Homework #5

1 Let X be an infinite set and d be the discrete metric. Determine, with proof, the connected sets in the metric space (X, d).

2 Let (X, d) be an arbitrary metric space. Is the intersection of two connected sets in (X, d) necessarily connected? *Prove or give a counterexample.*

3 Prove that if S is a connected subset of the metric space (X, d) then its closure $\overline{S}$ is also connected.

4 Let (X, d_X) be any metric space and (Y, d_Y) be any metric space *with the discrete metric*. Determine, with proof,

(a) all continuous functions $f : Y \to X$ and

(b) all continuous functions $f : \mathbb{R} \to Y$, where the metric on $\mathbb{R}$ is the usual Euclidean one.