

Homework #2

- 1 Let (X, d) be a metric space. Define the function d_* by

$$d_*(x, y) = \frac{d(x, y)}{1 + d(x, y)}.$$

Prove that (X, d_*) is also a metric space.

- 2 Determine (with justification) whether the set $\{(x, y) : x, y > 0\}$ is open and/or closed in the metric space (\mathbb{R}^2, d_2) .
- 3 Determine (with justification) whether the set $\{(x, y) : xy = 1\}$ is open and/or closed in the metric space (\mathbb{R}^2, d_2) .
- 4 Determine (with justification) whether the set $\{(x, y) : |x - y| > 1\}$ is open and/or closed in the metric space (\mathbb{R}^2, d_2) .