NOTE: Be sure to bubble the answers to questions 1–24 on your scantron.

Questions 1 – 19 are worth 4 points each.

1. Let \( f(x) = \begin{cases} \frac{2x - 2}{\sqrt{2x - 1} - \sqrt{x}} & x \neq 1 \\ Kx + 2 & x = 1 \end{cases} \).

Find the value of \( K \) so that \( f \) will be continuous at \( x = 1 \).

a. \( K = 1 \)  
   b. \( K = 2 \)  
   c. \( K = 3 \)  
   d. \( K = 4 \)  
   e. \( K = 5 \)

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Solution

\( \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \)

\[ \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{2x - 2}{\sqrt{2x - 1} - \sqrt{x}} \]

\[ = \lim_{x \to 1^-} \frac{2(x - 1)}{\sqrt{2x - 1} - \sqrt{x}} \cdot \frac{\sqrt{2x - 1} + \sqrt{x}}{\sqrt{2x - 1} + \sqrt{x}} \]

\[ = \lim_{x \to 1^-} \frac{2(x - 1)(\sqrt{2x - 1} + \sqrt{x})}{2x - 1 - x} \]

\[ = \lim_{x \to 1^-} \frac{2(x - 1)(\sqrt{2x - 1} + \sqrt{x})}{x - 1} \]

\[ = 2(\sqrt{2} + 1) \]

\[ = 2(1 + 1) = 4 \]

(Use same function)

\[ \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{2x - 2}{\sqrt{2x - 1} - \sqrt{x}} = 4 \]

\[ f(1) = k(1) + 2 \]

\[ k + 2 = 4 \rightarrow k = 2 \]
3. If \( f(x) = \frac{x^2 - 5x + 4}{x - x^3} \), which of the following is/are true?

P. \( f(x) \) has a removable discontinuity at \( x = 1 \).

Q. \( \lim_{x \to 0^+} f(x) = -\infty \).

R. \( f(x) \) can be made continuous at \( x = 1 \) by defining \( f(1) = \frac{3}{2} \).

a. P only \hspace{1cm} b. R only \hspace{1cm} c. P and R only

d. Q and R only \hspace{1cm} e. P and Q only

Solution

P: \( f(x) \) has a removable discontinuity at \( x = 1 \) \( \quad (T) \)

1. Notice from \( \frac{x^2 - 5x + 4}{x - x^3} = \frac{x^2 - 5x + 4}{x(1-x)} \)

We cross out term \((1-x)\), so \( x = 1 \) is removeable.

2. You can check by finding \( \lim_{x \to 1^-} \frac{x^2 - 5x + 4}{x - x^3} \) and \( \lim_{x \to 1^+} \frac{x^2 - 5x + 4}{x - x^3} \)

make sure they are equal.

Q: \( \lim_{x \to 0^+} f(x) = -\infty \) \( \quad (F) \)

R: \( f(x) \) can be made continuous at \( x = 1 \) by defining \( f(1) = \frac{3}{2} \) \( \quad (T) \)

\( \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{x^2 - 5x + 4}{x - x^3} = \lim_{x \to 1^-} \frac{1(1-4)}{1(1+1)} = \frac{1}{2} \)

\( \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{x^2 - 5x + 4}{x - x^3} = \lim_{x \to 1^+} \frac{1(1-4)}{1(1+1)} = \frac{1}{2} \)

Therefore, \( f(1) = \frac{3}{2} \)
4. Let \( p = \lim_{x \to 0} \left( \frac{x - 4}{x^2 - 2x} \cdot \frac{2}{x} \right) \) and \( q = \lim_{x \to 2^+} \frac{x^2 + x - 6}{|2 - x|} \).

Find \( p \) and \( q \).

a. \( p = 0 \) and \( q = 5 \)  
   b. \( p = \infty \) and \( q = -5 \)  
   c. \( p = -\frac{1}{2} \) and \( q = -5 \)  
   d. \( p = \frac{1}{2} \) and \( q = 5 \)  
   e. \( p = 0 \) and \( q = \infty \)

Solution

\[
p = \lim_{x \to 0} \left( \frac{x - 4}{x^2 - 2x} \cdot \frac{2}{x} \right) = \lim_{x \to 0} \left( \frac{x - 4}{x(x - 2)} \cdot \frac{2}{x} \right)
\]

\[
= \lim_{x \to 0} \frac{(x - 4) - 2(x - 2)}{x(x - 2)}
\]

\[
= \lim_{x \to 0} \frac{x - 4 - 2x + 4}{x(x - 2)}
\]

\[
= \lim_{x \to 0} \frac{-x}{x(x - 2)} = \frac{-1}{(0 - 2)} = \frac{1}{2}
\]

\[ \text{plug in } x = 0 \]

\[ Q : \lim_{x \to 2^+} \frac{x + x - 6}{12 - x} \]

\[
|2 - x| = \begin{cases} 
2 - x & \text{if } 2 - x \text{ is positive} \\
-(2 - x) & \text{if } 2 - x \text{ is negative}
\end{cases}
\]

As \( x \to 2^+ \), try \( x = 2.1 \), so \( 2 - x = 2 - 2.1 = -0.1 \) \( \leftarrow \) negative.

\[ 2 - x = -(2 - x) \]

\[
= \lim_{x \to 2^+} \frac{x^2 + x - 6}{(2 - x)} = \lim_{x \to 2^+} \frac{(x + 3)(x - 2)}{(x - 2)} = 2 + 3 = 5
\]