

1.2.7. The meaning of the equation $f = g$ is that $f(x) = g(x)$ for all $x \in S$. In this problem, we have $S = \{0, 1\}$. We compute:

$$\begin{aligned}f(0) &= 2 \cdot 0 + 1 = 1, \\g(0) &= 1 + 4 \cdot 0 - 2 \cdot 0^2 = 1, \\f(1) &= 2 \cdot 1 + 1 = 3, \\g(1) &= 1 + 4 \cdot 1 - 2 \cdot 1^2 = 3.\end{aligned}\tag{1}$$

Thus, $f(0) = g(0)$ and $f(1) = g(1)$, so $f = g$.

By definition of addition of functions, $f + g$ is the function which sends an element $x \in S$ to the real number $f(x) + g(x)$. We compute:

$$\begin{aligned}(f + g)(0) &= f(0) + g(0) = 1 + 1 = 2, \\(f + g)(1) &= f(1) + g(1) = 3 + 3 = 6, \\h(0) &= 5^0 + 1 = 2, \\h(1) &= 5^1 + 1 = 6.\end{aligned}\tag{2}$$

Thus, $(f + g)(0) = h(0)$ and $(f + g)(1) = h(1)$, so $f + g = h$.

1.3.5. For any $n \times n$ matrix M , let $M_{i,j}$ denote the entry of M in row i and column j , for $1 \leq i, j \leq n$. Then $(M^t)_{ij} = M_{ji}$ by definition, so it is immediate from the definition of symmetric matrices that M is symmetric if and only if $M_{ij} = M_{ji}$ for all $1 \leq i, j \leq n$. Now let A be an arbitrary square matrix and let n be the number of rows (or columns) of A . Then by the definition of matrix addition and the definition of A^t , we have for all $1 \leq i, j \leq n$,

$$(A + A^t)_{ij} = A_{ij} + (A^t)_{ij} = A_{ij} + A_{ji} = A_{ji} + A_{ij} = A_{ji} + (A^t)_{ji} = (A + A^t)_{ji}.\tag{3}$$

Thus, $A + A^t$ is symmetric.