$\boxed{1}$ Check that the function

$$y_h = C(1+t) + De^t$$

solves the homogeneous equation

$$ty'' - (1+t)y' + y = 0.$$

Then find a general solution to the non-homogeneous equation

$$ty'' - (1+t)y' + y = t^2 e^{2t}.$$

Flip over for another problem...

$\boxed{2}$ In an initial value problem, you are given a differential equation, together with a value of y and a value of y'. In a *boundary value problem*, on the other hand, you are given a differential equation and two values of y (we think of these as the values of y "on the boundary"). The following questions concern the boundary value problem

$$y'' + \lambda^2 y = \sin t; \quad y(0) = 0; \quad y(\pi) = 1.$$

a. Find the general solution to the given differential equation for all $\lambda \neq \pm 1$ (ignoring the boundary conditions for now).

b. Find the general solution to the given differential equation when $\lambda = \pm 1$ (again ignoring the boundary conditions).

c. Show that the boundary value problem has a solution if and only if λ is not an integer.