Homework #1

1. Suppose that \( f : X \to Y \) is uniformly continuous and that \((a_n)\) is a Cauchy sequence from \(X\). Prove that the sequence \((f(a_n))\) is Cauchy in \(Y\).

2. Prove the **Intermediate Value Theorem**: If the function \( f : [a, b] \to \mathbb{R} \) is continuous, then it takes on every value between \( f(a) \) and \( f(b) \) at some point within the interval \([a, b]\). (In other words, for every value \( y \) between \( f(a) \) and \( f(b) \), there is some point \( x \in [a, b] \) such that \( f(x) = y \).)