Multiple Choice Questions

1. Find $\frac{dy}{dx}$ if $y = \sin(\tan(3x))$

   (a) $\frac{dy}{dx} = 3 \cos(3x) \sec^2(3x)$

   (b) $\frac{dy}{dx} = 3 \cos(\sec^3(3x)) + \tan(3x) \cos(3x)$

   (c) $\frac{dy}{dx} = \cos(\tan(3x)) \sec(3x) \tan(3x)$

   (d) $\frac{dy}{dx} = 3 \cos(\tan(3x)) \sec^2(3x)$

   (e) $\frac{dy}{dx} = 3 \cos(\sec^2(3x))$

2. Find $f'(1)$ if $f(x) = \frac{2 \pi x \pi}{\pi} - 4^{x^2+1}$.

   (a) $2 - 32 \ln 4$

   (b) $\frac{2}{\pi} - 2 \ln 4$

   (c) $-6$

   (d) $\frac{2}{\pi} - 16$

   (e) $2 - 16 \ln 4$

3. An oscillating sprinkler makes an angle of $\theta$ radians with the ground, the range covered (horizontal distance in feet) is given by the formula $R = \frac{1}{2} \sin \theta \cos \theta$. At what rate the range $R$ is changing with respect to $\theta$ when $\theta = \frac{\pi}{6}$

   (a) The range is decreasing by 0.25 feet per radian.

   (b) The range is decreasing by 1 foot per radian.

   (c) The range is increasing by 0.25 feet per radian.

   (d) The range is decreasing by $\sqrt{3}/8$ feet per radian.

   (e) The range is increasing by $\sqrt{3}/8$ feet per radian.
4. Write the equation of the normal line to the graph of the function \( f(x) = \frac{x^{4/3} + 2e^x}{x} \) at \( x = 1 \).

(a) \( y = 3x + (4 + 2e) \)
(b) \( y = 3x - (4 + 2e) \)
(c) \( y = -3x + (4 - 2e) \)
(d) \( y = -3x - (4 - 2e) \)
(e) \( y = -3x + (4 + 2e) \)

5. At which value(s) does the function \( g(x) = x\sqrt{4x + 3} \) have a horizontal tangent line?

(a) \( x = -2/3 \) and \( x = -3/4 \)
(b) \( x = -1/2 \) only
(c) \( x = 0 \) only
(d) \( x = -2/3 \) only
(e) \( x = -1/2 \) and \( x = -3/4 \) only

6. Find the slope of the tangent line to \( x^3 + y^2 + xy + e^{x-1} = 8 \) at the point \((1, -3)\)

(a) \( -7/5 \)
(b) 2
(c) \( 4/5 \)
(d) \( 3/8 \)
(e) \( 1/5 \)

7. Let \( h(x) = [xf(x) - 4]^2 \). Find \( h'(2) \) if \( f(2) = 1/2 \) and \( f'(2) = -1 \).

(a) 9
(b) -6
(c) 6
(d) -5
(e) 15

8. Let \( f(x) = x^2 - \cos x + 2e^{2x} \) and \( g \) be the inverse of \( f \), find \( g'(1) \).

(a) 0.25
(b) 0.5
(c) 1
(d) 1.25
(e) 2
9. If \( g(x) = \tan^{-1}(e^x) \), find \( g''(x) \).

(a) \( g''(x) = \frac{e^x}{1 + e^{2x}} \)

(b) \( g''(x) = \frac{2e^{2x}}{(1 + e^{2x})^2} \)

(c) \( g''(x) = \frac{e^x - e^{3x}}{(1 + e^{2x})^2} \)

(d) \( g''(x) = \frac{2e^{2x} + e^x}{(1 + e^x)^2} \)

(e) \( g''(x) = \frac{-e^{2x}}{(1 + e^{2x})^2} \)

10. A balloon is at a height of 20 meters, and is rising at the constant rate of 5 m/sec. A bicyclist passes beneath it, traveling in a straight line at the constant speed of 20 m/sec. How fast is the distance between the bicyclist and the balloon increasing 2 seconds later?

(a) 21 m/sec

(b) 19 m/sec

(c) 15 m/sec

(d) 17/2 m/sec

(e) 11 in/sec

11. A particle is moving according to the position function \( S(t) = (4t + 1)^{3/2} \), where \( S(t) \) is measured in centimeters and \( t \) in seconds. Find the acceleration of the particle after 6 seconds.

(a) 3/20 cm/sec\(^2\)

(b) 5/6 cm/sec\(^2\)

(c) 3/5 cm/sec\(^2\)

(d) 12/5 cm/sec\(^2\)

(e) 1/5 cm/sec\(^2\)
12. Match the graph of the following function with the graph of its derivative

(a) (b) (c)
(d) (e)

13. Use the definition of the derivative to evaluate the limit

\[ \lim_{x \to \pi/3} \frac{\tan x - \sqrt{3}}{x - \pi/3} \]

(a) −4
(b) −1/4
(c) 1/4
(d) 4
(e) 2

14. The dollar cost of producing \( x \) bagels is \( C(x) = 300 + 6.25x - 0.5 \left( \frac{x}{1000} \right)^3 \). Determine the marginal cost in dollars per unit of producing 2000 bagels.

(a) 3.25
(b) 6.244
(c) 6.301
(d) 6.294
(e) −3.294
15. Use linearization for a suitable function to approximate \( \frac{1}{\sqrt{0.8}} \).

(a) 1.2
(b) 1.05
(c) 0.9
(d) 1.01
(e) 1.1

16. (BONUS) If a function \( f \) is continuous at \( x = a \), then it is differentiable at \( x = a \)

(a) TRUE
(b) FALSE

17. (BONUS) If a function \( f \) has a vertical tangent line at \( x = a \), then \( f' \) has a vertical asymptote at \( x = a \)

(a) TRUE
(b) FALSE

18. (BONUS) If \( f(x) = \ln|x| \), then \( f'(x) = \frac{1}{x} \)

(a) TRUE
(b) FALSE

19. (BONUS) If a function \( f \) is differentiable at \( x = a \), then \( f' \) is continuous at \( x = a \)

(a) TRUE
(b) FALSE

20. If a one to one function \( f \) is differentiable at \( a \), then its inverse, \( f^{-1} \) is differentiable at \( f(a) \).

(a) TRUE
(b) FALSE
Free Response Questions

1. Suppose that when a ball is thrown in the air with an initial velocity of 32 feet per second, its height in feet above the ground after \( t \) seconds is given by \( s(t) = -16t^2 + 32t + 48 \).

   (a) Use the limit definition of derivative to find a formula for the velocity of the ball at any time \( t \).

   (b) Find the velocity of the ball when it hits the ground. Include units in your answer.
2. Find the derivative of the following functions. Simplify as much as possible.

(a) \( f(x) = xe^x \sin^{-1}(x) \)

(b) \( f(x) = \frac{x^2 + 1}{\tan^{-1}(\sin x)} \)

3. The demand and cost functions for a product are, respectively, \( p(x) = 120 - 0.01x \) and \( C(x) = 40x + 1200 \), where \( x \) is the number of units produced weekly. If the manufacturer decides to increase the production by 70 units per week, find the rate at which the profit is changing with respect to time when the weekly production is 3600 units.
4. (a) Use logarithmic differentiation to find the slope of the tangent line to $f(x)$ at $x = 1$ for

$$f(x) = \frac{e^{x^2-2} \sqrt{6 + 3x}}{(3x - 1)^4}$$

(b) Find the slope of the tangent line to $y = (\tan x)^{1/x}$ at $x = \pi/4$

5. A toy rocket launched vertically into the air has position function $s(t) = 64 \sqrt{t} - 8t^2$, where $s = 0$ is the ground level, $s$ is measured in meters, and $t > 0$ is measured in seconds.

(a) How many seconds does the rocket spend travelling upward?

(b) After how many seconds does the rocket return to the ground?

(c) What is the total distance travelled by the rocket after it launched until it hit the ground?

(d) What is the maximum height reached by the rocket?
## Answer Key

### Multiple Choice Questions: 4 points each


### Bonus Questions: 1 point each

16. FALSE  17. TRUE  18. TRUE  19. FALSE  20. TRUE

For Free Response Questions Refer to the worked out solutions