

The exam will cover sections 1.2, 1.3, 2.2, 2.3, 2.4, and 2.6. All topics from this review sheet or from the suggested exercises are fair game.

- 1 Give explicit solutions to the initial value problem $\frac{dy}{dx} = xy^3$ with $y(0) = 1$, $y(0) = 1/2$, and $y(0) = -2$. Then determine the domains of each of these solutions.
- 2 Show that every separable first-order differential equation can easily be converted into an exact equation.
- 3 For each of the following differential equations, indicate whether they are separable, linear, or can easily be converted into an exact equation. *Note that some equations may be more than one type, while others may not be any of these types.* Then, solve the equations which are separable, linear, or exact.

a. $\frac{dy}{dx} = \frac{-2xy}{x^2+y^2}.$

b. $\frac{dy}{dx} = xy \sin x.$

c. $\frac{dy}{dx} = \sin(x + y^2).$

d. $\frac{dy}{dx} = \frac{x-y}{2x}.$

e. $\frac{dy}{dx} = \frac{5x^4}{\cos y + e^y}.$

- 4 For each of the following initial value problems, determine if they have zero, one, or more than one solution(s). *You do not need to solve these equations.*

a. $y \frac{dy}{dx} + x = 0; y(1) = 0.$

b. $\frac{dy}{dx} = 3y^{2/3}; y(0) = 0.$

c. $y \frac{dy}{dx} = \arctan(x + y); y(1) = 1.$

- 5 Make an appropriate substitution in order to solve the following differential equations.

a. $\frac{dy}{dx} = \frac{2y}{x} - x^2y^2.$

b. $x^2 \frac{dy}{dx} = xy - y^2.$

c. $\frac{dy}{dx} = \frac{1}{(2x+y)e^{2x+y}} - 2.$