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Modeling the Chesapeake Bay and Tributaries: a Synopsis[1]

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ABSTRACT: The last decade has seen the development and application of a spectrum of physical and numerical hydrographic models of the Chesapeake Bay and its tributaries.

The success of the James River Hydraulic Model has initiated the construction of an estuarine hydraulic model of the entire Chesapeake System.

Numerical analogues for hydrographic behavior and contaminant dispersion in one-, two-, and three-dimensional model estuaries exist for various regions of the Bay. From an engineering viewpoint, one-dimensional models are sufficiently advanced to be routinely employed in aiding management decisions. Bay investigators are playing leading roles in the development of two- and three-dimensional models of estuarine flows.

Introduction

One of the main goals of research on the Chesapeake Bay is the eventual ability to quantitatively predict the effects of a given perturbation upon the estuarine system. In the hydrodynamic sense such prediction implies the use of models—physical, conceptual, or numerical analogues of the prototype dynamics.

The past seven years has witnessed a flurry of modeling activity on the Bay by the various research agencies. Unfortunately, however, there has been only limited exchange of information between the groups doing the work. In the interest of fostering better communication within the modeling community of the Bay, I have endeavored to assemble a qualitative summary of the major hydrographic and water quality simulations which were concerned with Chesapeake Bay.

Each modeling effort in the Bay area has been unique in many of its aspects. Nevertheless, in order to maintain some semblance of coherence, all endeavors have been grouped into five narrative sections.

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PHYSICAL MODELS

Before the advent of large numerical computers, the only hope of any reasonable analogue of Bay hydrography was to build small physical analogues of the area one wished to study. The U. S. Army Corps of Engineers has over 40 years of experience with such models at the Waterways Experiment Station (WES) in Vicksburg, Mississippi, and their thinking and methodology have dominated physical modeling in this area.

Normally, the Corps analogues are distorted Froude models with a 1000:1 horizontal reduction and 100:1 vertical reduction. As its name implies, the Froude model maintains the same ratio between inertial and gravitational forces as in the prototype, thus accurately simulating gravity wave phenomena.

Full dimensional similarity can never be achieved, however, and to accommodate this fact the models are "tuned" to correspond to tidal heights, tidal currents, and salinities. Even after such maneuvers, there is bound to remain a certain error in many variables, such as salinity (Hargis 1968, Hyer 1972).

To date the major effort in physical modeling of a Bay tributary has been coordinated by the Virginia Institute of Marine Science (Hargis 1965, 1966, 1968). To assess the influence of the deepening of the James River channel on the oyster production in the estuary, a model was constructed, tuned, and run at WES. Among other things, the model showed that the 0.1% change in cross-sectional area occasioned by the dredging project could increase stratification and thereby reduce the tidal flow by as much as 20% in certain reaches of the river. The reduction in upstream bottom flow was hypothesized to diminish the dispersal of seaward spawned oyster larvae (Nichols 1972).

After the completion of the James River project, the model was to be destroyed, but the Commonwealth of Virginia requested that the Army retain the model and use it to evaluate other intended James River projects. Since then at least eight separate research programs have been conducted in which the model was used to simulate perturbations caused by heated effluent from nuclear power plants, sewage treatment plants, spoil disposal and fill projects, and pollutant dispersal (Mason and Biggs 1969).

Flushed by success and public response to the James River model, the Corps is constructing a similar model of the entire Chesapeake Bay and tributaries at Mattapeake, Maryland (USACE 1970). Acquisition of prototype data (viz. Klepper 1972a, b) is complete.

Probably the major advantage of the physical models is the ease of visual realization of phenomena. The ability to observe tidal phenomena first-hand at a guaranteed time advantage, over a precise morphometry, is a distinct help to the researcher trying to formulate a hypothesis, or a politician trying to decide an issue.

The case against physical models is usually fought on economic grounds, and there is no denying that the capital involved in construction and maintenance is great. Other concerns, however, bother the researcher, such as the difficulty in obtaining copious quantitative data from the model. The most significant concern, however, remains the adequacy of dimensional similarity. Froude (gravitational-inertial) similarity is good. Reynolds (inertial-viscous) similarity is probably adequate in the horizontal dimensions, but not in the vertical. Prandtl (diffusive-viscous) and Grasshof (convective-inertial) similarity are questionable, and since many of the applications of physical models involve pollutant dispersal, one should be forewarned not to accept such model results as quantitatively definitive.

Analytical Models

The nonlinearities of the governing equations and complex geometries of the estuary normally preclude any analytical (i.e., paper and pencil) solution to estuarine hydrodynamic models. Nonetheless, there do exist a few examples of rudimentary models which are not numerical in nature.

Such models still provide the quickest and most economical way of estimating mesoscale phenomena such as plume-dispersion. Early efforts by Pritchard and Carter (1965) to predict the excess temperature distribution from a heated discharge were semiempirical in nature, consisting of a similarity transform (the heat-mass transfer analogy) on dye-dispersion data. Continued experience allowed for a more deductive approach (Pritchard and Carter 1972) whereby the nearfield plume was modeled by a patchwork of dimensional considerations, rough continuity calculations, and empirical relations garnered from earlier dye studies. Recently, Carter and Regier (1974) have extended this semiempirical approach to study the three-dimensional heated surface jet in a cross-flow.

An example of a more refined analytical model is Carter's (1974) effort to model the two-dimensional longshore dispersion of a sewage outfall. Carter predicts concentrations by applying the Okubo-Pritchard model for the horizontal distribution of a tracer from an instantaneous vertical line source in a fluid field with a constant longitudinal velocity.

There are limits, however, to what can be accomplished by analytical techniques. One should not expect more than on order-of-magnitude precision of any prediction from such a model. In the absence of any previous estimates, however, they provide a fast, inexpensive starting point for further study or action.

One of the more ambitious efforts at employing analytical schemes in modeling estuaries was made by Harrison and Fang (1971) when they used a modification of the Streeter-Phelps equation (Camp's equation) for oxygen depletion to study the effect of waste dumping at West Point, Virginia. Actually, this effort represents a finite-element approach to modeling wherein the estuary is divided into several macroelements within which the differential equations are solved and boundary conditions matched at segment interfaces. It stands as an example of the transition between analytical models and the numerical integration schemes of the next section.

Numerical Modeling

An analytical solution of the full equations of mass, momentum, and energy conservation for an estuary is generally out of the question. Fortunately, computers now offer an alternative method of mathematical simulation. The estuary is divided into a chain or lattice of small segments and an algebraic balance equation for mass, momentum, or energy is written for each segment for a small increment of time. The system of such algebraic equations forms the finite-difference approximation of the basic differential equations and constitutes what is generally accepted as estuarine modeling.

The numerical integration of the balance equations in all three dimensions is an extremely arduous (though, as we shall see, not impossible) task. It is not surprising, then, that the first models to evolve involved simplifying assumptions. Following the lead of the highly successful review on estuarine modeling (Ward and Espey 1971), we shall categorize modeling research as one-dimensional, two-dimensional, or three-dimensional according to the degree of spatial averaging employed. Pritchard (1958, 1971) has provided a review of the basic equations, simplifying assumptions, and averaging techniques employed in most estuarine models.

One-Dimensional Modeling

Often, one is interested only in the changes in a given variable along the longitudinal axis of an estuary. One is content to speak in terms of a property which is averaged in the vertical and lateral sense. In such a case the governing equations contain a single spatial independent variable.

The treatment of one-dimensional, well-mixed systems with a uniform geometry is amenable to analytical techniques, and the history of models in riverine systems is long (e.g., Streeter and Phelps 1925). The estuary, however, possesses complex geometry and inhomogeneities which generally preclude such a simplified approach. Okubo (1964) presents the assumptions and averaging techniques necessary to treat the estuary as a one-dimensional system.

Numerical techniques and computer technology had advanced during the early sixties, to where one-dimensional estuarine systems were being numerically modeled elsewhere. Then, in the late sixties, Hetling (1968) applied a model developed by Thomann (1963) to predict the longitudinal distribution of chloride concentrations in the upper Potomac estuary to determine whether that area could be used as a freshwater reservoir. The model is extremely simple, describing the tidally averaged rate of change of chloride in terms of advection by the freshwater input and diffusion according to a semiempirical dispersion coefficient.

Simple though the Thomann Model was, it nevertheless proved to be a highly successful engineering tool in conducting engineering analyses. Driven by their initial success, the EPA Laboratory at Annapolis, Maryland, began a concerted effort at modeling the Potomac. Data for such models were amassed (Jaworski and Clark 1972). The model itself was extended downstream to Pope's Creek and expanded to predict dissolved oxygen concentrations resulting from increased sewage loadings into the upper estuary (Hetling 1969).

The original Thomann Model was limited in its predictive capabilities because it only treated the tidally averaged continuity equation. To interject the rudiments of the hydrodynamic behavior to the estuary, Crim (1972) introduced a set of computer models that had been successfully applied to estuaries in the San Francisco-Delta area. Based on a one-dimensional momentum equation for velocity as a function of time along the river, these models could, nevertheless, take advantage of the rapid propagation of momentum in an estuary via inertial mechanisms (in comparison to advective processes) to simulate two-dimensional, vertically averaged, flow patterns via a network of one-dimensional channels. The flow pattern predicted by this "Dynamic Estuary Model" (DEM) could in turn (by invoking a quasi-steady-state assumption) be plugged into the species continuity equation by a passively transported substance.

Clark and Feigner (1972) offer a comparison of the Thomann and the EPA-DEM and include a detailed sensitivity analysis of the effects of various inputs on water quality predictions. Because of it's simplicity, the Thomann Model offers economy and speed. On the other hand, the EPA-DEM, because it includes tidal flow, can better predict the distribution of constituents with fast reaction times, e.g., nitrification.

Clark and Jaworski (1972) used the DEM to predict nitrogen, phosphorous, algae, and dissolved oxygen distributions throughout the Potomac estuary. Clark, Donnelly, and Villa (1973) likewise modeled the nutrient loadings to the upper Bay from the Susquehanna and Baltimore Metropolitan Region employing the DEM. The chemical kinetics of these models (with the exception of phosphorous in Clark's model) are assumed to be first-order. Although the observed data can be fit by linear schemes, this is no quarantee that the kinetics are, in fact, first-order, i.e., the model has been tuned, but not verified. The application of the models to situations outside the range of calibration data is, thus, questionable.

Crim and Lovelace (1973) presented two programs (the AUTØQUAL system) which consisted of a simplication of their previous work to strictly one-dimensional regimens along with a refined input/output technique. An explanation of the model derivation and accompanying program listings make the package easily accessible for engineering analysis of water quality problems in a variety of systems. In the continuity equations for dissolved oxygen, they included terms for carbonaceous and nitrogenous oxygen demand as well as a benthic respiration sink and photosynthesis-respiration processes. The hydrodynamics of the river were included via an empirical functionality between the depth and the flow through the segment. The particular functionality chosen allows for smooth transition between riverine flow and estuarine flow, where the constants involved are evaluated according to Manning's frictional hypothesis.

The two programs listed include AUTØSS, which predicts the final steady-state distributions of water quality to constant BOD loadings, and AUTØQD, which yields the quasi-dynamic response to the system to time varying inputs.

Lovelace (1973) expanded the planning and engineering utility of the AUTØQUAL system by modifying it to include non-point source loadings.

Pheiffer and Lovelace (1973) and Pheiffer (1974) employed Crim's models of the Patuxent River system to predict the water quality under projected wasteloadings of 1980 and 2000. The results of the simulation indicated that Maryland water quality standards in the riverine areas would be violated in those years without removal of the oxidizable forms of nitrogen. AUTØSS was also calibrated and verified for the Rappahannock estuary (T. Pheiffer, pers. commun.) where it was used by EPA in establishing wasteload allocations for the Fredericksburg, Va. area.

Hyer et al. (1971) used a similar variation of the Thomann Model to refine the model of Hyer (mentioned earlier) treating the O_2 response of the York, Pamunkey, and Mattaponi estuaries to wasteloadings at their confluence at West Point. Subsequently, the groups expanded both the geographical and technical scope of their one-dimensional efforts. Over the next two years, eight models were written for the Rappahannock (Fang et al. 1972) and the James (Fang et al. 1973) estuaries. The separate models were intended to compare the attributes of tidally averaged vs. real-time representations as well as explicit vs. implicit schemes for time integration of the governing differential equations.

Water quality models of more detailed biological-chemical systems in one-dimensional estuaries have appeared in the literature. Jaworski et al. (1971) extended the Orlob et al. (1969) model for the Potomac estuary to include the behavior of phosphate, ammonia and organic nitrogen, nitrate and nitrite, organic carbon, dissolved oxygen, and algae. The representation of algal growth, respiration, and death kinetics was empirical. This reference also summarizes all the previous Potomac River modeling and makes recommendations for future loading restrictions. Schofield and Krutchkoff (1973) undertook the ambitious task of creating a twelve-species ecosystems model of an estuary which includes seven chemical species as well as algae, protozoa, zooplankton, carnivores, and bacteria. Although it is billed as a stochastic model, the major emphasis of this work is upon the creation of a corresponding deterministic model to assess the mean values of the 12 parameters as a function of time and distance along the estuary. The driving hydrodynamics in the continuity equations are quite simplistic and verification (in actuality, calibration) is, as the authors acknowledge, quite tenuous, since it is based on data for only six of the parameters and involves an intuitive parameter regression scheme. Bard and Krutchkoff (1974) applied this model to the freshwater region of the James estuary and proceeded with a sensitivity analysis of the various parameters. No attempt was made to verify the model.

A major environmental question on the Chesapeake of late was how the increased diversion of freshwater by the newly enlarged Chesapeake and Delaware Canal would affect the longitudinal salinity gradient in the main stem. To answer this question, Boicourt (1969) developed a one-dimensional, linear, parobolic equation for the salt balance. The equation was integrated numerically over one-kilometer segments of the upper Bay. Recognizing that the amount of freshwater input to the head of the Bay has a strong effect on the "lumped" parameter for longitudinal dispersion, Boicourt derived an empirical relationship for the dispersion coefficient as a function of the river inflow using extensive Chesapeake Bay Institute salinity data and Susquehanna flow measurements.

Just as the longitudinal dispersion in the upper Bay is dominated by river input, many of the embayments around the edges of the Bay have their longitudinal flushing characteristics dominated by the salinity of the main stem at the mouth of the embayment. Recognizing this, Han (1974) simulated salt exchange in the Rhode River estuary using the same continuity equation as Boicourt, only allowing the dispersion coefficient to be proportional to the time rate of change of salinity at the mouth of the estuary.

Those who question the *a-posteriori* assumptions of Boicourt and Han are referred to the model of Kuo and Fang (1972) of salinity intrusion into the York-Mattaponi-Pamunkey confluence. Kuo and Fang employ the *a-priori* assumption that the boundary condition at the seaward end of the model can be estimated for the new time increment by a linear extrapolation of the salinities in the lower two segments at the present time. Unfortunately, this leads to an unstable "washout" condition under high flow pulses; i.e., once the salinity disappears from the lower end of the model, there is no mechanism by which it can return.

The complement of one-dimensional models on the Bay is rounded out by two efforts of a purely hydrological nature. Rives (1973) expressed the equations of momentum and continuity using the tidal surface displacement and the tidal flux as dependent variables. Integrating these equations over 15 segments of the Potomac estuary, he simulated the tidal behavior and pointed out the presence of standing wave phenomena. The node of the wave appeared on the model about 5 km downstream of Maryland Point and coincided with the area of calculated maximum energy dissipation.

Gardner and Pritchard (1974) integrated the hydrodynamic equations in the Chesapeake and Delaware Canal with the expressed purpose of estimating the increased flux of water due to canal enlargement. They used their results as input to Boicourt's (1969) model to predict the salinity changes in the main stem of the Bay.

Two-Dimensional Modeling

As is well known, the Chesapeake Bay is a partially mixed estuary possessing a significant vertical gradient in practically all hydrographic parameters. The cross-sectional averaging necessary in one-dimensional models blurs a great deal of significant physical phenomena and limits the degree to which the chemists and biologists can apply such model output to their systems, which usually exhibit strong vertical differences. Thus, in many applications it would be worthwhile to expend the additional effort and funds to employ a two-dimensional hydrodynamic simulation.

As a first approximation, Pritchard (1969) suggested the development of an estuarine model which is segmented normally in the longitudinal direction and bisected vertically into upper and lower segments. Wilson (1970) developed such a model for the Patapsco estuary to elucidate the flushing characteristics of Baltimore Harbor.

Previous measurements in the Patapsco indicated that the tidally averaged longitudinal currents changed direction at about 3 meters. This depth was therefore chosen to divide the upper and lower segments. Both longitudinal and vertical advection was considered, but measurements in the James River estuary indicated that only vertical diffusion was of relative importance. A tidally averaged continuity equation including the above terms was integrated with varying boundary conditions of seaward salinity and freshwater input. Four qualitatively different types of flows were identified, and the model predicted the steady-state discharge of contaminants of the four modes.

In a few years Wilson's (1973) technique had progressed, and he was able to represent accurately the tidal dynamics of the lower Potomac estuary over a more realistic two-dimensional grid. A simplification which allowed for economy of solution effort was the linearization of the longitudinal momentum equation by assuming that the density-induced pressure field is constant with time (having values determined by field measurement). It is fortunate that this assumption seems valid over the spatial and temporal domains involved. It is not universally valid, of course, and the modeling community would benefit from a dimensional argument to define the limits of the assumption. The key to the agreement between model and prototype dynamics lay in allowing the eddy viscosity to be a rudimentary function of depth. A somewhat surprising result was that the very same set of eddy viscosities could be used to reproduce both tidal and nontidal flows in the estuary.

To this time all effort at two-dimensional modeling has been directed toward the development of laterally averaged simulations. The appreciable vertical gradients and large length-to-width ratios of the major Chesapeake estuaries make this a reasonable line of development. To date there have been no efforts to create vertically averaged, two-dimensional models applicable to such Chesapeake locales as Mobjack Bay, Pocomoke Sound, Eastern Bay, Susquehanna Flats, etc., where horizontal differences are pronounced.

Three-Dimensional Modeling

The transition from two-dimensional to three-dimensional models involves an increase in programming effort and computer capacity well in excess of that required for the similar step from one- to two-dimensional representations. While three-dimensional simulations of meteorological, limnological, and marine systems have appeared in the literature over the past couple of years, the inaccessibility to estuarine investigators of the computers required for such studies had delayed the appearance of three-dimensional estuarine models until the past year.

Although many estuarine problems can be adequately dealt with using the directionally averaged models, others are truly three-dimensional in scope and require corresponding treatment.

Ironically, the conceptual framework for three-dimensional models is more straightforward than that of the spatially averaged models. This arises from the fact that not all phenomena can be adequately integrated into one or two dimensions. It is not too surprising, then, that the recent three-dimensional estuarine model of Caponi (1974a, b) is also quite comprehensive in its inclusion of phenomena. The nonlinear equations of motion are integrated over a gridwork of arbitary lateral geometry and bottom topography. Boundary conditions include the representation of river input, oceanic driving forces at the mouth, wind stress, and atmospheric pressure distributions. Density is considered a linear function of salinity, and Coriolis Force is included. The continuity of a number of hydrodynamically passive species can be included with little increase in storage requirements.

Unlike Wilson's model, the pressure field is determined dynamically by the algorithm. For a large, shallow estuary like the Chesapeake, the computational time expended to iterate upon the dynamical pressure field may be unnecessary. For certain particular applications, however, like the simulation of Langumir circulation cells, the dynamic pressure field is crucial (C. D. Mobley, pers. commun.).

Because he proceeded almost entirely on *a-priori* grounds, Caponi expressed the turbulent Reynolds stresses as simple Fickian-type functions of the velocities. Other assumptions would be hard to justify on *a-priori* grounds, but will eventually be necessary. The diffusion of momentum and of salt were assumed to be characterized by the same coefficients (turbulent Prandtl No. = 1). This assumption obviously needs refinement.

The model has been run on a simplified geometry (rectangular-box estuary) and an approximation of the main stem of the Chesapeake Bay. At present, however, the model has not been calibrated or verified (due in part to the unavailability of comprehensive data). Initial published trials (Caponi 1974a) on the Bay model did not predict two-layer flow or the east-west salinity gradient due to Coriolis Force, but this has recently been traced to defects in a *subroutine* coding. Predicted velocities and tidal amplitudes are still too large, and there is some question as to whether the large, horizontal, tidally averaged eddies are real phenomena or artifacts of the model. A finer gridwork (and large computer) are necessary to keep the edge effects from becoming so dominant throughout the water column.

While salinity tends to dominate over temperature as the cause of densimetric flows in the estuary, the Caponi model, nonetheless, suffers from the lack of a temperature field. There are seasonal times (e.g., late summer) and local conditions (large power plants) where temperature is a key element in determining meso-scale flow patterns.

Ongoing Projects

Numerical modeling of the Bay is presently a very widespread activity, and it is not the purpose of this summary to catalog all such research. The subject would seem incomplete, however, without brief mention of some of the projects underway which should substantially alter the state of the art of Chesapeake modeling within the next several months.

With regard to one-dimensional modeling, it is only a short time before all the tributaries of the Bay system will have been modeled at least once for water quality. Hydroscience, Inc. (Salas and Thomann, 1975) is developing a steady-state model of phytoplankton-nitrogen, phosphorus interaction throughout the entire Bay system for the State of Maryland Water Resources Administration. Although the model is one-dimensional in concept, a two-dimensional lattice of junctions and channels is used to attempt some simulation of lateral transport in the main stem of the Bay. In an independent effort, the modelers at the Virginia Institute of Marine Science are developing a series of one-dimensional models of the smaller Virginia tributaries.

At Johns Hopkins, A. F. Blumberg (pers. commun.) is developing a time-dependent, nonlinear, two-dimensional model for the dynamics in laterally homogeneous estuaries. The model accounts for the influences of tides, winds, turbulent mixing, and freshwater inflow, and it is being tested against data obtained in the Potomac.

A. J. Elliott, also of the Chesapeake Bay Institute, is wedding a one-dimensional model of pollutant concentrations in the upper, low-salinity portion of the Potomac estuary to a two-dimensional, laterally averaged network in the lower estuary.

At the level of three-dimensional simulations, Caponi is including a temperature field into this model and actively pursuing a more realistic representation of sub-grid-scale phenomena.

J. R. Hunter, Chesapeake Bay Institute, is working on a three-dimensional, steady-state kinematic model of the upper Bay. The model will have provision for a passive contaminant with a settling velocity so that the fate of sediment-borne pollutants can be predicted.

Finally, J. J. Lendertse (pers. commun.) relates that his group at RAND in Santa Monica, California, is developing a three-dimensional hydrodynamic model of the entire Chesapeake Bay System. The model includes a salinity and temperature field over a fine 3-km horizontal net. The depth includes as many as eleven grid points, and when finished should become the most detailed estuarine model to date.

Summary and Recommendations

The state of the art of hydrodynamic modeling in the Chesapeake Bay has made rapid strides in the past seven years. The mix of simulation efforts to be found in the area shows considerable breadth, so that anyone searching for a model to employ in a given situation has a wide choice. This breadth, however, tends to be quite compartmentalized; i.e., there is almost a one-to-one correspondence between the categories of models presented above and the several research organizations in the area. As a division of labor this might serve well. From an academic viewpoint, however, it is apparent that everyone involved in the field would benefit from greater cross-fertilization of ideas. Some informal mechanism to accomplish better communication among investigators is in order.

It is encouraging to note that in relation to modeling efforts elsewhere, the combined efforts of Chesapeake workers had, despite a late start, advanced to the forefront of research. This is especially obvious in regard to the two- and three-dimensional numerical models cited. There is reason for concern, however, in that this surge of accomplishment has been built upon a lead of copious hydrographic data acquisition. The present decline in field activity on the Bay could, if not reversed, eventually jeopardize continued modeling progress.

Finally, there remains the question as to what track future efforts will take. While the academic questions, such as those concerning sub-grid-scale phenomena, could remain formidable for decades to come, it is quite likely that, from an engineering viewpoint, hydrodynamic models will achieve a high degree of realism and precision within the next ten years. It is quite conceivable that development efforts will then shift to coupled chemical, biological, geological, and hydrographic models. Thus, those involved in writing hydrographic algorithms could expedite the advance of modeling by anticipating the eventual coupling of such phenomena.

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