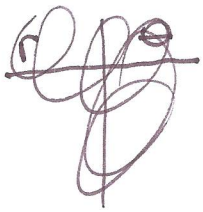
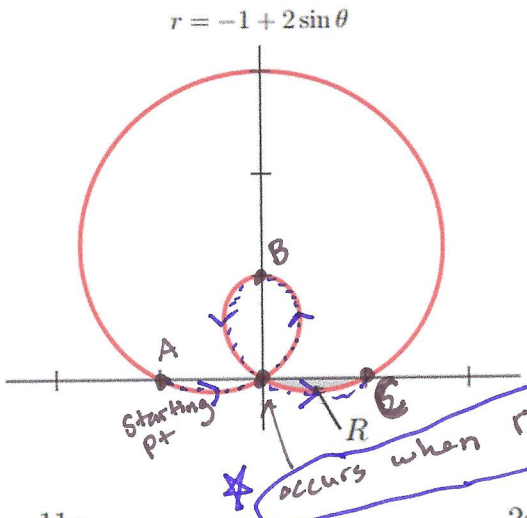
FROM SPRING 2016

The following problems were pulled from exams and could potentially be on Discussion Quiz 6:

12. If $\int_a^b \frac{1}{2}(-1+2\sin\theta)^2\,d\theta$ presents the area of the shaded region R below, find a and b.





θ	r	x	y	label
0	-1	-1	0	A
$\frac{\pi}{6}$	0	0	0	
$\frac{\pi}{2}$	1	0	1	B
$\frac{5\pi}{6}$	0	0	0	
π	-1	1	0	C

occurs when $r=0$

Set $r=0$ and solve for θ

$0 = -1 + 2\sin\theta$

$1 = 2\sin\theta$

$\frac{1}{2} = \sin\theta$

$\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

This info alone would rule out all choices except for E

a. $a = \frac{11\pi}{6}$ and $b = 2\pi$

b. $a = \frac{2\pi}{3}$ and $b = \pi$

c. $a = \frac{5\pi}{3}$ and $b = 2\pi$

d. $a = \frac{3\pi}{2}$ and $b = 2\pi$

e. $a = \frac{5\pi}{6}$ and $b = \pi$

Point C is $(1,0)$ on rectangular coordinate plane

$y=0$ means

$y = r\sin\theta = 0$

which happens at $\theta = 0, \pi, 2\pi$

From Spring 2017 EXAM

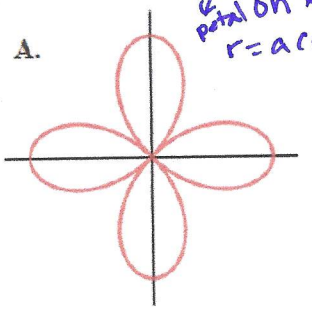
For the next 3 problems, worth 2 points each, match the given polar equations to the graphs. (Graphs may not be drawn to scale.)

12. $r = \cos(3\theta)$

13. $r = \sin(2\theta)$

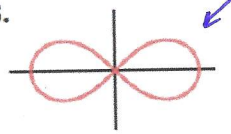
14. $r^2 = \cos(2\theta)$

A.



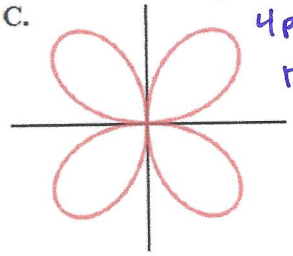
petal family $n=2$
petal on x-axis $\Rightarrow$ cos
$r = a\cos(2\theta)$

B.



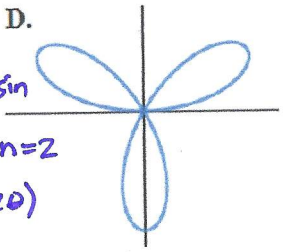
lemniscate
petal on x-axis $\Rightarrow$ cos
$r^2 = \cos(2\theta)$

C.



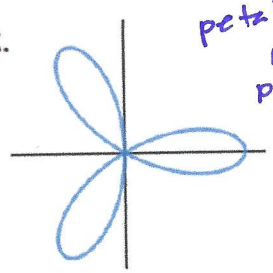
petal on 1st quad $\Rightarrow$ sin
4 petals $\Rightarrow n=2$
$r = \sin(2\theta)$

D.



petal family
$n=3$
petal on 1st Quadrant
$\Downarrow$
sin
$r = a\sin(3\theta)$

E.



petal family
$n=3$
petal on x-axis $\Rightarrow$ cos
$r = \cos(3\theta)$

12. r=cos(3θ) matches with graph E

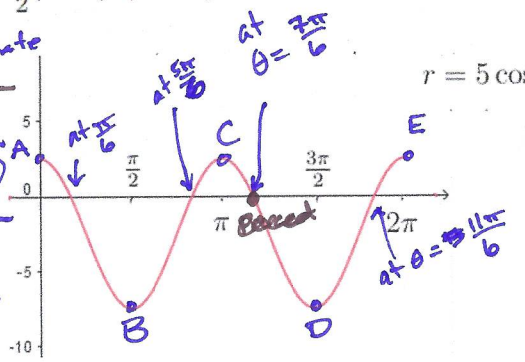
13. r= sin(2θ) matches with graph C

14. r^2 =cos(2θ) matches with graph B

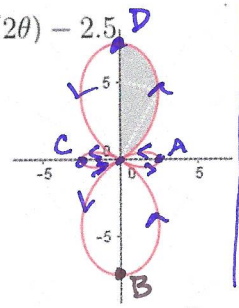
FROM SPRING 2017

15. $\int_a^b \frac{1}{2}(5\cos(2\theta) - 2.5)^2\, dx$ indicates the area of the shaded region. Find a and b.

label	polar (r, θ)	x-y coordinate (x, y)
A	$(2.5, 0)$	$(2.5, 0)$
B	$(-7.5, \frac{\pi}{2})$	$(0, -7.5)$
C	$(2.5, \pi)$	$(-2.5, 0)$
D	$(-7.5, \frac{3\pi}{2})$	$(0, 7.5)$



$r = 5\cos(2\theta) - 2.5$



Step 1: LABEL PEAKS and Valleys

Step 2: Convert to rectangular form and label points on graph

Step 3: Figure out which points correspond to shaded region

A. $a = \frac{\pi}{6}$, $b = \frac{5\pi}{6}$ B. $a = \pi$, $b = 2\pi$ C. $a = \frac{7\pi}{6}$, $b = \frac{3\pi}{2}$

D. $a = \frac{\pi}{2}$, $b = \frac{5\pi}{6}$ E. $a = \frac{\pi}{6}$, $b = \frac{\pi}{2}$

For Step 3: we know that point D and the origin point proceeding D were corresponding to the shaded region!

★ Additional Step: Find Origin Points (Set $r = 5\cos(2\theta) - 2.5$ equal to 0 and solve for θ) ★

★ Also for Step 2 I like to draw arrows indicating how the graph travels from A to B, B to C, so on and so forth ★

The proceeding origin point occurs when $\theta = \frac{7\pi}{6}$

Point D occurs when $\theta = \frac{3\pi}{2}$

So $a = \frac{7\pi}{6}$ and $b = \frac{3\pi}{2}$

From FALL 2016

13. Draw the curves on the same plane. At how many distinct point(s) do the curves intersect?

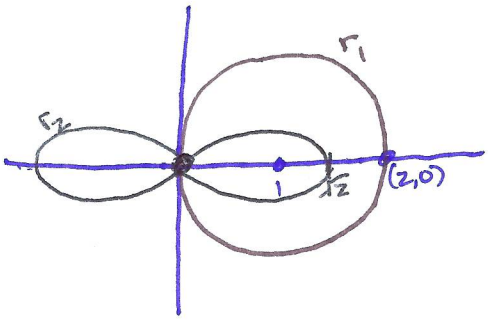
$$r_1 = 2\cos\theta, \qquad r_2^2 = 2\cos(2\theta)$$

A. 0 B. 1 C. 2 D. 3 E. 4

$r_1 = 2\cos\theta$ circle of radius 1 centered at $(1, 0)$

$r_2^2 = 2\cos(2\theta)$ lemniscate petals on x-axis

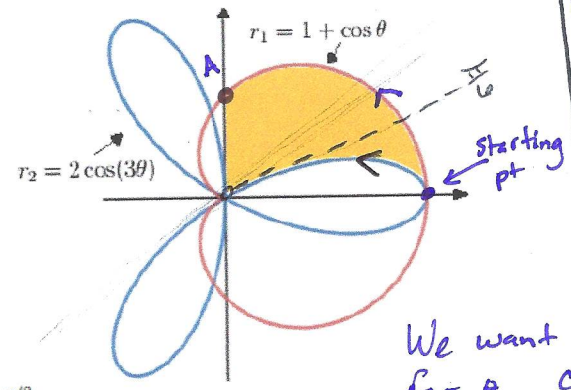
Size of petals will be $\sqrt{2}$



r_1 and r_2 only intersects at the origin (at one pt)

From FALL 2018 EXAM

9. Set up an integral that represents the area of the shaded region.



Note $r_1 = 2$ at $\theta = 0$ and $r_2 = 2$ at $\theta = 0$ Meaning both r_1 and r_2 have the same starting pt Tells us that the lower integral limit should be 0 for both r_1 & r_2

Formula for Area

$$\frac{1}{2}\int^{\beta} r^2\, d\theta$$

r_1 is top curve so we would want $r_1^2 - r_2^2$

A. $\frac{1}{2}\int_0^{\pi/2} (r_1^2 - r_2^2)\, d\theta$

B. $\frac{1}{2}\int_0^{\pi/2} r_1^2\, d\theta - \frac{1}{2}\int_0^{\pi/3} r_2^2\, d\theta$

C. $\frac{1}{2}\int_0^{\pi/2} r_1^2\, d\theta - \frac{1}{2}\int_0^{\pi/6} r_2^2\, d\theta$

D. $\frac{1}{2}\int_0^{\pi/3} r_1^2\, d\theta - \frac{1}{2}\int_0^{\pi/4} r_2^2\, d\theta$

E. $\frac{1}{2}\int_0^{\pi/2} r_1^2\, d\theta - \frac{1}{2}\int_0^{\pi/4} r_2^2\, d\theta$

We want to find at what ~~point~~ value for θ causes $r_1 = 1 + \cos(\theta)$ to hit A? For rectangular coordinate A the x value is 0 meaning

$x = r\cos\theta = 0$ which occurs when $\theta = \frac{\pi}{2}$

If we wanted to find the area of r_1 on the 1st Quadrant we would then want $\frac{1}{2}\int_0^{\frac{\pi}{2}} r_1^2\, d\theta$

θ corresponding to A

Starting point is on x-axis

*To find upper limit for r_2's integral, find value θ when r_2 first hits the origin (ie when $r_2 = 0$)

$r_2 = 2\cos(3\theta) = 0 \Rightarrow \cos(3\theta) = 0 \Rightarrow \theta = \frac{\pi}{6}$ $\quad \frac{1}{2}\int_0^{\pi/6} r_2^2\, d\theta$

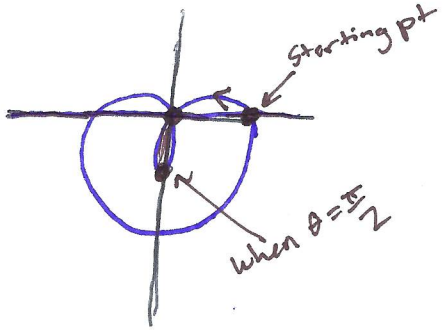
From FALL 2016

9. Let $A = 2\int_{\alpha}^{\pi/2} \frac{1}{2}(2 - 4\sin\theta)^2\, d\theta$ be the area of the inner loop of $r = 2 - 4\sin\theta$. What is the value of α?

limacon

A. $\alpha = \frac{\pi}{6}$ B. $\alpha = \frac{5\pi}{6}$ C. $\alpha = \frac{\pi}{3}$ D. $\alpha = 0$ E. $\alpha = \frac{\pi}{4}$

Using symmetry about the y axis



We need to find when $r = 2 - 4\sin\theta$ first hits the origin

Set $r = 0$ and solve for θ

$0 = 2 - 4\sin\theta$

$4\sin\theta = 2$

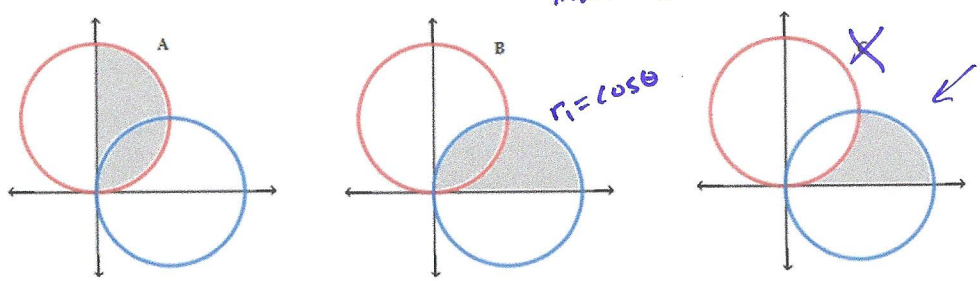
$\sin\theta = 1/2$ $\quad \theta = \frac{\pi}{6} \leftarrow \alpha$

FROM FALL 2018

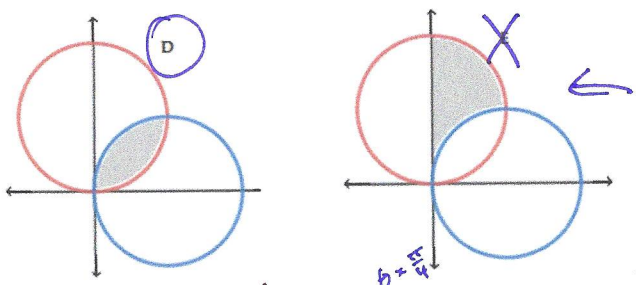
8. Given two polar curves, $r_1 = \cos\theta$, $r_2 = \sin\theta$. Which shaded region below is represented by the integral below?

$$\frac{1}{2}\int_0^{\pi/4} (\sin\theta)^2\, d\theta + \frac{1}{2}\int_{\pi/4}^{\pi/2} (\cos\theta)^2\, d\theta$$

Adding 2 areas together meaning we're not trying to find the area of region C + E

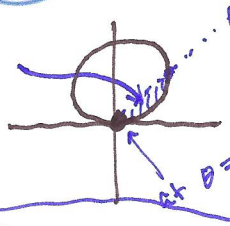


looks like $\frac{1}{2}\int_\alpha^\beta r_1^2\, d\theta \overset{\text{minus}}{-} \frac{1}{2}\int_a^b r_2^2\, d\theta$

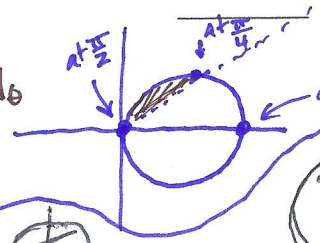


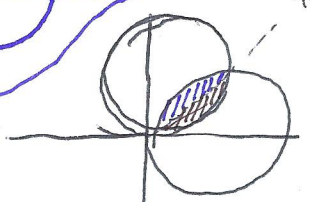
looks like $\frac{1}{2}\int_\alpha^\beta r_2^2\, d\theta \overset{\text{minus}}{-} \frac{1}{2}\int_a^b r_1^2\, d\theta$

$\frac{1}{2}\int_0^{\frac{\pi}{4}} (\sin\theta)^2 d\theta$

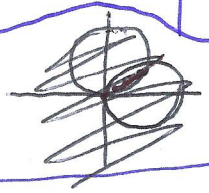


$\frac{1}{2}\int_{\frac{\pi}{4}}^{\pi/2} (\cos\theta)^2 d\theta$





So $\frac{1}{2}\int_0^{\frac{\pi}{4}} (\sin\theta)^2 d\theta + \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\cos\theta)^2 d\theta =$



FROM FALL 2018

6. If $\int_a^b \frac{1}{2}(r_1^2 - r_2^2)\, d\theta$ presents the area of the region inside the curve $r_1^2 = 50\cos(2\theta)$ and outside the curve $r_2 = 5$, in the first quadrant, find a and b.

lemniscate $a = \sqrt{50}$

circle radius 5 center (0,0)

A. $a = \frac{\pi}{3}$, $b = \frac{\pi}{2}$ B. $a = \frac{\pi}{6}$, $b = \frac{\pi}{4}$ C. $a = 0$, $b = \frac{\pi}{3}$

D. $a = 0$, $b = \frac{\pi}{2}$ E. $a = 0$, $b = \frac{\pi}{6}$

$a = 0$

$b \leftarrow$ point θ when $r_1^2 = r_2^2$

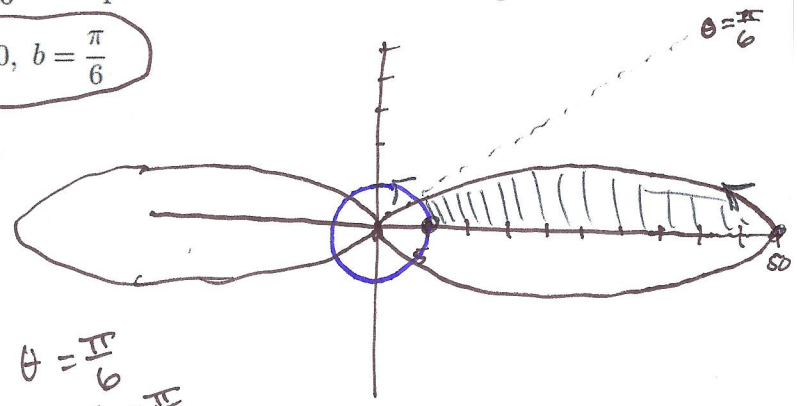
$25 = 50\cos(2\theta)$

$\frac{1}{2} = \cos(2\theta)$

$\cos^{-1}\left(\frac{1}{2}\right) = 2\theta$

$\frac{\pi}{3} = 2\theta \Rightarrow \theta = \frac{\pi}{6}$

so $b = \frac{\pi}{6}$



FALL 2016

14. Which choice below represents the area of the region that lies inside the first curve and outside the second curve?

circle radius $\frac{\sqrt{3}}{2}$ center $(0, \frac{\sqrt{3}}{2})$

$0 \le \theta \le \pi$ $\quad r_1 = \sqrt{3}\sin\theta, \quad r_2 = \cos\theta$ $\quad 0 \le \theta \le \pi$

circle radius $1/2$ center $(1/2, 0)$

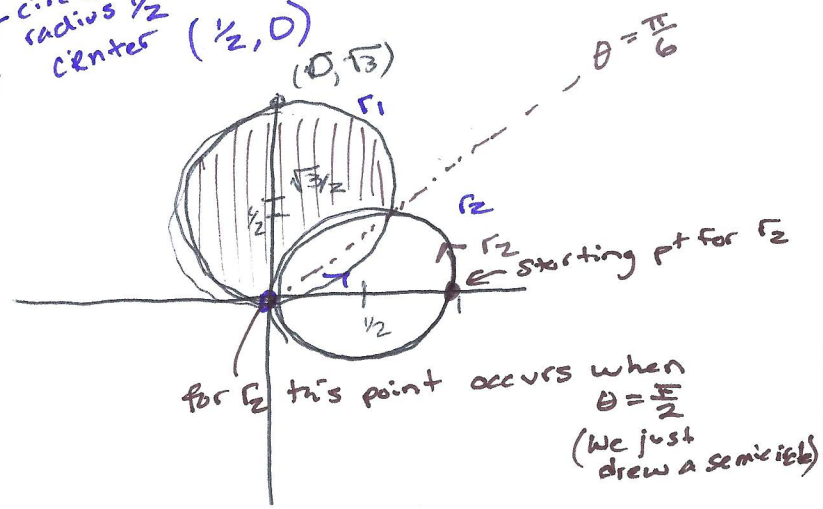
A. $\frac{1}{2}\int_{\pi/6}^{\pi/2} r_1^2\, d\theta - \frac{1}{2}\int_{0}^{\pi/6} r_2^2\, d\theta$

B. $\frac{1}{2}\int_{\pi/6}^{\pi} r_1^2\, d\theta - \frac{1}{2}\int_{\pi/6}^{\pi/2} r_2^2\, d\theta$

C. $\frac{1}{2}\int_{\pi/6}^{2\pi} r_2^2\, d\theta - \frac{1}{2}\int_{\pi/6}^{\pi} r_1^2\, d\theta$

D. $\frac{1}{2}\int_{\pi/6}^{\pi/2} r_1^2\, d\theta - \frac{1}{2}\int_{\pi/6}^{\pi} r_2^2\, d\theta$

E. $\frac{1}{2}\int_{\pi/6}^{\pi} r_1^2\, d\theta - \frac{1}{2}\int_{0}^{\pi/6} r_1^2\, d\theta$



Point of intersection : At $\theta = \frac{\pi}{6}$

$r_1 = r_2$

$\sqrt{3}\sin\theta = \cos\theta$

$\sqrt{3} = \frac{\cos\theta}{\sin\theta}$

$\sqrt{3} = \cot\theta$

$\frac{1}{\sqrt{3}} = \tan\theta$

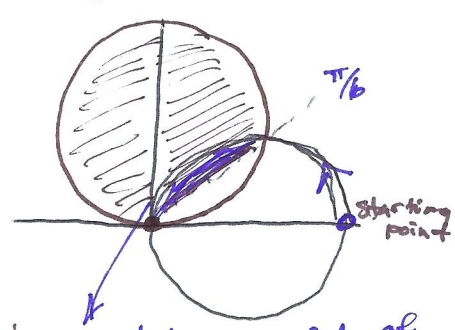
$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \tan\theta$

when $\sin(\theta) = \frac{1}{2}$ and $\cos(\theta) = \frac{\sqrt{3}}{2}$ $\quad \theta = \frac{\pi}{6}$

for r_1 θ ranges from 0 to π

to find area of r_1 use $\frac{1}{2}\int_{\frac{\pi}{6}}^{\pi} r_1^2\, d\theta =$

$\frac{\pi}{6}$ ← intersection pt



To get rid of purple region we find $\frac{1}{2}\int_{\frac{\pi}{6}}^{\theta} r_2^2\, d\theta$ ← when r_2 hits point of origin i.e. when $\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{2}$

Remember ~~r_2 ranges from~~ for r_2, θ ranges from 0 to π and starts at point $(1,0)$ to draw a semicircle stop @ $\frac{\pi}{2}$

We want to get rid of purple area.