Write your proofs using *complete English sentences* as well as mathematical formulae.
Bonus points may be awarded for particularly well-argued proofs.
In this exam $\mathbb{R}$ denotes the field of real numbers.

Name: ______________________________

1. Give examples of the following.
   (a) (2 points) A linearly dependent set of vectors in $\mathbb{R}^3$ such that at least one of the vectors is not a linear combination of the others.

   (b) (3 points) A vector space that has no finite generating set. Justify your answer.

2. (5 points) Determine whether the following set is a basis of $P_2(\mathbb{R})$.

$$\{1 + 2x + x^2, 3 + x^2, x + x^2\}$$

3. Let V be the space of 2×2 matrices with real entries, and let W be the subset defined by
$$W = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_{2\times 2}(\mathbb{R}) : a + b + c = 0\}.$$

   (a) (4 points) Prove that W is a subspace of V.

   (b) (4 points) Find a basis of W.

   (c) (2 points) What is the dimension of W ?