MAC 2312
Spring 2017

EXAM 3

A. Sign your scantron on the back at the bottom in ink.

B. In pencil, write and encode in the spaces indicated:

1) Name (last name, first initial, middle initial)
2) UF ID Number
3) Section Number

C. Under "special codes", code in the test ID number 3, 1.

1 2 ● 4 5 6 7 8 9 0

● 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code", encode A.

● B C D E

E. 1) There are eleven 4-points multiple choice questions, three 2-points multiple choice questions, one bonus 3 points multiple choice question plus two free response questions worth 20 points for a total of 73/70 points.

2) The time allowed is 90 minutes.

3) You may write on the test.

4) Raise your hand if you need more scratch paper, if you have a problem with your test or any emergency. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. **KEEP YOUR SCANTRON COVERED AT ALL TIMES.**

G. When you are finished:

1) Before turning in your test, check for transcribing errors. Any mistakes you leave in are there to stay.

2) Bring your test, scratch paper, and scantron to your proctor to turn them in. Be prepared to show your UF ID card.

3) Answers will be posted in E-Learning after the exam.

H. **On my honor, I have neither given nor received unauthorized aid doing this exam.**

Signature: _____

Questions 1–11 are worth 4 points each.

1. Starting with the geometric series $\sum_{n=0}^{\infty} x^n$, find the sum of the series $\sum_{n=1}^{\infty} \frac{n}{2^n}$.

   (hint: use differentiation)

   A. $\frac{1}{8}$ B. $\frac{1}{4}$ C. 2 D. $\frac{1}{2}$ E. 4

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2. For θ in the interval $[0, 2\pi)$, what is the interval of θ required to trace out the inner loop of the limacon $r = 1 - 2\sin\theta$?

   A. $\theta = 0$ to $\theta = \pi/6$

   B. $\theta = 0$ to $\theta = \pi/2$

   C. $\theta = 5\pi/6$ to $\theta = 2\pi$

   D. $\theta = \pi/6$ to $\theta = 5\pi/6$

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3. First find a power series centered at 0 for the function $f(x) = 8x + e^{-4x}$. Use the Taylor polynomial T_2 to approximate $f(0.1)$.

   A. 1 B. 1.469 C. 1.4 D. 1.48 E. 1.47

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4. Find a power series for the integral and its radius of convergence R.

$$\int \frac{\arctan(x^2) - x^2}{x^2}\, dx$$

A. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^{4n+1}}{(2n+1)(4n+1)} + c,\ R = 1$

B. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^{4n+1}}{(2n+1)(4n+1)} + c,\ R = \infty$

C. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{(2n+1)(4n+1)} + c,\ R = \infty$

D. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^{4n}}{(2n+1)(4n)} + c,\ R = \infty$

E. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{(2n+1)(4n+1)} + c,\ R = 1$

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5. Find the length of the path over the given interval.

$$c(t) = (e^t + e^{-t}, 8 - 2t), \quad 0 \le t \le 2$$

A. $e - 1$ B. $e - e^{-1}$ C. $e^2 - e^{-2}$ D. $e + e^{-1}$ E. $e^{-1} - e$

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6. Which of the following is <u>not</u> a polar coordinate representation for the point $(x, y) = (0, 1)$?

A. $(-1, -\pi/2)$ B. $(1, -3\pi/2)$ C. $(1, \pi/2)$

D. $(-1, \pi/2)$ E. $(-1, 3\pi/2)$

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7. Find a Taylor series for $\ln(1+4x)$ centered at 1.

A. $\ln 5 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}4^n(x-1)^n}{5^n n}$

B. $\ln 5 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}4^n(x-1)^n}{5^n}$

C. $\ln 5 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}4^n(x-1)^n}{5^n n!}$

D. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}4^n x^n}{n}$

E. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}4^n(x-1)^n}{5^n n!}$

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8. What is the interval of convergence of the power series $\int \frac{1}{x^4+1}\,dx$, center at 0?

A. $[-1,1]$ B. $(-4,4]$ C. $[-1,1)$ D. $(-4,4)$ E. $(-1,1]$

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9. Find the Maclaurin series for $f(x) = \sin(x^2)$ and use it to determine $f^{(18)}(0)$.

A. $\frac{18!}{9!}$ B. $\frac{9!}{7!}$ C. $\frac{18!}{5!}$ D. $\frac{9!}{18!}$ E. $\frac{18!}{7!}$

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10. Let $\sum_{n=0}^{\infty} c_n(x-6)^n$ be a power series with finite radius of convergence $R > 0$. Which of the following is possible?

A. the series $c_0 - 5c_1 + 25c_2 - 125c_3 + \cdots$ diverges but $c_0 + 5c_1 + 25c_2 + 125c_3 + \cdots$ converges

B. the series $c_0 - 5c_1 + 25c_2 - 125c_3 + \cdots$ diverges but $c_0 - 6c_1 + 36c_2 - 216c_3 + \cdots$ converges

C. the series $c_0 - 5c_1 + 25c_2 - 125c_3 + \cdots$ diverges but $c_0 + 6c_1 + 36c_2 + 216c_3 + \cdots$ converges

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11. Find the point (x, y) where the tangent line to the curve $c(t) = (3t^2 - 2, 3t^2 + 2t)$ is <u>horizontal</u>.

A. $\left(-\frac{14}{9}, \frac{52}{9}\right)$ B. $(10, 16)$ C. $\left(-\frac{5}{3}, -\frac{1}{3}\right)$

D. $\left(-\frac{23}{12}, -\frac{1}{4}\right)$ E. DNE

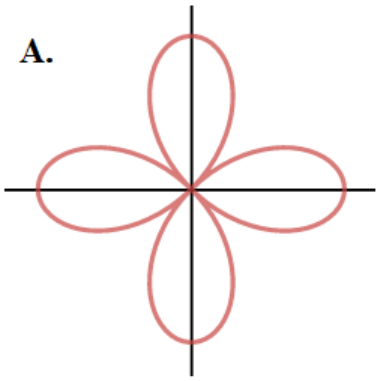
For the next 3 problems, worth 2 points each, match the given polar equations to the graphs. (Graphs may not be drawn to scale.)

12. $r = \cos(3\theta)$

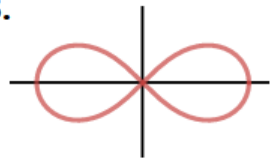
13. $r = \sin(2\theta)$

14. $r^2 = \cos(2\theta)$

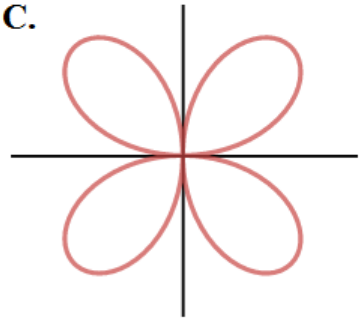
A.



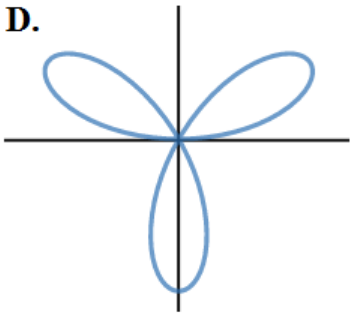
B.



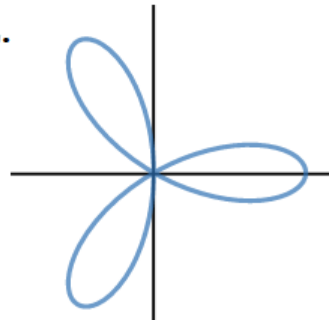
C.



D.

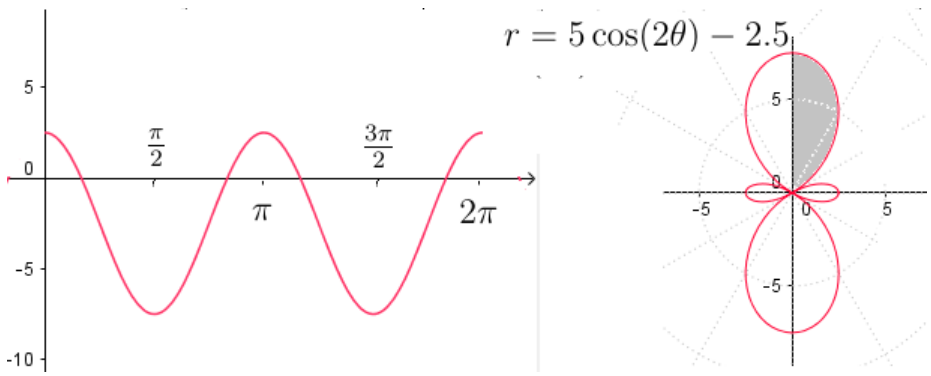


E.



Bonus Questions worth 3 point.

15. $\displaystyle\int_a^b \frac{1}{2}(5\cos(2\theta) - 2.5)^2\, dx$ indicates the area of the shaded region. Find a and b.



A. $a = \dfrac{\pi}{6},\ b = \dfrac{5\pi}{6}$

B. $a = \pi,\ b = 2\pi$

C. $a = \dfrac{7\pi}{6},\ b = \dfrac{3\pi}{2}$

D. $a = \dfrac{\pi}{2},\ b = \dfrac{5\pi}{6}$

E. $a = \dfrac{\pi}{6},\ b = \dfrac{\pi}{2}$

End of Multiple Choice problems

Section # _____UFID_____NAME _____
On my honor, I have neither given nor received unauthorized aid doing this exam.

Signature:_____

FR Scores	
1	/10
2	/10
FR Total	/20

SHOW ALL WORK TO RECEIVE FULL CREDIT.

1. (7 pt) (a) Find a Taylor series for $f(x) = \sqrt{x-2}$, centered at 6.

n	$f^{(n)}(x)$	$f^{(n)}(\ \ )$	c_n

Taylor Series for $\sqrt{x-2} =$ _____

(3 pt) (b) Use the Taylor polynomial T_2 to approximate the value $\sqrt{4.1}$.
(leave your answer as sums of fractions)

$\sqrt{4.1} \sim$ _____

2. Let $r_1 = 6\cos\theta, \quad r_2 = 4 - 2\cos\theta.$

(a) (4pt) Solve for $r_1 = 0,\ 0 \le \theta_1 < 2\pi$

$\theta_1 =$ _____

Solve for $r_2 = 0,\ 0 \le \theta_2 < 2\pi$

$\theta_2 =$ _____

Determine the angle(s) $\theta,\ (0 \le \theta < 2\pi)$ where the two curves intersect.

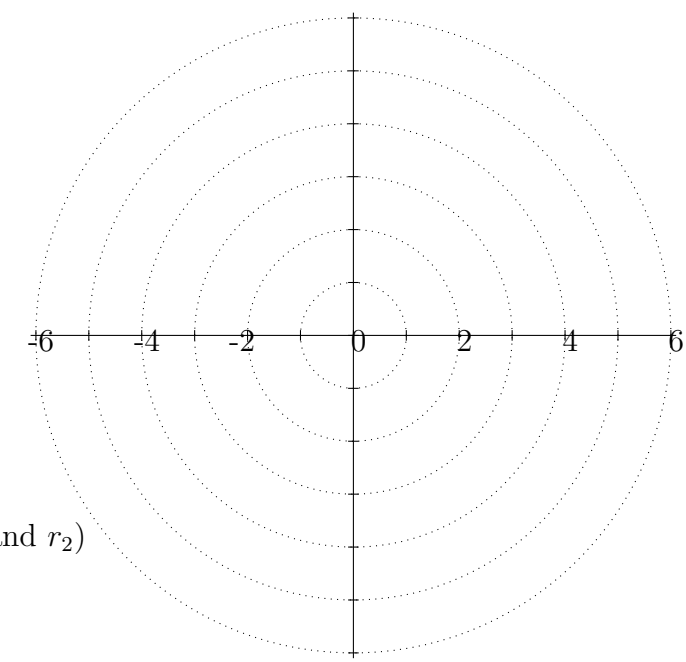
$\theta =$ _____

Determine the value of r_1 when $\theta = 0$. $(r_1, 0) = (\quad, 0)$

Determine the value of r_2 when $\theta = 0$. $(r_2, 0) = (\quad, 0)$

(b)(3pt) Sketch both curves on the same coordinate plane, identify the curves as $r_1,\ r_2$ on the graph.

Label the line(s) of intersections on the graph.



(c) (3pt) Set up an integral for the area of the region in the **first quadrant** lies **inside both** r_1 and r_2. (Do not evaluate.) (You may leave the integral in terms of r_1 and r_2)

Area=_____