Homework #5

1. Let $X$ be an infinite set and $d$ be the discrete metric. Determine, with proof, the connected sets in the metric space $(X, d)$.

2. Let $(X, d)$ be an arbitrary metric space. Is the intersection of two connected sets in $(X, d)$ necessarily connected? Prove or give a counterexample.

3. Prove that if $S$ is a connected subset of the metric space $(X, d)$ then its closure $\overline{S}$ is also connected.

4. Let $(X, d_X)$ be any metric space and $(Y, d_Y)$ be any metric space with the discrete metric. Determine, with proof,
   
   (a) all continuous functions $f : Y \to X$ and
   
   (b) all continuous functions $f : \mathbb{R} \to Y$, where the metric on $\mathbb{R}$ is the usual Euclidean one.