Homework #2

1. Let \((X, d)\) be a metric space. Define the function \(d_*\) by

\[
d_*(x, y) = \frac{d(x, y)}{1 + d(x, y)}.
\]

Prove that \((X, d_*)\) is also a metric space.

2. For the following subsets of the metric space \((\mathbb{R}^n, d_2)\), determine (with justification) whether they are open and/or closed (you may use any results we have proved in class):

\[
\{(x, y) : x, y > 0\}, \quad (x, y) : xy = 1\}, \quad \{(x, y) : |x - y| > 1\}.
\]