Fourier analysis on finite abelian groups and uncertainty principles

Qing Xiang

University of Delaware

(Joint work with Tao Feng and Henk D. L. Hollmann)

Let G be a finite abelian group. If $f : G \to \mathbf{C}$ is a nonzero function with Fourier transform $\hat{f}$, the Donoho-Stark uncertainty principle states that $|\mathrm{supp}(f)||\mathrm{supp}(\hat{f})| \geq |G|$. The purpose of this talk is twofold. First, we present the shift bound for abelian codes with a streamlined proof. Second, we use the shifting technique to prove a generalization and a sharpening of the Donoho-Stark uncertainty principle. In particular, the sharpened uncertainty principle states, with notation above, that

$$|\mathrm{supp}(f)||\mathrm{supp}(\hat{f})| \geq |G| + |\mathrm{supp}(f)| - |H(\mathrm{supp}(f))|,$$

where $H(\mathrm{supp}(f))$ is the stabilizer of $\mathrm{supp}(f)$ in G.