MAP 2302 Elementary Ordinary Differential Equations

Homework Assignment # 2, Due January 20, 2016

1) Show that the functions

$$y(x) = \begin{cases} 0, & \text{for } x \leq c \\ \frac{(x-c)^2}{4} & \text{for } x > c, \end{cases}$$

for any fixed real number c, are solutions of the differential equation $y' = y^{1/2}$ on the entire real axis. (Do not forget to show that the function is differentiable everywhere, particularly at $x = c$.) Do the same for the function $y(x) \equiv 0$. Of these functions, which are solutions of the initial value problem $y' = y^{1/2}$, $y(0) = 0$ on the real axis?

2) Solve $4xy + (x^2 + 1)y' = 0$ with $y(1) = 2$. What is the interval of definition of the solution?

3) Solve the differential equation $dQ/dt = k(a - Q)(b - Q)$ with constants $k, a, b > 0$, which arises in the description of chemical reactions. What will be the asymptotic value of Q as $t \to \infty$? (In other words, which value does $Q(t)$ approach as $t \to \infty$?)

4) Compute the function $y(x)$ whose graph has a slope at any point $(x, y(x))$ of the curve equal to $y^3(x)$ and which passes through the point $(0, 1)$.

5) The *Bernoulli equation* is the differential equation $y' + a(x)y = b(x)y^n$, with $n \neq 0, 1$. Show that the transformation $w = y^{1-n}$ reduces the Bernoulli equation to the following linear ODE: $w' + (1 - n)a(x)w = (1 - n)b(x)$, which you can solve. By inverting the transformation $w = y^{1-n}$, you can therefore solve the original Bernoulli equation. Carry out this procedure for the following ODE: $y' - \frac{1}{x}y = -\frac{1}{2y}$. (In other words, solve this equation.)

Also from the text:

Section 1.1: Problems 19 (ignore graphing), 25

Section 2.2: Odd problems 1–29