

An Introduction to Sharp Permutation Groups

Let G be a finite group and X a G -set with associated permutation character θ . If $L = \{\theta(g) \mid g \in G - \{1\}\}$, then an elementary character theoretic argument yields

$$|G| \left| \prod_{k \in L} (\theta(1) - k) \right|.$$

When $|G| = \prod_{k \in L} (\theta(1) - k)$, G is said to be a *sharp permutation group of type $(L, |X|)$ on X* , and refer $|L|$ as the *sharp permutation rank* of G . This result is a special case of a result of Blichfeldt, in which θ is replaced by any (virtual) complex character of G . Characters for which equality hold (in the analogous inequality) are referred to as sharp characters of the group G .

In this talk, I'll provide an overview of results related to sharp permutation groups (with emphasis on the cases $|L| = 1, 2$), including recent work with Professor Peter K. Sin as well as problems for further investigation.