Please write your proofs carefully and in complete English sentences. If you wish to use theorems from the text, make it clear which theorem you are using, by stating or describing it. Be careful to avoid using mathematical notation incorrectly. When in doubt, use English. Anything that the grader cannot understand may receive no credit.

Name: _____

1. (6 points) Let G be a group and K a normal subgroup. If H is a subgroup of G, prove that the set $\overline{H} := \{hK \mid h \in H\}$ is a subgroup of G/K.

2. (6 points) State Lagrange's Theorem. Suppose a group G has order pq, where p and q are prime. Show that every proper subgroup of G must be cyclic.

3. True or false? If you think the statement is true, give a proof, stating any theorems you need. If false, provide a concrete counterexample.

   (a) (3 points) If N is a normal subgroup of H and H is a normal subgroup of G then N is a normal subgroup of G.

   (b) (3 points) If G is a group of permutations of a set S and $s \in S$, then the stabilizer $Stab_G(s)$ is a normal subgroup of G.

   (c) (3 points) If G is an abelian group of odd order, then the map $\sigma : G \to G$, $\sigma(g) = g^2$ is an isomorphism of G with itself.