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Growth and development: Variational principles reconsidered

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Abstract: Variational descriptions of macroscopic systems have gradually fallen into disuse. However, networks of ecosystem transfers are seen to grow and develop in a fashion best described as the optimization of a whole-system attribute called the 'ascendency'. This variational principle appears to synthesize and reconcile the observations of Lotka, Jaynes, Prigogine and Odum on how self-organizing systems evolve. It enhances the theory of dissipative structures by providing a framework for order in the universe that can accomodate the features of uniqueness, history, freedom and irreversibility.

Keywords: Organization, calculus of variations, networks, information theory, ecology

Background

During the early days of irreversible thermodynamics complex systems were considered to behave in a holistic fashion describable in thermodynamic terms (Prigogine, 1945). More specifically, the process of self-organization near thermodynamic equilibrium was thought to reflect the LeChâtalier–Braun principle in that evolving systems tended towards a state of minimum entropy production. However, the generalized 'forces', which must be identified in conjunction with any generalized, observed flow, usually remained obscure in highly dissipative ensembles. Who, for example, can quantify with any meaningful generality the force that carries the mouse into the stomach of the fox?

Attention then turned toward the significance of microscopic events as they affect and/or effect macroscopic dissipative structures. Perhaps the most revolutionary outcome was the discovery of a freedom from the determinism of the universal laws of physics. Prigogine almost rhapsodically declared that the way was now open to a new 'dialogue with nature'.

While it would be unfair to say that the Brussels school has abandoned macroscopic description, many earlier attitudes certainly now seem out-of-favor:

"More generally, the 'overall' behavior cannot ... be taken as dominating in any way the elementary processes constituting it". (176) [1]
"Optimization models thus ignore both the possibility of radical transformations ... and the inertial constraints that may eventually force a system ..." (207)

Reconsiderations

Nevertheless, the universal laws of thermodynamics are not to be denied, as Prigogine himself is quick to point out. Obviously, the Second Law may be cast in variational form. It is entirely compatible with the theory of dissipative structures. It is eminently universal and certainly macroscopic. But is it unique in these regards?

I think not. There are other universal phenomenological statements most conveniently expressed

[1] All direct quotations in this text are taken from Prigogine and Stengers (1984). The number in parentheses at the end of each quote refers to the page in which the excerpt appears.

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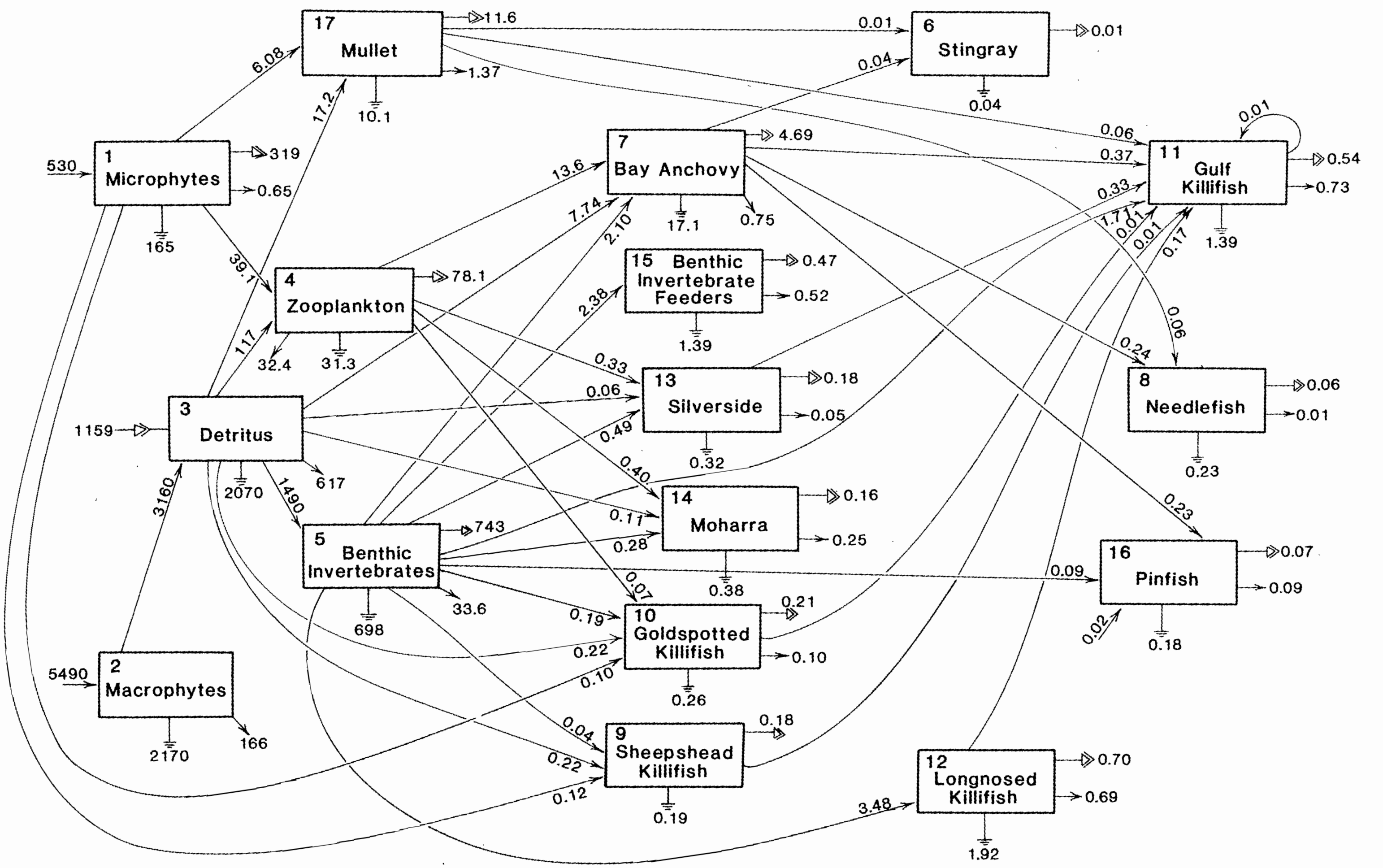

Figure 1. A schematic of carbon flows among the taxa of a marsh gut ecosystem, Crystal River, Florida. All flows are in (mg carbon)/m^2 day. The ground symbols represent respiratory losses. The linked arrows depict returns to the detritus. (Figure from Ulanowicz (1983)), data from M. Homer and W.M. Kemp, unpublished ms.)

as optimization principles, for example, "Living systems grow and develop". However, the thermodynamic interpretation of this observation is precisely what had foundered earlier. Prigogine provides a clue to a way out of this dilemma when he reiterates Whitehead and Heraclitus:

" 'The elucidation of the meaning of the sentence 'everything flows' is one of metaphysics' main tasks.' Physics and metaphysics are indeed coming together today in a conception of the world in which process, becoming, is taken as a primary constituent of physical existence ..." (303)

Could it be that the explicit invocation of thermodynamic forces was unnecessary for (or, worse still an impediment to) the articulation of growth and development?

It should not surprise many that efforts have been long underway in economics (Leontief, 1951), transportation, sociology (see Renfew this conference) and ecology (Platt et al., 1981; Fasham, 1984; Patten et al., 1976) to describe evolving systems in terms of their networks of transitions and flows (e.g., Figure 1). Henceforth, a quantified network of flows (or a temporal series thereof) will be considered to contain sufficient data with which to describe the evolution of a system.

An alternative description

When viewed solely in terms of flows, the growth and development of a system come to appear as two aspects of a unitary process. In order to see this it is helpful to distinguish four categories of flow that can occur in a system (See Figure 2): (a) The flows between any two compartments i and j within an n-compartment system are designated by T_{ij}. (b) The inputs to compartment i coming from outside the ensemble boundaries become T_{0i}. (c) The exports of still-usable medium from i out of the system are $T_{i,n+1}$. (d) Finally, the amount of medium that is dissipated (i.e., becomes unusable by any other compartment) by compartment i is $T_{i,n+2}$.

Now growth may be thought of as an increase in size. As the description is now limited to flows, the most natural way to gauge the size of a particular compartment is by the total amount of flow through that node. In general, one may either

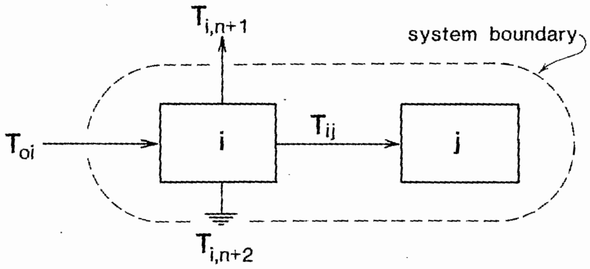

Figure 2. Representations of the four categories of flow that may occur in an n-compartment ecosystem. Flows between arbitrary compartments i and j within the system are labelled T_{ij}. Inputs to the system are treated as coming from a virtual compartment O. Exports of useable medium are assumed to flow to hypothetical compartment $n+1$, and dissipation of medium to $n+2$.

sum all the inputs,

$$T_i' = \sum_{j=0}^{n} T_{ji}, \quad i = 1, 2, \ldots, n+2,$$

or collect all the outputs,

$$T_i = \sum_{j=1}^{n+2} T_{ij}, \quad i = 0, 1, 2, \ldots, n.$$

Either way, the unique size of the entire system becomes the sum of the individual compartmental throughputs,

$$T = \sum_{i=1}^{n+2} T_i' = \sum_{i=0}^{n} T_i.$$

Growth is thereby represented as an increase in the total system throughput, T. The familiar gross natural product (GNP) in economics is calculated in virtually this same manner.

On the other side of the coin, development may be taken as an increase in organization. Quantifying the factor of organization requires some finesse, and space does not permit a full derivation here (see Hirata and Ulanowicz, 1984; Ulanowicz, 1986). Suffice it to say that an organized system is assumed to be highly articulated in that a flow issuing from any given node will engender flow in only a narrow subset of other locii. Rutledge et al. (1976) quantified such articulation by equating it to the average mutual information inherent in the flow network:

$$A = K \sum_{i=1}^{n} \sum_{j=1}^{n} (T_{ji}/T) \log(T_{ji}T/T_jT_i'),$$

where K is a scalar constant of proportionality. In Figure 3a each node exchanges medium equally with all other nodes, and articulation is minimal. In Figure 3b transfers are slightly more decisive, and in Figure 3c the network is maximally articulated.

The scale factor K is usually ignored by those who apply information theory, but here it becomes of paramount importance in establishing the size of a system. The most natural choice for K is to equate it to the total system throughput, T. Then the quantity A becomes the product of a factor of size and an index of organization and is given the name 'ascendency'. I submit that growth and development are cogently quantified by any increase in system ascendency.

Ascendency was not originally developed in epistemological fashion (Ulanowicz, 1980). Rather, its roots are phenomenological. Odum (1969) presented a summary of some 24 attributes thought to characterize mature ecosystems. Under appropriate conditions almost all the trends can contribute to a higher network ascendency. Whence, ecosystems appear to evolve so as to optimize the ascendency of their underlying network of transformations.

The unifying power of ascendency transcends ecology. Facets of other well-known hypotheses of self-organization are evident in the drive toward optimal ascendency. This may be illustrated, and simultaneously the limits to increasing ascendency may be described, by decomposing A into four terms:

$$A = C - (E + S + R),$$

where

$$C = -T \sum_{i=1}^{n} (T_i/T) \log(T_i/T),$$

$$E = -\sum_{i=1}^{n} T_{i,n+1} \log(T_i/T),$$

$$S = -\sum_{i=1}^{n} T_{i,n+2} \log(T_i/T),$$

$$R = -\sum_{i=1}^{n} \sum_{j=1}^{n} T_{ji} \log(T_{ji}/T_i').$$

In this form the ascendency may be increased by maximizing C and/or by minimizing any of the three terms in parentheses. C has the mathematical form of an informational 'entropy'. It serves as an upper bound on A, and is called the development capacity. One way C may increase is for the total system throughput, T, to rise. This will occur when species are maximizing their power throughput, a non-conservative strategy for survival advocated by Alfred J. Lotka and later by H.T. Odum (and Pinkerton, 1955). However, finite input flows and mandatory dissipations at each node serve ultimately to limit the rise of T. C also may be augmented by maximizing the informational 'entropy' factor (Jaynes, 1957). Network 'entropy' is increased by ever-finer partitioning among an increasing number of nodes, however, the finite availability of resources implies that some finely-partitioned nodes inevitably will become too small to persist in the face of chance environmental perturbations.

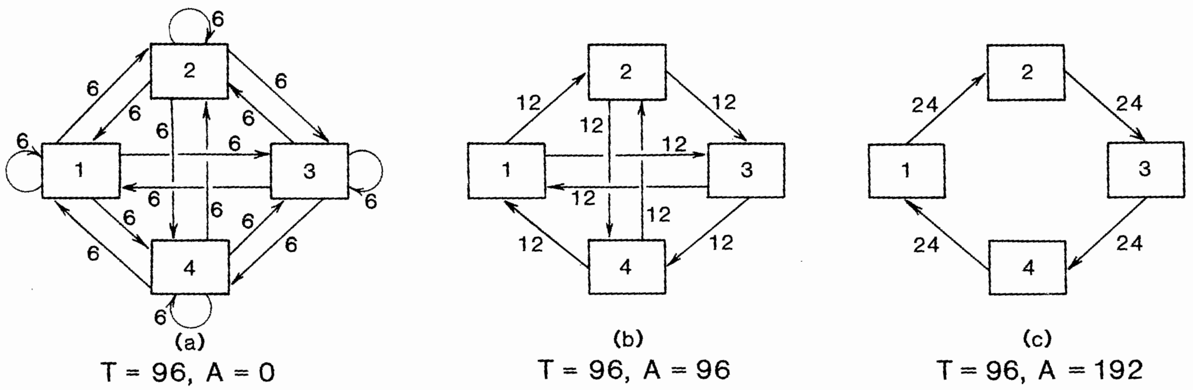

Figure 3. Three hypothetical, closed networks with increasing degrees of articulation. All three systems have identical total systems throughputs ($T = 96$ units). (a) The maximally connected and minimally articulated configuration. (b) The same compartments with an intermediate level of articulation. (c) The maximally articulated configuration of flows.

The three terms in parentheses collectively constitute a conditional uncertainty, referred to here as the systems' overhead. The first overhead term, E, is generated by transfers to higher hierarchical levels. Minimizing E fosters internalization. However, if the exports and imports of a given system both happen to be elements in a positive cybernetic loop at some higher level, then decreasing the exports from the given system might eventually diminish its own sustenance.

Minimizing the dissipation term, S, is an obvious analog to the early Prigogine (1945) principle. So long as resources are abundant, A is preferentially increased by a growing T or by a widening gap between capacity and overhead. Minimizing S under such conditions (e.g., embryonic growth) would be counter-productive. Later, however, after limitations become more prominent, minimizing S becomes an appropriate stategy to increasing A in mature systems.

The final term, R, rises with the number of redundant or parallel pathways in the network. Decreasing R results in a more streamlined and efficient network topology. However, it can also make for a more fragile structure. In systems with insufficient R perturbations at any point are propagated directly to downstream nodes, whereas a modicum of redundant pathways will allow for compensatory flows to affected compartments along less impacted lines of communication (Odum, 1953).

Reconciliation

In closing I wish to stress that this theory of macroscopic influence does not necessitate a return to Newtonian determinism. Ascendency optima can be both manifold and virtual. In many instances the process of increasing ascendency resembles less a fixed approach to a globally defined maximum as it does a movement up the local topography defined by the ascendency of the instantaneous network configuration. As the system progresses it is subjected to chance events, which in turn may alter the local topography. Optimal ascendency quantifies one-half of the dialogue which appears to be occurring between the macroscopic and the microscopic world (Allen, 1985).

Nothing about optimal ascendency directly contradicts the concrete results of the theory of dissipative structures. Uniqueness, history, irreversibility, freedom — all survive and may be celebrated within the context of optimal ascendency. However, while this new macroscopic view recognizes the open character of the world, it implies that such openness is not absolute. It is an exaggeration to speak of "fluctuations *compelling* the system to evolve towards a new state" (141) in the same sense that it was a mistake to believe that LaPlace's demon could control all events. Uniqueness, history, freedom, even irreversibility have their limits. But whatever degrees may have been lost from these attributes of the cosmos are more than compensated for by the grandeur in the vision of the universe as an interrelated unit, by the renewed dignity attached to human endeavor as a non-reductionistic behavior, and by the hope springing from a rediscovered sense of an order that pervades the world.

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