MAP 2302
Final Exam

Please write your answers in full detail.

1. (6 points) Let $f(t)$ be the periodic function of period 2 such that

$$f(t) = \begin{cases} t, & 0 \le t < 1 \\ 2-t, & 1 \le t < 2 \end{cases}.$$

Graph this function and its windowed version.
Find the Laplace transform of $f(t)$.

2. (5 points) Find the general solution of the equation

$$y''' + 2y'' - 9y' - 18y = -18x^2 - 18x + 22.$$

3. (6 points) Given that $f(x) = e^x$ is a solution of the equation

$$xy'' - (x+1)y' + y = 0, \quad x > 0,$$

find a second linearly independent solution.

4. (5 points) If $\mathcal{L}\{f(t)\}(s) = F(s)$ and $\mathcal{L}\{g(t)\}(s) = G(s)$ express

$$\mathcal{L}\{e^t\,[f'(t) * (g(t-5)u(t-5))]\}(s)$$

in terms of $F(s)$ and $G(s)$, explaining clearly which properties of the Laplace transform you use in each step. (Pay attention to the brackets!)

5. (6 points) Find the inverse Laplace transform of

$$F(s) = \frac{7s^2 + 23s + 30}{(s-2)(s^2+2s+5)}.$$

6. (6 points) Solve the initial value problem

$$y'' - y = 4\delta(t-2) + t^2; \quad y(0) = 0, y'(0) = 2,$$

where $\delta(t)$ is the Dirac delta function.

7. (6 points) Solve the initial value problem

$$y'' + 3y' + 2y = e^{-3t}u(t-2), \qquad y(0) = 2, \quad y'(0) = 0.$$

Formulae

$$\mathcal{L}\{e^{at}f(t)\}(s) = F(s-a) \tag{1}$$

$$\mathcal{L}\{f'(t)\}(s) = sF(s) - f(0) \tag{2}$$

$$\mathcal{L}\{tf(t)\}(s) = -\frac{d}{ds}F(s) \tag{3}$$

$$\mathcal{L}\{f(t-a)u(t-a)\}(s) = e^{-as}F(s) \tag{4}$$

$$\mathcal{L}\{(f*g)(t)\}(s) = F(s)G(s) \tag{5}$$