Limits

Limit # 1. \( \lim_{n \to \infty} n \sin \frac{1}{n} = \lim_{x \to \infty} x \sin \frac{1}{x} \)

\[ = \lim_{x \to \infty} \frac{\sin \frac{1}{x}}{\frac{1}{x}} = 1 \]
by L’Hopital’s Rule.

Definition 1. A sequence is a function whose domain is the set of all positive integers.

Definition 2. Let \( \{a_n\} \) be a sequence of real numbers and let \( L \) be a real number. Then

(a) \( \lim_{n \to \infty} a_n = L \) if the following condition holds:

For each open interval \( U \) centered at \( L \),

\( a_n \) is in \( U \) from some index \( N \) onward.

(b) \( \lim_{n \to \infty} a_n = \infty \) if the following holds:

For each interval \( U = (M, \infty] \),

\( a_n \) is in \( U \) from some index onward.

(c) \( \lim_{n \to \infty} a_n = -\infty \) if the following holds:

For each interval \( U = [-\infty, M) \),

\( a_n \) is in \( U \) from some index onward.

Theorem 6*. \( \lim_{n \to \infty} a_n = 0 \) if \( \lim_{n \to \infty} |a_n| = 0 \).

Definition Let \( \{a_n\} \) be a sequence of real numbers. Then the sequence \( \{S_n\} \) given by

\[ S_n = a_1 + a_2 + a_3 + \cdots + a_n \] for \( n = 1, 2, 3, \cdots \)

is called an infinite series and is denoted by \( \sum_{n=1}^{\infty} a_n \).