The exam will cover sections 8.1–8.4, 4.1, and 4.8–4.10.

Terms to know: underdamped, overdamped, critically damped, angularly frequency, natural frequency, period, quasifrequency, quasiperiod, steady-state solution, frequency gain, resonance.

$\boxed{1}$ Find all singular points of the following differential equations.

a. $(x^2 - 1)y'' + xy' + 2y = 0$.

b. $x^3(x^2 + 1)y'' + xy' - y = 0$.

c. $(x^2 - 2)y'' + \sqrt{2}y' - (\sin x)y = 0$.

d. $(\sin x)y'' + \pi y' - (\sin x)y = 0$.

e. $xy'' + (\sin x)y = 0$.

$\boxed{2}$ Find the first four terms in a power series expansion at $x = 0$ for a general solution to the given differential equation.

a. $y' + (x + 2)y = 1$.

b. $y' - (\sin x)y = 0$.

c. $e^{2x}y' - y = e^x$.

d. $y'' - xy' + x^4y = \sin x$.

$\boxed{3}$ Find the general solution for the following differential equations. (Your answer should contain a recurrence relation for the power series coefficients.)

a. $y'' + 4y = 0$.

b. $2y' + (x - 2)y = 0$.

c. $y'' + x^2y = 0$.

$\boxed{4}$ The function $f(t)$ is said to be *eventually bounded* if there is a constant M such that $|f(t)| < M$ for all sufficiently large t. Use the mass-spring analogy to determine whether all solutions to each of the following differential equations are eventually bounded.

a. $y'' + t^2 y = 0$.

b. $y'' - t^2 y = 0$.

c. $y'' + y^6 = 0$.

d. $y'' + (4 + 2\cos t)y = 0$ (Mathieu's equation).

$\boxed{5}$ The following differential equations represent the movement of a mass-spring system. For each, determine if it is underdamped, overdamped, or critically damped.

a. $y'' + 5y' + 6y = 0$.

b. $y'' + 12y' + 36y = 0$.

c. $y'' + \frac{9}{2}y' + 2y = 0$.

d. $y'' + 4y' + 13y = 0$.

$\boxed{6}$ A $1/8$ kg mass is attached to a spring with stiffness 16 N/m. The damping constant (friction coefficient) for the system is 2 N-sec/m. If the mass is moved $3/4$ m to the left of equilibrium and given an initial leftward (negative) velocity of 2 m/sec, determine the equation of motion of the mass and give its damping factor, quasiperiod, and quasifrequency.

$\boxed{7}$ At what time does the mass in the previous problem first return to equilibrium?

$\boxed{8}$ A 10 kg weight is attached to a vertical spring with damping constant 2 kg-s/m. At rest, the spring is stretched 2 m. What is the spring constant of this spring? (Acceleration due to gravity near the surface of the Earth is approximately 9.8 m/s^2.)

$\boxed{9}$ Determine the equation of motion for an undamped system at resonance governed by

$$y'' + 16y = 2\cos 4t, \quad y(0) = 1, \quad y'(0) = 0.$$

$\boxed{10}$ A 1 kg mass is attached to a horizontal spring with damping constant 2 kg-s/m and spring constant 1 N/m. Does this system have a resonance frequency?