Some Limits to Know

1. $\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^x = e^a$

2. Let k be a positive integer and let $a > 0$.

$$\lim_{x\to\infty}\frac{x^k}{e^{ax}} = \lim_{x\to\infty}\frac{k\,x^{k-1}}{a\,e^{ax}} = \ldots = \lim_{x\to\infty}\frac{k!}{a^k\,e^{ax}} = 0$$

3. Let k be a positive integer.

$$\lim_{x\to\infty}\frac{(\ln x)^k}{x} = \lim_{x\to\infty}\frac{k(\ln x)^{k-1}\left(\frac{1}{x}\right)}{1} = \ldots = \lim_{x\to\infty}\frac{k!}{x} = 0.$$

4. $\lim_{n\to\infty}\sqrt[n]{n} = \lim_{n\to\infty} n^{\frac{1}{n}} = \lim_{x\to\infty} x^{\frac{1}{x}} = \lim_{x\to\infty} e^{\frac{lnx}{x}} = e^0 = 1$ by #3

5. If c is a positive constant, then

$$\lim_{n\to\infty}\sqrt[n]{c} = \lim_{n\to\infty} c^{\frac{1}{n}} = \lim_{x\to\infty} c^{\frac{1}{x}}$$

$$= \lim_{x\to\infty} e^{\frac{\ln c}{x}} = e^0 = 1$$

6. $$\lim_{x\to\infty}\frac{a_\circ x^k + a_1 x^{k-1} + \cdots + a_k}{b_\circ x^j + b_1 x^{j-1} + \cdots + b_j} = \begin{cases} \frac{a_\circ}{b_\circ} \text{ if } j = k \\ 0 \text{ if } j > k \\ \pm\infty \text{ if } j < k \end{cases}$$

7. $\lim_{n\to\infty} n\,\sin\frac{1}{n} = \lim_{x\to\infty} x\,\sin\frac{1}{x}$

$$= \lim_{x\to\infty}\frac{\sin\frac{1}{x}}{\frac{1}{x}} = \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$ by Red #2, p 190