1) Show that the functions

\[ y(x) = \begin{cases} 
0, & \text{for } x \leq c \\
\frac{(x-c)^2}{4}, & \text{for } x > c,
\end{cases} \]

for any fixed real number \( c \), are solutions of the differential equation \( y' = y^{1/2} \) on the entire real axis. (Do not forget to show that the function is differentiable everywhere, particularly at \( x = c \).) Do the same for the function \( y(x) \equiv 0 \). Of these functions, which are solutions of the initial value problem \( y' = y^{1/2} \), \( y(0) = 0 \) on the real axis?

2) Solve \( 4xy + (x^2 + 1)y' = 0 \) with \( y(1) = 2 \). What is the interval of definition of the solution?

3) Solve the differential equation \( \frac{dQ}{dt} = k(a - Q)(b - Q) \) with constants \( k, a, b > 0 \), which arises in the description of chemical reactions. What will be the asymptotic value of \( Q \) as \( t \to \infty \)? (In other words, which value does \( Q(t) \) approach as \( t \to \infty \)?)

4) Compute the function \( y(x) \) whose graph has a slope at any point \( (x, y(x)) \) of the curve equal to \( y^3(x) \) and which passes through the point \( (0, 1) \).

5) The Bernoulli equation is the differential equation \( y' + a(x)y = b(x)y^n \), with \( n \neq 0,1 \). Show that the transformation \( w = y^{1-n} \) reduces the Bernoulli equation to the following linear ODE: \( w' + (1-n)a(x)w = (1-n)b(x) \), which you can solve. By inverting the transformation \( w = y^{1-n} \), you can therefore solve the original Bernoulli equation. Carry out this procedure for the following ODE: \( y' - \frac{1}{2}y = -\frac{1}{2y} \). (In other words, solve this equation.)

Also from the text:

Section 1.1: Problems 19 (ignore graphing), 25
Section 2.2: Odd problems 1–29