The exam will cover sections 1.2, 1.3, 2.2, 2.3, 2.4, and 2.6. All topics from this review sheet or from the suggested exercises are fair game.

1. Give explicit solutions to the initial value problem \( \frac{dy}{dx} = xy^3 \) with \( y(0) = 1 \), \( y(0) = \frac{1}{2} \), and \( y(0) = -2 \). Then determine the domains of each of these solutions.

2. Show that every separable first-order differential equation can easily be converted into an exact equation.

3. For each of the following differential equations, indicate whether they are separable, linear, or can easily be converted into an exact equation. Note that some equations may be more than one type, while others may not be any of these types. Then, solve the equations which are separable, linear, or exact.
   a. \( \frac{dy}{dx} = -\frac{2xy}{x^2+y^2} \).
   b. \( \frac{dy}{dx} = xy \sin x \).
   c. \( \frac{dy}{dx} = \sin(x + y^2) \).
   d. \( \frac{dy}{dx} = \frac{x-y}{2x} \).
   e. \( \frac{dy}{dx} = \frac{5y^4}{\cos y + e^y} \).

4. For each of the following initial value problems, determine if they have zero, one, or more than one solution(s). You do not need to solve these equations.
   a. \( y \frac{dy}{dx} + x = 0 \); \( y(1) = 0 \).
   b. \( \frac{dy}{dx} = 3y^{2/3} \); \( y(0) = 0 \).
   c. \( y \frac{dy}{dx} = \arctan(x + y); y(1) = 1 \).

5. Make an appropriate substitution in order to solve the following differential equations.
   a. \( \frac{dy}{dx} = \frac{2y}{x} - x^2y^2 \).
   b. \( x^2 \frac{dy}{dx} = xy - y^2 \).
   c. \( \frac{dy}{dx} = \frac{1}{(2x+y)e^{2x+y}} - 2 \).