Review 4

1: \( f \) is continuous on \([0,2]\) and differentiable on \((0,2)\).

\( f'(x) = 6x^2 - 4 \).

Apply MVT. So, there exists \( c \in (0,2) \) s.t.

\[
\frac{f(2) - f(0)}{2 - 0} = f'(c) = f(2) - f(0).
\]

6\(c^2 - 4 = \frac{11 - 3}{2} = 4 \)

6\(c^2 = 8 \)

\( c = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \).

Not in the domain: \((0,2)\).

So, \( c = \frac{2\sqrt{3}}{3} \). (C)

2) \( f'(x) = 2 - \frac{18}{x^2} = \frac{2x^2 - 18}{x^2} \)

= \frac{2(x^2 - 9)}{x^2} = \frac{2(x - 3)(x + 3)}{x^2}.

C.N. are \( x = 0, 3, -3 \).

But 0' is not in the domain of \( f(x) \).

So, only critical numbers are \( x = -3, 3 \).

\( f' \):

\[
\frac{1}{2} \frac{3}{e^3} \frac{e^2}{e^3} \frac{3}{e^3} \frac{3}{e^3}.
\]

Note that \( e^2 > 3 \).

So, \( \frac{e^2}{e^3} > \frac{3}{e^3} \).

That means, \( \frac{1}{e} > \frac{3}{e^3} \).

So, Abs: max: is \( \frac{1}{e} \). (E)

3) To find critical numbers:

\[
f'(x) = \frac{x(x') - \ln(x)}{x^2} = \frac{1 - \ln(x)}{x^2}.
\]

1 - \( \ln(x) = 0 \implies x = e \).

\( x^2 = 0 \implies x = 0 \).

But 0' is not in the domain.

So, only critical number is \( x = e \).

\( f(e) = ye \)

\( f(e^3) = \frac{3}{e^3} \).

To find the largest among \( ye \) and \( \frac{3}{e^3} \),

\[
\frac{e^2}{e^3} \frac{3}{e^3} \frac{3}{e^3}.
\]

Take the common denominator.

So, \( \frac{e^2}{e^3} > \frac{3}{e^3} \).

Local max @ \( x = -3 \)

Local min @ \( x = 3 \). (D)
4) \( f(x) = 3x^5 - 10x^3 \)
   \[ f'(x) = 15x^4 - 30x^2 \]
   \[ f''(x) = 60x^3 - 60x = 60x(x^2 - 1) = 60x(x - 1)(x + 1) \]

   \[ f''(x) = 0 \Rightarrow x = 0, 1, -1 \]

   \[ f'' \begin{array}{c|c|c|c|c|c}
   - & + & - & + & - & + \\
   -1 & 0 & 1 & 1 & 0 & 1
   \end{array} \]

   \( f \) is concave up on 
   \((-1, 0) \cup (1, \infty)\)

5) \( P:\) False
   \( x=0 \) is not a critical number, \( b^2 \cos 2 \theta \) is not in the domain of \( f(x) \).
   
   \( f(x) = \frac{(x-4)^2}{x} \)

   Domain of \( f(x) = (-\infty, 0) \cup (0, \infty) \).

6) First note that \( f(x) \) is continuous.
   
   \[ f'(x) = \frac{2x}{x^2} - 5 \cos x \]
   \[ f''(x) = \frac{1}{2} - \cos x \]
   \[ f''(x) = 0 \Rightarrow \cos x = \frac{1}{2} \]
   \[ x = \frac{\pi}{3}, \frac{\pi}{2}, \frac{5\pi}{3}, \frac{7\pi}{3} \]

   So \( x = \frac{\pi}{3}, \frac{5\pi}{3} \) are reflection points.

7) \( P:\) False.

   In the above graph \( f'(x) = 0 \) but \( f(x) \) is not a local max or min.

   \( Q:\) True
   \( f(x) = 0 \) when \( x = 12 \).
   So \( f(x) \) has a horizontal tangent line at \( x = 12 \).

   \( R:\) True
   Critical values are \( x = 4, 12 \)

   \[ f' \begin{array}{c|c|c|c|c|c|c}
   0 & 12 &\text{max} & 12 &\text{min} & 12 &\text{max} & 12
   \end{array} \]
   \( f \) is increasing on \((4, 12)\)
R: False

\[ f'(x) = 4 - 3x^2 \]
so \( f'(1) = 4 - 3 = 1 > 0 \)

so \( f \) is increasing at \( x = 1 \)

\[ f''(x) = -6x \]
\[ f''(1) = -6 < 0 \]
so \( f \) is concave down at \( x = 1 \)

so \( f \) is increasing at \( x = 1 \),
but not concave up

\[ f(x) = xe^x \]
\[ f'(x) = xe^x + e^x = (x+1)e^x \]
\[ f''(x) = xe^x + e^x + e^x = (x+2)e^x \]

Thus, interval where \( f' < 0 \) and \( f'' > 0 \) is
\((-2, -1)\).

A: TRUE

Given that \( f \) is continuous.

\[ f'(x) = 20x(x-1)(x+1) \]
\[ f''(x) = 0 \Rightarrow x = 0, \pm 1 \]

so \( x = 0 \) is an inflection point.

B: False
\[ f'(\sqrt{2}) = 0 \]

But \( f''(\sqrt{2}) = 20(\sqrt{2})(\sqrt{2}-1) > 0 \)
so \( f \) has a local min
at \( x = \sqrt{2} \)

C: False
\[ f'(-\sqrt{2}) = 0 \]
\[ f''(-\sqrt{2}) = 20(-\sqrt{2})(-\sqrt{2} - 1) < 0 \]
so \( f \) has a local max at
\( x = -\sqrt{2} \).
10. Evaluate the limit: \( \lim_{x \to 0} \left( 1 + \frac{1}{x} \right)^{3x} \)

**Indeterminate power** \( 1^\infty \)

**Step 1:** Suppose the limit exists

**Step 2:** Take \( \ln \) of both sides

**Step 3:** Use log rule to pull out exponent.

**Step 4:** LHS is now indeterminate product \( \infty \cdot 0 \).

**Step 5:** (Convert \( 3x \ln \left( 1 + \frac{1}{x} \right) \) into different indeterminate form)

**Step 6:** Use L’Hospital’s Rule

\[
\lim_{x \to 0} \frac{3 \ln \left( 1 + \frac{1}{x} \right)}{3x} = \ln L
\]

\[
\lim_{x \to 0} \frac{1 + \frac{1}{x}}{-\frac{1}{x^2}} = \ln L
\]

\[
\lim_{x \to 0} \frac{3}{1 + \frac{1}{x}} = \ln L
\]

\[
3 = \ln L
\]

Exponentiate to find \( L \):

\[
e^3 = e^{\ln L}
\]

\[
L = e^3 \Rightarrow \lim_{x \to 0} \left( 1 + \frac{1}{x} \right)^{3x} = e^3
\]
II. Let $1 \leq f'(x) \leq 4$ for all $x$. Use MVT to find the largest $m$ and smallest $M$ values such that

$$m \leq f(b) - f(z) \leq M$$

What is $m + M$?

a. 4    b. 16    c. 20    d. 24    e. 30

MVT says that if $f$ is continuous on $[a, b]$ and differentiable on $(a, b)$ then there is some number $c$ in $[a, b]$ s.t.

$$f'(c) = \frac{f(b) - f(a)}{b - a} \Rightarrow f'(c)(b-a) = f(b) - f(a)$$

\[ f \text{ is continuous on } [2, 6] \text{ and differentiable on } (2, 6). \]

We have $1 \leq f'(x) \leq 4$. By mean value theorem, there is a $c$ in $[2, 6]$ such that

$$f'(c) = \frac{f(6) - f(2)}{6 - 2}$$

So

$$1 \leq \frac{f(6) - f(2)}{6 - 2} \leq 4$$

$$1 \leq \frac{f(6) - f(2)}{4} \leq 4$$

$$4 \leq f(6) - f(2) \leq 16$$

So $m = 4$ and $M = 16$

$$m + M = 20$$
Evaluate the limit:
\[ \lim_{{x \to 1^+}} \frac{1}{\ln(x)} - 1 \]

Indeterminate form \( \infty - \infty \)

Step 1: Find common denominator.

\[
\begin{align*}
1. & \quad \lim_{{x \to 1^+}} \frac{1}{\ln(x)} \frac{(x-1)}{(x-1)} - \frac{1}{x-1} \frac{(\ln(x))}{(\ln(x))} \\
2. & \quad \lim_{{x \to 1^+}} \frac{x-1}{\ln(x)(x-1)} = \frac{\ln x}{\ln(x-1)} \\
3. & \quad \lim_{{x \to 1^+}} \frac{x-1-\ln x}{\ln(x)(x-1)} \to \frac{0}{0}
\end{align*}
\]

Step 2: Step 1 gives us two indeterminate form \( \frac{0}{0} \)

Step 3: Use L'Hopital's Rule

\[
\lim_{{x \to 1^+}} \frac{1-0 - \frac{1}{x}}{\frac{x}{x(x-1)} + \ln x(1-0)} = \frac{0}{0}
\]

Step 4: Get indeterminate form \( \frac{0}{0} \)

Use L'Hopital's Rule again

\[
\lim_{{x \to 1^+}} \frac{0 + \frac{1}{x^2}}{0 + \frac{1}{x^2} + \frac{1}{x}} = \frac{1}{2}
\]
3. The intensity of a light source at a distance $D$ from an observer is given by the equation $D = \frac{100}{\sqrt{I}}$ where $I$ is given in lumens.

Find the relative error if the intensity $I = 16$ lumens is used if the intensity may be off by 0.8 lumens.

- $a. 2.5\%$
- $b. 5\%$
- $c. 1.25\%$
- $d. 2\%$
- $e. 1\%$

**Formula for Percentage Error**

$\text{Relative error} = 100\%$

**Formula for relative error**

$$\frac{dy}{y} = f'(x)dx$$

In this case we want to find $\frac{dD}{D}$

$$dD = 100 \left( \frac{1}{2} \right) I^{-3/2} dI$$

we want to find $dD$ when $I = 16$ and note $dI = \pm 0.8$

$$dD = 100 \left( \frac{1}{2} \right) 16^{-3/2} (\pm 0.8)$$

$$= -50 \left( \frac{1}{16} \right) (\pm 0.8)$$

$$= -50 \left( \frac{1}{64} \right) (\pm 0.8)$$

$$= -50 \left( \frac{1}{6} \right) (\pm 0.1)$$

$$= \frac{5}{6} (\pm 0.1)$$

relative Error:

$$\frac{dD}{D} = \frac{\frac{5}{6}}{100} = \frac{\frac{5}{6}}{100} = \frac{\frac{5}{6}}{100} = \frac{5}{100} = \frac{1}{40} = -\frac{1}{0.04} = -4 \cdot 0.1$$

$$\text{relative error} = \pm \frac{1}{4} (0.1)$$

$$\text{percent error} = \frac{1}{4} (0.1) \cdot 100\%$$

$$0.25(0.1) \cdot 100\%$$

[0.25%]
The area of a circle is expanding at a rate of $32\pi$ in$^2$/sec. How fast is the radius increasing at the instant when $r = 2$ in? 

- a. 16 in/sec, b. 2 in/sec, c. 6 in/sec, d. 4 in/sec, e. 8 in/sec

**Formula** $A = \pi r^2$

**Given** $\frac{dA}{dt} = 32\pi$

**Want to Find** $\frac{dr}{dt}$ when $r = 2$

**Use Implicit Differentiation**

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$32\pi = 2\pi r \frac{dr}{dt}$$

$$32\pi = 2\pi (2) \frac{dr}{dt}$$

$$8 = \frac{dr}{dt}$$

$$\frac{dr}{dt} = 8 \text{ inches per second}$$

[\text{e.}]

The remaining multiple choice problems answer the following questions:

15. If $f$ is differentiable on $(-\infty, \infty)$ and has two roots, then $f'$ has at least one root.
   (a) True  
   (b) False

16. If $f$ is continuous on $[a, b]$, differentiable on $(a, b)$ and $f(a) = f(b)$, then there is a $c$ in $(a, b)$ such that $f'(c) = 0$.
   (a) True  
   (b) False

17. If $f'(c) = 0$, then $f$ has a local maximum or minimum at $c$.
   (a) True  
   (b) False

18. If $f$ has an absolute minimum value at $c$, then $f'(c) = 0$.
   (a) True  
   (b) False

Or we could think about cases where $f'$ is undefined, such as $f(x) = \sqrt{x}$, which has an absolute min at $x = 0$ but $f'(x) = \frac{1}{2\sqrt{x}}$ is undefined at $x = 0$. We could look at this in two ways. We could think about how in some cases of extremum value theorems, we might have an abs. min or absolute max at one of the endpoints $[a, b]$ even though $f'(x)$ or $f''(x)$ might not be equal to 0.
19. If $f$ is continuous on $(a,b)$, then $f$ attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers $c$ and $d$ in $(a,b)$.

(a) True  
(b) False

20. If $f$ is differentiable everywhere and $f(-1) = f(1)$, then there is a number $c$ such that $|c| < 1$ and $f'(c) = 0$.

(a) True  
(b) False

- $|c| < 1$ so $c$ is in $(-1, 1)$.

21. If $f''(x) = 0$, then $(2, f(2))$ is an inflection point of the curve $y = f(x)$.

(a) True  
(b) False

22. If $f$ and $g$ are increasing on and interval $I$, then $f + g$ is increasing on $I$.

(a) True  
(b) False

23. If $f$ and $g$ are increasing on an interval $I$, then $f - g$ is increasing on $I$.

(a) True  
(b) False

24. If $f$ and $g$ are increasing on an interval $I$, then $fg$ is increasing on $I$.

(a) True  
(b) False

25. If $f$ and $g$ are positive increasing function on an interval $I$, then $fg$ is increasing on $I$.

(a) True  
(b) False

$f$ and $g$ are increasing on $I = [-3, 0]$ but

$$(fg)'(x) = (x)(x-2) + (x-2)x$$

$$(fg)'(x) = (x-2)x$$

So $2x-2$ will be negative on $[3, -1]$.
26. If \( f \) is increasing and \( f(x) > 0 \) on \( I \), then \( g(x) = \frac{1}{f(x)} \) is decreasing on \( I \).

(a) True
(b) False

We have \( f(x) > 0 \) on \( I \) and \( f \) increasing implies that \( f'(x) > 0 \) also.

\[
g(x) = \frac{1}{f(x)}
\]

\[
g'(x) = -\frac{f'(x)}{f^2(x)}
\]

\[
g'(x) = -\frac{f'(x)}{f^2(x)}
\]

Note that \( f^2(x) > 0 \) on \( I \) and \( f'(x) > 0 \) on \( I \) but the minus sign forces \( g'(x) < 0 \) on \( I \).

So \( g \) is decreasing!
Let \( f(x) = (x + 1)(x - 1)^{1/3} \). Then \( f'(x) = \frac{2(x - 1)}{3(x - 1)^{2/3}} \) and \( f''(x) = \frac{4(x - 2)}{9(x - 1)^{5/3}} \).

(a) Determine the following about \( f \), if none, write "none".

i. Domain of \( f \): \((\infty, \infty)\)

ii. \( \lim_{x \to \infty} f(x) = \infty \)

iii. Asymptotes of \( f \): None

iv. \( x\)-intercept(s): \( x = \pm 1 \)

v. \( f'(x) = \frac{2(x - 1)}{3(x - 1)^{2/3}} \)

vi. \( f''(x) \) undefined when denominator = 0

vii. \( f'(x) \) undefined at \( x = 1 \)

viii. \( f'(x) \) when numerator = 0

ix. \( f'(x) \) when \( x = \frac{1}{2} \)

\[ f'(x) = \frac{4(x - 2)}{9(x - 1)^{5/3}} \]

(b) Local maxima (as an ordered pair): None

(c) Local minima (as an ordered pair): Local min at \( x = \frac{1}{2} \) by 1st derivative test

\( f'(x) = \frac{2(x - 1)}{3(x - 1)^{2/3}} = \frac{2}{3} (\frac{1}{2})^{2/3} = -\frac{3}{2 \cdot \sqrt[3]{2}} \) at \( (\frac{1}{2}, \frac{3}{2 \sqrt[3]{2}}) \)

(d) Inflection point(s) (as an ordered pair): From viii, we have inflection points at \( x = 1, x = 2 \)

\[ f'(1) = 0 \]
\[ (1, 0) \]
\[ f'(2) = 3 \]
\[ (2, 3) \]
(e) Sketch the graph of \( y = (x + 1)(x - 1)^{1/3} \), clearly labeling all the important features of the graph.

Note that \( f(1/2) \approx -1.2 \).
2. (9 points) A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 3 ft/s, find: \( \frac{dx}{dt} = 3t^{1/2} \)

(a) How fast is the top of the ladder sliding down the wall when the bottom of the ladder is 8 ft from the wall? Include units in your answer.

We want to find \( \frac{dy}{dt} \) when \( x=8 \)

Use pythagorean theorem

\[ x^2 + y^2 = 10^2 \]

Implicitly derive

\[ 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \]

We have \( \frac{dx}{dt} = 3 \frac{t}{2} \)

Note when \( x=8 \)

\[ y = 6 \]

by pythagorean theorem

\[ 2 \times (8)(3) + 2(6) \frac{dy}{dt} = 0 \]

Solve for \( \frac{dy}{dt} \)

\[ \frac{dy}{dt} = -\frac{48}{12} = -4 \]

It's possible that we may need to write \( \frac{dy}{dt} \) in second sense we wrote falling but I wouldn't worry too much for now.

It is negative because the height of \( y \) is decreasing

The top of the ladder is falling at a rate of \( -4 \) \( \text{ft/s} \).

(b) How fast is the angle between the ladder and the ground changing when the bottom of the ladder is 8 ft from the wall? Include units in your answer.

Use \( \tan \theta = \frac{y}{x} \)

Implicitly derive

\[ \sec^2 \theta \frac{d\theta}{dt} = \frac{x \frac{dy}{dt} - y \frac{dx}{dt}}{x^2} \]

When \( x=8 \)

\[ y = 6 \]

\[ \sec \theta = \frac{1}{\cos \theta} = \frac{1}{\left( \frac{10}{10} \right)^2} = \frac{1}{\left( \frac{8}{10} \right)^2} = \left( \frac{10}{8} \right)^2 = \left( \frac{5}{4} \right)^2 = \frac{25}{16} \]

\[ \frac{d\theta}{dt} = \frac{25}{16} \times \frac{-4}{6} = \frac{-5}{2} \]

\[ \frac{d\theta}{dt} = \frac{-50}{Z^2} = \frac{-1}{2} \]

The angle is increasing/decreasing at a rate of \( \frac{\pi}{2} \) rad/sec

(circle the correct choice)

\[ \frac{d\theta}{dt} = \frac{\pi}{2} \text{ rad/sec} \]

When writing decreasing first you don't write \( -\frac{\pi}{2} \) rad/sec

It the question said the angle is changing at a rate of \( \frac{d\theta}{dt} \) then put \( -\frac{\pi}{2} \) rad/sec
3. (8 points) Let \( f(x) = \sqrt{1 + 2x} \).

(a) Find the linearization of \( f(x) \) at \( a = 4 \).

\[
L(x) = f(a) + f'(a) (x-a)
\]
\[
L(x) = f(4) + f'(4) (x-4)
\]
\[
L(x) = 3 + \frac{1}{3} (x-4)
\]

\[
f'(x) = \frac{1}{2} (1 + 2x)^{-\frac{1}{2}}
\]
\[
f'(x) = \frac{2}{2\sqrt{1+2x}}
\]
\[
f'(x) = \frac{1}{\sqrt{1+2x}}
\]
\[
f'(a) = f'(4) = \frac{1}{\sqrt{9}} = \frac{1}{3}
\]
\[
f'(4) = \sqrt{1+2(4)} = \sqrt{9} = 3
\]
\[
L(x) = 3 + \frac{1}{3} (x-4)
\]

(b) Use part (a) to approximate the number \( \sqrt{9.06} \).

Don't find \( L(9.06) = 3 + \frac{1}{3} (9.06-4) \)

We want to find \( L(x) \) when \( \sqrt{1+2x} = \sqrt{9.06} \)
\[
1+2x = 9.06
\]
\[
x = 4.03
\]
\[
L(4.03) = 3 + \frac{1}{3} (4.03-4)
\]
\[
= 3 + \frac{1}{3} (0.03)
\]
\[
= 3 + 0.01
\]
\[
\sqrt{9.06} \approx L(4.03) = 3.01
\]
4. (10 points) If \( f(x) = x - 3x^{1/3} \), then \( f'(x) = \frac{x^{2/3} - 1}{x^{2/3}} \) and \( f''(x) = \frac{2}{3x^{5/3}} \). Find the following (if none, write “none”):

(a) \( f' \) and \( f'' \) number lines:

\[ \text{Critical points: } x = -1, 0, 1 \]

\[ \text{Increasing intervals: } (-\infty, -1) \cup (1, \infty) \]

\[ \text{Decreasing interval: } (-1, 1) \]

(b) \( x \)-intercept(s): \( x = 0, \pm 3\sqrt{3} \);

\( \text{Set } f(x) = 0 \text{ and solve for } x \)

\( y \)-intercept: \( y = \frac{0}{f(0)} \)

(c) relative minimum at \( (x, y) = (1, -2) \):

relative maximum at \( (x, y) = (-1, 2) \)

(d) inflection point(s) at \( (x, y) = (0, 0) \)

(e) Sketch the graph of \( y = f(x) \) using the above information. Clear label relative extrema and inflection point(s) on the graph.

Note: \( 3^{2/3} = 3\sqrt{3} \approx 5.2 \) and \( -3^{2/3} = -3\sqrt{3} \approx -5.2 \).
For I messed up on typing up the problem.

It should have said, evaluate \( \lim_{x \to 0} \frac{\sin^2 x}{e^{4x}-1-4x} \).

If you evaluated \( \lim_{x \to 0} \frac{\sin^2 x e^{4x}-1-4x} {e^{4x}-1-4x} \) then the answer is \(-1\) and you don't need L'Hôpital's Rule.

For Finding \( \lim_{x \to 0} \frac{\sin^2 x}{e^{4x}-1-4x} \) we'll need to use L'Hôpital's Rule twice.

\[
\lim_{x \to 0} \frac{\sin^2 x}{e^{4x}-1-4x} = \lim_{x \to 0} \frac{2\sin x \cos x}{4e^{4x}-4} = 0
\]

Indeterminate form.

Let's use L'Hôpital's rule again.

\[
\lim_{x \to 0} \frac{2\sin x (-\sin x) + 2\cos x \cos x}{16 e^{4x}} = \lim_{x \to 0} \frac{-2\sin^2 x + 2\cos^2 x}{16 e^{4x}} = \frac{1}{8}
\]

So the \( \lim_{x \to 0} \frac{\sin^2 x}{e^{4x}-1-4x} = \frac{1}{8} \).
(b) Use L'Hopital's Rule to evaluate \( \lim_{x \to 0} (\cos x)^{\frac{1}{x^2}} \)

1. Suppose the limit exists.
2. Take natural log at both sides.
3. Use log rule to pull out exponent to the front.
4. Try seeing what indeterminate form this problem is now.
5. \( \frac{0}{0} \) For this problem we got \( \frac{0}{0} \).
   So apply L'Hopital's Rule.
6. Apply L'Hopital's Rule again.
7. Exponentiate to find \( L \).

\[
\begin{align*}
L &= \lim_{x \to 0} (\cos x)^{\frac{1}{x^2}} \\
\ln L &= \lim_{x \to 0} \ln \left( (\cos x)^{\frac{1}{x^2}} \right) \\
\ln L &= \lim_{x \to 0} \frac{1}{x^2} \ln (\cos x) \\
&= \lim_{x \to 0} \frac{(\ln(\cos x))'}{x^2} \\
&= \lim_{x \to 0} \frac{\frac{1}{\cos x} \cdot (-\sin x)}{2x} \\
&= \lim_{x \to 0} \frac{-\sin x}{2 \cos x}
\end{align*}
\]

\[\ln L = \lim_{x \to 0} \frac{-\sin x}{2 \cos x} = \frac{-1}{2}\]

\[e^{\ln L} = e^{-\frac{1}{2}}\]

\[L = e^{-\frac{1}{2}}\]

\[\lim_{x \to 0} (\cos x)^{\frac{1}{x^2}} = e^{-\frac{1}{2}}\]
(c) Suppose that \( f(0) = -3 \) and \( f'(x) \leq 5 \) for all values of \( x \). What is the largest possible value for \( f(2) \)?

Idea: Use Mean Value Theorem

Since \( f \) is differentiable on \([0, 2]\) and \( f \) is continuous on \([0, 2]\), by Mean Value Theorem we have that there is a \( c \) in \((0, 2)\) such that

\[
f'(c) = \frac{f(2) - f(0)}{2 - 0} \leq 5
\]

\[
f'(c) = \frac{f(2) - (-3)}{2} \leq 5
\]

\[
f(2) + 3 \leq 10 \Rightarrow f(2) \leq 7
\]

So the largest possible value is \( \frac{7}{7} \).