

Quantitative measures of organization for multiagent systems

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Received 31 July 2002; accepted 8 November 2002

Abstract

A set of “information theoretic” measures has been developed to quantify the degree of constraint inherent in the organization of a multiagent system. Separate measures can be provided to quantify spatial organization, trophic organization and, more generally, the overall structure of interactions. The additive character of these quantities allows them to be distributed in various fashions among species and places in a way that allows one to assign an “Importance Index” to those taxa and places. In addition, a measure to gauge the degree of adaptation of a species to a particular environment is proffered. The proposed measures allow one to formulate the following hypotheses in quantitative fashion: (1) that any disturbance of an ecosystem at a location associated with a high spatial Importance Index will exert a greater impact on the population dynamics than will a similar disturbance aimed at a place where the values of these indexes are lower; (2) that any disturbance in an ecosystem affecting a particular species with high individual Importance Indexes will cause a greater impact on the overall population dynamics than will a disturbance aimed at a species with a lower values of these indexes; (3) that the ascendancy of evolving system has a propensity to increase. The precise quantitative formulation of these hypothesis would permit them to be tested via multiagent simulation. Estimating the probabilities pertaining to these hypotheses presents a number of problems that merit discussion.

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Keywords: Multiagent systems; Average mutual constraints; Ascendancy; Spatial organization; Structural organization; Adaptation

1. Introduction

We wish to introduce quantitative measures of organization applicable to multiagent systems. Computer scientists and sociologists have used the term “agent-based modeling”, while ecologists prefer “individual-based modeling” to describe this modeling approach, however, both terms refer to essentially similar approaches. The earliest review

on individual-based models was by [Huston et al. \(1988\)](#), and the subject has been reviewed a number of times thereafter ([Hogeweg and Hesper, 1990](#); [Lomnicki, 1992](#); [DeAngelis et al., 1994](#); [Judson, 1994](#); [Uchmanski and Grimm, 1996](#); [Grimm, 1999](#); [Grimm et al., 1999](#); [Berec, 2002](#); [Breckling, 2002](#)). The multiagent modeling approach was reviewed in a series of papers that appeared in [Sichman et al. \(1998\)](#).

Organization is a fundamental concept that is highly relevant to many disciplines. To an ecologist, an economist or a sociologist the definitions of organization may appear to be different, but there is an underlying common essence. Among the first to describe this essence was [Simon \(1973\)](#), whose concept of development derived from a theory of complex

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hierarchical systems. He saw the core of organizational structures in administrative and economic systems as framed around the notions of efficiency, control, and personal contributions by the individual (Simon, 1952; Mintzberg, 1983). Although the importance of the concept of organization has been widely acknowledged by economists and social scientists (Lomi and Larsen, 2001; Hannoun et al., 1998; Prietula et al., 1998; Matutinovic, 2002), to the best of our knowledge, a clear, formal theory of organization, so crucial to the development of a generalized systems science, remains largely wanting.

The problem of describing organization seems to involve two types of questions—those qualitative and others that are quantitative. Examples of the first kind of questions are: What is organization? How is an organized system different from one that is disorganized? Questions of the second sort center around how to measure organization? We begin with the most general question of the first sort, namely, “What is organization?”.

Since the work of Ashby (1962), *system organization* has been understood as a dynamic equation that binds the parameters of the system. Such an approach is perfectly appropriate for those systems that can be described in terms of differential equations, and it has long been an implicit part of General Systems Theory. While this definition captures the functional aspects of organization, it ignores the structural aspects, and these limitations have been discussed by Krivov et al. (2002).

Recent studies in complexity have generated a variety of models, such as Cellular Automata, Boolean Networks, and Multiagent Systems, that address particular aspects of the behavior of complex systems. These models have made significant contributions to our understanding of the notion *organization*. It became apparent that very complex organization could be generated by the interaction of uncomplicated objects under extremely simple behavioral rules. For example, Reynolds (1987) demonstrated that patterns in flocking birds would emerge if individuals were to follow just two simple rules: (1) keep a close distance to one’s two nearest neighbors, and (2) avoid obstacles.

Studies in complexity have lent new accents to our understanding of organization. Organization has implicitly become associated with the set of all unique patterns contained in a system. Crutchfield (1994)

was among the first to explicitly define organization as set of patterns, and formulated a program of study of organization following this avenue. This work goes under the rubric of *computational mechanics* (Crutchfield, 1994; Shalizi and Crutchfield, 2001; Shalizi et al., 2003). Crutchfield’s approach is computational, algebraic, and stochastic, all at the same time (Shalizi and Crutchfield, 2001). Computational mechanics, as it has developed to date, pertains only to one-dimensional, stochastic processes that are modeled by symbolic sequences. It remains unclear how this theory might be extended to describe the organization of complex multiagent systems or networks.

Another formal approach based on an understanding of organization as a set of patterns contained in a system is called Logic Modeling of System Dynamics (LMSD) (Krivov et al., 2002). LMSD is based upon the premise that states of a system can be adequately described as *models* in the sense of predicate logic (Ershov and Palutin, 1989; Suppes, 1969). Such models provide semantics for the formulae from predicate calculus—boolean expressions with variables. Patterns are associated with local self-similarities in the system. The existence of an isomorphism between two subsystems is formally equivalent a local self-similarity between them. Krivov maintained that *patterns* within a system may be described using formulae containing free variables to provide a uniform description of those parts of the system that are similar. LMSD is a powerful way to describe patterns in systems and can be applied to arbitrarily complex systems—such as ecologies and economies. The notion of pattern that it provides, however, is not *stochastic*. Furthermore, LMSD does not in itself provide a direct means for quantifying organization. Whence, the quantitative treatment of organization developed in this present paper might be considered a modest first step toward developing a stochastic version of LMSD.

The other fundamental type of question pertaining to organization are those describing its quantitative measurement. Every purely quantitative aspect of organization is seen, however, to exhibit multiple facets. Most quantitative studies of organization are focussed upon trying to measure complexity,¹ but there are other

¹ See extensive bibliography on “complexity” at <http://www.fmb.mmu.ac.uk/~bruce/combib/>.

important aspects to organization that are too often ignored.

It is intuitively obvious that certain purely structural aspects of a system can impact the character of its dynamics. For networks one such aspect is the connectivity, which is defined as the average number of connection per node. Kauffman (1991, 1993) related the connectivity of boolean networks to their overall dynamics and demonstrated that only within a certain limited range of connectivities can a system exhibit self-organized behavior. One important aspect of flow networks is the number of distinct functional roles that exist within a given network. Zorach and Ulanowicz (2003) demonstrated that real ecosystems are confined to a narrow window of parameter space, with connectivities falling between 1 and 3.25 and the number of distinct functional roles between 2 and 5. Randomly generated networks, by contrast, often lie far outside this window.

One of the most essential aspects of organization is the coherence of the elements or the mutual constraints that hold the system together. Under the rubric of *Network Analysis*, Ulanowicz (1980, 1986, 1997), Hirata and Ulanowicz (1984), Ulanowicz and Norden (1990) has developed a set of information theoretic measures to quantify the degree of constraint inherent in the organization of any ecological or economic system. These measures can be generalized and extended so as to pertain to multiagent systems. To achieve this, we represent a multiagent system as a statistical ensemble. Following Ulanowicz (1980, 1986), we use an analog of the information-theoretic index, Average Mutual Information (AMI), to quantify spatial and structural correlations within multiagent models of ecological systems. We also define an index of adaptation to be the correlation between the internal demands of agents and the subsequent processes that satisfy these demands. We also discuss the possible application of these indexes in quantitative studies of ecosystems dynamics.

2. Information theoretic measures of trophic networks organization

The introduction of “information” theory to ecology can be traced back to MacArthur (1955). Margalef (1968), who considered the dynamic processes in an

ecosystem to be a channel, that projects information into the future. It was Rutledge et al. (1976) who first used the AMI as an intermediate in their derivation of a measure to quantify the redundancy of ecological networks. Ulanowicz (1986, 1997) later used the AMI to quantify the degree of constraint inherent in trophic flow networks and for formulating the theory of “ascendency”. These later works will be reviewed briefly in this section.

2.1. Quantifying constraints

We begin by considering two sets of events $\{A_i\}$ and $\{B_j\}$, which presumably occur in some statistical ensemble. At the moment we are not concerned with the character of these events. The question that we want to ask is how does one quantify the magnitude of overall constraint between these two sets of events? The sets may be tightly correlated or could be totally uncorrelated. Our objective is to quantify the transition from a very loosely coupled, highly indeterminate collection of events into one in which events are more constrained. We begin, as did Boltzmann (1872) who anticipated information theory, by quantifying the *indeterminacy*, h_j , of event B_j ,

$$h_j = -k \log P(B_j)$$

where $P(B_j)$ is the marginal probability that event B_j will happen, and k is a scalar constant. Roughly speaking, h_j is correlated with how surprised the observer will be when B_j occurs. If B_j is almost certain to happen, $P(B_j)$ will be a fraction near 1, and h_j will be quite small. Conversely, if B_j happens only rarely, $P(B_j)$ will be a fraction very near zero, and h_j will become a large positive number. In the latter instance the observer is very surprised to encounter B_j .

Constraint removes indeterminacy. Therefore, the indeterminacy of a system with constraints should be less than what it would be in unconstrained circumstances. Suppose, for example, that an a priori event A_i exerts some constraint upon whether or not B_j subsequently occurs. The probability that B_j will happen in the wake of A_i is by definition the conditional probability, $P(B_j|A_i)$, so that the (presumably smaller) indeterminacy of B_j under the influence of A_i (call it $g_{j,i}$), will be measured by the Boltzmann formula as

$$g_{j,i} = -k \log P(B_j|A_i)$$

It follows that one may use the decrease in indeterminacy, $h_j - g_{j,i}$, as one measure² of the intensity of the constraint that A_i exerts upon B_j . Call this constraint h_{ij} , where

$$h_{ij} = h_j - g_{j,i} = [-k \log P(B_j)] - [-k \log P(B_j|A_i)] \\ = k \log \left[\frac{P(B_j|A_i)}{P(B_j)} \right]$$

We note here for future reference that the constraint that A_i exerts upon B_j is formally equal to the constraint that B_j exerts on A_i . Using Bayes' theorem we see that

$$h_{ij} = k \log \left[\frac{P(B_j|A_i)}{P(B_j)} \right] = k \log \left[\frac{P(B_j, A_i)}{P(A_i)P(B_j)} \right] \\ = k \log \left[\frac{P(A_i|B_j)}{P(A_i)} \right] = h_{ji}$$

Hence, one may speak of the *mutual* constraint that A_i and B_j exert on each other.

One may use this measure of constraint between any arbitrary pair of events A_i and B_j to calculate the amount of constraint inherent in the system as a whole: one simply weights the mutual constraint of each pair of events by the associated joint probability, $P(A_i, B_j)$ that the two will co-occur and then sums over all possible pairs. This yields the expression for the expectation value of the mutual constraint, or the *average mutual constraint* (AMC), as

$$\text{AMC}(\{A_i\}, \{B_j\}) = k \sum_i \sum_j P(A_i, B_j) \cdot \log \left[\frac{P(A_i, B_j)}{P(A_i)P(B_j)} \right] \quad (1)$$

Here notations $\{A_i\}$ and $\{B_j\}$ describe, respectively the set of *all* A_i and that of *all* B_j . We intentionally use the same dummy indexes in the right and left hand sides of equation. If we deal with complete sets of events, then AMC becomes the familiar AMI function. Since we do not assume the completeness of the sets of events considered, however, it is more appropriate to use the new term AMC.

² We assumed that $h_j > g_{j,i}$. In those case where this assumption does not work we may need to consider $|h_j - g_{j,i}|$ instead of $h_j - g_{j,i}$.

The following idea, then, constitutes the cornerstone of our theoretical constructions:

AMC($\{A_i\}, \{B_j\}$) quantifies the strength of coupling between two sets of events $\{A_i\}$ and $\{B_j\}$.

If a system is totally random, i.e. $P(A_i, B_j) = P(A_i)P(B_j)$, then $\text{AMC}(\{A_i\}, \{B_j\}) = 0$. In any non-random system, $\text{AMC}(\{A_i\}, \{B_j\}) > 0$. Depending on how we select the two sets of events, $\{A_i\}$ and $\{B_j\}$, AMC could be used to quantify various aspects of system organization. If events $\{A_i\}$ and $\{B_j\}$ happen to be related to spatial movement, then AMC measures the correlation between spatial events and provides an index of spatial organization. If the events $\{A_i\}$ and $\{B_j\}$ pertain to trophic interactions, AMC provides an index of trophic organization. A particularly fecund use of the expression arises when it is applied to describe the strength of coupling between the demands of an organism and the processes that satisfy those demands.

2.2. Trophic networks and ascendancy theory

Each node of a trophic network represents a particular taxonomic species, and the links between the nodes depict the feeding relations among the taxa. The nodes of a trophic network are often referred to as compartments. The feeding relations usually are assumed to consist of flows among the compartments of biomass, carbon, or energy. Both the topology of a trophic network and the intensities of its flows are essential factors in the dynamics of ecosystems.

Quantifying the intensive process of the development of an ecosystem is a somewhat indirect process. The object here is to quantify the transition from a very loosely coupled, highly indeterminate collection of exchanges to one in which exchanges are more constrained along fewer specific pathways. If we focus upon trophic exchanges, a convenient interpretation of A_i is “a quantum of medium leaves compartment i ” and of B_j , “a quantum enters compartment j ”. Thus, we define T_{ij} as the amount of some conservative medium that flows from i to j in a given interval of time. The total amount of flow transpiring in the system is then reckoned by summing over both i and j , i.e. $T = \sum_{i,j} T_{ij}$. The joint probability that flow

will both leave i and enter j is therefore estimated by the quotient T_{ij}/T , and the marginal probabilities that flow either leaves i or enters j become the normalized sums of T_{ij} over the appropriate index:

$$P(A_i) \sim \frac{\sum_j T_{ij}}{T}$$

and

$$P(B_j) \sim \frac{\sum_i T_{ij}}{T}$$

In terms of these measurable exchanges, the estimated AMC takes the form

$$\text{AMC} = k \sum_i \sum_j \left(\frac{T_{ij}}{T} \right) \log \left[\frac{T_{ij}T}{(\sum_k T_{ik})(\sum_l T_{lj})} \right] \quad (2)$$

It is important to note that this expression is scale invariant. Should we change the units of time, the values of the flows will change, but the value of expression for AMC will remain the same.

As for the scalar factor that gives the ascendancy its dimensions, it is most convenient to identify the scalar constant, k , with the total amount of flow, T . Accordingly,

$$A = \sum_i \sum_j T_{ij} \log \left[\frac{T_{ij}T}{(\sum_k T_{ik})(\sum_l T_{lj})} \right] \geq 0 \quad (3)$$

This *ascendancy* is a measure derived from networks of trophic interactions. It refers to the combined effects of mutualism occurring among the networks of transfers. In the absence of major destructive influences, mutualism (or strict positive feedback) causes the magnitudes of its constituent processes to grow and eventually to dominate those of non-participating processes. The value of A is a homogeneous function of the time scale in exactly the same way as do the values of the flows. For example, if the units of flow are changed from kg/s to kg/min, the numerical values of flow will increase 60 times, as will those for A .

Ascendancy provides an integrative assessment of both the (topological) complexity of trophic organization and the intensity of interactions within the system. The ascendancy has been suggested as an index of the development process (Ulanowicz, 1980,

1997), i.e. the development of a system could be associated with an increase in ascendancy. It now appears that the range of applications of the principle of increasing ascendancy to various ecosystem behaviors has been expanding. Ulanowicz and Baird (1999), for example, have demonstrated that the familiar Liebig's Law of the Minimum (which holds that the growth of an organism is limited by that element which is available in least proportion to the respective amount required by that organism) can be deduced analytically from the ascendancy hypothesis. Ascendancy theory provides a direct connection between the diversity of ecological *processes* and system persistence (Ulanowicz, 2000a,b). In addition, one of the latest hot issues in Complexity Theory is how processes constituting some self-organizing systems are arrayed statistically according to power-laws, rather than according to the more well-behaved exponential distributions. Ulanowicz and Wolff (1991) studied the occurrence of power-law distributions in ecosystems over a decade ago by applying the ascendancy measure to a catalog of ecological networks. It has also been suggested that ascendancy can be extended to incorporate spatial and temporal dimensions and, therefore, could be used as index of organization in space and time (Pahl-Wostl, 1992; Ulanowicz, 2000b).

Many complex networks that occur in nature share certain statistical features. Milo et al. (2002), for example, found that ostensibly distinct objects, such as networks of gene transcription, neuronal synaptic connections, ecological feeding relations, and the World Wide Web all appear to exhibit similar motifs. The topological properties of cellular networks share surprising similarities with networks of social interactions. Oltvai and Barabasi (2002) have hypothesized that certain universal organizing principles may apply to all networks, from cells to the World Wide Web. Such observations and speculations suggest that results deriving from the study of trophic networks could possibly be extended to, or reinterpreted in context of, other kinds of natural networks. In fact, this paper emerged from an effort to reinterpret the ideas reviewed in this section in the context of multiagent systems; and, as the endeavor unfolded, the specific requirements of the task affected the subsequent evolution of our discourse.

3. The multiagent system as a statistical ensemble

AMC can be used to quantify spatial, causal, and/or structural constraints. Before we do that, however, we need to cast the ecosystem as a statistical ensemble. In this section we will try to represent ecological interactions and other ecological happenings as stochastic *events*, which occur in space and time with certain *probabilities*. The notations we develop in this section are synthetic to a degree. Insofar as we talk about the properties of agents and their interactions, our notations resemble those employed in LMSD (Krivov et al., 2002). Furthermore, inasmuch as we need to sum up and multiply the probabilities of related events, we have found an indexing system similar to the one used in tensor analysis to be most convenient. Unfortunately, experience has shown that mathematically rigorous notations can often obscure critical ideas. We developed our notations, therefore, under the objective of maximizing the clarity with which the correlations of events in multiagent systems can be described. We acknowledge; however, that, from standpoint of pure mathematics, the chosen formalism is neither rigorous, nor complete.

3.1. Agents

We shall assume that our universe of discourse consists of agents of various types: plants, animals, human, enterprises, etc., and that all can be treated as agents. We will use the letters a, b, c, d with indexes (or without) as the unique names for agents. Further, we assume that the agents we deal with are divided into categories C_1, \dots, C_n , which may also have specific names. The notation $C_i(a)$ will imply that agent a belongs to category C_i .

3.2. Time and space

We will assume that the space of our ecosystem is organized as a two dimensional grid of *sites*. Each site has its own characteristic dimensions and can host a certain number of agents. The sites are indexed by x, y . The numbers of agents that a site can host depend on the categories of the agents. We assume that time is discrete, that is, it is a linear sequence of intervals, each of a certain characteristic duration.

The length of the time intervals corresponds to the characteristic length of *events* that take place in the ecosystem. Since many processes in ecosystems exhibit a periodic character, we will assume that time can be divided into phases (such as hours of the day). Furthermore, we will use as subscripts the letters t, t_1, t_2 to denote the time-phases. We will assume that all time phases consist of an equal number of atomic time-intervals. We will call the duration of the total cycle interval the *total period*. The description of events in our ecosystem will be considered in terms of specific discrete partitions of time and space. The values of the probabilities of events will depend upon the properties of the partitions we employ. All the while, we acknowledge that different partitions of time and space are possible and could be used for other purposes.

3.3. Events

We will consider three kinds of events:

- (a) The natural demands of agents.
- (b) The processes and interactions between the agents. Some of these interactions can lead to satisfactions of agents' demands.
- (c) Spatial events.

These three types of events were chosen to correspond to the types of organization being considered. The notation for events is chosen so as to facilitate immediate recognition of which events are involved in the given probabilistic computations.

3.4. Natural demands of agents

We assume that *some* categories of agents have certain sets of demands and that these demands differ from category to category. We denote $Q_i = \{Q_i^d\}$ as the set of demands that agents of category i can have. This can include demands for food, water, rest, mating, etc. The upper index d enumerates the demands that category i can have. The number of demands differs from a category to category. The notation $Q_{i,t}^d(a)$ will imply that agent a has demand Q_i^d at time-phase t . The expression $Q_{i,t}^d(a)$ is an atomic Boolean formula which can be either true or false, depending upon the particular values of a , and the indexes d ,

i , and t . For instance, $Q_{i,t}^d(a)$ is false for any agent a that does not belong to category C_i . As is often done, we will associate the probabilities of events with their relative frequencies. Probabilities of events are measured over a long interval of time which must include at least a few total periods. We will assume that the probability $P(Q_{i,t}^d(a))$ is the ratio between the number of time-intervals for which $Q_{i,t}^d(a)$ is true to the total number of time-intervals in time-phase t that fall within the interval of measurement. Issues relating to the counting of probabilities in multiagent systems will be discussed in Section 8.

3.5. Processes

We assume that agents always participate in various processes. The processes are divided into two large categories—*interactions* and *non-transitive processes*. We assume that for each agent of category i and for each demand Q_i^d there exists an interaction that leads to the satisfaction of that demand. If this happens to be the interaction of an agent a of category i with an agent b of category j , we will denote the interaction as $R_{i,j}^d(a, b)$. Often a particular demand (for example, the consumption of food) can only be satisfied via interactions with agents from several categories. In such cases, we have to consider the full set of interactions $R_i^d(a, -) = \{R_{i,j}^d(a, b)\}_{j,b}$. Certain demands (such as sexual demands) can be satisfied only via interactions with agents of the same category. We assume that there exists an indexing of agent interactions of the form given above that is consistent with the indexing of demands. Whenever interactions happen in space and time, we can attach indexes to denote the temporal and spatial locations of those events as well. The notation that will be used for this purpose is $R_{i,j,t,x,y}^d(a, b)$. We will not differentiate the non-interactive processes and use symbol $R_i(a)$ to denote any non-interactive process for agents of category C_i . To accommodate spatial and temporal information we will use symbol $R_{i,t,x,y}(a)$.

We further relate the probabilities of processes with their relative frequencies. As we did earlier, we assume the probability $P(R_{i,j,t,x,y}^d(a, b))$ will be the ratio between the number of time-intervals for which $R_{i,j,t,x,y}^d(a, b)$ is true to the total number of time-intervals in time-phase t that fall into the interval of measurement.

3.6. Spatial events

We will consider four kinds of spatial events: (1) $A_{i,t,x,y}(a)$ will indicate that an agent a of category i has *arrived* at location (x, y) during time phase t ; (2) $D_{i,t,x,y}(a)$ will denote that an agent a of category i has *departed* from location (x, y) during time phase t ; (3) $P_{i,t,x,y}(a)$ will signify that an agent a of category i has remained *stationary* within the location (x, y) during time phase t ; and (4) $Ab_{i,t,x,y}(a)$ will mean that an agent a of category i has been *absent* in location (x, y) during time-phase t . A generic indexing of all the three categories will be required to be able to treat spatial events. We will use notation $S_{i,t,x,y}^k$, where index k takes on the values 1, 2, 3, 4 corresponding to spatial events of the first, second, third and the fourth kinds, respectively. The calculations of probabilities for spatial events are based on the same assumptions as in the case of demands and processes.

3.7. Complex and atomic events

It should be noted that events constitute sets, and hence set theoretical operations are defined on the events. The following notations will be important:

$$R_{i,-,t,x,y}^d(a) = \bigcup_{j,b} R_{i,j,t,x,y}^d(a, b),$$

$$R_{-,j,t,x,y}^d(b) = \bigcup_{i,a} R_{i,j,t,x,y}^d(a, b)$$

It should be noted here, however, that a degree of subjectivity is inherent in these definitions. The modeler must describe the set of all possible demands and interactions that an agent of species C_i can exhibit. Also the modeler has the freedom to consider or not to consider certain actions. Thus, the entire picture will depend on the initial identifications made by the modeler.

4. Quantifying spatial organization

In Sections 4–6 we use AMC for quantifying spatial, structural, and behavioral correlations in multiagent models of ecological systems. For abbreviation, we introduce the function $C(X, Y)$ on the set of all events.

$$C(X, Y) = P(X, Y) \log \left[\frac{P(X, Y)}{P(X)P(Y)} \right] \quad (4)$$

We invoke the convention that summation is being carried out over all distinct indexes appearing on the left hand side, while non-distinct indexes remain fixed. The following illustrates this convention and the above abbreviation:

$$\text{AMC}(\{A_{i,l}\}, \{B_{j,l}\}) = k \sum_{i,j} C(A_{i,l}, B_{j,l}) \quad (5)$$

The repeated index l is fixed both in right and left side of expression (5). Distinct indexes are used as summation indexes on the right and as indexes for elements of the set on the left side of the expression. They are intentionally left the same on both sides of expression. When we use this convention, the expression from AMC follows immediately from the notations on left hand side of the expression.

We now consider correlations between the spatial events $S_{i,t,x,y}^k$. To begin, consider the coupling of the set of all spatial events pertaining to agents of species C_i that happened at site (x, y) at time t :

$$\begin{aligned} \text{SC}_i(t, x, y) &= \text{AMC}(\{S_{i,t,x,y}^d(a)\}, \{S_{i,t,x,y}^k(b)\}) \\ &= k \sum_{d,k,a,b} C(S_{i,t,x,y}^d(a), S_{i,t,x,y}^k(b)) \end{aligned} \quad (6)$$

This value represents the correlation between the arrival, staying put, departure and absence from the site (x, y) of the agents of species C_i at time t . The intuitive interpretation of AMI suggests that, if these events are correlated, then the behavior of species C_i is organized, and the value of $\text{SC}_i(t, x, y)$ will be high. We may illustrate this by an example. Assume that C_i consists of N birds. In the first case, we assume that the motion of the birds is entirely random. In this case all correlation coefficients $C(S_{i,t,x,y}^d(a), S_{i,t,x,y}^k(b))$ will be 0 and thus $\text{SC}_i(t, x, y)$ is also 0 for all t, x, y . In the second case we can assume that birds move as an ideal flock; that is, they enter, stay, and depart from each location simultaneously. In the latter case we have

$$\begin{aligned} P(S_{i,t,x,y}^d(a), S_{i,t,x,y}^k(b)) \\ = P(S_{i,t,x,y}^d(a) | S_{i,t,x,y}^k(b)) P(S_{i,t,x,y}^k(b)) \end{aligned}$$

and

$$P(S_{i,t,x,y}^d(a) | S_{i,t,x,y}^k(b)) = \begin{cases} 1 & \text{if } d = k \\ 0 & \text{otherwise} \end{cases}$$

This justifies writing

$$\begin{aligned} \text{SC}_i(t, x, y) &= k \sum_{d,k,a,b} P(S_{i,t,x,y}^d(a), S_{i,t,x,y}^k(b)) \cdot \\ &\quad \log \left[\frac{P(S_{i,t,x,y}^d(a), S_{i,t,x,y}^k(b))}{P(S_{i,t,x,y}^d(a)) P(S_{i,t,x,y}^k(b))} \right] \\ &= k \sum_{d,a,b} P(S_{i,t,x,y}^d(a)) \log \left[\frac{1}{P(S_{i,t,x,y}^d(a))} \right] \\ &= -kN \sum_{d,a} P(S_{i,t,x,y}^d(a)) \log[P(S_{i,t,x,y}^d(a))] \\ &= kN \sum_a H_i(t, x, y, a) \end{aligned}$$

where $H_i(t, x, y, a)$ is the Shannon entropy that quantifies the uncertainty associated with the presence of agent a at location x, y during time-phase t . Thus, in the example of ideal flocking, the index $\text{SC}_i(t, x, y)$ is a positive number, which (in case of ideal organization) will grow in proportion to the size of the organization N . The bigger the size of the flock, the larger its index. These observations justify our choice of $\text{SC}_i(t, x, y)$ as our index of spatial organization. The reader is invited to prove that, when the population is divided into two completely independent ideal flocks F_1 and F_2 , with sizes N_1 and N_2 , respectively, then the flocking movement index will be

$$\begin{aligned} \text{SC}_i(t, x, y) \\ = kN_1 \sum_{a \in F_1} H_i(t, x, y, a) + kN_2 \sum_{a \in F_2} H_i(t, x, y, a) \end{aligned}$$

The next index measures the same spatial organization as it pertains to the whole community, rather than just to a single species:

$$\begin{aligned} \text{SC}(t, x, y) &= \text{AMC}(\{S_{j,t,x,y}^d(a)\}, \{S_{i,t,x,y}^k(b)\}) \\ &= k \sum_{i,j,d,k,a,b} C(S_{j,t,x,y}^d(a), S_{i,t,x,y}^k(b)) \end{aligned} \quad (7)$$

Spatial organization SC is a function of site (x, y) and time-phase t . At some sites $\text{SC}(t, x, y)$ will attain a high value, at other sites it will have low value. It is intuitively obvious that the points of high SC will indicate some kind of crossroads in an ecosystem. That may be feeding, resting, and mating sites, i.e. sites with great significance for the functioning of the

ecosystem. If we are considering plant populations, then large values of SC will possibly indicate sites with higher degrees of patchiness. It remains possible that $SC(t, x, y)$ will yield high values on some sites other than those wherein feeding, resting, mating or sites of high patchiness occur. In fact, we anticipate the existence of such sites; and, if they occur, then they must be considered as important for the functioning of the ecosystem as the sites where the aforementioned processes occur. The expression for $SC(t, x, y)$ could be expressed in the form

$$SC(t, x, y) = \sum_i SC S_i(t, x, y)$$

where

$$SC S_i(t, x, y) = k \sum_{j,d,k} C(S_{j,t,x,y}^d, S_{i,t,x,y}^k)$$

The last expression represents that the part of the magnitude of $SC(t, x, y)$ that is associated with the species C_i . Hence, the overall value of spatial organization will be composed of values contributed by different species. The value $\langle SC S_i(t, x, y) \rangle$ —the average of $SC S_i(t, x, y)$ over all sites and all time-phases tells how well the spatial movement or location of species C_i is organized in relation to other species. The intuitive interpretation of AMC suggests that species of animals and plants with a higher degree of spatial adaptation, and which are well connected to various spatial events, will have relatively higher values of $\langle SC_i(t, x, y) \rangle$.

The interesting question to consider is whether $SC(t, x, y)$ depends on the spatial and temporal scales? That is, does it depend on the duration of the time intervals and the size of the sites? In general, multi-agent systems are not scale invariant. Each interaction pattern has its own characteristic spatial and temporal dimensions. Change the scale and the pattern is no longer observed. The question of whether the system is organized or not always depends on the spatial and temporal scales. Since $SC(t, x, y)$ is our chosen measure of organization, we will presume that this value will always depend on the sizes of temporal and spatial grids.

We expect that the magnitude of $SC(t, x, y)$ tends to zero whenever spatial and temporal scales become either too large or too small. We anticipate the existence of characteristic dimensions of space and time

for which $SC(t, x, y)$ will be maximal. Such dimensions, we will call as the characteristic scales of organization.

5. Quantifying structural organization

The network of interactions between species is defined by the set of relations $R_{i,j}^d$. Index d represents the type of interaction. These types could be concrete instances of well-known classes of interactions, such as predation, competition, symbiosis, etc.

The same logical sequence that led to the quantification of the structural organization of trophic networks in the section titled *Trophic Networks and Ascendency Theory* can be invoked again to formulate a measure for the structural organization of networks of interactions.

The AMI for structural organization with one fixed index d is

$$\begin{aligned} STC^d(t, x, y) &= AMC(\{R_{i,-,t,x,y}^d(a)\}, \{R_{-,j,t,x,y}^d(b)\}) \\ &= k \sum_{i,j,a,b} C(R_{i,-,t,x,y}^d(a), R_{-,j,t,x,y}^d(b)) \end{aligned} \quad (8)$$

This is a reformulation of Eq. (2) for trophic networks.

Here

$$\begin{aligned} R_{i,-,t,x,y}^d(a) &= \bigcup_{j,b} R_{i,j,t,x,y}^d(a, b), \\ R_{-,j,t,x,y}^d(b) &= \bigcup_{i,a} R_{i,j,t,x,y}^d(a, b) \end{aligned} \quad (9)$$

which are analogous to sets A_i and B_j . The measure $STC^d(t, x, y)$ can be used whenever we want to focus our attention on the interactions of only a certain kind—for instance, interactions associated with the consumption of food.

If we desire to consider more than one type of interaction, the total AMC for the whole structural organization becomes

$$\begin{aligned} STC(t, x, y) &= AMC(\{R_{i,-,t,x,y}^d(a)\}, \{R_{-,j,t,x,y}^k(b)\}) \\ &= k \sum_{d,k,i,j,a,b} C(R_{i,-,t,x,y}^d(a), R_{-,j,t,x,y}^k(b)) \end{aligned} \quad (10)$$

This expression can be rewritten in the form

$$\text{STC}(t, x, y) = \sum_i \text{STCS}_i(t, x, y)$$

where

$$\begin{aligned} \text{STCS}_i(t, x, y) &= \frac{1}{2}k \sum_{d,k,j,a,b} C(R_{i,-,t,x,y}^d(a), R_{-,j,t,x,y}^k(b)) \\ &+ \frac{1}{2}k \sum_{d,k,j,a,b} C(R_{j,-,t,x,y}^d(a), R_{-,i,t,x,y}^k(b)) \end{aligned}$$

The last expression represents that part of the value of $\text{STC}(t, x, y)$ that is associated with the species C_i . The value $\langle \text{STCS}_i(t, x, y) \rangle$, the average of $\text{STCS}_i(t, x, y)$ over all sites and all time-phases, tells how well the species C_i is connected to other species. The intuitive interpretation of AMC suggests that species of animals and plants with stronger connections with only certain other species will have relatively higher value of $\langle \text{STCS}_i(t, x, y) \rangle$.

6. The combined measures of structural, spatial, and temporal organization

The combined measure of structural and spatial organization at the site $\text{CC}(t, x, y)$ can be estimated in terms of the following AMC:

$$\begin{aligned} \text{AMC}(\{R_{i,-,t,x,y}^d(a)\} \cup \{S_{i,t,x,y}^k(a)\}, \\ \{R_{-,j,t,x,y}^n(b)\} \cup \{S_{j,t,x,y}^l(b)\}) \end{aligned}$$

$$\begin{aligned} \text{CC}(t, x, y) &= k \sum_{i,j,d,n,a,b} C(R_{i,-,t,x,y}^d(a), R_{-,j,t,x,y}^n(b)) \\ &+ k \sum_{i,j,d,l,a,b} C(R_{i,-,t,x,y}^d(a), S_{j,t,x,y}^l(b)) \\ &+ k \sum_{i,j,k,n,a,b} C(S_{i,t,x,y}^k(a), R_{-,j,t,x,y}^n(b)) \\ &+ k \sum_{i,j,k,l,a,b} C(S_{i,t,x,y}^k(a), S_{j,t,x,y}^l(b)) \end{aligned}$$

This expression can be rewritten in the following form, illustrating the additive character of our measure of

organization:

$$\begin{aligned} \text{CC}(t, x, y) &= \text{STC}(t, x, y) \\ &+ k \sum_{i,j,d,l,a,b} C(R_{i,-,t,x,y}^d(a), S_{j,t,x,y}^l(b)) \\ &+ k \sum_{i,j,n,k,a,b} C(S_{i,t,x,y}^k(a), R_{-,j,t,x,y}^n(b)) \\ &+ \text{SC}(t, x, y) \end{aligned} \quad (11)$$

As was done above, this expression can be written in the form

$$\text{CC}(t, x, y) = \sum_i \text{CCS}_i(t, x, y)$$

The expression $\text{CCS}_i(t, x, y)$ represents that part of value of $\text{CC}(t, x, y)$ that is associated with species C_i . Thus, the overall combined value of importance will be composed of values contributed by different species. The average value $\langle \text{CCS}_i(t, x, y) \rangle$ reveals how much species C_i contributes to the total organization of the ecosystem.

The combined measure of total structural, spatial, and temporal organization at site $\text{TC}(t, x, y)$ can be estimated as

$$\begin{aligned} \text{TC}(t, x, y) &= \text{AMC}(\{R_{i,-,t1,x,y}^d(a)\} \cup \{S_{i,t1,x,y}^k(a)\}, \\ &\{R_{-,j,t2,x,y}^n(b)\} \cup \{S_{j,t2,x,y}^l(b)\}) \end{aligned}$$

Here, we consider the correlations of events that can possibly happen at different time phases. This measure has an additive character similar to $\text{CC}(t, x, y)$. In fact, if F is the total set of time phases, then

$$\text{TC}(t, x, y) = \sum_{t \in F} \text{CC}(t, x, y) + \Delta$$

where Δ represents a complicated term that involves only temporal correlations.

7. The index of adaptation

We define the adaptation $F(i, t)$ of species C_i as the ability of the members of that species to satisfy their natural needs in the given environment at time t . It is important to underline the difference between adaptation and adaptability. High adaptation of an agent to given environment may be accompanied by low adaptability (ability of system to adapt to new environment)

and vice versa. [Conrad's Adaptability \(1983\)](#) provided a formal treatment of adaptability, however his entire exposition was cast in the framework of Markovian processes, which are not commonly invoked in ecosystem modeling. The subtleties of the relationship between adaptation and adaptability were discussed in [Ulanowicz \(2002\)](#) in the context of trophic networks. Here we will focus on the operational measure of adaptation, which may be applicable to multiagent systems.

The needs of agents, as well as interactions which lead to satisfaction of those needs, are events that occur in space and time. They occur with certain frequencies, and it is possible to associate probabilities with the related demands and satisfactions. We want to describe the strength of causal coupling between the demands $Q_{i,t}^d(a)$ and the interactions $R_{i,\dots,t+\Delta}^d(a)$ that satisfy them. An underscore ($_$) as a subscript should be understood in the same way as in [Eq. \(9\)](#), i.e. we are allowing interactions with agents of any species that satisfy the demand d . It follows from our previous discussion of AMC that to describe the strength of coupling between the sets of events $\{Q_{i,t}^d(a)\}$ and $\{R_{i,\dots,t}^d(a)\}$, we should use the expression $\text{AMC}(\{Q_{i,t}^d(a)\}, \{R_{i,\dots,t}^d(a)\})$.

We can now provide a quantitative expression for our definition of adaptation, namely:

$$F_i(a, t) = \text{AMC}(\{Q_{i,t}^d(a)\}, \{R_{i,\dots,t}^d(a)\}) \\ = k \sum_d C(Q_{i,t}^d(a), R_{i,\dots,t}^d(a)) \quad (12)$$

This measure of adaptation reflects the ability of agents to satisfy their needs; however, it also quantifies the diversity and unpredictability of those needs. Consider the hypothetical case wherein agents are able to satisfy their needs instantly. In this case, $P(R_{i,\dots,t}^d(a) | Q_{i,t}^d(a)) = 1$; so that

$$F_i(a, t) = -k \sum_d P(Q_{i,t}^d(a)) \cdot \log(Q_{i,t}^d(a))$$

From this expression we see that the richer the set of needs that the agent must satisfy, the higher the required index of adaptation. That is, more highly developed species with manifold needs have a higher index of adaptation. This is a major point in which our index of adaptation contrasts with the prevailing concept of fitness. The term “fitness” has a long history ([Iseda, 1996](#)), having been introduced by Herbert

Spencer in 1864. The fitness concept, however, remained a vague, unquantified notion until the 1930s. [Fisher \(1930\)](#) was among the first to connect the term to reproductive success, but it was [Waddington \(1939\)](#) who proposed to measure the “fitness” of an organism according to the number of offspring it leaves. This has now become the common meaning of the term. An individual of a higher species, such as a human being that produces one or no offspring over its lifetime would be assigned a very low index of fitness. Therefore, the term “fitness” does not reflect evolutionary status of a species. By way of contrast, the index of adaptation we are proposing differentiates between primitive and higher species, assigning a higher adaptation level to species with manifold demands.

In this treatment we have assumed that all demands need to be satisfied in the near-term. It is not a complicated task, however, to incorporate both immediate and long-term demands (those that do not need to be satisfied immediately), and the reader is invited to modify expression (12) so as to incorporate long-term demands into the index of adaptation.

The average adaptation of *all* species can be taken as the sum of the adaptation of each species weighted by the population/biomass fraction of the entire ecosystem constituted by that species. Although the adaptation measure does not provide complete account of agent capacities, the significance of such measure should not be underestimated. The availability of a numerical estimate for adaptation in the context of a multiagent simulation will be the key to quantifying the processes of evolution in a specific ecosystem.

8. Estimating the probabilities

In this section, we consider briefly the problem of estimating the probabilities of various events occurring in multiagent systems. We discuss the issue in rather general terms, not always seeking to tie it to the particular indexes of organization that we have introduced here. Although the concept of probability has a precise mathematical meaning ([Kolmogorov, 1956](#)), its application to real, physical systems is not free from certain ambiguities. To begin with, we will revisit the notion of probability as it pertains to the physical sciences, noting several issues associated with the concept. We will then consider various approaches for calculating

probabilities as they pertain to multiagent systems. It turns out there are several alternative avenues to estimating such probabilities, and each of them tacitly involves a lack of organization of some kind—either spatial, temporal, or behavioral. We proceed to show that the restrictions of this sort are inherent in the concept of probability itself. We then formulate a *principle of incompleteness for the probabilistic measures of organization*, which constitutes the central idea in this section.

Properly speaking, a probability of an event's happening can be defined only if that occurrence is a possible outcome of some repeatable, self-similar observation. "By the 'probability' of a particular outcome of an observation we mean our most likely estimate of the fraction of a number of repeated observations that will yield that particular outcome" (Feynman et al., 1963). For example, the probability that a molecule of an ideal gas has a velocity v in the range $v_1 < v < v_2$ would be the fraction of molecules which have their velocities in this range: $P(v_1 < v < v_2) = N(v_1 < v < v_2)/N$. It is not necessary to actually make all the observations. What is important is our ability to estimate the number of observations which yield the outcome in which we are interested.

The problem arising here is how can we think of a set of identical observations in the context of real physical systems? The resolution is to find some self-similarity or homogeneity in the system we are considering. If our hypothetical repeated observation is taken to be the measurement of the speed of all molecules in the ensemble, such observation can be considered self-similar if and only if all the molecules are identical. This example leads us to an important conclusion: "Any assumption involving the homogeneity of a system also implies a certain lack of organization at that level." For example, when we assume that all molecules are identical, it becomes impossible to imagine a unique spatial behavior or a unique location in a certain framework as it might pertain to a selected molecule. While the assumption of homogeneity is perfectly appropriate for the molecules of a natural gas, it would usually be an unwarranted simplification if applied to the agents in a multiagent system. For instance, if we assume all the agents of a category C_i are identical, and the differences between them are random, then we are unable to assume

any unique pattern of spatial behavior. Territorial or flocking behavior would become inconceivable.

Any method leading to the estimation of probabilities in multiagent systems will necessarily be based on some well-defined source of homogeneity in the system. We, therefore, consider four options concerning which type of homogeneity is assumed and proceed to point out the strengths and the weaknesses of each choice as follows.

- *Time*: We can assume that the system is homogeneous in *time*. If we assume that all instants of time instances are equivalent, we can use the frequency of an event as the measure of its probability. For example, the probability, $P_{i,x,y}(a)$, that an agent a in category i remains *stationary* within location (x, y) is estimated to be the fraction of the units of time that the agent remains *stationary* within this location, divided by the total interval of time considered. In similar manner, we can calculate the probabilities of interactions. Such probabilities must be associated with a large interval of time over which observations are made. It becomes meaningless to consider such things as the probability of an event A happening at a particular moment t_1 . As a consequence, causal correlations like $C(A_{t_1}, B_{t_2})$ would fail to have meaning. Furthermore, we would not be able to identify any patterns of periodic behavior, such as periodic migrations. Thus, we conclude that the assumption of the equivalence of all instants of time prevents us from quantifying any organization imposed by casual links. We note in passing that some shortcomings of this approach may be cured if we attach to each instant of time a window of width $2d$ and calculate the probabilities using only those instants of time that fall in the range $[t - d, t + d]$. This would allow us to associate events with approximate moments of time, which may be useful in some situations.
- *Periodic time*: We can divide time into periods, such as 24 h, or 48 periods of half an hour each, and then equate all periods that belong to the same classes. This approach would allow us to quantify some aspects of temporal periodic organization and would be natural for those systems where agents exhibit periodic behavior that is essential to the problem being considered. The scope of this approach is limited, however, to only those patterns of periodic

behavior that are synchronized to our fundamental period of time. This approach also requires a considerably longer interval of observation compared with the first method. The indexes of organization introduced in this paper were designed around this particular approach, and it is probably the most appropriate way to treat multiagent systems with periodic time.

- *Total fluxes*: We can assume that all quanta of biomass pertaining to species C_i are equivalent. Ulanowicz (1986, 1997) used this approach to calculate his indexes of structural organization. It potentially could be used to quantify the organization imposed by causal links. Considering what was said above about ideal gases, however, the method cannot be used to address any aspects of spatial organization. Furthermore, this approach seems appropriate only for trophic interactions.
- *Spatially explicit fluxes*: We can associate biomass fluxes with both compartments C_i and spatial locations (x, y) . This approach was explored by Pahl-Wostl (1992). This approach is far more potent than considering only total fluxes. It allows one to quantify certain aspects of spatial organization, but only those pertaining to the spatial organization of separate species. From the foregoing discussion, it is evident that it does not allow one to deal with interspecies patterns of organization, such as flocking, competition for space, etc. To deal with these elements of organization we must consider each agent as an individual.

The above four options do not constitute an exhaustive list of possible methods to estimate probabilities in multiagent systems. They should be sufficient, however, to demonstrate that fundamental restrictions on the probabilistic measures of organization do exist. Accordingly, we elaborate the following principle of incompleteness pertaining to probabilistic measures of organization: *“There is no probabilistic measure of organization which can simultaneously and exhaustively quantify all the aspects of structural, spatial, and causal organizations in a system.”* As should be obvious, this restriction derives from the necessity to make at least one assumption about self-similarity or homogeneity existing in a system before one can make any practical estimate of the probabilities of events in that system.

9. Discussion

We have introduced indexes that quantify various aspects of ecosystem organization. We have distinguished special indexes for spatial and structural organization and for any combination thereof. Finally, there is the index that quantifies the adaptation of a species. All the indexes of organization that we have introduced (save adaptation) are summarized in Table 1.

These definitions may raise certain theoretical concerns. One such concern might be how these indexes of organization depend upon spatial and temporal scales. It is important to note that most of the attributes associated with the organization of multiagent systems are not scale invariant, and our indexes most certainly depend upon spatial and temporal scales. Since the notion of the organization of the system itself depends on the scales of time and space, it might be illustrative to consider explicitly the dependence of these expressions on time-space. We anticipate that the magnitude of *each index* will tend towards zero whenever spatial and temporal scales become either too large or too small. We expect that for each index there will be a magnitude that will characterize both the temporal interval and the spatial grid-work for which *the index* will be maximal. Such a magnitude we call the characteristic scale of organization.

As long as the probabilities of events depend on a characteristic dimensions of space and time, all attributes of organization that depend on such probabilities will also depend on scale. One way of obtaining scale-invariant expressions is to work with probability densities and with densities of AMC. The definition of such densities would require extra caution, however, for in general we are not able to resolve infinitely small areas or infinitely small intervals of time via traditional procedures.

The fact that there are many ways to partition an ecosystem into taxa and to denote a set of interac-

Table 1
Indexes of organization as defined in the text

Organization type	One species	All species
Spatial	SC_i, SCS_i	SC
Structural	$STCS_i$	STC
Spatio-structural	CCS_i	CC
Spatio-structural-periodical	TCS_i	TC

tions between them raises another theoretical concern, namely how the partitioning of the ecosystem into species, or the selection of certain finite set of agents interactions affects the defined indexes. How the notion of organization depends upon the underlying frame of reference (i.e. the language used to describe the system) seems to be fundamental. We envisage that such dependence will be essential; and, again, we anticipate a “most appropriate” partition that captures the most essential processes and produces the greatest magnitudes in the measures of organization.

Can the indexes we have defined be put to any practical use? If they indeed reflect the degrees of organization, then the following three hypotheses could be entertained.

Hypothesis 1. Any disturbance of an ecosystem at a location associated with a high spatial Importance Index $SC(t, x, y)$, $TC(t, x, y)$, $CC(t, x, y)$ will exert a greater impact on the population dynamics than will a similar disturbance occurring at a place where the values of these indexes are lower.

Hypothesis 2. Any disturbance in an ecosystem affecting a particular species C_i with high individual Importance Indexes SC_i , STC_i , CC_i will cause a greater impact on the overall population dynamics than will a disturbance impacting a species with a lower value of that index.

Furthermore, the availability of a measure of adaptation should allow us to identify evolving systems as those with an increase in average adaptation. Whence the following surmise concerning system ascendancy.

Hypothesis 3. The ascendancy of an evolving system has a propensity to increase.

The general character of the proposed hypotheses suggests the necessity to test them under a wide range of circumstances. It is possible that it may not be feasible to test the hypotheses on actual data, at least at this stage. The simulation of ecosystems behavior *in silico*, however, affords the unique opportunity to test the efficacy of our indexes by studying the evolution and development of complex systems under various “what if” scenarios. Through simulation we can readily compare the dynamics of ecosystems

under disturbance with their counterparts that are free from such perturbations. Moreover, while studying a (multiagent) system *in silico*, all the data about the system in action necessary to calculate our indexes can be made readily available.

Numerous funding agencies lately have devoted significant resources to the development of ecological indicators. By and large these initiatives all have been of an ad-hoc nature—witness the popular IBI indexes. By contrast, if the hypotheses just formulated should survive scrutiny, a set of indexes that reveals the status of the system at a fundamental dynamical level would then be available for managerial applications. The implications for using such indexes to achieve more penetrating and relevant environmental impact assessments to help in guiding management decisions should be patent to all.

Acknowledgements

S. Krivov was supported by National Science Foundation's Program grant (NSF award no. 9982938). R. Ulanowicz was supported in part by the National Science Foundation's Program on Biocomplexity (Contract No. DEB-9981328), and the US Geological Survey Program for Across Trophic Levels Systems Simulation (ATLSS; Contract 1445CA09950093). The authors are grateful to Valery Perminov for his helpful comments on the manuscript. Three anonymous reviewers provided suggestions that significantly improved the final version.

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