Homework #4

$\boxed{1}$ Let X be a compact metric space and $\mathcal{F}$ a family of closed subsets of X. Prove that if

$$\bigcap_{C \in F} C \neq \emptyset$$

for all finite subfamilies $F \subseteq \mathcal{F}$ (this condition is called the *finite intersection property*) then actually

$$\bigcap_{C \in \mathcal{F}} C \neq \emptyset.$$

Hint: take complements, create an open cover, invoke the definition of compactness.

$\boxed{2}$ State and prove the converse to the previous problem.

$\boxed{3}$ Prove that every sequentially compact metric space is also complete. (*This is Proposition 6.15 in the lecture notes.*)