Midterm 3 will cover sections 7.2–7.8.

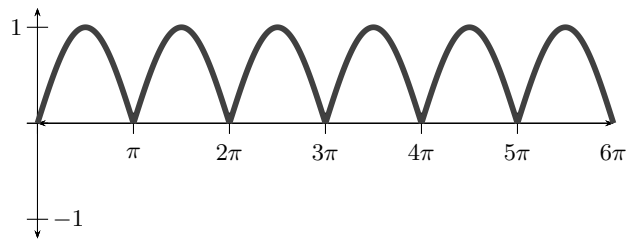
$\boxed{1}$ Solve for $\mathscr{L}\{y\}$ given the following initial value problems.

a. $y'' - 4y' + 8y = e^{2t}\cos 3t$; $y(0) = 1$; $y'(0) = 3$.

b. $y'' + 2y' - 3y = e^t + t + 1$; $y(0) = 9$; $y'(0) = -3$.

c. $y'' - 4y = \begin{cases} \sin t & 0 < t < \pi, \\ -\sin t & t > \pi. \end{cases}$; $y(0) = y'(0) = 0$.

d. $y'' + y' - 2y = f(t)$, where $f(t)$ is the fully-rectified sine wave below; $y(0) = y'(0) = 1$.



e. $y'' - 4y' + ty = 0$; $y(0) = 1$; $y'(0) = 0$. (Find a differential equation satisfied by $\mathscr{L}\{y\}$.)

f. $y'' + 4y = \delta(t-2)$; $y(0) = y'(0) = 0$. (Here δ is the Dirac delta function.)

g. $y'' + 5y' - y = e^{\sin t}\delta(t-3)$; $y(0) = 0$, $y'(0) = 3$.

$\boxed{2}$ Compute the following inverse Laplace transforms.

a. $\mathscr{L}^{-1}\left\{\dfrac{2s^2 - 1}{s^3 + s^2 - 6s}\right\}$.

b. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^2 - 8s + 17}\right\}$.

c. $\mathscr{L}^{-1}\left\{\dfrac{9 - s^2}{(s^2+9)^2}\right\}$.

d. $\mathscr{L}^{-1}\left\{\dfrac{3s}{s^2 + 4s + 6}\right\}$.

e. $\mathscr{L}^{-1}\left\{\dfrac{(1 - e^{-s})^2}{s^3}\right\}$.

f. $\mathscr{L}^{-1}\left\{\dfrac{e^{-\pi s}}{s^2 + 2s + 5}\right\}$.

g. $\mathscr{L}^{-1}\{7\}$.

h. $\mathscr{L}^{-1}\left\{\dfrac{s^2 + 2s}{s^2 + 4}\right\}$.

i. $\mathscr{L}^{-1}\left\{\dfrac{s\mathscr{L}\{g\}}{s^2 + 4}\right\}$.

j. $\mathscr{L}^{-1}\left\{\dfrac{\mathscr{L}\{g\} + s}{s}\right\}$.

$\boxed{3}$ Solve the integro-differentential equation

$$y(t) + \int_0^t e^{t-v} y(v)\, dv = \sin t.$$