The exam will cover sections 8.1–8.4, 4.1, and 4.8–4.10.

Terms to know: underdamped, overdamped, critically damped, angularly frequency, natural frequency, period, quasifrequency, quasiperiod, steady-state solution, frequency gain, resonance.

1 Find all singular points of the following differential equations.
   a. \((x^2 - 1)y'' + xy' + 2y = 0\).
   b. \(x^3(x^2 + 1)y'' + xy' - y = 0\).
   c. \((x^2 - 2)y'' + \sqrt{2}y' - (\sin x)y = 0\).
   d. \((\sin x)y'' + \pi y' - (\sin x)y = 0\).
   e. \(xy'' + (\sin x)y = 0\).

2 Find the first four terms in a power series expansion at \(x = 0\) for a general solution to the given differential equation.
   a. \(y' + (x + 2)y = 1\).
   b. \(y' - (\sin x)y = 0\).
   c. \(e^{2x}y' - y = e^x\).
   d. \(y'' - xy' + x^4y = \sin x\).

3 Find the general solution for the following differential equations. (Your answer should contain a recurrence relation for the power series coefficients.)
   a. \(y'' + 4y = 0\).
   b. \(2y' + (x - 2)y = 0\).
   c. \(y'' + x^2y = 0\).
The function \( f(t) \) is said to be *eventually bounded* if there is a constant \( M \) such that \( |f(t)| < M \) for all sufficiently large \( t \). Use the mass-spring analogy to determine whether all solutions to each of the following differential equations are eventually bounded.

a. \( y'' + t^2y = 0 \).

b. \( y'' - t^2y = 0 \).

c. \( y'' + y^6 = 0 \).

d. \( y'' + (4 + 2 \cos t)y = 0 \) (Mathieu’s equation).

The following differential equations represent the movement of a mass-spring system. For each, determine if it is underdamped, overdamped, or critically damped.

a. \( y'' + 5y' + 6y = 0 \).

b. \( y'' + 12y' + 36y = 0 \).

c. \( y'' + \frac{9}{2}y' + 2y = 0 \).

d. \( y'' + 4y' + 13y = 0 \).

A 1/8 kg mass is attached to a spring with stiffness 16 N/m. The damping constant (friction coefficient) for the system is 2 N-sec/m. If the mass is moved \( \frac{3}{4} \) m to the left of equilibrium and given an initial leftward (negative) velocity of 2 m/sec, determine the equation of motion of the mass and give its damping factor, quasiperiod, and quasifrequency.

At what time does the mass in the previous problem first return to equilibrium?

A 10 kg weight is attached to a vertical spring with damping constant 2 kg-s/m. At rest, the spring is stretched 2 m. What is the spring constant of this spring? (Acceleration due to gravity near the surface of the Earth is approximately 9.8 m/s\(^2\).)

Determine the equation of motion for an undamped system at resonance governed by

\[
y'' + 16y = 2 \cos 4t, \quad y(0) = 1, \quad y'(0) = 0.
\]

A 1 kg mass is attached to a horizontal spring with damping constant 2 kg-s/m and spring constant 1 N/m. Does this system have a resonance frequency?