Write your proofs using *complete English sentences* as well as mathematical formulae.

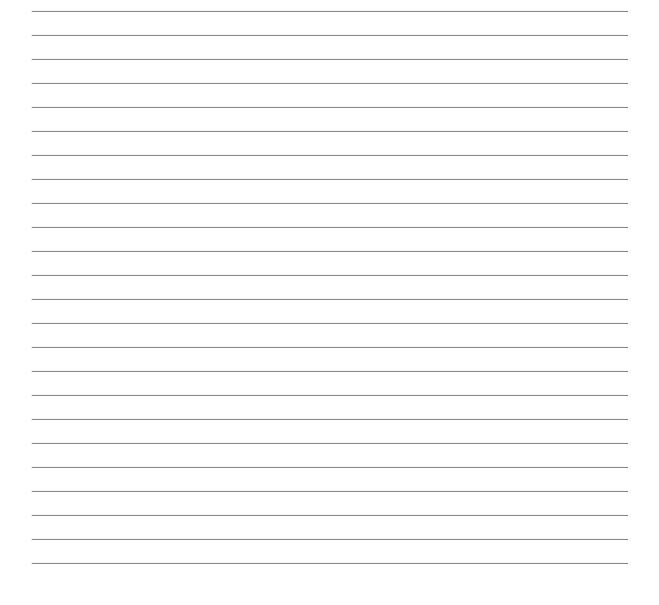
Work which the grader cannot follow may receive a grade of zero. In this exam F denotes a field and \mathbb{R} denotes the field of real numbers.

Name: _____

1. (8 points) In $M_{2\times 2}(\mathbb{R})$, consider the set

$$X = \{ \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 2 & 0 \\ 4 & -2 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \}$$

of four matrices. Find a subset of X which is a basis for the span of X.



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Final exam

- 2. Let the linear map $T: P_2(\mathbb{R}) \to P_2(\mathbb{R})$ be given by T(f(x)) = f(x) + f'(x) + f''(x).
 - (a) (4 points) By computing with the matrix of T or otherwise, prove that T is invertible.
 - (b) (4 points) Compute $T^{-1}(x^2 x + 2)$.

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3. (8 points) Determine whether the matrix

$$A = \begin{pmatrix} -1 & -2 & 2 & 2\\ 0 & 1 & -2 & -2\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1 \end{pmatrix} \in M_{4 \times 4}(\mathbb{R})$$

is diagonalizable or not. If so, find a basis of \mathbb{R}^4 consisting of eigenvectors of A.

- 4. Give a proof or a counterexample for each of the following statements.
 - (a) (2 points) The determinant is a linear map from $M_{n \times n}(F)$ to F.
 - (b) (2 points) The union of two subspaces of a vector space is a subspace.
 - (c) (2 points) The sum of two surjective linear maps is surjective.
 - (d) (2 points) If $T: V \to W$ and $U: W \to Z$ are linear maps such that T is injective and U is surjective, then UT is either injective or surjective.



5. (8 points) Use the Gram-Schmidt procedure to find an orthonomal basis of the subspace of \mathbb{R}^4 spanned by (1, -1, 0, 0), (0, 0, 1, -1), (0, 1, -1, 0).