Math 115  Section 7.1  Law of Sines

We use law of sines and cosines for triangles that are not right triangles.

Oblique triangles - triangles that have no right angles.
- acute triangle - all angles are between 0° and 90°.
- obtuse triangle - one angle is between 90° and 180°.

For standard notation - we label the angles A, B, and C and their opposite sides are a, b, and c.

\[
\text{SUM of Three Angles} \\
\text{in a Triangle} \\
\text{is } 180°
\]

Laws of sines are for triangles where we know
1) side, angle, angle (SAA)
2) angle, side, angle (ASA)
3) side, side, angle (SSA)---the tricky case. Can lead to 0, 1, or 2 triangles

We will use proportions (or ratios) to find the missing parts.

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]

NOTE: Remember the longest side lies opposite the largest angle; the shortest side is opposite the smallest angle.

Example: Solve each triangle. Round lengths of sides to the nearest tenth and angle measurements to the nearest degree. (So set the calculator in

\[\text{Degrees}\]

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

Pugging in the values we know, we have:

\[
\frac{16}{\sin 44°} = \frac{b}{\sin 54°} \quad \rightarrow \quad b = \frac{16 \sin 54°}{\sin 44°} \approx 18.6
\]

\[
\frac{16}{\sin 44°} = \frac{c}{\sin 82°} \quad \rightarrow \quad c = \frac{16 \sin 82°}{\sin 44°} \approx 22.8
\]
\[ A = 180^\circ - (102.3^\circ + 28.7^\circ) = 49^\circ \]

b) \( C = 102.3^\circ, B = 28.7^\circ, b = 27.4 \text{ ft.} \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
\frac{a}{\sin 49^\circ} = \frac{27.4}{\sin 28.7^\circ} = \frac{c}{\sin 102.3^\circ}
\]

\[ a = \frac{27.4 \sin 49^\circ}{\sin 28.7^\circ} \approx 43.1 \text{ ft} \]

\[ c = \frac{27.4 \sin 102.3^\circ}{\sin 28.7^\circ} \approx 55.7 \text{ ft} \]

If two sides and one opposite angle are given, three cases exist: (SSA)

1) no triangle exists
2) only 1 triangle exists
3) 2 triangles exists

**Example:** Solve each triangle. Round lengths of sides to the nearest tenth and angle measurements to the nearest degree.

a) \( a = 15, b = 25, \) and \( A = 85^\circ \)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} \rightarrow \frac{15}{\sin 85^\circ} = \frac{25}{\sin B} \rightarrow \frac{\sin B}{15} = \frac{25}{\sin 85^\circ}
\]

\[ \sin B = \frac{25 \sin 85^\circ}{15} \approx 1.6603 \ldots, \text{ which is greater than 1.} \]

**AN IMPOSSIBLE CASE.** So, no solution.

b) \( a = 22, b = 12, \) and \( A = 42^\circ \)

\[
\frac{\sin 42^\circ}{22} = \frac{\sin B}{12}
\]

\[ \sin B = \frac{12 \sin 42^\circ}{22} \rightarrow B = \sin^{-1} \left( \frac{12 \sin 42^\circ}{22} \right) \approx 21^\circ \text{ (if } B \text{ is acute)} \]

or \[ B = 180^\circ - \sin^{-1} \left( \frac{12 \sin 42^\circ}{22} \right) \approx 159^\circ \text{ (if } B \text{ is obtuse)} \]

If \( B = 159^\circ \) then \[ C = 180^\circ - (A + B) = 180^\circ - (42^\circ + 159^\circ) = 180^\circ - 201^\circ = -21^\circ \]