Limits

Limit # 1. $\lim_{n\to\infty} n \sin\frac{1}{n} = \lim_{x\to\infty} x \sin\frac{1}{x}$

$= \lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}} = 1$ by L'Hopital's Rule.

Definition 1. A **sequence** is a function whose domain is the set of all positive integers.

Definition 2. Let $\{a_n\}$ be a sequence of real numbers and let L be a real number. Then

(a) $\lim_{n\to\infty} a_n = L$ iff the following condition holds:

For each open interval U centered at L,

a_n is in U from some index N onward.

(b) $\lim_{n\to\infty} a_n = \infty$ iff the following holds:

For each interval $U = (M, \infty]$,

a_n is in U from some index onward.

(c) $\lim_{n\to\infty} a_n = -\infty$ iff the following holds:

For each interval $U = [-\infty, M)$,

a_n is in U from some index onward.

Theorem 6*. $\lim_{n\to\infty} a_n = 0$ iff $\lim_{n\to\infty} |a_n| = 0$.

Definition Let $\{a_n\}$ be a sequence of real numbers. Then the sequence $\{S_n\}$ given by

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n \quad \text{for } n = 1, 2, 3, \cdots$$

is called an **infinite series** and is denoted by $\sum_{n=1}^{\infty} a_n$.