Homework #3

1. By carefully negating the definition of convergence, state what it means for the sequence \((a_n)\) from the metric space \((X, d)\) to diverge. Your statement should begin “The sequence \((a_n)\) from \((X, d)\) diverges if and only if for every \(L \in X\)....”.

2. Suppose \((a_n)\) is a sequence from \((\mathbb{R}, d_1)\). Show that if \((a_n)\) converges to \(L\) then its sequence of Cesaro means \((s_n)\) defined by

\[
s_n = \frac{1}{n+1} \sum_{i=0}^{n} a_i
\]

also converges to \(L\). Also give an example of a divergent sequence \((a_n)\) whose Cesaro means converge.