, V_J) \cong \begin{cases} K, & \text{if } J = N \setminus \{i, i + 1\}, \\ 0, & \text{otherwise.} \end{cases}$$

Proof. (I am indebted to Stephen Siegel for pointing out an oversight in an earlier version of this proof.) By Lemma 1, we have

$$\begin{aligned} \text{Ext}_{KG}^1(V_{N \setminus \{i\}}, V_J) &\cong \text{Hom}_{KG}(JP_{N \setminus \{i\}}, V_J) \\ &\cong \text{Hom}_{KG}(J(V_i \otimes V_N), V_J) \\ &\cong \text{Hom}_{KG}(J((V_i \otimes V_i) \otimes V_{N \setminus \{i\}}), V_J). \end{aligned}$$

Now the head of $V_i \otimes V_i$ is isomorphic to K , by Lemma 2, and the head of $(V_i \otimes V_i) \otimes V_{N \setminus \{i\}} \cong P_{N \setminus \{i\}}$ is of course simple and isomorphic to $V_{N \setminus \{i\}}$. It follows that

$$JP_{N \setminus \{i\}} \cong J(V_i \otimes V_i) \otimes V_{N \setminus \{i\}},$$

so the lemma is equivalent to the statement that $\text{Hom}_{KG}(J(V_i \otimes V_i) \otimes V_{N \setminus \{i\}}, V_J)$ is zero unless $J = N \setminus \{i, i + 1\}$, in which case it is one-dimensional.

By Lemma 2, $J(V_i \otimes V_i)$ is a $U(V_{i+1}, K, V_{i+2}, K, V_{i+1}, K)$.

First, consider the submodule $J^4(V_i \otimes V_i) \cong \Lambda^2(V_i)$. We have

$$J^4(V_i \otimes V_i) \otimes V_{N \setminus \{i\}} \cong \Lambda^2(V_i) \otimes V_{N \setminus \{i\}},$$

which, being a quotient of the projective indecomposable module $(V_i \otimes V_i) \otimes V_{N \setminus \{i\}}$, has a simple head. This implies that

$$J^5(V_i \otimes V_i) \otimes V_{N \setminus \{i\}} \subseteq J^2 P_{N \setminus \{i\}}.$$

The lemma will therefore be proved if we can show that $(J(V_i \otimes V_i) / J^5(V_i \otimes V_i)) \otimes V_{N \setminus \{i\}}$ has a simple head isomorphic to $V_{N \setminus \{i, i+1\}}$. By the remark following Lemma 4, we have

$$J(V_i \otimes V_i) / J^5(V_i \otimes V_i) \cong U(V_{i+1}, K, V_{i+2}, K).$$

From the uniqueness and Lemma 5(b), we see that this module is a quotient of $\Lambda^2(V_i) \otimes V_{i+1}$, which in turn is a quotient of $V_i \otimes V_i \otimes V_{i+1}$, by (2.1). Thus, $(J(V_i \otimes V_i) / J^5(V_i \otimes V_i)) \otimes V_{N \setminus \{i\}}$ is a quotient of $(V_i \otimes V_i \otimes V_{i+1}) \otimes V_{N \setminus \{i\}} = V_{\{i, i+1\}} \otimes V_N$.

If $m > 3$, Lemma 1(b) states that $V_{\{t, t+1\}} \otimes V_N \cong P_{N \setminus \{t, t+1\}}$, which of course has simple head $V_{N \setminus \{t, t+1\}}$, so we are done in this case.

If $m = 3$, Lemma 1(b) states that $V_{\{t, t+1\}} \otimes V_N \cong P_{t+2} \oplus V_N \oplus V_N$. Since V_t is not a composition factor of $J(V_t \otimes V_t)/J^5(V_t \otimes V_t)$, it is easy to see (for example, by iterating Lemma 2 at the level of composition factors) that $(J(V_t \otimes V_t)/J^5(V_t \otimes V_t)) \otimes V_{N \setminus \{t\}}$ does not have V_N as a composition factor, so must be a quotient of P_{t+2} , which again is the desired conclusion. The lemma is now proved.

LEMMA 8. *Let $J \subseteq N$, and suppose $i \notin J$.*

(a) *If $i + 1 \notin J$ then $\text{soc}^2(V_t \otimes V_t \otimes V_J)$ is a $U(V_{J \cup \{t+1\}}, V_J)$.*

(b) *If $i + 1 \in J$ then $\text{soc}^2(V_t \otimes V_t \otimes V_J)$ is a $U(V_{J \setminus \{t+1\}}, V_J)$.*

Proof. Since, by Lemma 7, the module

$$\text{soc}^2(P_{N \setminus \{t\}}) \cong \text{soc}^2(V_t \otimes V_N) \cong \text{soc}^2((V_t \otimes V_t \otimes V_J) \otimes V_{N \setminus (J \cup \{t\})})$$

is uniserial, and since $\text{soc}(V_t \otimes V_t \otimes V_J) \otimes V_{N \setminus (J \cup \{t\})} \cong V_J \otimes V_{N \setminus (J \cup \{t\})} = V_{N \setminus \{t\}}$ is simple, it follows that $\text{soc}^2(V_t \otimes V_t \otimes V_J)$ must also be uniserial. It remains to determine the simple module in the second layer. Since $\Lambda^2(V_t)$ is a submodule of $V_t \otimes V_t$, we have a submodule $\Lambda^2(V_t) \otimes V_J$ of $V_t \otimes V_t \otimes V_J$, so we need only identify a simple module in the second socle layer of $\Lambda^2(V_t) \otimes V_J$.

(a) If $i + 1 \notin J$, then the answer is provided by Lemma 5(c).

(b) If $i + 1 \in J$, then $\text{soc}^2(\Lambda^2(V_t) \otimes V_{t+1}) = U(K, V_{t+1})$ by Lemma 5(b) and Lemma 4, and since $\text{soc}(\Lambda^2(V_t) \otimes V_{t+1}) \otimes V_{J \setminus \{t+1\}} \cong V_J$ is simple, it follows that the simple module in the second socle layer of $\Lambda^2(V_t) \otimes V_J$ is $V_{J \setminus \{t\}}$ as claimed.

3. Proof of the Theorem

The argument we shall use is identical to the one used by Alperin for $\text{SL}_2(2^n)$.

First we note that the hypotheses and conclusions of the Theorem depend only on $I \cup J$ and $I \cap J$. Moreover,

$$\text{Ext}_{KG}^1(V_I, V_J) \cong \text{Ext}_{KG}^1(V_{I \cup J}, V_{I \cap J}).$$

Hence, replacing I and J with $I \cup J$ and $I \cap J$, we may assume without loss of generality that I contains J . If $I = N$, then V_N is projective, so the cohomology group in question vanishes, so the Theorem is true in this case. Hence, we may also assume that I is a proper subset of N . By Lemma 3, we may also assume that $|I| \leq |J| + 1$.

Suppose I and J satisfy the conditions of the Theorem. This means that $I = J \cup \{r\}$ for some element $r \in N$ such that neither r nor $r - 1$ lies in J . So Corollary 6 shows $\text{Ext}_{KG}^1(V_I, V_J) \neq 0$.

If $|I| = m - 1$, then $I = N \setminus \{i\}$ for some $i \in N$, so the Theorem is true in this case by Lemma 7. We may therefore assume from now on that $|I| < m - 1$.

To complete the proof of the Theorem, it is sufficient to show that there is some $s \in N \setminus I$ such that

$$\dim_K \text{Ext}_{KG}^1(V_I, V_J) \leq \dim_K \text{Ext}_{KG}^1(V_{I \cup \{s\}}, V_{J \cup \{s\}})$$

and such that I and J satisfy the conditions of the Theorem if and only if $I \cup \{s\}$ and $J \cup \{s\}$ do. Indeed, once we show all this, the Theorem will follow from an immediate downward induction, together with the nonvanishing we have already proved.

Suppose that $\dim_K \text{Ext}_{KG}^1(V_i, V_j) = d$, so that there exists a module X which has a unique maximal submodule Y such that $X/Y \cong V_i$ and Y is isomorphic to a direct sum of d copies of V_j . For $s \in N \setminus I$, define $X' = X \otimes V_s$, $Y' = Y \otimes V_s$, $I' = I \cup \{s\}$ and $J' = J \cup \{s\}$. Then $X'/Y' \cong V_i$ and Y' is isomorphic to a direct sum of d copies of V_j . If, for some choice of s , Y' is the unique maximal submodule of X' , we shall have $\dim_K \text{Ext}_{KG}^1(V_{I'}, V_{J'}) \geq d$, as desired. Now the statement that Y' has this property is equivalent to the statement that $\dim_K \text{Hom}_{KG}(X', V_{J'}) = 1$ if $I' = J'$, and is zero otherwise. Since X has Loewy length at most 2, we have

$$\begin{aligned} \text{Hom}_{KG}(X', V_{J'}) &\cong \text{Hom}_{KG}(X \otimes V_s, V_s \otimes V_j) \\ &\cong \text{Hom}_{KG}(X, (V_s \otimes V_s) \otimes V_j) \\ &= \text{Hom}_{KG}(X, \text{soc}^2((V_s \otimes V_s) \otimes V_j)). \end{aligned}$$

We shall consider the cases $I = J$ and $|I| = |J| + 1$ in turn, and show that we can find a suitable s in each case.

First, if $I = J$ then I and J do not satisfy the conditions of the Theorem, and neither do I' and J' , for any choice of $s \in N \setminus I$, since $I' = J'$. Also, Lemma 8 shows that $\dim_K \text{Hom}_{KG}(X, \text{soc}^2(V_s \otimes V_s \otimes V_j)) = 1$, which is what we wanted since $I = J$. Thus any $s \in N \setminus I$ will do in this case.

Finally, suppose $|I| = |J| + 1$, so that $I = J \cup \{k\}$ for some $k \in N \setminus J$. Since $I \neq J$, we must try to find $s \in N \setminus I$ such that $\text{Hom}_{KG}(X, \text{soc}^2(V_s \otimes V_s \otimes V_j)) = 0$ and such that I' and J' satisfy the conditions of the Theorem if and only if I and J do. By Lemma 8, there will be a nonzero homomorphism from X to $\text{soc}^2(V_s \otimes V_s \otimes V_j)$ if and only if $I = J \cup \{s+1\}$, that is $s+1 = k$ (we must be in case (a) because of the assumption that $J \subseteq I$). Thus, fulfilling the requirement of no homomorphisms is equivalent to choosing $s \neq k-1$.

If I and J do not satisfy the conditions of the Theorem, then $k-1 \in J$. Thus, any choice of $s \in N \setminus I$ will automatically be different from $k-1$. Since $I' = J' \cup \{k\}$ and $k-1 \in J$, we see also that I' and J' do not satisfy the conditions of the Theorem, and we are done.

If I and J do satisfy the conditions of the Theorem, then $k-1 \notin J$. Since $|I| < m-1$, we may choose $s \in N \setminus I$ different from $k-1$. For such a choice of s , the requirement of no homomorphisms is satisfied. Moreover, $k-1 \notin J'$ so that I' and J' also satisfy the conditions of the Theorem. This completes the proof of the Theorem.

References

1. J. L. ALPERIN, 'Projective modules for $SL(2, 2^n)$ ', *J. Pure Appl. Algebra* 15 (1979) 219–234.
2. H. H. ANDERSEN, 'Extensions of simple modules for finite Chevalley groups', *J. Algebra* 111 (1987) 388–403.
3. G. BELL, 'Cohomology of degree 1 and 2 of the Suzuki groups', *Pacific J. Math.* 75 (1978) 319–329.
4. J. CHASTKOFSKY and W. FEIT, 'On the projective characters in characteristic 2 of the groups $Suz(2^n)$ and $Sp_4(2^n)$ ', *Publ. Math. IHES* 51 (1980) 9–35.
5. E. CLINE, B. PARSHALL and L. SCOTT, 'Cohomology of finite groups of Lie type, I', *Publ. Math. IHES* 45 (1975) 169–191.
6. R. P. MARTINEAU, 'On 2-modular representations of the Suzuki groups', *Amer. J. Math.* 94 (1972) 55–72.
7. C.-H. SAH, 'Cohomology of split extensions II', *J. Algebra* 45 (1977) 17–68.
8. P. SIN and W. WILLEMS, 'On G -invariant quadratic forms in characteristic two', preprint, 1989.

Department of Mathematics
University of Florida
Gainesville, FL 32611
USA