

No Calculators. Answer the questions in the spaces provided on the question sheets. **Please write your answers in full detail.** If you run out of room for an answer, continue on the back of the page.

Name: _____

1. (8 points) Find the inverse Laplace transform of

$$\frac{7s^2 - 41s + 84}{(s - 1)(s^2 - 4s + 13)}.$$

2. (8 points) Solve the initial value problem $y'' + 4y = g(t)$, $y(0) = 0$, $y'(0) = 0$, where

$$g(t) = \begin{cases} \sin t, & \text{if } 0 \leq t < 2\pi \\ 0, & \text{if } 2\pi \leq t. \end{cases}$$

3. (8 points) Find the first four nonzero terms of the power series expansion about $x = 0$ for the general solution of the initial value problem.

$$(x+1)y'' - y = 0, \quad y(0) = 0, \quad y'(0) = 1.$$

Formulae

$$\mathcal{L}\{\sin bt\}(s) = \frac{b}{s^2 + b^2} \quad (1)$$

$$\mathcal{L}\{e^{at}f(t)\}(s) = F(s - a) \quad (2)$$

$$\mathcal{L}\{f(t - a)u(t - a)\}(s) = e^{-as}F(s) \quad (3)$$

$$\mathcal{L}\{g(t)u(t - a)\}(s) = e^{-as}\mathcal{L}\{g(t + a)\}(s) \quad (4)$$

$$\mathcal{L}\{f'(t)\}(s) = sF(s) - f(0) \quad (5)$$

$$\mathcal{L}\{tf(t)\}(s) = -\frac{d}{ds}F(s) \quad (6)$$

$$\mathcal{L}\{(f * g)(t)\}(s) = F(s)G(s) \quad (7)$$