Music concert tickets — as well as many other admission-based events — are often sold in bundles, and the existing literature does not adequately address the composition of an optimal bundle this setting. This paper decomposes substitutability of component events into both the degree of horizontal differentiation between components and the additivity of consumers preferences across them. It considers a monopoly setting in which two component events may be offered separately or as a pure bundle (i.e., mixed bundling is treated as infeasible). The paper considers both the optimal pricing and bundling strategy when the degree of differentiation between the components is exogenously determined and the construction of the optimal bundle when the degree of differentiation may be selected by the monopolist. I find that, in terms of the degree of horizontal differentiation between the components, very similar and very different components are sold separately, whereas moderately differentiated components are offered as a bundle. I also demonstrate that capacity constraints affect the incentive to bundle — reducing (intensifying) it when the constraint applies to unbundled components separately (jointly). Moreover, capacity constraints decrease the level of differentiation between bundled components. Lastly, I find that duopolists offer more similar events separately than a monopolist would optimally offer as a bundle.

1 Introduction

Music concerts, like any other good, arrive at the marketplace after numerous strategic decisions by their producers. One such strategic decision, or practice, that appears ubiquitous is the concert arena is bundling.
This commonly observed pricing strategy, in which firms offer several unique component products as a single combined bundle, is in place every time that multiple acts are placed on the same ticket (e.g., opening acts, joint tours, and festivals). Previous inquiry into this pervasive business practice focuses on the incentive to bundle two (or more) products, but fails to address the composition of an optimal bundle. Deciding whether to bundle two existing products is an important business decision, but a common question in the concert ticket business is: which acts should be bundled? This paper examines the incentive to bundle, but its more important contribution surrounds the identification of an optimal bundle.

Music concerts demonstrate the importance of identifying an optimal bundle. Concerts are routinely bundled into multiple act events, and it stands to reason that the composition of these bundles is not random. Do event promoters bundle similar acts to increase consumers’ willingnesses to pay, or do they bundle more differentiated events to draw a larger crowd?\(^1\) This paper answers that question and demonstrates how consumers preferences and capacity constraints may affect the composition of the profit-maximizing bundle.

Bundling can take many different practical forms, but two broad categories of bundling strategies have been identified. Bundles are classified as either pure bundles or mixed bundles. Pure bundles consist of component products that are not available for separate purchase. Mixed bundles consist of components that can be purchased as a discounted bundle, or separately. Much of the bundling literature focuses on mixed bundling and demonstrates that mixed bundling is preferred (by a monopolist) to pure bundling for many different product types. Mixed bundling is an effective price discriminating strategy. This practice, however, is rarely observed in concert ticket pricing.

Music concerts are typically offered as a bundle or separately; there is no discount offered to consumers who purchase a ticket to more than one.\(^2\) Mixed bundling may be undesirable in this particular market for two reasons. First, if we are considering sequential acts in a single venue, it may be too costly or infeasible — or even inconvenient to consumers — to clear out the venue between acts and re-check tickets. This explanation is consistent with observing mixed bundling in classical music series, where acts play on separate evenings. Another reason that mixed bundling is not regularly observed in concert ticket sales may be the existence of binding capacity constraints. When capacity is constrained, there is no need to offer tickets to bundled component events separately, because the consumers with the highest willingnesses to pay are already filling

\(^1\)Consistent with Krueger (2005), I consider promoters to be the profit maximizers. The analysis, however, does not depend on selecting a particular profit maximizer.

\(^2\)I acknowledge that a discount could be offered for consumers that attend one concert on Friday night and another on Saturday night, but this practice is rarely observed outside of classical music series.
all of the available seats. Both practical feasibility and binding capacity constraints may explain the lack of mixed bundling in concert ticket sales.

This paper explores the incentive to bundle events when mixed bundling is not feasible and examines the determinants of an optimal bundle. The remainder of the paper is organized as follows. Section 2 presents a survey of the extant literature. Section 3 develops a model of horizontally differentiated products, in which consumers exhibit concave transportation costs. It considers a monopolist's optimal pricing and bundling strategies when the component products locations are exogenously assigned, and then, considers the same incentives when the component products locations are endogenously selected. I demonstrate that the incentives to bundle products are dependent both on the level of differentiation between the components and on the degree of additivity of consumers utility functions. Lastly, I examine the effect of capacity constraints on the incentive to bundle, finding that the unique structure of a particular constraint determines whether it reduces or intensifies the incentive to bundle.

2 Extant Literature

Much of the previous literature on bundling incentives focuses on the relationship between the products, or more specifically, on the relationship between consumers’ willingnesses to pay for them. Early work by Adams and Yellen (1976) and Schmalensee (1984) demonstrate that a monopolist's incentive to bundle is stronger when consumers’ willingnesses to pay for the two component products are inversely related. Schmalensee (1982) also shows that mixed bundling is a better price discrimination mechanism than pure bundling. When consumers’ valuations for two products are inversely related, the monopolist can incentivize more consumers to purchase both products by bundling them into a single product. Moreover, by offering a discounted bundle, alongside separately sold component products, the monopolist can enjoy the price discriminating effect of bundling without losing consumers whose net valuation for one of the components is positive, but whose net valuation of the bundle is negative.

The incentive to bundle can also be greatly affected by whether the products are complements, substitutes, or unrelated goods. Reservation values for complements are superadditive, while reservation values for substitutes are subadditive, or inversely related. Guiltinan (1987) shows that firms are more likely to (purely) bundle complementary goods, because their reservation values are strictly superadditive. Vankastesh and Kamakura (2003), however, present more nuanced results. They demonstrate that pure bundling is optimal
for strong complements, and selling components separately is optimal with strong substitutes. They find that pure bundling is strictly suboptimal (when also considering mixed bundling) for substitutable goods.

Each of the above papers describes circumstances under which pure bundles, mixed bundles, or individually sold components are optimal strategies for the monopolist, but none consider the construction of an optimal bundle. Some more recent marketing research (See, for example, Bradlow and Rao (2000) and Chung and Rao (2003)) has attempted to apply product design and attribute theory to bundle design, but their approaches have difficulty measuring the degree of heterogeneity between bundle components and characterizing the substitutability between components. The current paper examines the incentive to bundle horizontally differentiated components, identifies an optimal bundle design, and describes the role of both product differentiation and additivity of consumers’ utility in determining these incentives.

This paper makes a number of meaningful contributions to the theory of horizontally differentiated products and bundling. First, the model examines both the incentive to bundle two component products and the design of an optimal bundle. It also focuses on products that cannot be sold as a mixed bundle, such as admission to events. As most studies point to the desirability of mixed bundling as the profit-maximizing strategy, this approach shines light on a group of products that are often neglected by the bundling literature. This paper also considers the potential for multi-purchasing and concave transport costs in a Hotelling model, which have unfortunately been understudied. Section 3 continues with the construction an analysis of the model.

3 The Model

The model examines the incentive to bundle horizontally differentiated products when mixed bundling is not feasible. When bundling is desirable, and a continuum of components is available, the model identifies the seller’s profit maximizing bundling strategy. The analysis in this section is presented in the context of concert bundling, but it generalizes to any bundling situation in which mixed bundling is not possible. In particular, it generalizes to any scenario in which admission fees are charged for entry into an event such as film festivals and wine tastings.

The market is characterized by a monopolist seller who can choose between hosting and selling admission to two separate events, $E_1$ and $E_2$, or hosting a single combined event, $E_{12}$. The combined event $E_{12}$ is simply a bundle of component acts $E_1$ and $E_2$. There are no marginal costs of production included in
the model. For entertainment events (e.g., concerts or film festivals), zero marginal cost is a reasonable assumption. Fixed costs are not explicitly treated in the model, but as they are likely to be higher when the two events are offered separately, including them would only intensify the incentives to bundle that are examined throughout the remainder of the paper.

The seller maximizes profits in a static context by choosing whether to bundle the two events, selecting the (horizontal) location of the two events, and choosing the price for the product(s) offered simultaneously. Analytically, the decision or, incentive to bundle the two events is determined by which option offers the largest profits, given that the locations of the events and the admission prices would maximize the seller’s profits in either case. It is, thus, irrelevant to consider whether the bundling choice is made \textit{ex ante}. The focus of the proceeding section is to identify whether the monopolist has an incentive to bundle and if so, what component events it should optimally bundle.

Consumers differ in their horizontal preferences over the potential events. For example, in the context of music concerts, some consumers prefer rock music while others prefer folk music. Consumers’ preferences over the potential events are uniformly distributed along a Hotelling Line, and the utility derived by a consumer with preference location $\theta_i$ from attending a single-component event $E_j$ located at $l_j$ is given as:

$$U_{i,j} = V - |l_j - \theta_i|^5,$$

while a consumer with preference location $\theta_i$ that consumes both events $E_1$ and $E_2$, either as a bundle or separately derives utility:

$$U_{i,j} = (1 + \alpha)V - |l_1 - \theta_i|^5 - |l_2 - \theta_i|^5,$$

where $V$ is the gross valuation that a consumer receives from attending any single-component event, and $\alpha \in [0, 1]$ measures the desirability of attending a second event. The above utility function assumes that the consumers’ gross valuation of attending multiple events is weakly subadditive. That is, the gross valuation of attending a second event is either less than or equal to that of the second. This specification is adopted to capture the notion that a portion of the utility derived from a attending a particular event is not location-specific — consumers may simply enjoy attending an event — and that portion of the gross valuation is likely declining in the number of events a consumer attends. The disutility that a consumer receives from attending a non-ideal event (i.e., an event that is not perfectly aligned with the consumer’s preferences) is assumed to be concave in the difference between the consumer’s preferences and the horizontal location of
the event.

The concavity of “transport costs” assumption is supported by two related, practical considerations. First, consumers are more aware of differences between events that are closer to their own preferences. A fan of American folk music is more likely to distinguish between a Bob Dylan and a Willie Nelson concert than between a Madonna and Kylie Minogue. Fans of 1980s pop music would likely make the reverse distinction. Second, when two differentiated products are bundled, concavity of transport costs implies that consumers whose ideal component forms part of the bundle is more likely to consume the product than a consumer whose ideal component is located between the two components that form the bundled event. That is, with concave transport costs, an avid fan of one component act is more likely to attend a bundled concert than a moderate fan of both acts.

Thoroughly characterizing the firm’s optimal price and bundling strategies requires an investigation under both exogenous and endogenous location assumptions. These two assumptions address different practical situations; neither is more realistic than the other. Under an assumption of exogenous locations, I investigate bundling choices when the product offering is already determined. For example, a winery that produces Pinot Noir and Cabernet Franc faces the choice of selling tastings for each separately (as is common at Canadian wineries) or as a “flight” (as is more common at California wineries). Similarly, an entertainment firm that manages two acts whose concert tours will pass Tampa, Florida on the same weekend must choose whether to host separate events or combine the shows into a bundled event. Under both scenarios, the level of horizontal differentiation between the two products has already been selected when the price and bundling strategies are selected. Under an assumption of endogenous locations, I consider bundling decisions when the products variants can be selected from a pool of potential products. For instance, the winery may have a large number of different varietals (or even different offering within a single varietal) from which to design its tasting. It would be able to choose the level of differentiation between the two wines that is offers and whether to price them separately or together. Similarly, the entertainment firm may manage many acts and be able to select which acts to offer in Tampa. The ability of the firm to select the level of differentiation between its product offerings determines the applicability of an exogenous or endogenous locations assumption. Below I consider price and bundling strategies under both assumptions.

*Exogenous Product Locations: Components Sold Separately*

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3 The disutility that a consumer receives from attending a non-ideal event is referred to as “transport costs” in the literature related to horizontal product differentiation à la Hotelling. See, for example, d’Aspremont, Gabszewicz, and Thissse (1979), Economides (1986), and Cremer, Marchand, and Thisse (1991).
Identifying the monopolist’s incentive to bundle differentiated products requires a characterization of the profits associated with selling the components separately. Bundling is only desirable if it yields higher profits than selling the components separately. This subsection describes the monopolist’s profit-maximizing pricing strategy when it does not bundle under the assumption of exogenous product locations. I modify the traditional Hotelling Model to allow for the potential of multi-purchasing by a subset of consumers, identifying those consumers who will purchase both events even when they are priced and sold separately.

To begin, I identify the demand for each unbundled component. The demand for a particular component event is dependent on the location of both component events, because consumers can engage in multi-purchasing. If the components are not sufficiently different, some consumers choose to attend both events. When the components are sufficiently differentiated, however, no consumer chooses to attend both events. Figure 1 presents the two cases graphically.

If the events are sufficiently similar, some consumers will attend both events even when they are offered separately. Depending on the level of differentiation between the two events, it is possible that those consumers for whom one of the components represents their ideal event engage in multi-purchasing (Panel 1a), or only a group of consumers whose ideal event lies between the two component events engage in multi-purchasing (Panel 1b). In both cases, the consumer whose preferences place them at the midpoint of all purchasing consumers are the most likely to engage in multi-purchasing. Following the intuition laid out by Anderson
et. al. (2010), multi-purchasing in a Hotelling setting arises when a consumer may receive positive utility from consuming a second variant after they have purchased their most preferred variant. The independent choices to purchase each variant are only links through the degree of additivity of utilities. When preferences are strictly subadditive (i.e., $\alpha < 1$), the choice to purchase the “second” variant is only affected by consumption of the “first” variant through the reduction in the gross valuation (i.e., from $V$ to $\alpha V$).\(^4\) In the context of attending an event, this implies that if a consumer has a valuation of attending an event, and if that valuation is decreasing in the number of events they attend, the first event that a consumer chooses to attend only affects consumption of the second by reducing the consumer’s interest in attending an event. Prices, however, do not have a competitive effect between the events.

The potential for multi-purchasing gives rise to a discontinuity in the demand function when the preferences across the two components are strictly subadditive. When multi-purchasing occurs, some consumers of each event have the reduced gross valuation of attending that event, because it is their second event. As a result, the demand for component event $E_i$ is given as:

$$D_i(p_i, p_j, d) = \begin{cases} 
(V - p_i)^2 + (\alpha V - p_i)^2, & \text{if } d < (\alpha V - p_i)^2 + (\alpha V - p_j)^2 \\
2(V - p_i)^2, & \text{if } d \geq (\alpha V - p_i)^2 + (\alpha V - p_j)^2
\end{cases} \quad (3)$$

where $d$ is the degree of horizontal differentiation between $E_1$ and $E_2$ (i.e., $d \equiv l_2 - l_1$).\(^5\) When the components are sold separately, cross-price effects only arise through the effect of pricing on multi-purchasing. If the price of one event increases, it becomes less likely that any consumer chooses to purchase both products. Increasing the degree of horizontal differentiation between the two events has a similar effect. Additionally, if there are some consumers that do attend both events, decreasing $\alpha$ below 1 reduces the demand for both events. That is, whenever the two events share some attendees, and the gross valuation of attending an event is strictly subadditive, demand for each event is decreasing in $\alpha$.

In order to characterize the monopolist’s pricing strategy, I must consider their behavior both when the level of differentiation between the two events is low enough to facilitate multi-purchasing and when it is not. I pay particular attention to the monopolist’s pricing behavior around the multi-purchasing threshold.

Multi-purchasing only arises when 1) the component events are sufficiently similar, and 2) consumers’ valuation of attending a second event is sufficiently large. As products become more differentiated, and as

\(^4\)Here, “second” means less preferred, and “first” means more preferred. It is not necessarily linked to the order in which the products are consumed.

\(^5\)For simplicity, assume $E_2$ is located to the right of $E_1$ on the Hotelling line. That is, $l_2 - l_1 > 0$. 

consumers derive less utility from attending a second event, the monopolists’ ability to sell both events to a subset of consumers becomes less likely. The effect of differentiation is straightforward and follows directly from the conditional statements in the piece-wise demand function. The pricing strategy, conditional on the level of horizontal differentiation between the two events, is characterized as follows:

\[
p_i^* = \begin{cases} 
\frac{2 + 2\alpha - \sqrt{2(4\alpha - 1 - \alpha^2)}}{6} V, & \text{if } d < (\alpha - \frac{1}{3})^2 V^2 \\
\frac{V}{3}, & \text{if } d \geq (\alpha - \frac{1}{3})^2 V^2
\end{cases}
\] (4)

We can see that the monopolist sets its per-component price at \( p_i = \frac{V}{3} \) whenever \( \alpha \leq \frac{1}{3} \). For such strongly subadditive preferences, the monopolist maximizes profits by separating consumers into two distinct marketplaces. No consumers engage in multi-purchasing, regardless of the similarity between the two component events.

The effect of \( \alpha \) on the profit-maximizing pricing strategy in the presence of multi-purchasing is less straightforward, as it introduces an interesting tradeoff for the monopolist to evaluate when selecting its pricing strategy. The monopolist benefits from multi-purchasing behavior, as it provides the monopolist with a larger base of potential consumers. It, however, also faces the consequence that the gross valuation of an event is reduced for mass of its customers. That is, the number of potential consumers of each event is increasing, but the average gross willingness to pay is falling. This tradeoff gives rise to some an interesting relationship between the marginal gross valuation of attending a second event and the profit-maximizing pricing strategy. Figure 2 presents this relationship.

It can be seen easily that \( p_i^* = \frac{V}{3} \) for \( \alpha < \frac{1}{3} \). More interestingly, for \( \alpha \in \left[\frac{1}{3}, 2 - \sqrt{2}\right] \), \( p_i^* \) is decreasing in \( \alpha \). Over this range, the profit-maximizing pricing strategy, given that the multi-purchasing condition is satisfied, is characterized by falling prices as consumers’ willingnesses to pay for a second event increase. While a greater desire to attend a second event increases the number of potential consumers for both events through more common multi-purchasing, the average valuation of an attendee falls, placing downward pressure on prices. Once \( \alpha > 2 - \sqrt{2} \), however, further increases in the valuation of attending a second event place upward pressure on the price of both separately sold events.

**Exogenous Product Locations: Bundled Components**

Identifying the monopolist’s incentive to bundle differentiated products requires a characterization of the optimal bundling strategy and a comparison of the profits associated with that strategy to the profits
that are earned when the events are sold separately. Above, I identify the monopolists' profit-maximizing behavior when events are priced and sold separately. I must now identify the optimal pricing decision when the component events are offered as a bundle. I characterize the incentive to bundle as a function of both the degree of differentiation between the component events and the level of additivity of consumers' utilities.

To begin, I characterize the relationship between consumers’ preferences for the bundled product and the degree of horizontal differentiation between the component events. The two consumers whose ideal events are included in the bundle (i.e., the consumers located at $l_1$ and $l_2$) receive the greatest utility from consuming the bundled product.\(^6\) This implies that demand is only positive when the bundled price, $p_{12}$, and degree of differentiation are such that $(1 + \alpha)V - \sqrt{d} - p_{12} > 0$. Additionally, of the consumers with tastes located between $l_1$ and $l_2$, the consumer located at $\frac{l_1 + l_2}{2}$ receives the least utility from consuming the bundled product, implying that subset of consumers located between $l_1$ and $l_2$ abstain from purchasing the bundled product when $(1 + \alpha)V - 2\sqrt{d} - p_{12} > 0$. As a consequence, when the price or degree of differentiation increases above a particular threshold, some consumers located between $l_1$ and $l_2$ choose not to purchase the bundled product, but those consumers located at and near $l_1$ and $l_2$ continue to purchase the bundle. As a

\(^6\)his feature follows from the assumption of concave transport costs.
result, the demand for the bundled event is given by the following piece-wise function:

\[
D_{12}(p_{12}, d) = \begin{cases} 
\frac{d^2}{2[(1+\alpha)V-p_{12}]^2} + \frac{[(1+\alpha)V-p_{12}]^2}{2}, & \text{if } p_{12} \leq (1 + \alpha)V - 2\sqrt{\frac{d}{2}} \\
\frac{d^2}{2[(1+\alpha)V-p_{12}]^2} + \frac{[(1+\alpha)V-p_{12}]^2}{2} - [(1 + \alpha)V - p_{12}]\sqrt{2d - [(1 + \alpha)V - p_{12}]^2}, & \text{if } p_{12} > (1 + \alpha)V - 2\sqrt{\frac{d}{2}}
\end{cases}
\]  

(5)

Figure 3 provides a graphical representation of the relationship between consumers’ tastes and component characteristics. Panel A depicts the case in which the component events are not sufficiently differentiated to deter consumption by a set of consumers located between \(l_1\) and \(l_2\). Panel B depicts the case in which the component events are sufficiently differentiated. Both panels graph the location of the bundled product (L) and consumers’ individual preferences over the horizontally differentiated product attributes in the \(l_1, l_2\)-plane. Consumers are uniformly distributed along the 45-degree line, as the tastes of a given consumer i, \(\theta_i\), do not differ in their relation to the locations of the two component events. Those consumers that choose to purchase the bundle are identified by the darkened portion of the 45-degree line. Comparing the two panels, it is clear that increasing \(k\) above zero initially increases demand, but decreases after crossing a critical threshold.

![Panel 3-1: Continuous Demand](image1.png) ![Panel 3-2: Discontinuous Demand](image2.png)

Figure 3: Bundle Demand

The monopolist maximizes profits by selecting the bundled price, \(p_{12}\), according to:

\[
\max_{p_{12}} \pi_{12} = p_{12}D_{12}(p_{12}, d)
\]

(6)
The monopolist sets the bundled price such that the consumer whose preferences are located at the midpoint between the two component event locations receives no consumer surplus. The profit function is non-differentiable at the kink in the demand function. It is, however, monotonically increasing in \( p_{12} \) when \( p_{12} < (1 + \alpha) V - 2 \sqrt{\frac{d}{2}} \) and monotonically decreasing in \( p_{12} \) when \( p_{12} > (1 + \alpha) V - 2 \sqrt{\frac{d}{2}} \), implying that \( p_{12} = (1 + \alpha) V - 2 \sqrt{\frac{d}{2}} \) maximizes the monopolist’s profits.

If the monopolist chooses to bundle the two component events, it earns \( \pi_{12} = \frac{2d}{3} \left[ (1 + \alpha) V - 2 \sqrt{\frac{d}{2}} \right] \). Comparing the potential profits associated with bundling the component events to those earned when the events are offered separately, it can be seen that the incentive to bundle is dependent on the degree of horizontal differentiation between the two events and the marginal valuation of attending a second event. The following discussion regarding the incentives to bundle is presented as follows. For given ranges of \( \alpha \), the degrees of exogenously determined product differentiation that provide an incentive to bundle will be characterized. These relationships are also presented in Figure 4.

\( \alpha < 0.769 \): For low values of \( \alpha \), the monopolist always earns more profits selling admission to the two events separately. When, consumers do not place a high value on attending a second event, the monopolist is better off selling admission separately, regardless of the degree of product differentiation between the two events. Interestingly, this range of \( \alpha \) includes the range of \( d \) over which the profit-maximizing price of admission to each event is decreasing in \( \alpha \).

\( 0.769 < \alpha < 0.857 \) and \( \alpha > 0.884 \): Within each of these ranges of \( \alpha \), a single range of \( d \) over which the monopolist has an incentive to bundle the events arises. This range is bounded to the left and right, such that we may define it as \([d, \bar{d}]\), and \( d > 0 \). For very similar events (i.e., \( d < [d] \)), the monopolist earns greater profits by pricing admission separately and allowing a subset of consumers to engage in multi-purchasing. For sufficiently differentiated events (i.e., \( d > \bar{d} \)), the monopolist earns greater profits by pricing admission separately at \( p = \frac{V}{3} \) and eliminating the possibility for multi-purchasing.

\( 0.857 < \alpha < 0.884 \): Within this range of \( \alpha \), two unique ranges of \( d \) over which the monopolist has an incentive to bundle the events arise. Similar to the ranges described directly above, the monopolist is better off pricing two events that are characterized by very small and very large degrees of differentiation separately. Within this range of \( \alpha \), however, there is also a third range of \( d \) in which the monopolist does not choose to bundle the two events. This third range arises due to the upward jump in the profit-maximizing pricing strategy (when the component events are sold separately) when \( d \) is sufficiently large to prevent multi-purchasing. Within this range of \( \alpha \), that upward jump is large
enough to make bundling locally unprofitable.

When consumers do not place a high value on attending a second event (i.e., $\alpha$ is particularly small), bundling is not profitable. The monopolist earns higher profits selling the two products separately. Only when $\alpha > 0.769$ can bundling be profitable; the monopolist sells sufficiently similar or sufficiently different component events separately, but offers moderately differentiated events as a bundle.

The remainder of this section considers the effect of endogenizing the degree of product differentiation between the two events. This modification to the model addresses the issue of constructing the optimal bundle. I characterize the optimal degree of horizontal differentiation as a function of the additivity of consumers’ utilities.

*Endogenous Product Locations: Components Sold Separately*
As was demonstrated above, the monopolists’ profits are reduced when the products are sufficiently similar to give rise to multi-purchasing by a subset of consumers. The monopolist can avoid suffering these reduced profits when consumers’ preferences are not strictly additive by increasing differentiation. When \( \alpha < 1 \), the monopolist enjoys an increased willingness to pay for its component events if the degree of horizontal differentiation becomes sufficiently strong to deter any consumer from purchasing both products.\(^7\) That is, the monopolist increases its own profits by setting \( d > (\alpha - \frac{1}{3})^2V^2 \). The profit-maximization problem for the monopolist that sells component events separately is simplified to:

\[
\max_{p_i} \quad \pi_i = 2p_i(V - p_i)^2
\]

The firm sets a price of \( p_1 = p_2 = \frac{V}{4} \) for each of its events and sells \( D_1 = D_2 = \frac{8V^2}{3} \) units of each event. Consequently, the monopolist earns total profits (across the two events) equal to \( \pi = \frac{16V^3}{27} \). This level of profits arises for any degree of differentiation between the two events above a critical threshold maximizes profits. Profits are constant for any \( d \) that satisfies this threshold.

When consumers’ preferences across the two events are strictly additive, the resulting profits equal those obtained above. In this particular case, demand simplifies to \( D_i(p_i) = 2(V - p_i)^2 \), which is identical to the portion of the demand curve on which the monopolist operates above. The only distinction between strict subadditivity and strict additivity of preferences is that in the case of strict additivity, the monopolist does not have to satisfy a minimum level of differentiation between the two events. Any level of differentiation results in profits equal to \( \pi = \frac{16V^3}{27} \).

**Endogenous Product Locations: Bundled Components**

If the monopolist chooses to bundle the two events, regardless of the degree of differentiation between the component events, the monopolist maximizes profit by setting \( p_{12} = (1 + \alpha)V - 2\sqrt{\frac{d}{2}} \). Thus, the monopolist’ maximization problem can be stated as:

\[
\max_{d} \quad \pi_{12} = \frac{5d}{4} \left[(1 + \alpha)V - 2\sqrt{\frac{d}{2}}\right]
\]

The monopolist maximizes profit by setting \( d^* = \frac{2}{3} \left(\frac{(1+\alpha)V}{3}\right)^2 \) and \( p_{12}^* = \frac{(1+\alpha)V}{3} \) and earns a profit of \( \pi_{12}^* = \frac{10}{3} \left(\frac{(1+\alpha)V}{3}\right)^3 \). Comparing the maximum profits associated with bundling and those associated with

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\(^7\)I explain that the profits associated with the case of strict additivity of preferences (i.e., \( \alpha = 1 \)) results in the same level of profits when the events are not bundled below.
selling the component events separately, the incentive to bundle becomes quite simple. If the monopolist is able to select the degree of differentiation between the two component events, it chooses to bundle the two events when consumers sufficiently value attending a second event, when \( \alpha > 0.857 \). That is, the maximum profits that it can earn when bundling the two events exceeds those it can earn when the events are price separately if consumers exhibit a strong desire to attend a second event.

Interestingly, when the monopolist chooses to bundle the two events, it sets a degree of differentiation greater than that which it might select if it sells the component events separately. The profit maximizing degree of differentiation when the events are bundled is greater than the threshold level that was established above when considering separately priced events. This result implies that, depending on the values of consumers’ gross valuation of attending a second event, it is possible to see two events that are more closely aligned offered separately, while some more differentiated ones could be offered as a bundle.

In this section, I have characterized a monopolist’s pricing and bundling incentives when it offers two horizontally differentiated products and faces consumers with concave transport costs. Bundling only arises as a profit maximizing strategy when consumers derive a sufficiently large amount of utility from attending a second event, and that threshold level is most strict when the monopolist endogenously chooses the degree of differentiation between the two products. When the degree of product differentiation is exogenously determined, bundling is never observed when \( \alpha < 0.769 \). When consumers’ marginal gross valuation of attending a second event rises above this threshold, the monopolist chooses to bundle moderately differentiated products. Very similar and very dissimilar products are offered separately. When the degree of differentiation between the two products is endogenously chosen by the monopolist, bundling only arises if \( \alpha > 0.857 \). Interestingly, when this threshold is not met, the monopolist may choose to offer two events separately that are less differentiated than the bundle that is offered when the threshold is met.

### 4 Extension: Capacity Constraints

When considering events as the products being bundled, a logical extension is to consider the effect of capacity constraints. Courty (2003) claims that 39% of all shows sell out, implying that a binding capacity constraint has been met — unless, of course, the profit-maximizing quantity supplied just equals the size of the constraint. In the preceding section, bundling allowed the monopolist to extract additional surplus from each consumer and increased the overall number of consumers purchasing. A natural question to ask
is whether limiting the extent to which bundling can be used to attract additional consumers diminishes the
incentive to bundle. This section demonstrates that that is not the case; to the contrary, capacity constraints
may, in fact, intensify the incentive to bundle.

Capacity constraints can be applied to events in two distinct ways: 1) to each product (separately), or 2) to the
overall market quantity (jointly). The first method of applying a capacity constraint would entail constraining
each unbundled component at the same level as the bundled product, whereas the second would entail
constraining the sum of the unbundled components at the same level as the bundled product. Any binding
constraint on quantity, regardless of how it is applied to unbundled components, affects the pricing and
composition of the profit-maximizing bundle. The monopolist still sets the price and degree of differentiation
such that the consumer located at \( \frac{l_1 + l_2}{2} \) is indifferent between purchasing the bundle and not purchasing
the bundle (i.e., that consumer receives zero consumer surplus). If the capacity constraint is set at \( k \), this
implies that the profit-maximizing degree of differentiation is \( d^* = 4k/5 \), and price is \( p_{12}^* = (1 + \alpha)(V - 2\sqrt{2k/5}) \).
When the capacity constraint binds, the profits associated with bundling are given as \( k[(1 + \alpha)(V - 2\sqrt{2k/5})] \).
As a result, tightening the quantity restriction decreases the degree of differentiation and increases the price
of the bundled product.

As the total output associated with bundling is greater than that of unbundled components, capacity con-
straints initially affect only the profits associated with bundling. Capacity constraints that only bind when
the component events are bundled must reduce the incentive to bundle. It is less obvious what happens when
the constraint binds for unbundled components as well. By adopting one of the types of quantity restrictions
mentioned above, it is possible to fully characterize the effect of capacity constraints on the incentive to
bundle — including when capacity constraints affect the pricing of unbundled components.

*Capacity Constraints Imposed on Unbundled Components Separately*

Capacity constraints may be imposed such that each individual component faces the same constrained
quantity as the bundle. This type of quantity constraint is likely to arise with music concerts, in which the
constraint is imposed by the size of the venue. If two (bundled) acts are scheduled at the Tampa Theatre
on Friday night, the maximum quantity of tickets that may be sold is constrained by the number of seats.
If the two acts were offered as unbundled components, with one act playing Friday night and one playing
Saturday night, each event is independently constrained by the number of seats in the theatre. When each
act is offered separately, each may take advantage of the full size of the venue; the quantity sold of the
bundled event could not exceed that same occupancy limit.
For capacity constraints that are sufficiently large to bind only the bundled quantity, the incentive to bundle is maintained if: 

\[ k[(1 + \alpha)V - 2\sqrt{\frac{2k}{5}}] > \frac{16V^3}{27}. \]

When the constraint is restrictive enough to bind the quantity of unbundled components sold as well (i.e., \( k < \frac{8V^2}{9} \)), the monopolist chooses to bundle if:

\[
\max_d \pi_B - \pi_{NB} = k \left[ (1 + \alpha)V - 2\sqrt{\frac{2k}{5}} - 2V - 2\sqrt{\frac{k}{2}} \right] > 0. \tag{9}
\]

The relationship between the quantity restraint and the additivity of consumers utility is depicted in Figure 5. \(^8\) The shaded region is comprised of all combinations of \( \alpha \) and \( \hat{k} \) that maintain the incentive to bundle. Two interesting points arise out of this analysis. First, there is no binding constraint that does not reduce the bundling incentive, relative to the unconstrained equilibrium. The minimum level of additivity that maintains the incentive to bundle is \( \alpha > 0.859 \), compared to \( \alpha > 0.857 \) in the unconstrained equilibrium. Second, tightening the restraint beyond \( k = \frac{8V^2}{9} \) always reduces the incentive to bundle. That is, maintaining the incentive to bundle requires a higher value of \( \alpha \). This result is rather intuitive. If the monopolist is able to offer the two events as two separate products (unbundled components) and ease the effect of the capacity constraint, tightening the quantity restraint makes it more likely to do so.

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\(^8\) In order to simplify the graphical representation, I have placed an adjusted quantity constraint on the horizontal axis, where \( \hat{k} \equiv \frac{k}{V^2} \).
It is also reasonable to consider how the incentive to bundle is affected when the capacity constraint binds the sum of output (across the two unbundled components). Constraints may apply in this way for multi-venue or multi-stage events. Suppose, for example, that the Gasparilla Music Festival in Downtown Tampa can sell tickets to events are two different stages separately or jointly. Assuming that consumers will self-separate to fill each venue, the quantity constraint would apply not to the individual stages, but the total across the two. Regardless of whether admission to the two parks is sold as a bundle or unbundled components, a quantity constraint would apply to the sum of products sold.

When the overall market quantity of unbundled products is constrained at the same level of the bundle, and the constraint is restrictive enough to bind the quantity of unbundled components sold (i.e., $k < \frac{16V^2}{9}$), the monopolist chooses to bundle if:

$$\max_d \pi_B - \pi_{NB} = k \left[ (1 + \alpha)V - 2\sqrt{\frac{2k}{5}} - V + \sqrt{\frac{k}{4}} \right] > 0. \quad (10)$$

The relationship between the quantity restraint and the additivity of consumers utility is depicted in Figure 6. The shaded region is comprised of all combinations of $\alpha$ and $\hat{k}$ that maintain the incentive to bundle. Unlike the case of capacity constraints that bind unbundled components independently described above, there are some values for the capacity constraint that intensify the incentive to bundle, relative to the unconstrained equilibrium. Capacity constraints that apply to the sum of unbundled components can make bundling more likely. Additionally, tightening the constraint intensifies the incentive to bundle.

Figure 7 compares the two types of quantity restrictions and their effects on the incentive to bundle. It overlays Figures 5 and 6. It is clear that the incentive to bundle is always stronger with constraints that apply to unbundled components jointly. This result is intuitive; the monopolist is not able to avoid the constraint by splitting the bundle in this case.

Capacity constraints always weaken the incentive to bundle when they only bind on the (higher) bundled quantity. That is, when a quantity restriction that only affects the bundle is tightened, maintaining the incentive to bundle requires a higher value of $\alpha$. As the constrained capacity falls to the point that it affects the unbundled components, however, tightening the quantity restriction does not always reduce the incentive to bundle. If the restraint applies to unbundled components jointly, tightening a binding constraint increases the incentive to bundle.
Figure 6: Capacity Constraints Imposed On Unbundled Components Jointly

Figure 7: Capacity Constraints Compared

5 Conclusion

The preceding sections have examined the incentive to bundle events — focusing on the specific case of music concerts — and identified an optimal bundle. The extant literature focuses primarily on factors affecting the incentive to bundle components, but fails to characterize an optimal bundle adequately. Music concerts
(and more generally, admission-based events) often present the interesting question of what acts should be bundled. Additionally, the literature places much of its attention on mixed bundles, but this practice is not widely observed. Concert tickets are typically sold as a bundle or separately.

This paper addresses what products a monopolist would choose to bundle. Previous attempts (in the marketing literature) are incapable of simultaneously investigating differentiated products and maintaining a characterizable relationship between those products. That is, they describe how product attributes should differ, but they do not describe the degree of complementarity or substitutability between the product variants. Components, in this paper, are horizontally differentiated, and optimal bundles are characterized by the degree of differentiation between the component events. This setting allows for the identification of factors that affect both incentive to bundle and the composition of the optimal bundle (e.g., the additivity of consumers' preferences and capacity constraints).

This paper shows that both the incentive to bundle and the determination of the optimal bundle depend upon the level of additivity of consumers' utilities. When the monopolist cannot select the degree of differentiation between the component events (i.e., it cannot construct an optimal bundle), it chooses to offer very similar and very different components separately, but bundles moderately differentiated products. The monopolist bundles differentiated products to increase the demand for its bundle, but it also wants to limit the disutility that consumers suffer from attending an act that is substantially different from their individual tastes. Increasing the level of additivity exhibited by consumers' preferences typically increases the incentive to bundle, and results in a more differentiated optimal bundle. Capacity constraints also affect the incentive to bundle and the composition of the optimal bundle. Whether a capacity constraint reduces or intensifies the incentive to bundle depends on whether the constraint applies to unbundled components separately or jointly. If the constraint applies separately, the incentive to bundle is reduced, because the monopolist can offer the components separately and expand output. If the constraint applies jointly, the incentive to bundle is intensified.

Lastly, the paper demonstrates that duopolists may choose to offer less differentiated products than a monopolist that chooses to bundle. The duopolists differentiate their events enough to separate the markets — in the single firm setting, this level of differentiation corresponds to the critical level at which multi-purchasing is avoided. The monopolist, however, wishes to increase differentiation between bundled components above this level in order to extract additional surplus.


6 References


