WASHER Method

Rotated about x-axis OR any Horizontal line $(y = \#)$

$$V = \int_a^b \pi \left[OR^2 - IR^2 \right] \boxed{dx}$$

outer radius

inner radius

$a \le x \le b$

Rotated about y-axis or any vertical line $(x = \#)$

$$V = \int_a^b \pi \left[OR^2 - IR^2 \right] \boxed{dy}$$

$a \le y \le b$

Shell Method

Rotated about x-axis or any horizontal line $(y = \#)$

$$V = 2\pi \int_a^b \text{shell radius} \cdot \text{shell height} \, \boxed{dy}$$

$a \le y \le b$

shell radius $\perp$ to axis of rotation

shell height – parallel to axis of rotation

Rotated about y-axis or any other vertical line $(x = \#)$

$$V = 2\pi \int_a^b \text{shell radius} \cdot \text{shell height} \, dx$$

shell radius $\perp$ to axis of rotation

shell height – parallel to axis of rotation.

① Region: $y = x^2 + 1$
$y = 3 - x^2$

Washer $\pi \int_a^b OR^2 - IR^2 \, \boxed{dx}$

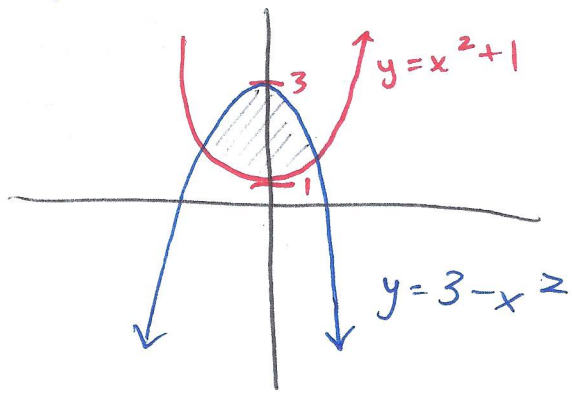
$\boxed{\text{Rotation: } x\text{-axis}}$

Shell $2\pi \int_a^b \text{shell radius} \cdot \text{shell height} \, \boxed{dy}$

(Canceling Choice B)

To find OR + IR
Graph functions to find top curve + bottom curve

OR = Top curve = $3 - x^2$
IR = bottom curve = $x^2 + 1$



$$V = \pi \int_a^b (3 - x^2)^2 - (x^2 + 1)^2 \, dx$$

Canceling choices C, D, and E

note
D would have been canceled also because the aren't squaring OR + IR

To find a + b set $x^2 + 1$ + $3 - x^2$ equal to each other and solve for x

$$x^2 + 1 = 3 - x^2$$

$$2x^2 - 2 = 0 \Rightarrow 2(x^2 - 1) = 0$$

$$2(x-1)(x+1) = 0$$

$$a = -1 \quad b = 1$$

Answer
A. $V = \pi \int_{-1}^{1} \left[(3 - x^2)^2 - (x^2 + 1)^2 \right] dx$

(2) $\displaystyle\int_0^1 \frac{5x+8}{x^2+3x+2}\,dx$

$$\frac{5x+8}{x^2+3x+2} = \frac{5x+8}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$\int_0^1 \frac{A}{x+1} + \frac{B}{x+2}\,dx = A\ln|x+1| + B\ln|x+2| \Big|_{x=0}^{x=1}$$

Use Cover Up method to find ~~a~~ A & B

$$A := \frac{5x+8}{x+2}\Big|_{x=-1} = \frac{3}{1} = 3$$

$$B := \frac{5x+8}{x+1}\Big|_{x=-2} = \frac{-10+8}{-1} = 2$$

$$\int_0^1 \frac{3}{x+1} + \frac{2}{x+2}\,dx = 3\ln|x+1| + 2\ln|x+2| \Big|_{x=0}^{x=1}$$

$$= 3\ln|2| + 2\ln|3| - \Big[\underbrace{3\ln|1|}_{=0} + 2\ln|2|\Big]$$

$$= 3\ln(2) + 2\ln(3) - 2\ln(2)$$

$$= \ln(2) + 2\ln(3)$$

$$= \ln(2) + \ln(3^2) = \ln(2) + \ln(9)$$

$$\cancel{= \ln(2)} \quad = \ln(2\cdot 9) = \ln(18)$$

Answer C: $\ln(18)$

$\boxed{3}$ $\displaystyle\int_{-\infty}^{0} \frac{e^{2x}}{e^x+1}\,dx$ u-sub!

1) Good First guess for u-sub is $u = \text{denominator}$ (might not always work but it's worth a try)

$u = e^x + 1 \longleftrightarrow$ $\boxed{\text{NOTE: } u - 1 = e^x}$

$du = e^x dx$

Changing limits of integration:

For $\boxed{b=0}$ $\int$ Stuff $\boxed{a=-\infty}$

(we either can change the limits of integration to be with respect to u) or (keep it like it is, integrate then resub and then take different of two values).

$u(b) = u(0) = e^0 + 1 = 1 + 1 = 2$

$u(a) = \lim_{x\to-\infty} u(x) = \lim_{x\to-\infty} e^x + 1 = 0 + 1 = 1$

$$\int_{-\infty}^{0} \frac{\overbrace{e^x}^{u-1}\,\overbrace{e^x dx}^{du}}{\underbrace{e^x+1}_{u}} = \int_1^2 \frac{u-1}{u}\,du = \int_1^2 1 - \frac{1}{u}\,du = u - \ln|u| \Big|_1^2$$

$= 2 - \ln(2) - 1 + \underbrace{\ln(1)}_{=0}$

$= \boxed{1 - \ln(2)}$

$\boxed{\text{Answer B}}$

4. $\displaystyle\int_0^1 \frac{x}{x^2+2x+2}\,dx = \int_0^1 \frac{x}{x^2+2x+1+1}\,dx = \int_0^1 \frac{x}{(x+1)^2+1}\,dx$

$$\boxed{\begin{aligned} &\text{Let } u = x+1 \\ &\quad du = dx \\ &\text{And } x = u-1 \end{aligned}}$$

$$\int_0^1 \frac{x}{x^2+2x+2}\,dx = \int_0^1 \frac{x}{(x+1)^2+1}\,dx = \int_{0+1}^{1+1} \frac{u-1}{u^2+1}\,du$$

$$= \int_1^2 \frac{u-1}{u^2+1}\,du$$

$$= \int_1^2 \frac{u}{u^2+1} - \int_1^2 \frac{1}{u^2+1}\,du$$

$$= \frac{1}{2}\ln|u^2+1| - \arctan(u)\Big|_1^2$$

$$= \frac{1}{2}\ln(4+1) - \arctan(2) - \frac{1}{2}\ln(2) + \underbrace{\arctan(1)}_{=\frac{\pi}{4}}$$

$$= \frac{1}{2}\ln(5) - \arctan(2) - \frac{1}{2}\ln(2) + \frac{\pi}{4}$$

$$= \boxed{\frac{1}{2}\ln\left(\frac{5}{2}\right) - \arctan(2) + \frac{\pi}{4}}$$

5) Compute $\int_0^{0.5} \arcsin x \, dx$ Yuck!

$$\int \arcsin(x)\,dx = \sqrt{1-x^2} + x\arcsin(x) + C$$

$$\int_0^{0.5} \arcsin x \, dx = \sqrt{1-x^2} + x\arcsin(x) \Big|_{x=0}^{x=0.5}$$

$$= \sqrt{1-0.5^2} + 0.5\arcsin(0.5) - \sqrt{1} - 0$$

$$= \sqrt{1-\tfrac{1}{4}} + \tfrac{1}{2}\arcsin\left(\tfrac{1}{2}\right) - 1$$

$$= \sqrt{\tfrac{3}{4}} + \tfrac{1}{2}\left(\tfrac{\pi}{6}\right) - 1$$

$$= \boxed{\frac{\sqrt{3}}{2} + \frac{\pi}{12} - 1}$$

$$= \frac{2-\sqrt{3}}{2} + \frac{\pi}{12}$$

What about

$\int_0^{1.5} \arcsin(x)\,dx = DNE$ (Well it ends up being an imaginary number of some sort)

$\arcsin(x)$ isn't defined on $x \geq 1$ or $x \leq -1$

$$\int_{-1}^{1} \frac{1}{x}\,dx =$$

6. $\int_0^{\frac{\pi}{2}} \cos^3(x)\sin^5(x)\,dx = \int_0^{\frac{\pi}{2}} \cos(x)\cos^2 x \sin^5 x\,dx$

$$= \int_0^{\frac{\pi}{2}} \cos(x)(1-\sin^2 x)\sin^5 x\,dx$$

$u = \sin x$

$du = \cos x\,dx$

$$\int_{\sin(0)}^{\sin(\frac{\pi}{2})} (1-u^2)u^5\,du$$

$$= \int_0^1 u^5 - u^7\,du = \frac{1}{6}u^6 - \frac{1}{8}u^8 \Big|_0^1$$

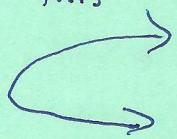
$$= \frac{1}{6}(1-0) - \frac{1}{8}(1-0)$$

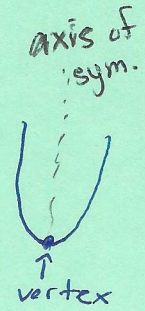
$$= \frac{1}{6} - \frac{1}{8} = \frac{8-6}{48} = \frac{2}{48}$$

$$= \frac{1}{24}$$

Answer A.

7. $x = \underbrace{y^2 - 2y}_{\text{Parabola}}$ but because it's in terms of y

It will look like this  rather than 



Recall: $f(x) = ax^2 + bx + c$ Vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

So for $x = \overbrace{y^2 - 2y}^{f(y)}$

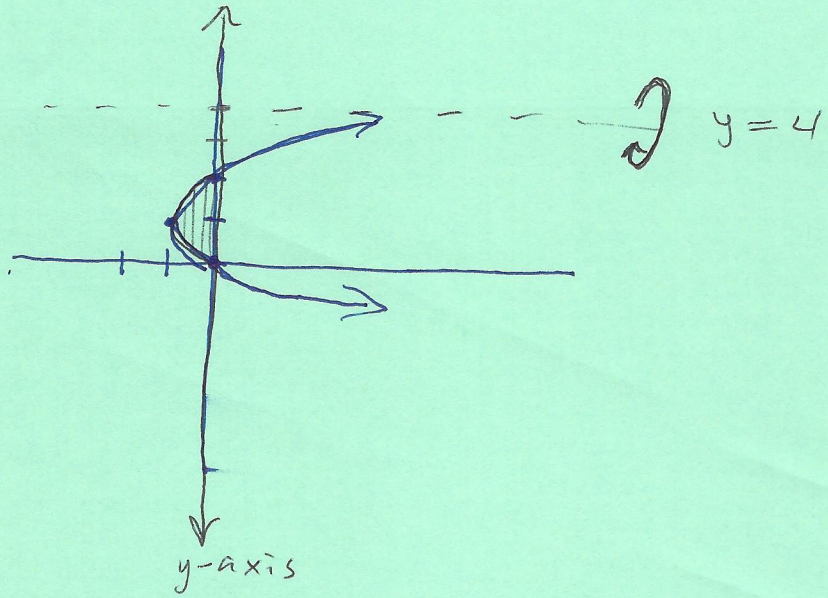
Vertex: $\left(f\left(-\frac{b}{2a}\right), -\frac{b}{2a}\right)$

swapping coordinates

Finding Vertex

$$-\frac{b}{2a} = -\frac{(-2)}{2 \cdot 1} = 1 \qquad y = 1$$

$$x = \overbrace{(1)^2 - 2(1)}^{f(1)} = -1 \qquad \text{Vertex: } (-1, 1)$$



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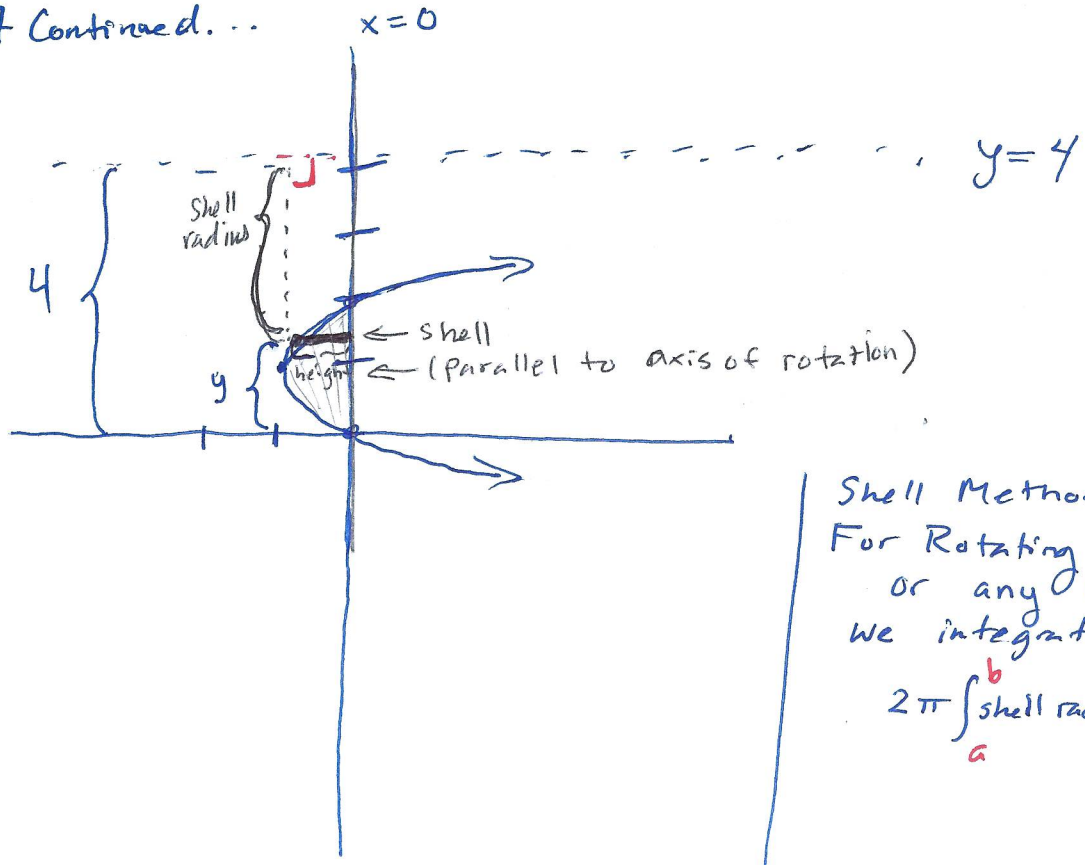


radius ⊥ to axis of rotation

Shell Method
For Rotating about x-axis or any horizontal line ($y = \#$)
we integrate with respect to y

$$2\pi \int_a^b \text{shell radius} \cdot \text{shell height} \, \boxed{dy}$$

$$a \le y \le b$$

Shell Radius: $4 - y$

Shell height: "Top" Curve − "Bottom" Curve
y-axis − Parabola

If we rotated picture 90° counter clockwise the top curve would by y-axis and bottom curve would be parabola $x = y^2 - 2y$

$$0 - (x) = 0 - (y^2 - 2y)$$
$$= 2y - y^2$$

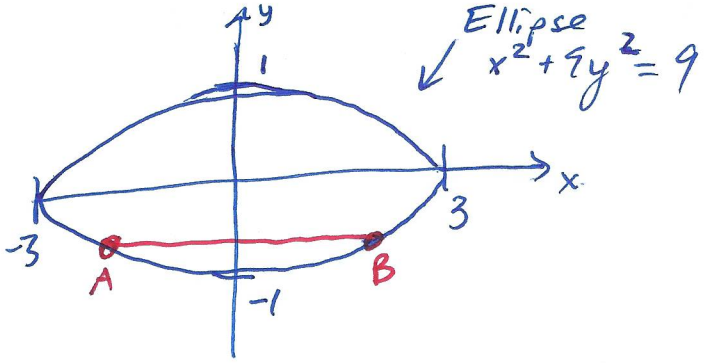
limits of integration: We're integrating with respect to y so we look at y bounds on the region

$$0 \le y \le 2$$

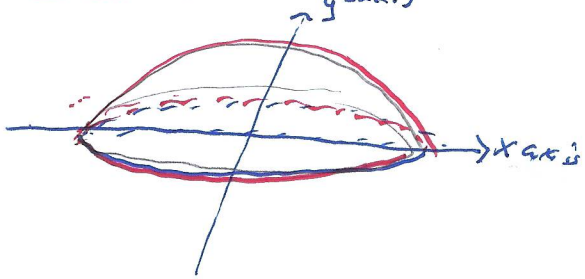
$$\boxed{V = 2\pi \int_0^2 (4-y)(2y-y^2)\,dy}$$

Answer is D

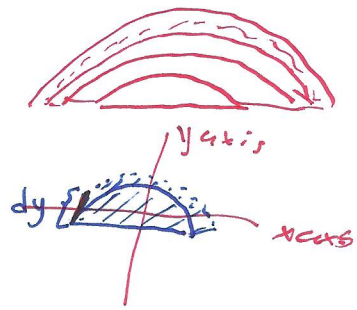
8.



3D Figure
Like a Football cut in half



"Cross-Sections $\perp$ to y-axis are semi circles"
Means we are slicing 3D figure horizontally
and Thickness of Cross-Sections will be dy



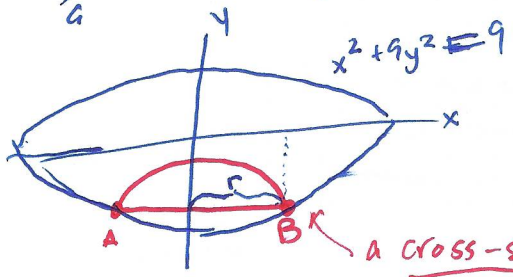
$$V = \int \text{area of cross-section} \cdot \text{thickness}$$

$$V = \int_a^b \text{area of semicircles} \cdot dy$$

Area of semicircle $= \frac{1}{2} \cdot$ Area of Circle
$= \frac{1}{2}\pi r^2$

$$V = \int_a^b \frac{1}{2}\pi r^2 \, dy$$

But What's r? And how do we write in terms of y?



$\boxed{\text{Diameter}}$ is length of line segment $\overline{AB}$.
Note points A and B lie on ellipse and the y-axis bisects line segment into two equal parts

Radius: Distance between y-axis and point B
Radius = x coordinate of B

Use $x^2+9y^2=9$ to write x in terms of y

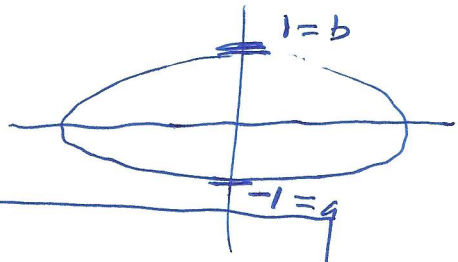
$x^2 = 9-9y^2 \Rightarrow x = \sqrt{9-9y^2}$

$$\boxed{r = \sqrt{9-9y^2}}$$

8. Continued

$$V = \int_a^b \frac{1}{2}\pi \left(\underbrace{\sqrt{9-9y^2}}_{r}\right)^2 dy$$

Bounds a, b will correspond to y bounds on Ellips



$$\boxed{\int_{-1}^{1} \frac{1}{2}\pi(9-9y^2)\,dy} = \int_{-1}^{1} \frac{1}{2}\pi\, 9(1-y^2)\,dy$$

Answer

$$= \frac{9\pi}{2}\int_{-1}^{1} 1-y^2\,dy$$

$$= \frac{9\pi}{2}\left[y - \frac{1}{3}y^3\right]\Bigg|_{y=-1}^{y=1}$$

$$= \frac{9\pi}{2}\left[1 - \frac{1}{3} - \left(-1 - \frac{1}{3}(-1)\right)\right]$$

$$= \frac{9\pi}{2}\left[1 - \frac{1}{3} + 1 - \frac{1}{3}\right]$$

$$= \frac{9\pi}{2}\left[2 - \frac{2}{3}\right]$$

$$= \frac{9\pi}{2}\left[\frac{4}{3}\right]$$

$$= \frac{3\pi}{2}\cdot 4 = \boxed{6\pi}$$