Homework Assignment \#6, Due February 24, 2016

1) Given that $y_{1}=x^{-1 / 2} \sin x$ is a solution of the equation

$$
x^{2} y^{\prime \prime}+x y^{\prime}+\left(x^{2}-\frac{1}{4}\right) y=0
$$

find the general solution of this equation.
2) Is $\left\{e^{x}, 3 e^{x}\right\}$ a fundamental set of solutions for the equation $y^{\prime \prime}-2 y^{\prime}+y=0$ ? What about $\left\{e^{x}, 3 e^{-x}\right\}$ ? And what about $\left\{e^{x}, x e^{x}\right\}$ ? Finally, what about $\left\{e^{x}, 3 x e^{x}\right\}$ ? Justify your answers.
3) Show that $\left\{x^{2}, x^{-1}\right\}$ is a fundamental set of solutions for the equation $x^{2} y^{\prime \prime}-2 y=0, x>0$. Find the solution of the initial value problem $x^{2} y^{\prime \prime}-2 y=0$, $y(1)=1, y^{\prime}(1)=2$.
4) Given that $y_{1}(x)=e^{2 x}$ is a solution of

$$
(2 x+1) y^{\prime \prime}-4(x+1) y^{\prime}+4 y=0,
$$

find the general solution of this equation.
5) Find the general solution of the ODE $3 y^{\prime \prime}-6 y^{\prime}+5 y=0$.

Also from the text:
Section 4.1: Problems 1, 3, 5, 23, 25, 27
Section 4.2: Problems 1-19 (odd)
Section 4.3: Problems 1-13 (odd)

