The following problems were pulled from exams and could potentially be on Discussion Quiz 6:

12. If \( \int_a^b \frac{1}{2}(-1 + 2 \sin \theta)^2 \, d\theta \) presents the area of the shaded region \( R \) below, find \( a \) and \( b \).

\[
r = -1 + 2 \sin \theta
\]

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( r )</th>
<th>( x )</th>
<th>( y )</th>
<th>label</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>A</td>
</tr>
<tr>
<td>( \frac{\pi}{2} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>B</td>
</tr>
<tr>
<td>( \pi )</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>C</td>
</tr>
</tbody>
</table>

- a. \( a = \frac{11\pi}{6} \) and \( b = 2\pi \)
- b. \( a = \frac{2\pi}{3} \) and \( b = \pi \)
- c. \( a = \frac{5\pi}{3} \) and \( b = 2\pi \)
- d. \( a = \frac{3\pi}{2} \) and \( b = 2\pi \)
- e. \( a = \frac{5\pi}{6} \) and \( b = \pi \)

This into alone would rule out all choices except for C.

Point C is \((1,0)\) on rectangular coordinate plane.

\( y = 0 \) means \( \cos \theta = 1 \)
\( y = r \sin \theta = 0 \)
which happens \( a + \theta = 0, \pi, 2\pi \).
For the next 3 problems, worth 2 points each, match the given polar equations to the graphs.
(Graphs may not be drawn to scale.)

12. $r = \cos(3\theta)$ matches with graph E
13. $r = \sin(2\theta)$ matches with graph C
14. $r^2 = \cos(2\theta)$ matches with graph B
15. \( \int_{a}^{b} \frac{1}{2} (5 \cos(2\theta) - 2.5)^2 \, d\theta \) indicates the area of the shaded region. Find \( a \) and \( b \).

<table>
<thead>
<tr>
<th>label</th>
<th>polar</th>
<th>x-y coordinate (x,y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(2.5,0)</td>
<td>(2.5,0)</td>
</tr>
<tr>
<td>B</td>
<td>(-7.5, \frac{\pi}{2})</td>
<td>(0, -7.5)</td>
</tr>
<tr>
<td>C</td>
<td>(2.5, \pi)</td>
<td>(-2.5, 0)</td>
</tr>
<tr>
<td>D</td>
<td>(-7.5, \frac{3\pi}{2})</td>
<td>(0, 7.5)</td>
</tr>
</tbody>
</table>

A. \( a = \frac{\pi}{6}, b = \frac{5\pi}{6} \)  
B. \( a = \pi, b = 2\pi \)  
C. \( a = \frac{7\pi}{6}, b = \frac{3\pi}{2} \)  
D. \( a = \frac{\pi}{2}, b = \frac{5\pi}{6} \)  
E. \( a = \frac{\pi}{6}, b = \frac{\pi}{2} \)

**Step 1:** Label Peaks and Valleys  
**Step 2:** Convert to rectangular form and label points on graph  
**Step 3:** Figure out which points correspond to shaded region

Additional Step: Find Origin Points (Set \( r = 5 \cos(2\theta) - 2.5 \) equal to 0 and solve for \( \theta \))

**Also for Step 2 I like to draw arrows indicating how the graph descends travels from A to B, B to C, so on and so forth.**

So \( a = \frac{11\pi}{6} \) and \( b = \frac{3\pi}{2} \)

From FALL 2016

13. Draw the curves on the same plane. At how many distinct point(s) do the curves intersect?

\( r_1 = 2 \cos \theta, \quad r_2 = 2 \cos(2\theta) \)

A. 0  
B. 1  
C. 2  
D. 3  
E. 4

- \( r_1 = 2 \cos \theta \) circle of radius 1 centered at \((1,0)\)
- \( r_2 = 2 \cos(2\theta) \) lemniscate petals on x-axis, size of petals will be \( \sqrt{2} \)

\( r_1 \) and \( r_2 \) only intersect at the origin (at one pt)
9. Set up an integral that represents the area of the shaded region.

Formula for Area
\[
\frac{1}{2} \int \theta \ r^2 \ d\theta
\]

\( r_1 \) is the top curve so we would want \( r_1^2 - r_2^2 \)

A. \( \frac{1}{2} \int_0^{\pi/2} (r_1^2 - r_2^2) \ d\theta \)
B. \( \frac{1}{2} \int_0^{\pi/2} r_1^2 \ d\theta - \frac{1}{2} \int_0^{\pi/2} r_2^2 \ d\theta \)
C. \( \frac{1}{2} \int_0^{\pi/2} r_1^2 \ d\theta - \frac{1}{2} \int_0^{\pi/2} r_2^2 \ d\theta \)
D. \( \frac{1}{2} \int_0^{\pi/2} r_1^2 \ d\theta - \frac{1}{2} \int_0^{\pi/2} r_2^2 \ d\theta \)
E. \( \frac{1}{2} \int_0^{\pi/2} r_1^2 \ d\theta - \frac{1}{2} \int_0^{\pi/2} r_2^2 \ d\theta \)

To find upper limit for \( r_2 \)’s integral, find value \( \theta \) when \( r_2 \) hits the origin (i.e., when \( r_2 = 0 \))
\( r_2 = 2 \cos(3\theta) = 0 \Rightarrow \cos(3\theta) = 0 \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6} \)

Note \( r_1 = 2 \) at \( \theta = 0 \) and \( r_2 = 2 \) at \( \theta = 0 \)
Measuring both \( r_1 \) and \( r_2 \) have the same starting pt
Tells us the lower integral limit should be 0 for both \( r_1 \) and \( r_2 \)

We want to find at what \( \theta \) for \( \theta \) causes \( r_1 = 1 + \cos(\theta) \) to hit \( A \)? For rectangular coordinates \( A \)
the \( x \) value is 0 meaning \( \chi = r \cos \theta = 0 \) which occurs when \( \theta = \frac{\pi}{2} \)

If we wanted to find the area of \( r_1 \) on the 1st quadrant we would then want \( \frac{1}{2} \int_0^{\pi/2} r_1^2 \ d\theta \)

From Fall 2018

9. Let \( A = \int_{\alpha}^{\pi/2} \frac{1}{2} (2 - 4 \sin \theta)^2 \ d\theta \)
be the area of the inner loop of \( r = 2 - 4 \sin \theta \).
What is the value of \( \alpha \)?

A. \( \alpha = \frac{\pi}{6} \)  B. \( \alpha = \frac{5\pi}{6} \)  C. \( \alpha = \frac{\pi}{3} \)  D. \( \alpha = 0 \)  E. \( \alpha = \frac{\pi}{4} \)

We need to find when \( r = 2 - 4 \sin \theta \) first hits the origin
Set \( r = 0 \) and solve for \( \theta \)
\( 0 = 2 - 4 \sin \theta \)
\( 4\sin \theta = 2 \)  \( \sin \theta = \frac{1}{2} \)  \( \theta = \frac{\pi}{6}, \frac{5\pi}{6} \)
8. Given two polar curves, \( r_1 = \cos \theta \), \( r_2 = \sin \theta \). Which shaded region below is represented by the integral below?

\[
\frac{1}{2} \int_0^{\pi/4} (\sin\theta)^2 \, d\theta + \frac{1}{2} \int_{\pi/4}^{\pi/2} (\cos\theta)^2 \, d\theta
\]

Adding 2 areas together means we're not trying to find the area of region C or E. It looks like \( \frac{1}{2} \int_{r_1}^{r_2} r^2 \, d\theta \).

\[
\frac{1}{2} \int_0^{\pi/4} (\sin\theta)^2 \, d\theta + \frac{1}{2} \int_{\pi/4}^{\pi/2} (\cos\theta)^2 \, d\theta = \frac{1}{2} \int_0^{\pi/2} (\cos\theta)^2 \, d\theta
\]

If \( \frac{1}{2} \int_a^b (r_1^2 - r_2^2) \, d\theta \) presents the area of the region inside the curve \( r_1^2 = 50 \cos(2\theta) \) and outside the curve \( r_2 = 5 \), in the first quadrant, find a and b.

A. \( a = \frac{\pi}{3}, b = \frac{\pi}{2} \)
B. \( a = \frac{\pi}{6}, b = \frac{\pi}{4} \)
C. \( a = 0, b = \frac{\pi}{3} \)
D. \( a = 0, b = \frac{\pi}{2} \)
E. \( a = 0, b = \frac{\pi}{6} \)

Point \( a = 0 \) when \( r_1 = r_2 \).

\[
25 = 50 \cos(2\theta)
\]

\[
\frac{1}{2} = \cos(2\theta)
\]

\[
\cos^{-1}\left(\frac{1}{2}\right) = 2\theta
\]

\[
\frac{\pi}{3} = 2\theta \Rightarrow \theta = \frac{\pi}{6}
\]

So, \( b = \frac{\pi}{6} \).
14. Which choice below represents the area of the region that lies inside the first curve and outside the second curve?

A. \( \frac{1}{2} \int_{\pi/6}^{\pi/2} r_1^2 d\theta - \frac{1}{2} \int_{0}^{\pi/6} r_2^2 d\theta \)

B. \( \frac{1}{2} \int_{\pi/6}^{\pi} r_1^2 d\theta - \frac{1}{2} \int_{\pi/6}^{\pi/2} r_2^2 d\theta \)

C. \( \frac{1}{2} \int_{\pi/6}^{\pi} r_1^2 d\theta - \frac{1}{2} \int_{\pi/6}^{\pi} r_1^2 d\theta \)

D. \( \frac{1}{2} \int_{\pi/6}^{\pi/2} r_1^2 d\theta - \frac{1}{2} \int_{\pi/6}^{\pi/2} r_2^2 d\theta \)

E. \( \frac{1}{2} \int_{\pi/6}^{\pi/2} r_1^2 d\theta - \frac{1}{2} \int_{0}^{\pi/6} r_1^2 d\theta \)

Point of intersection: \( A + \theta = \frac{\pi}{6} \)

\( r_1 = r_2 \)
\( \sqrt{3} \sin \theta = \cos \theta \)
\( \sqrt{3} = \frac{\cos \theta}{\sin \theta} \)
\( \sqrt{3} = \cot \theta \)
\( \frac{1}{\sqrt{3}} = \tan \theta \)
\( \frac{\frac{1}{2}}{\sqrt{3}} = \tan \theta \)

To find area of \( r_1 \) use \( \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} r_1^2 d\theta = \)

To get rid of purple region, we find \( \theta \) when \( r_2 \) hits point of origin, i.e., when \( \theta = \frac{\pi}{2} \)

Remember to start for \( r_2 \), \( \theta \) ranges from 0 to \( \pi \) and starts at point (1,0) and draw a semicircle of \( \theta = \frac{\pi}{2} \). We want to get rid of purple area.